



Global Fits to MSSM Models

by

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Talk outline^a

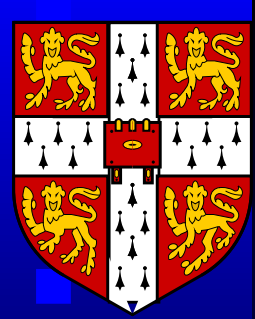
- LHC SUSY Searches
- SUSY fits to Indirect Data
- Effect of Searches

Please ask questions while I'm talking

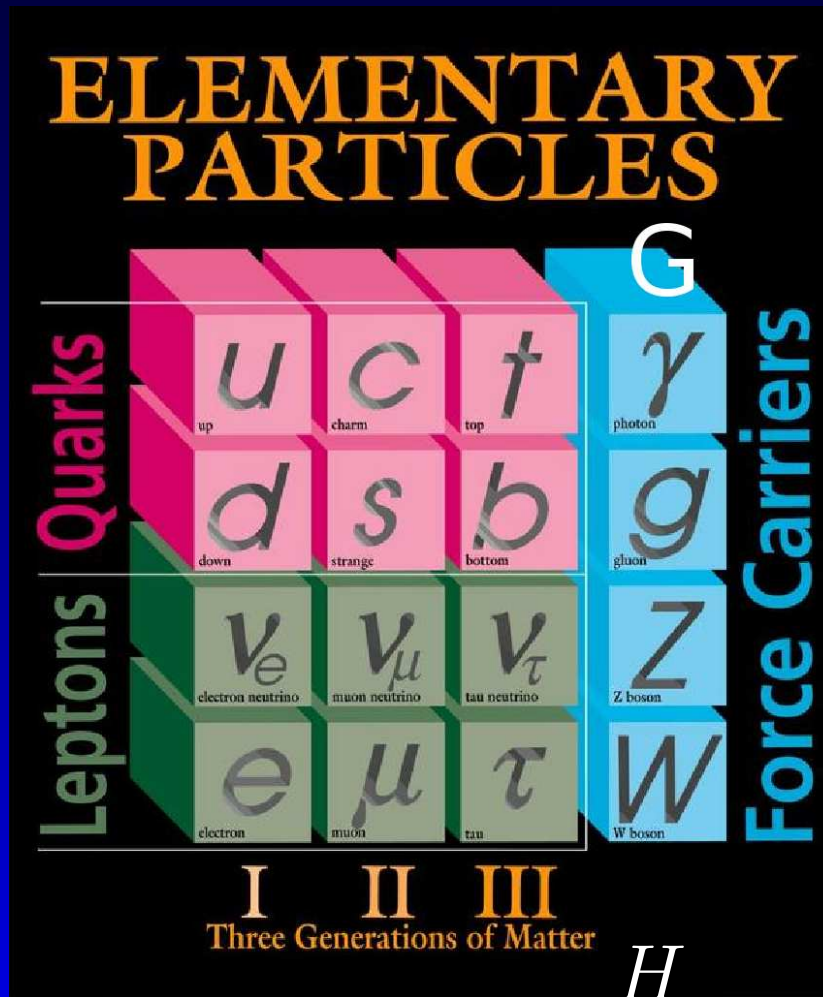
^aBCA, [arXiv:1102.3149](https://arxiv.org/abs/1102.3149); BCA, Khoo, Lester, Williams,

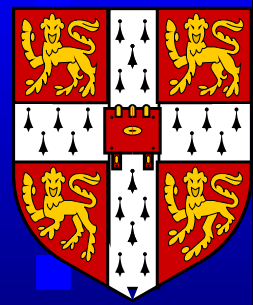
[arXiv:1103.0969](https://arxiv.org/abs/1103.0969)

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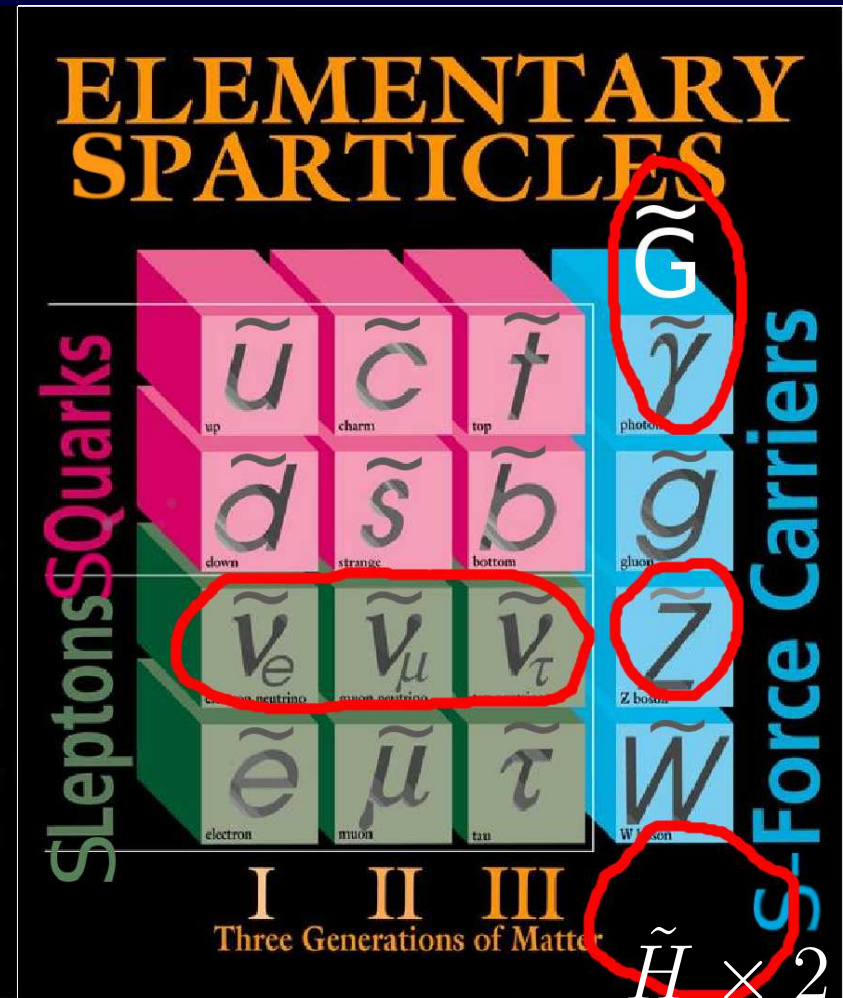
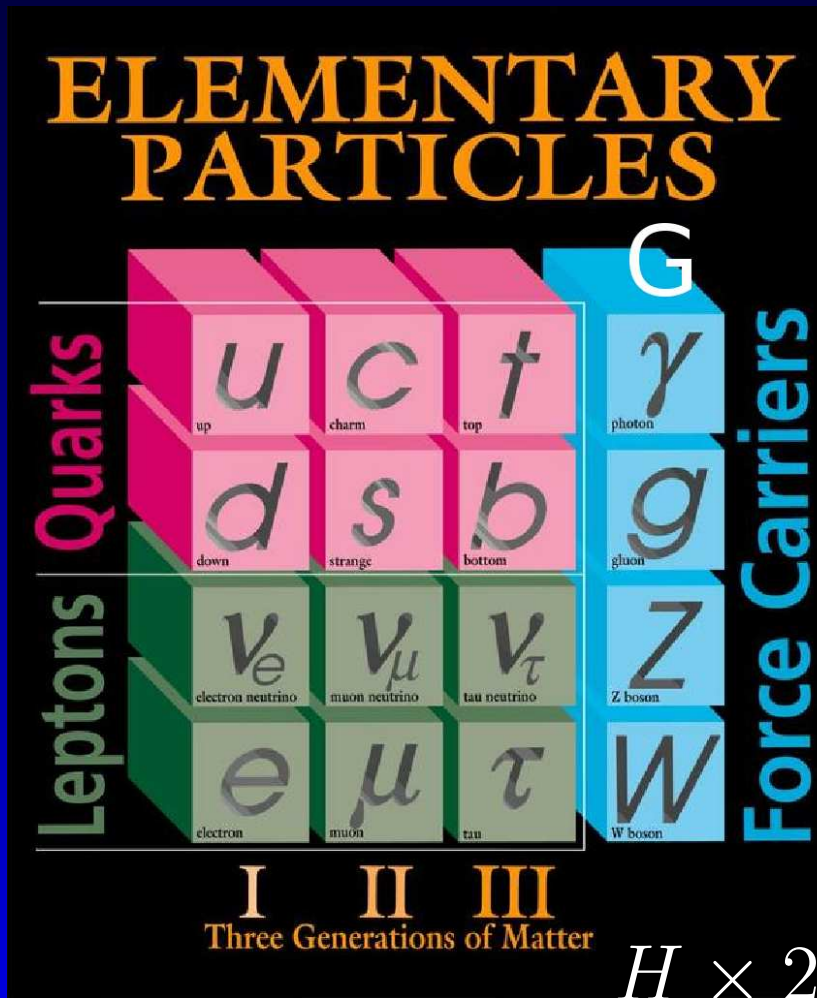
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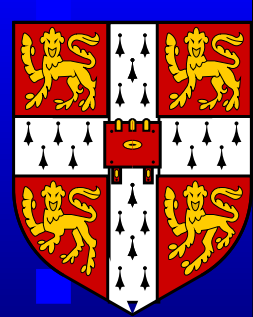




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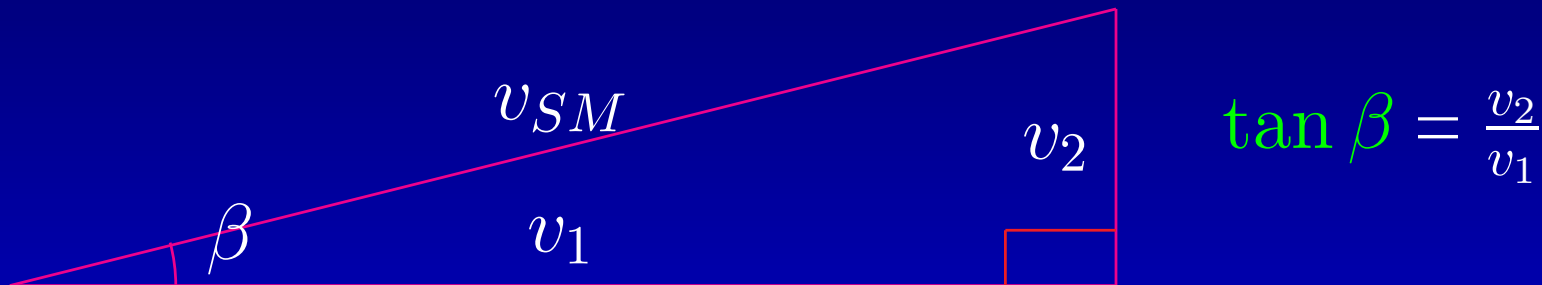


Electroweak Breaking

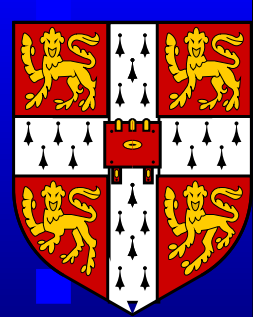
Both Higgs get vacuum expectation values:

$$\begin{pmatrix} H_1^0 \\ H_1^- \end{pmatrix} \rightarrow \begin{pmatrix} v_1 \\ 0 \end{pmatrix} \quad \begin{pmatrix} H_2^+ \\ H_2^0 \end{pmatrix} \rightarrow \begin{pmatrix} 0 \\ v_2 \end{pmatrix}$$

and to get M_W correct, match with $v_{SM} = 246$ GeV:



$$\mathcal{L} = h_t \bar{t}_L H_2^0 t_R + h_b \bar{b}_L H_1^0 b_R + h_\tau \bar{\tau}_L H_1^0 \tau_R$$
$$\Rightarrow \frac{m_t}{\sin \beta} = \frac{h_t v_{SM}}{\sqrt{2}}, \quad \frac{m_{b,\tau}}{\cos \beta} = \frac{h_{b,\tau} v_{SM}}{\sqrt{2}}.$$



CMS α_T Search

CMS: jets and missing energy arXiv:1101.1628

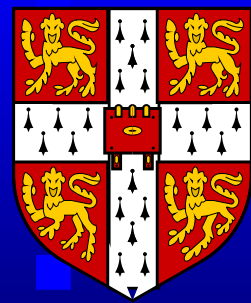
$$\mathcal{L} = 35 \text{ pb}^{-1}. H_T = \sum_{i=1}^{N_{jet}} |\mathbf{p}_T^{j_i}| > 350 \text{ GeV}.$$

$$(1) \quad \Delta H_T \equiv \sum_{j_i \in A} |\mathbf{p}_T^{j_i}| - \sum_{j_i \in B} |\mathbf{p}_T^{j_i}|.$$

One then calculates

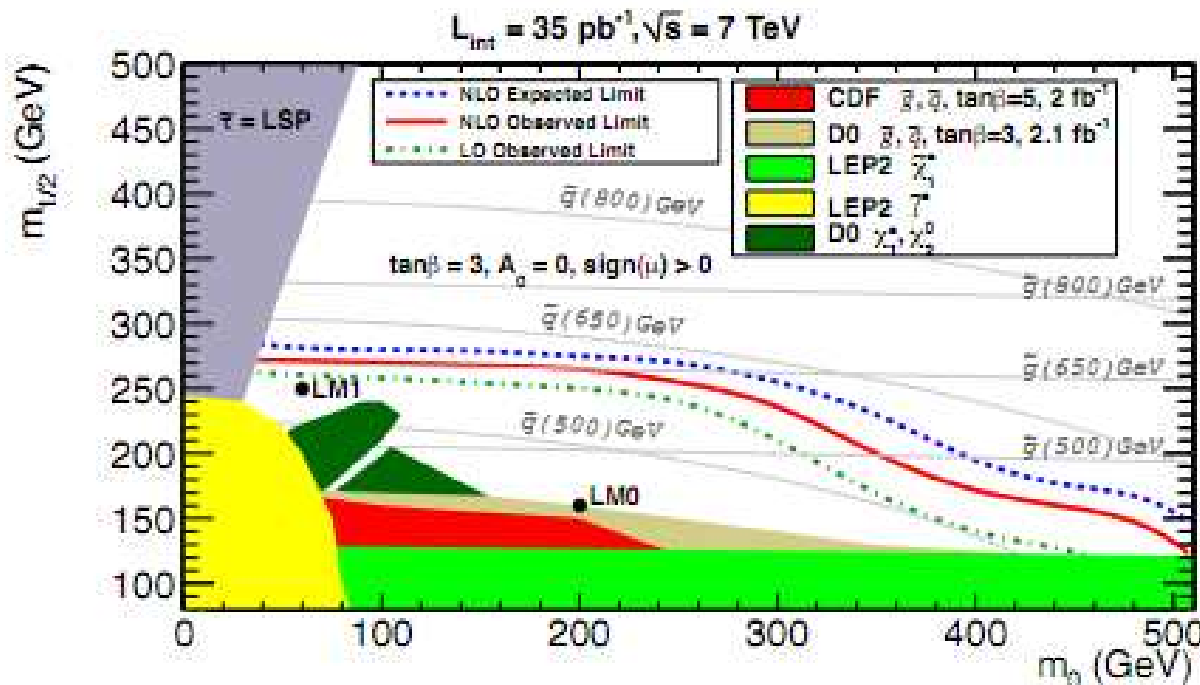
$$(2) \quad \alpha_T = \frac{H_T - \Delta H_T}{2\sqrt{H_T^2 - \cancel{H}_T^2}} > 0.55$$

$$\text{where } \cancel{H}_T = \sqrt{\left(\sum_{i=1}^{N_{jet}} p_x^{j_i}\right)^2 + \left(\sum_{i=1}^{N_{jet}} p_y^{j_i}\right)^2}.$$



Results

Selection	Data	SM	QCD multijet	$Z \rightarrow \nu\bar{\nu}$	W + jets	$t\bar{t}$
$H_T > 250 \text{ GeV}$	4.68M	5.81M	5.81M	290	2.0k	2.5k
$E_T^{j2} > 100 \text{ GeV}$	2.89M	3.40M	3.40M	160	610	830
$H_T > 350 \text{ GeV}$	908k	1.11M	1.11M	80	280	650
$\alpha_T > 0.55$	37	30.5 ± 4.7	19.5 ± 4.6	4.2 ± 0.6	3.9 ± 0.7	2.8 ± 0.1
$\Delta R_{\text{ECAL}} > 0.3 \vee \Delta\phi^* > 0.5$	32	24.5 ± 4.2	14.3 ± 4.1	4.2 ± 0.6	3.6 ± 0.6	2.4 ± 0.1
$R_{\text{miss}} < 1.25$	13	9.3 ± 0.9	0.03 ± 0.02	4.1 ± 0.6	3.3 ± 0.6	1.8 ± 0.1



ATLAS 0-lepton, jets and \cancel{p}_T

	A	B	C	D	
Pre-selection	Number of required jets	≥ 2	≥ 2	≥ 3	≥ 3
	Leading jet p_T [GeV]	> 120	> 120	> 120	> 120
	Other jet(s) p_T [GeV]	> 40	> 40	> 40	> 40
	E_T^{miss} [GeV]	> 100	> 100	> 100	> 100
Final selection	$\Delta\phi(\text{jet}, \cancel{p}_T^{\text{miss}})_{\text{min}}$	> 0.4	> 0.4	> 0.4	> 0.4
	$E_T^{\text{miss}}/m_{\text{eff}}$	> 0.3	–	> 0.25	> 0.25
	m_{eff} [GeV]	> 500	–	> 500	> 1000
	m_{T2} [GeV]	–	> 300	–	–

Table 1: Criteria for admission to each of the four overlapping signal regions A to D. All variables are defined in §4.

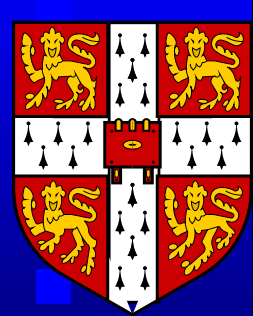
$$m_{\text{eff}} = \sum p_T^{(j)} + \cancel{p}_T,$$

$$m_T^{(i)2}(\mathbf{p}_T^{(i)}, \cancel{q}_T^{(i)}) \equiv 2 |\mathbf{p}_T^{(i)}| |\cancel{q}_T^{(i)}| - 2 \mathbf{p}_T^{(i)} \cdot \cancel{q}_T^{(i)}$$

where $\cancel{q}_T^{(i)}$ is the transverse momentum of particle (i) . For each event, it is a lower bound on $m(NLSP)$.

$$M_{T2}(\mathbf{p}_T^{(1)}, \mathbf{p}_T^{(2)}, \cancel{p}_T) \equiv \min_{\sum \cancel{q}_T = \cancel{p}_T} \left\{ \max \left(m_T^{(1)}, m_T^{(2)} \right) \right\}$$

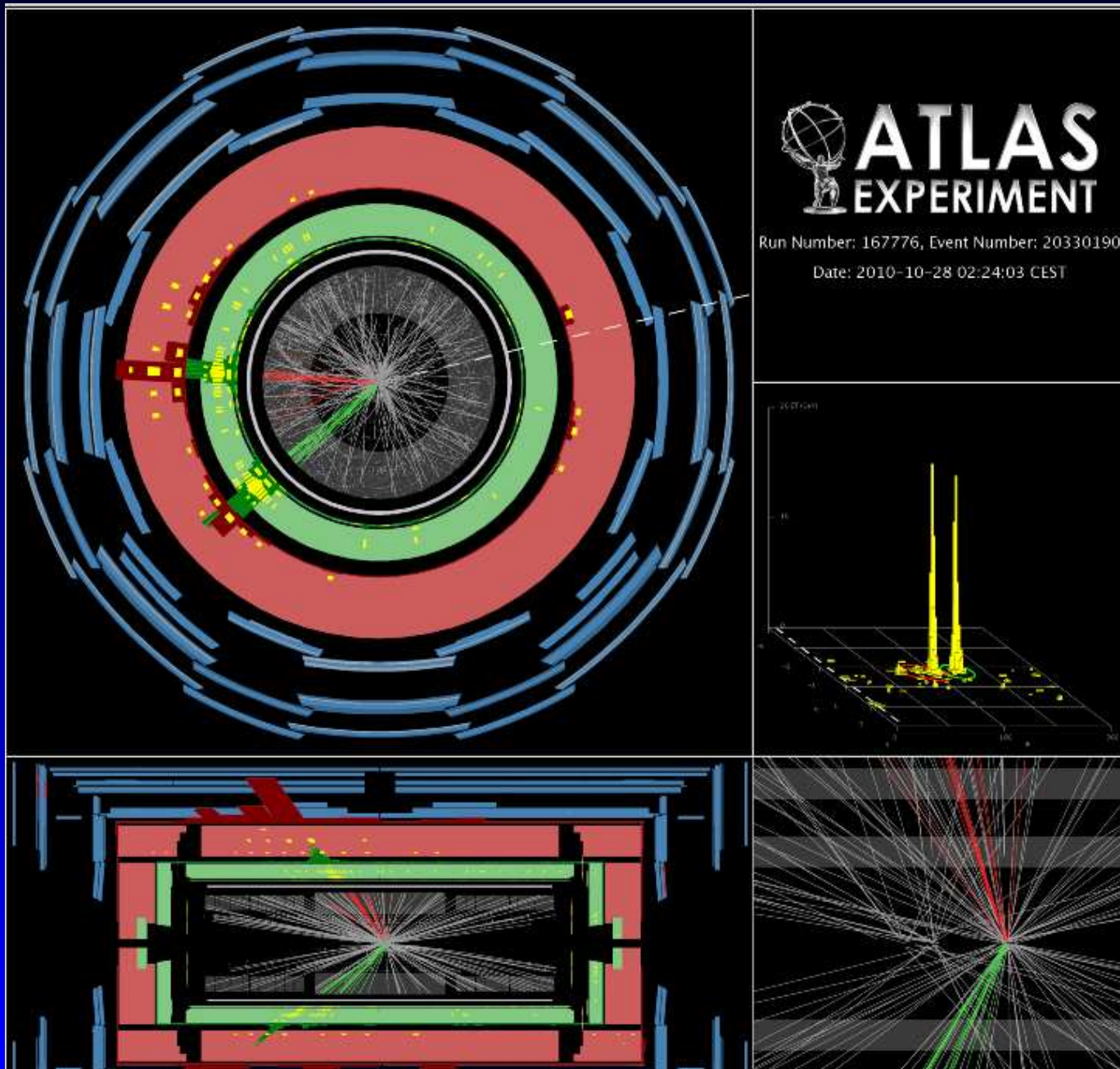
Candidate Event: High $E_T(j)$



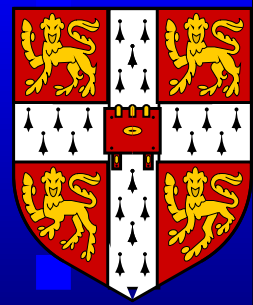
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Global Fits to MSSM Models

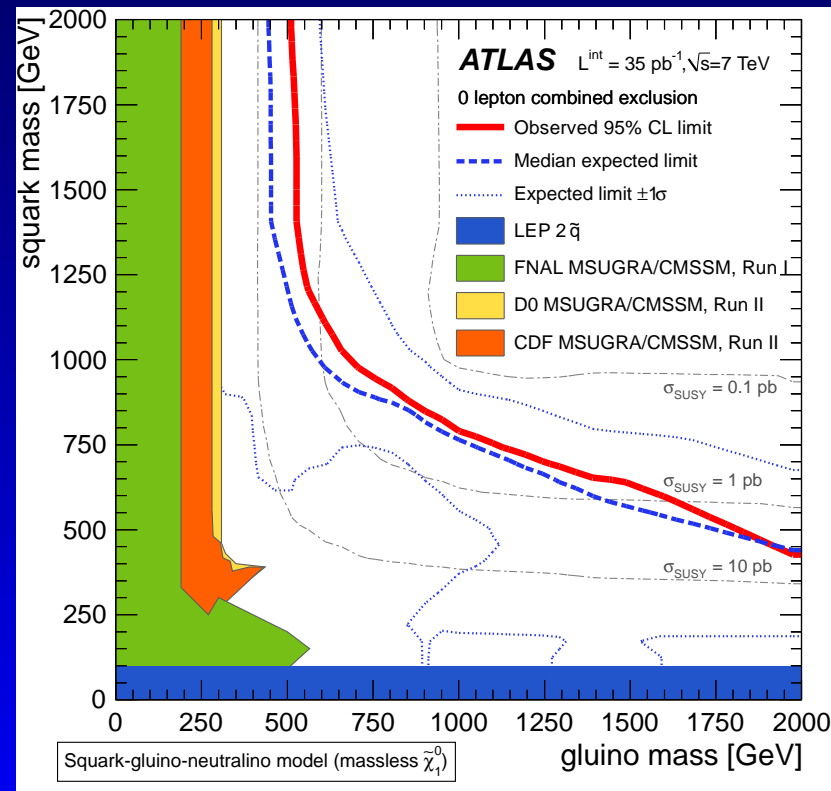


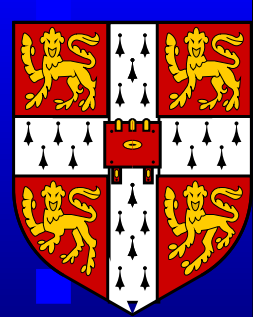
MSSM Exclusion: Simplified Model



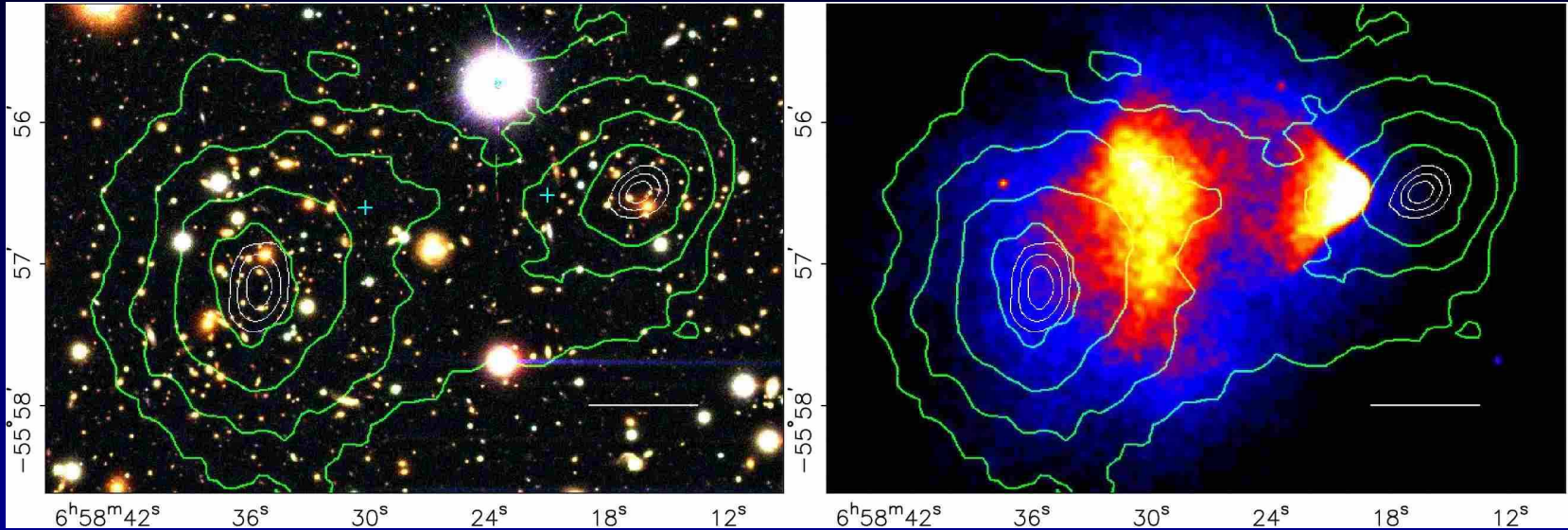
	Signal region A	Signal region B	Signal region C	Signal region D
QCD	$7^{+8}_{-7}[\text{u+j}]$	$0.6^{+0.7}_{-0.6}[\text{u+j}]$	$9^{+10}_{-9}[\text{u+j}]$	$0.2^{+0.4}_{-0.2}[\text{u+j}]$
W+jets	$50 \pm 11[\text{u}]^{+14}_{-10}[\text{j}] \pm 5[\mathcal{L}]$	$4.4 \pm 3.2[\text{u}]^{+1.5}_{-0.8}[\text{j}] \pm 0.5[\mathcal{L}]$	$35 \pm 9[\text{u}]^{+10}_{-8}[\text{j}] \pm 4[\mathcal{L}]$	$1.1 \pm 0.7[\text{u}]^{+0.2}_{-0.3}[\text{j}] \pm 0.1[\mathcal{L}]$
Z+jets	$52 \pm 21[\text{u}]^{+15}_{-11}[\text{j}] \pm 6[\mathcal{L}]$	$4.1 \pm 2.9[\text{u}]^{+2.1}_{-0.8}[\text{j}] \pm 0.5[\mathcal{L}]$	$27 \pm 12[\text{u}]^{+10}_{-6}[\text{j}] \pm 3[\mathcal{L}]$	$0.8 \pm 0.7[\text{u}]^{+0.6}_{-0.0}[\text{j}] \pm 0.1[\mathcal{L}]$
$t\bar{t}$ and t	$10 \pm 0[\text{u}]^{+3}_{-2}[\text{j}] \pm 1[\mathcal{L}]$	$0.9 \pm 0.1[\text{u}]^{+0.4}_{-0.3}[\text{j}] \pm 0.1[\mathcal{L}]$	$17 \pm 1[\text{u}]^{+6}_{-4}[\text{j}] \pm 2[\mathcal{L}]$	$0.3 \pm 0.1[\text{u}]^{+0.2}_{-0.1}[\text{j}] \pm 0.0[\mathcal{L}]$
Total SM	$118 \pm 25[\text{u}]^{+32}_{-23}[\text{j}] \pm 12[\mathcal{L}]$	$10.0 \pm 4.3[\text{u}]^{+4.0}_{-1.9}[\text{j}] \pm 1.0[\mathcal{L}]$	$88 \pm 18[\text{u}]^{+26}_{-18}[\text{j}] \pm 9[\mathcal{L}]$	$2.5 \pm 1.0[\text{u}]^{+1.0}_{-0.4}[\text{j}] \pm 0.2[\mathcal{L}]$
Data	87	11	66	2

Table 2: Expected and observed numbers of events in the four signal regions. Uncertainties shown are due to “MC statistics, statistics in control regions, other sources of uncorrelated systematic uncertainty, and also the jet energy resolution and lepton efficiencies” [u], the jet energy scale [j], and the luminosity [L].

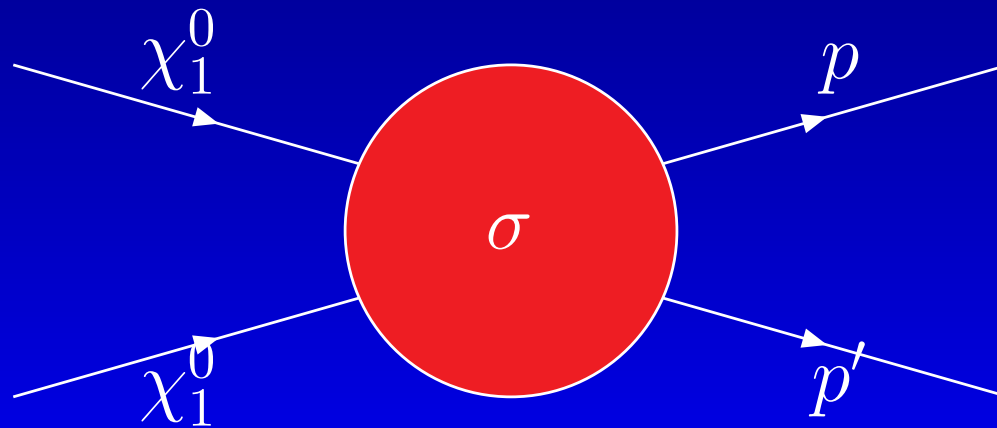




SUSY Dark Matter



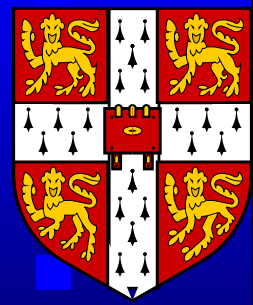
[astro-ph/0608407](https://arxiv.org/abs/astro-ph/0608407)



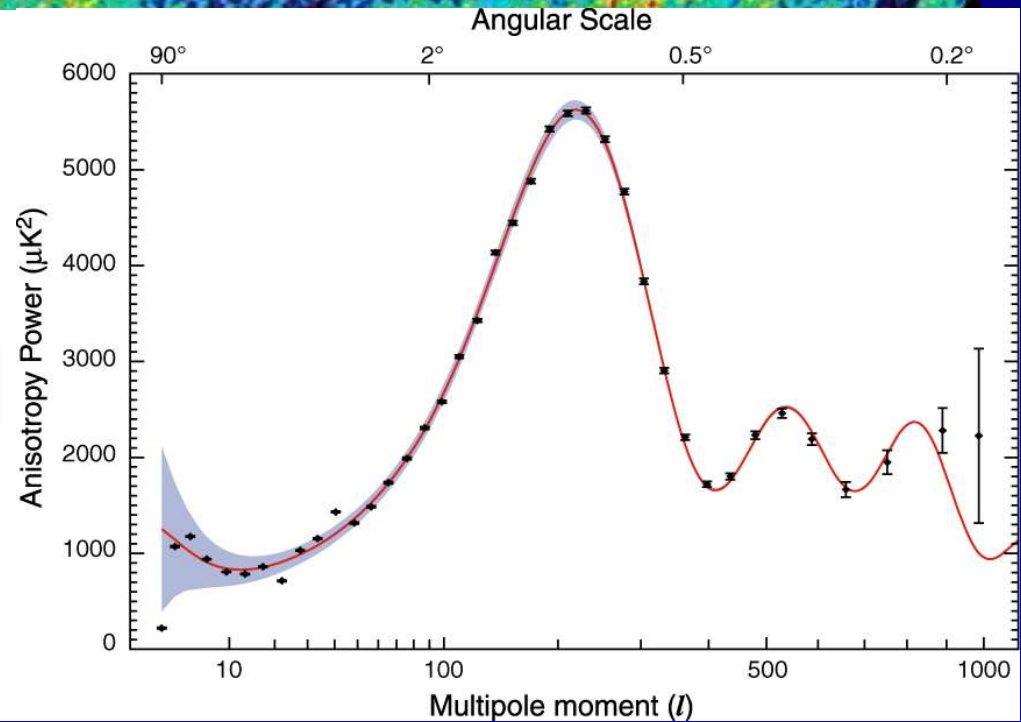
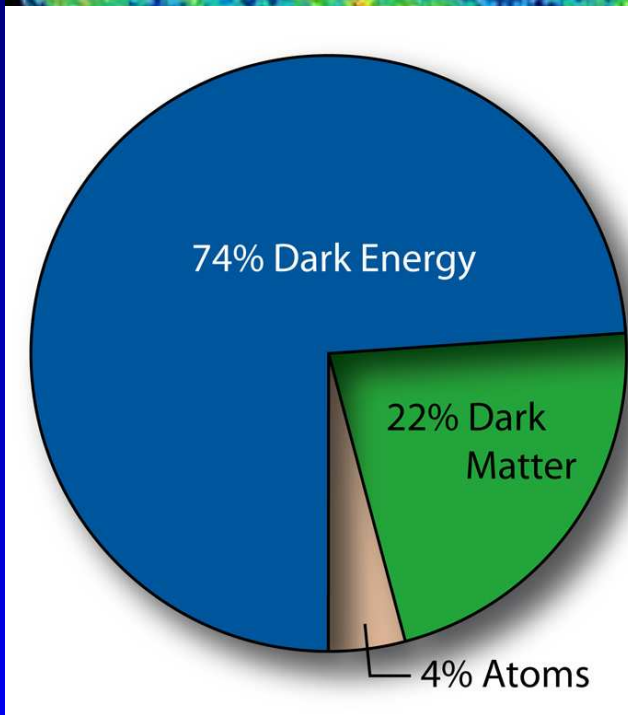
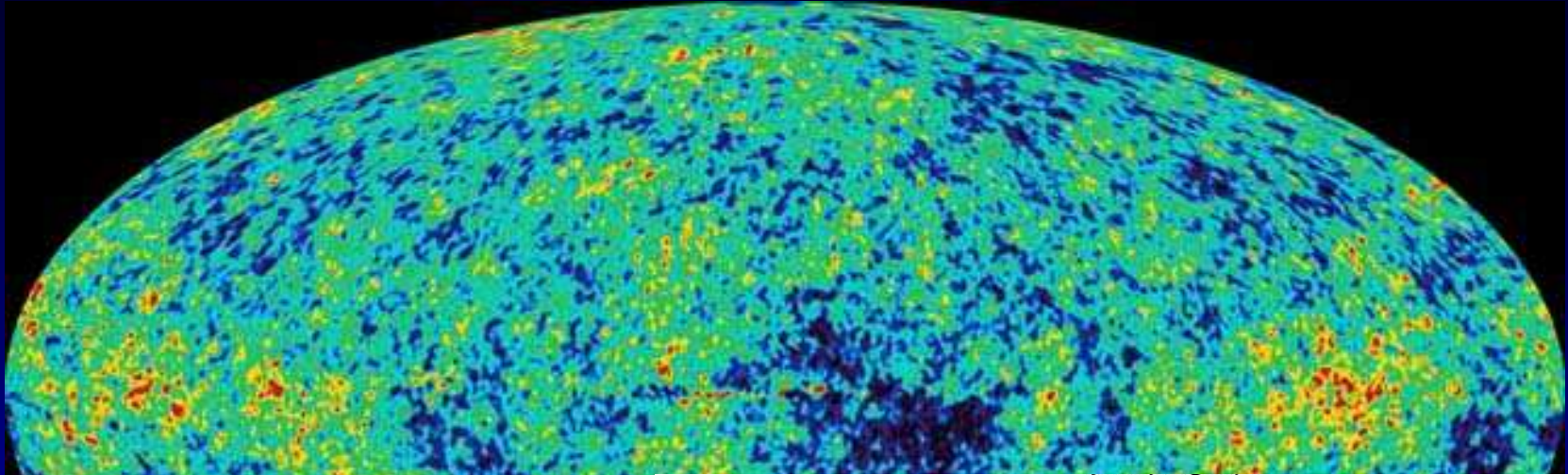


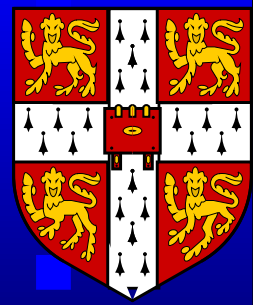
SUSY Prediction of Ωh^2

- Assume relic in thermal equilibrium with $n_{eq} \propto (MT)^{3/2} \exp(-M/T)$.
- Freeze-out with $T_f \sim M_f/25$ once **interaction rate** < **expansion rate** (t_{eq} critical)
- **micrOMEGAs** uses **calcHEP** to automatically calculate relevant Feynman diagrams for some given model Lagrangian: *flexible*.
- **darkSUSY**, **IsaRED** has MSSM annihilation channels hard-coded.
- Both **darkSUSY** and **micrOMEGAs** calculate (in-)direct predictions.



WMAP+BAO+Ia Fits

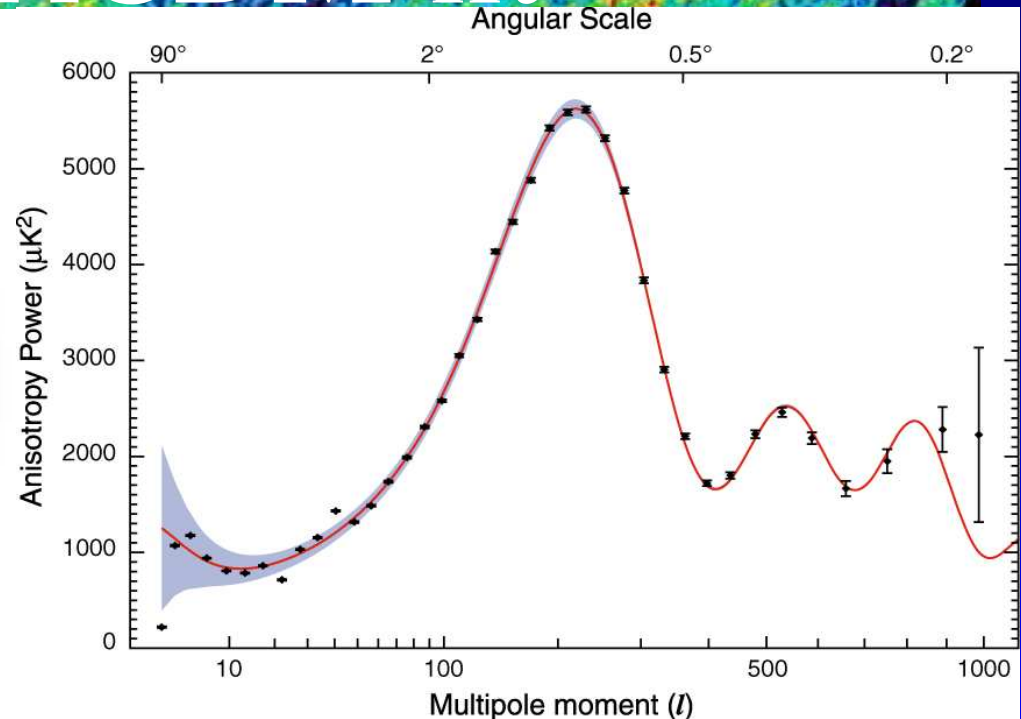
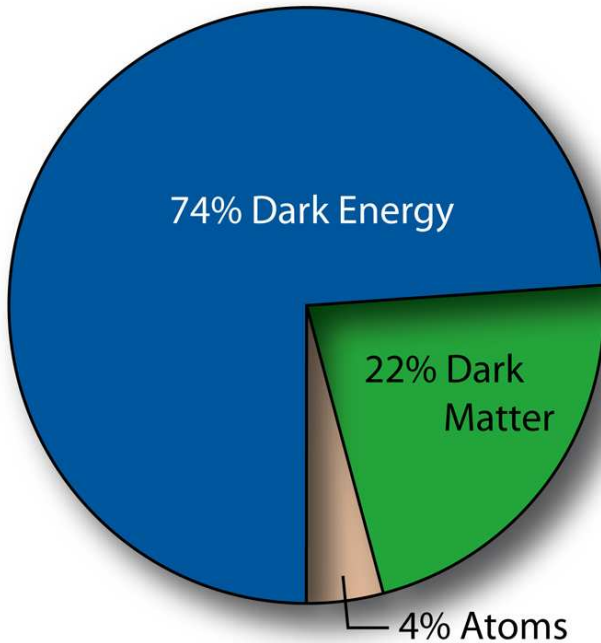
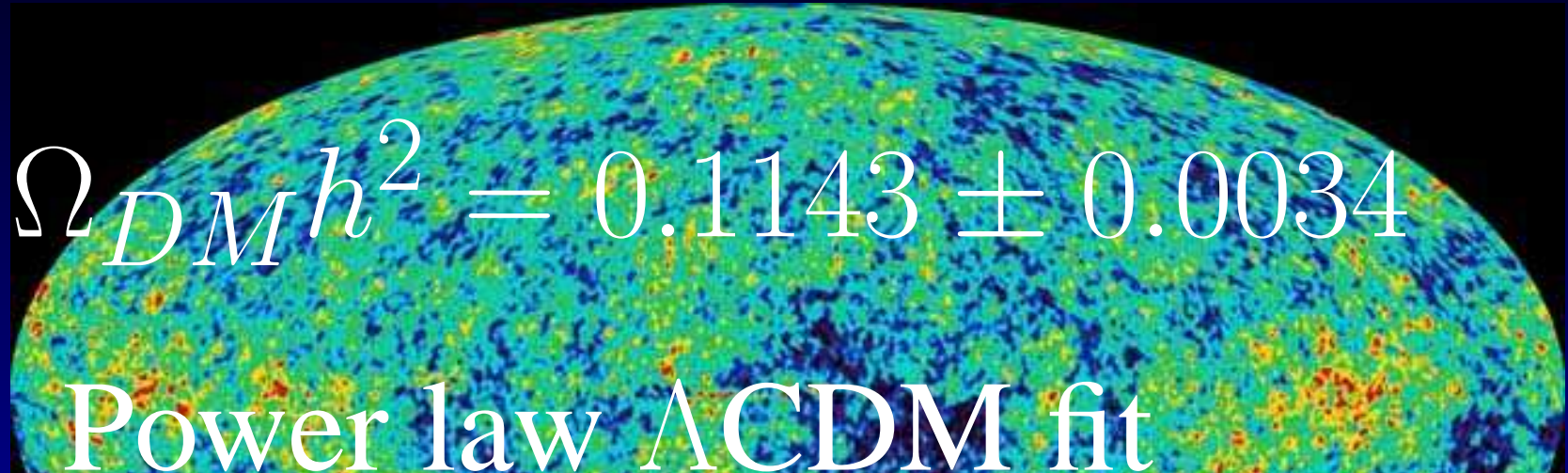


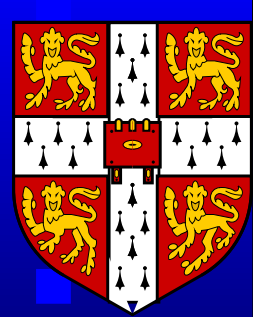


WMAP+BAO+Ia Fits

$$\Omega_{DM} h^2 = 0.1143 \pm 0.0034$$

Power law Λ CDM fit





Universality

Reduces number of SUSY breaking parameters from 100 to 3:

- $\tan \beta \equiv v_2/v_1$
- m_0 , the **common** scalar mass (flavour).
- $M_{1/2}$, the **common** gaugino mass (GUT/string).
- A_0 , the **common** trilinear coupling (flavour).

These conditions should be imposed at $M_X \sim O(10^{16-18})$ GeV and receive radiative corrections

$$\propto 1/(16\pi^2) \ln(M_X/M_Z).$$

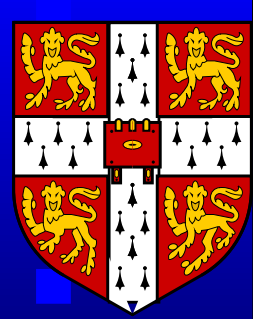
Also, Higgs potential parameter $\text{sgn}(\mu)=\pm 1$.

Implementation

We use

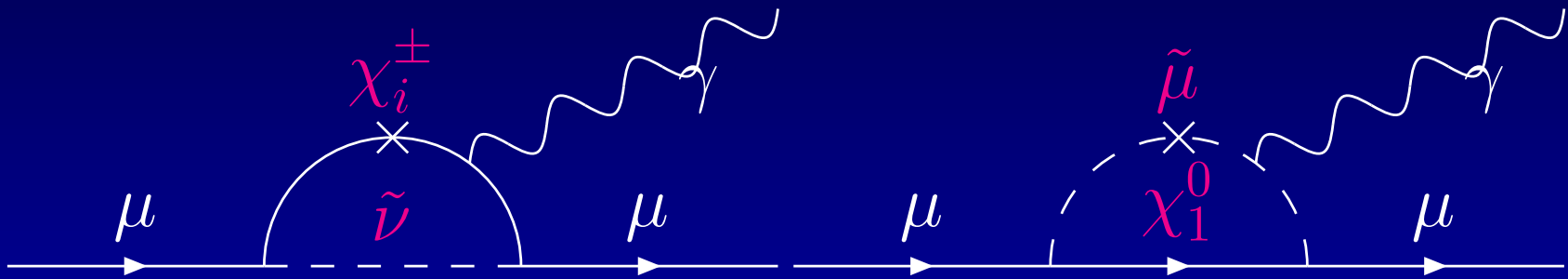
- 95% *C.L.* direct search constraints
- $\Omega_{DM} h^2 = 0.1143 \pm 0.02$ Boudjema *et al*
- $\delta(g - 2)_\mu/2 = (29.5 \pm 8.8) \times 10^{-10}$ Stöckinger *et al*
- *B*–physics observables including
 $BR[b \rightarrow s\gamma]_{E_\gamma > 1.6 \text{ GeV}} = (3.52 \pm 0.38) \times 10^{-4}$
- Electroweak data W Hollik, A Weber *et al*

$$2 \ln \mathcal{L} = - \sum_i \chi_i^2 + c = \sum_i \frac{(p_i - e_i)^2}{\sigma_i^2} + c$$

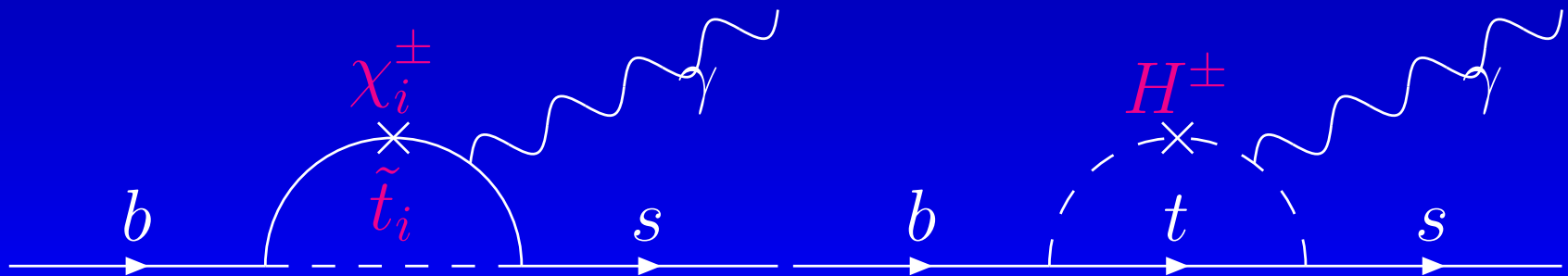


Additional observables

$$\delta \frac{(g-2)_\mu}{2} \sim 13 \times 10^{-10} \left(\frac{100 \text{ GeV}}{M_{SUSY}} \right)^2 \tan \beta$$



$$BR[b \rightarrow s\gamma] \propto \tan \beta (M_W/M_{SUSY})^2$$





mSUGRA Global Fits

There are 3 methodologies of doing these type of global fits:


- **Markov Chain Monte Carlo:** BCA *et al*; Ruiz de Austri *et al*: primary interpretation is *Bayesian*.
- **MultiNest:** Ruiz de Austri *et al*: Bayesian interpretation only.
- **Minimising χ^2 /Profile likelihood:** Buchmueller *et al*. Impressive array of electroweak observables. Moving to a *hybrid* approach. Frequentist interpretation only.



Application of Bayes'

$\mathcal{L} \equiv p(\underline{d}|\underline{m}, H)$ is pdf of reproducing data \underline{d} assuming pMSSM hypothesis H and model parameters \underline{m}

$$p(\underline{m}|\underline{d}, H) = p(\underline{d}|\underline{m}, H) \frac{p(\underline{m}, H)}{p(\underline{d}, H)}$$

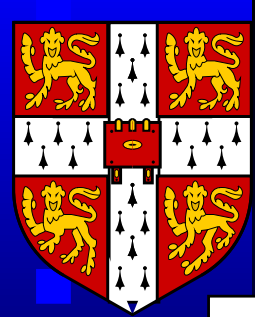


$p(\underline{m}|\underline{d}, H)$ is called the **posterior** pdf. We will compare $p(\underline{m}, H) = c$ with a **different** prior.

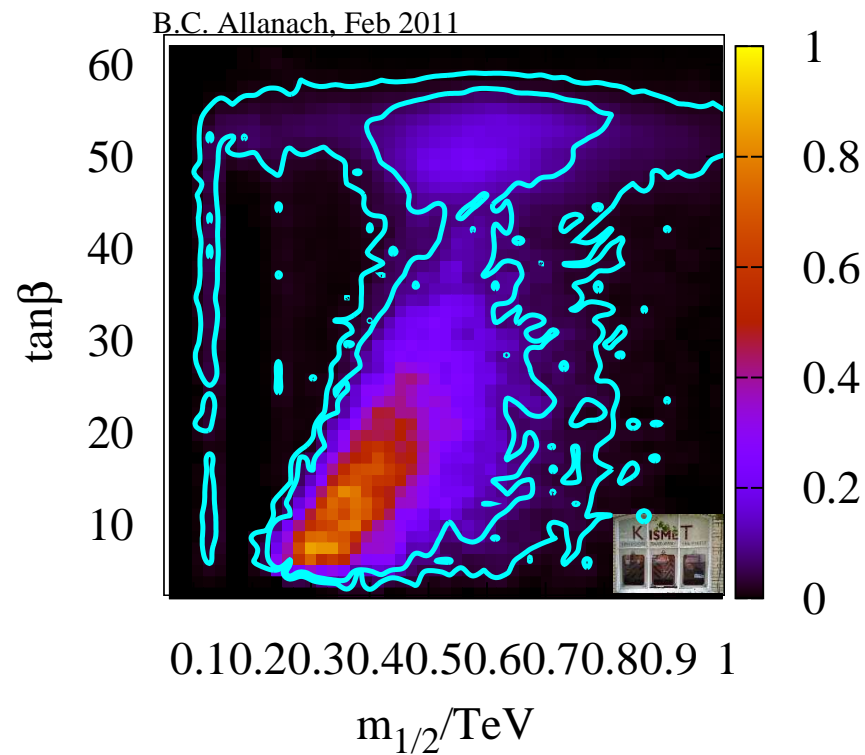
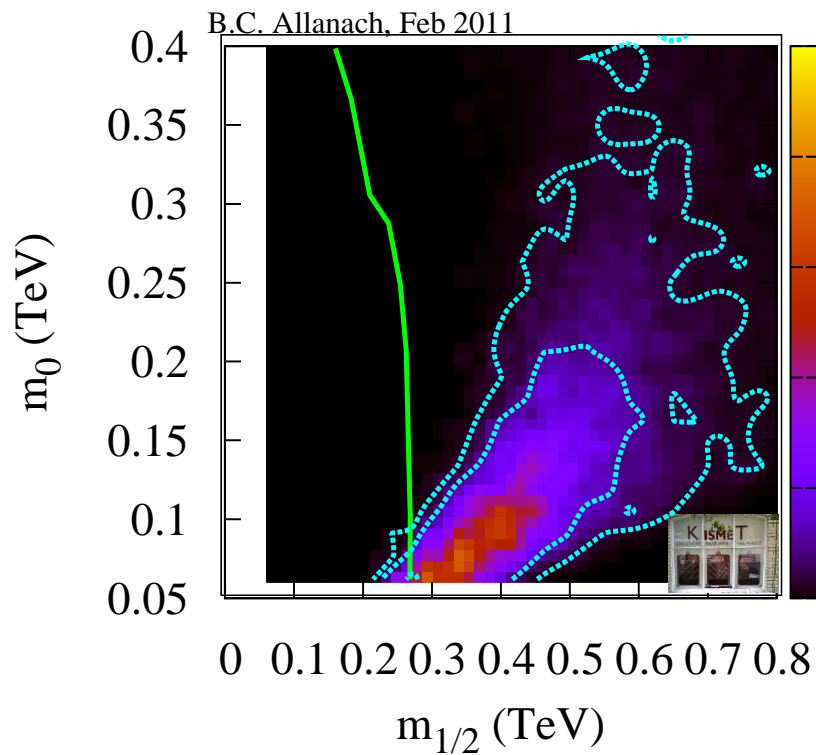
$$p(m_0, M_{1/2}|\underline{d}, H) = \int d\underline{o} p(m_0, M_{1/2}, \underline{o}|\underline{d}, H)$$

Called *marginalisation*.

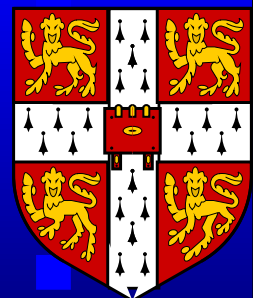




Log Fits



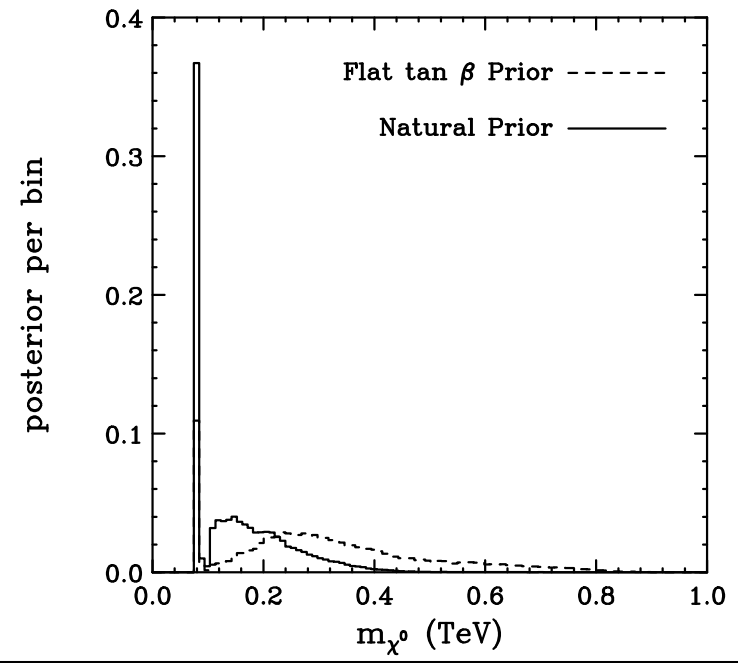
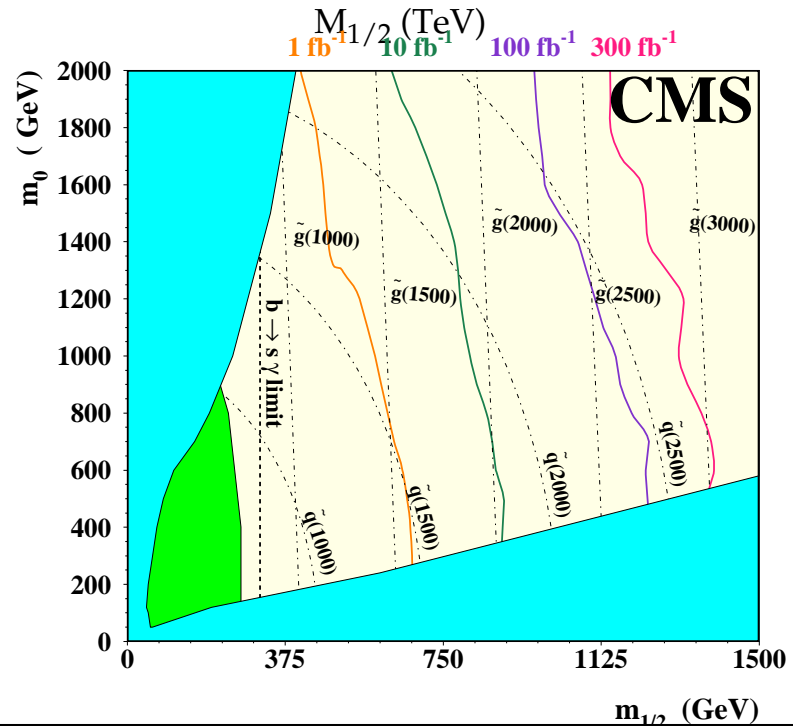
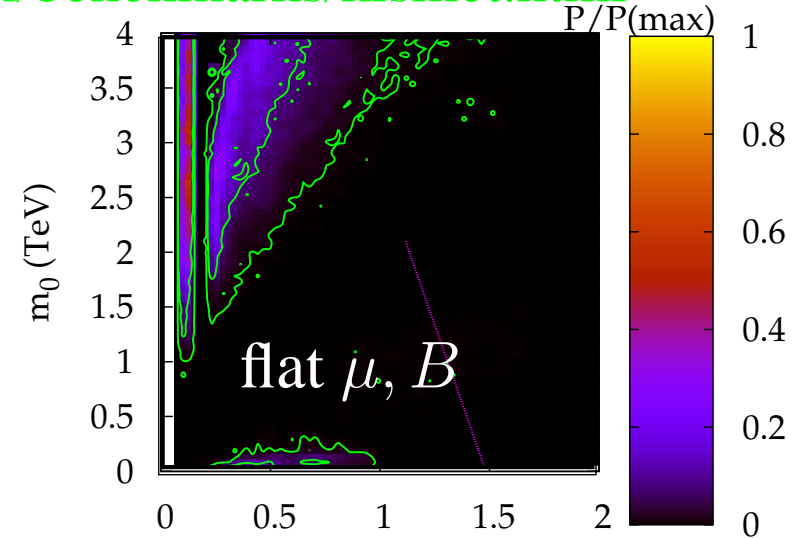
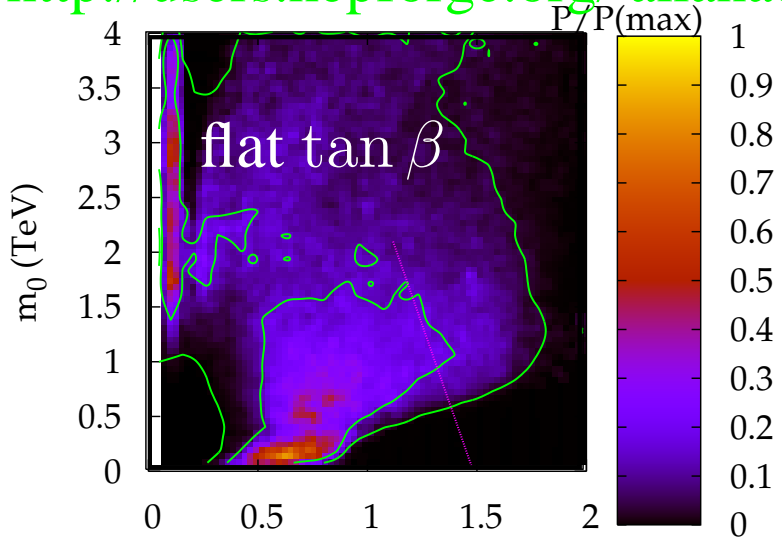
Choose priors on SM parameters set from data. Priors on SUSY parameters up to 4 TeV: flat in $\tan\beta$, A_0 , $\ln(m_0)$, $\ln(m_{1/2})$.



Killer Inference for Susy METeorology

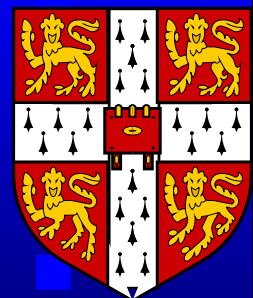
BCA, Cranmer, Weber, Lester, arXiv:0705.0487

<http://users.hepforge.org/~allanach/benchmarks/kismet.html>



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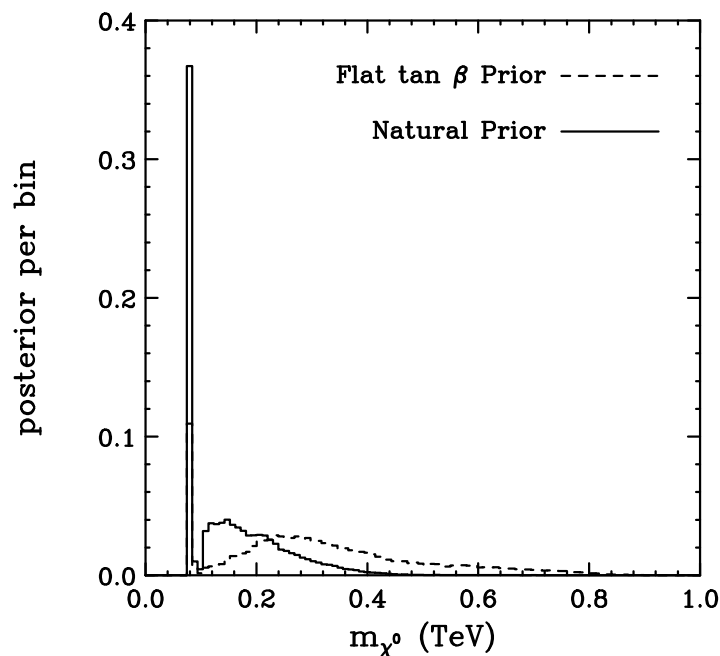
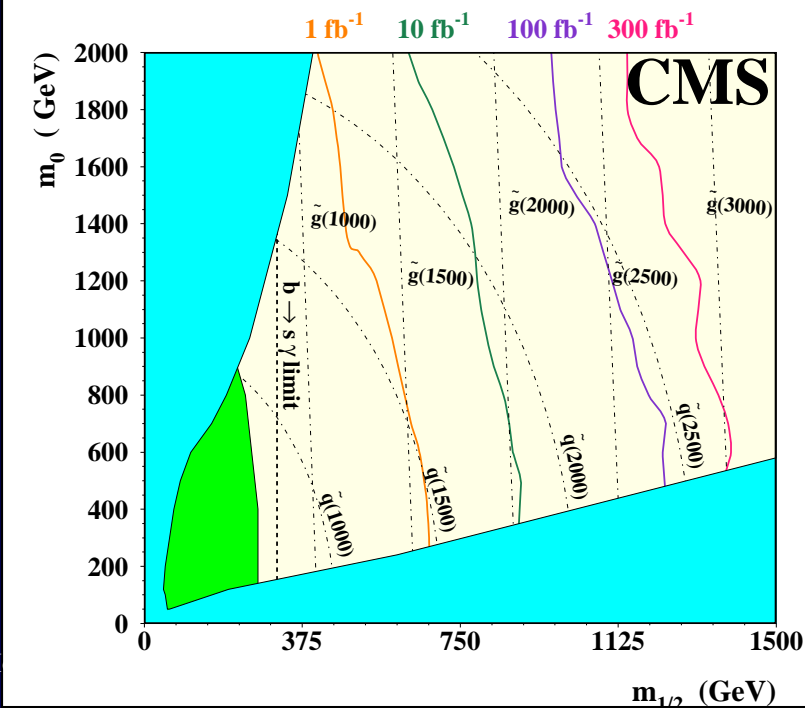


Killer Inference for Susy METeorology

BCA, Cranmer, Weber, Lester, arXiv:0705.0487

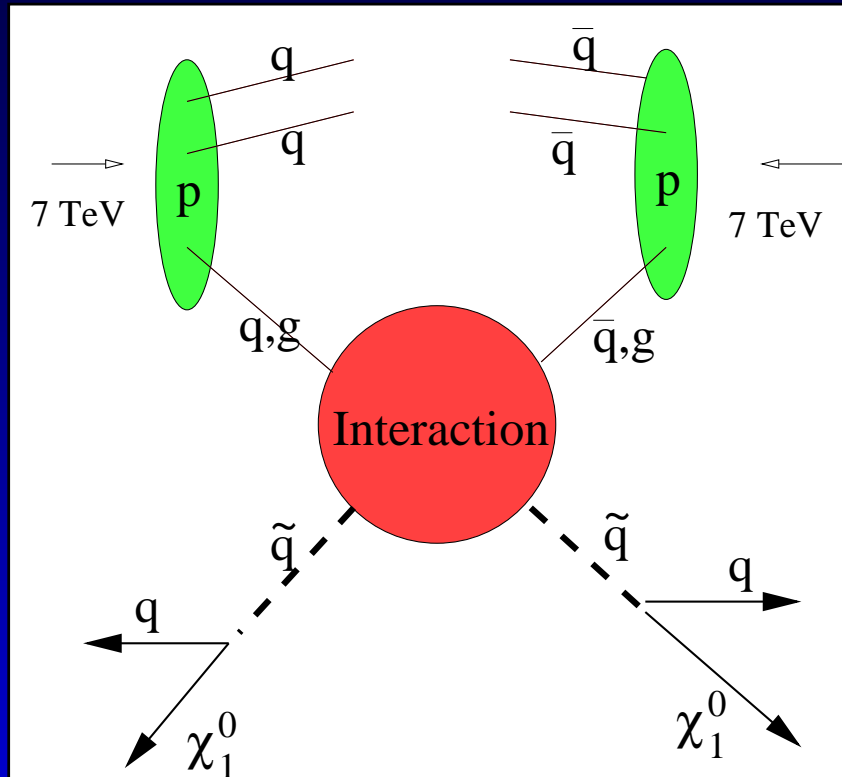


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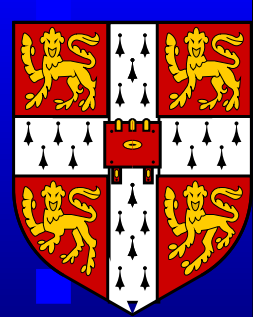


Collider SUSY Dark Matter Production

Strong sparticle production and decay to dark matter particles.

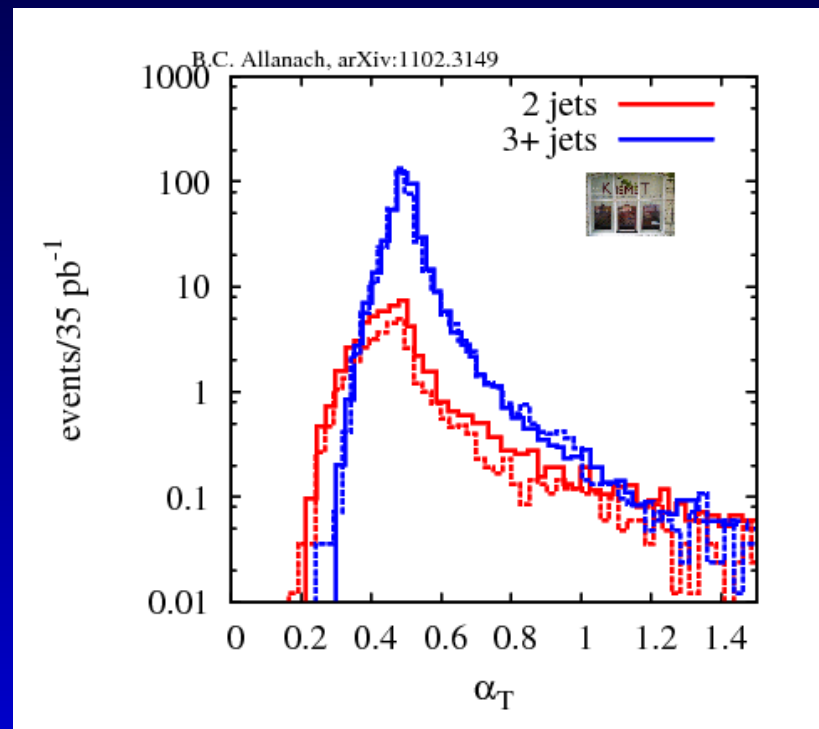


Any (light enough) dark matter candidate that couples to hadrons can be produced at the LHC



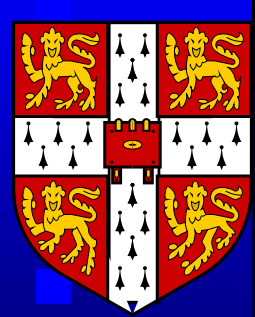
Validation of CMS Analysis

Used SOFTSUSY3.1.7, Herwig++-2.4.2 and fastjet-2.4.2 to simulate 10000 *signal* events
 α_T distributions with $H_T > 350$ GeV:

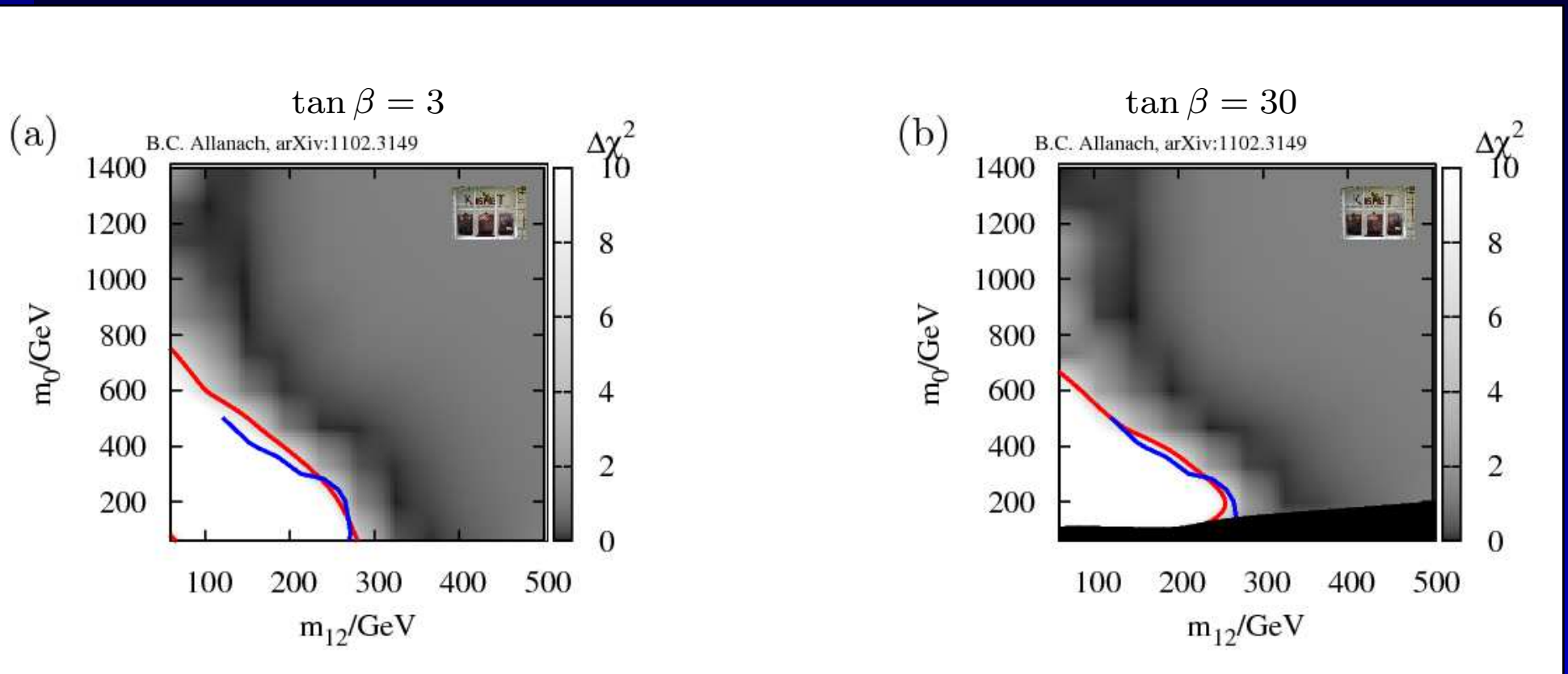


α_T distributions for SUSY point LM0

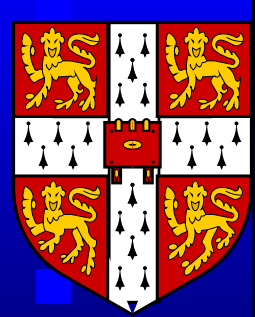
$m_0 = 200, m_{1/2} = 160, A_0 = -400, \tan \beta = 10$ by
my simulation (solid) and CMS' (dashed).



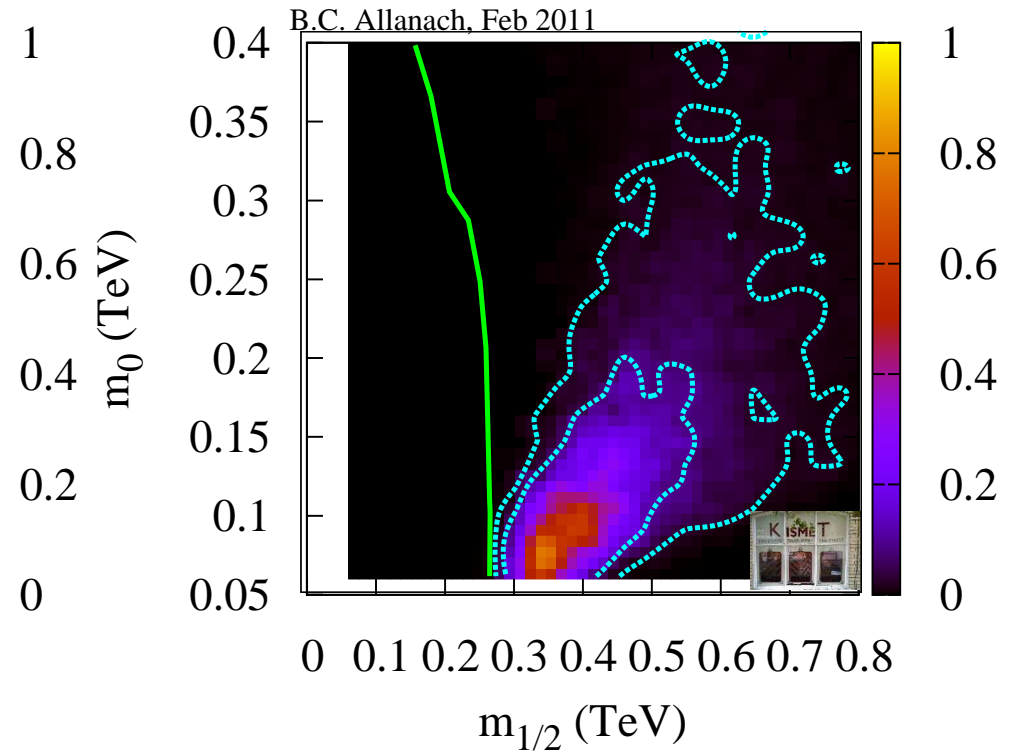
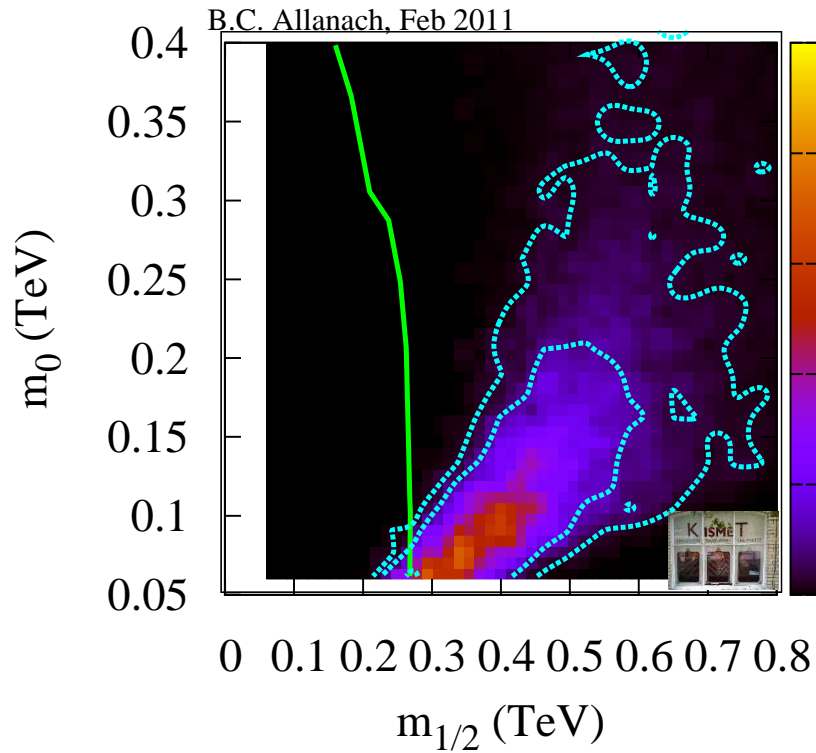
CMS Validation II

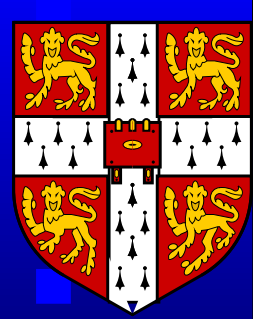


$\Delta\chi^2$ approx $\tan \beta$, A_0 independent \Rightarrow interpolate it across m_0 and $m_{1/2}$, then *re-weight fit with $\Delta\chi^2$* .



CMS Weighted Fits

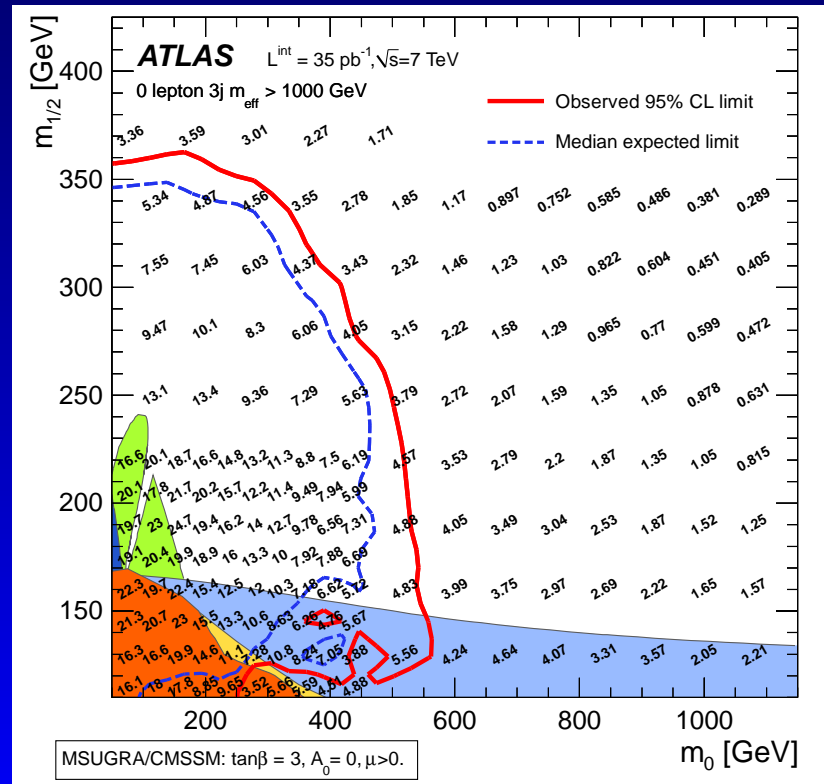


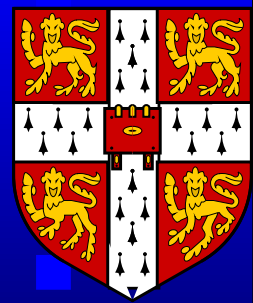


Validation of ATLAS Analysis

Information $\vec{\Sigma}^{(i)} = (n_s^{(i)}, n_b^{(i)}, \sigma_s^{(i)}, \sigma_b^{(i)})$, expected number of events past cuts

$$\lambda(\vec{\Sigma}^{(i)}, \delta_s, \delta_b) = n_s^{(i)}(1 + \delta_s \cdot \sigma_s^{(i)}) + n_b^{(i)}(1 + \delta_b \cdot \sigma_b^{(i)}),$$



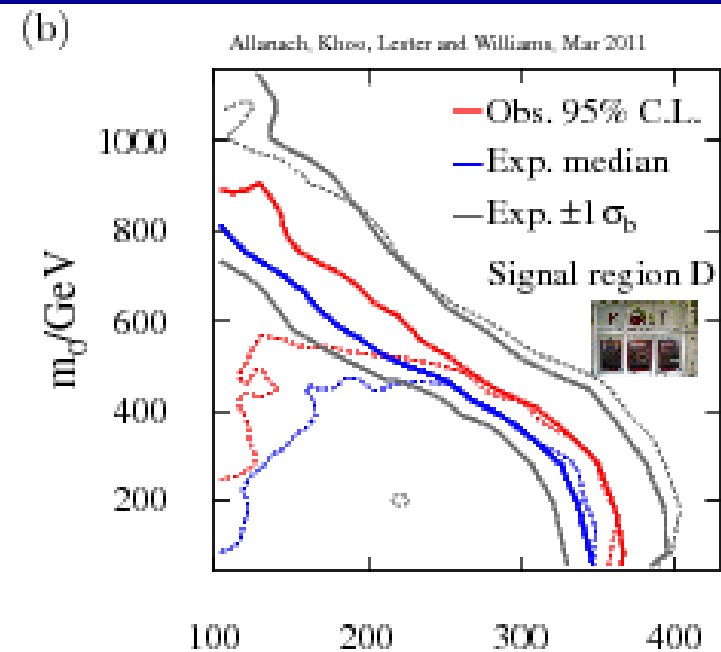
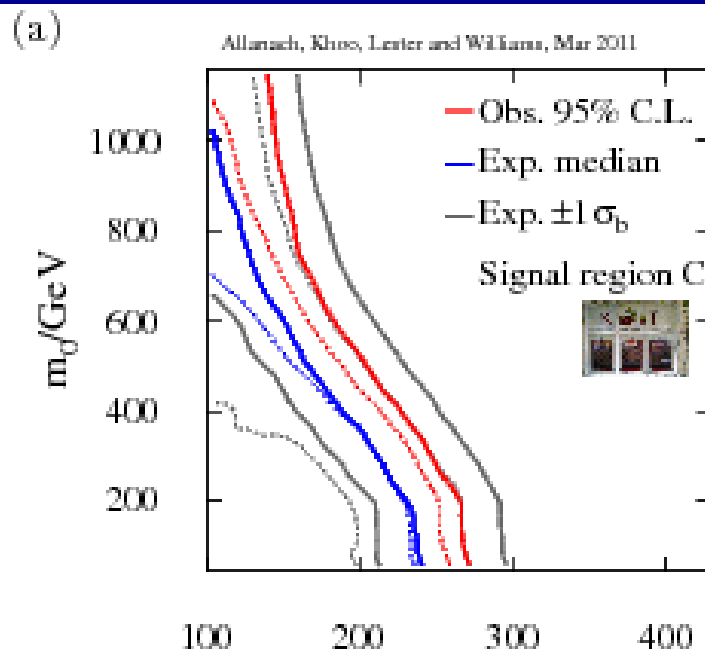


Validation of ATLAS Analysis II

For Poisson distribution p ,

$$P_{sys}(n_o^{(i)}, \delta_s, \delta_b | \vec{\Sigma}^{(i)}) = \frac{1}{N^{(i)}} p(n_o^{(i)} | \lambda) e^{-\frac{1}{2}(\delta_b^2 + \delta_s^2)},$$

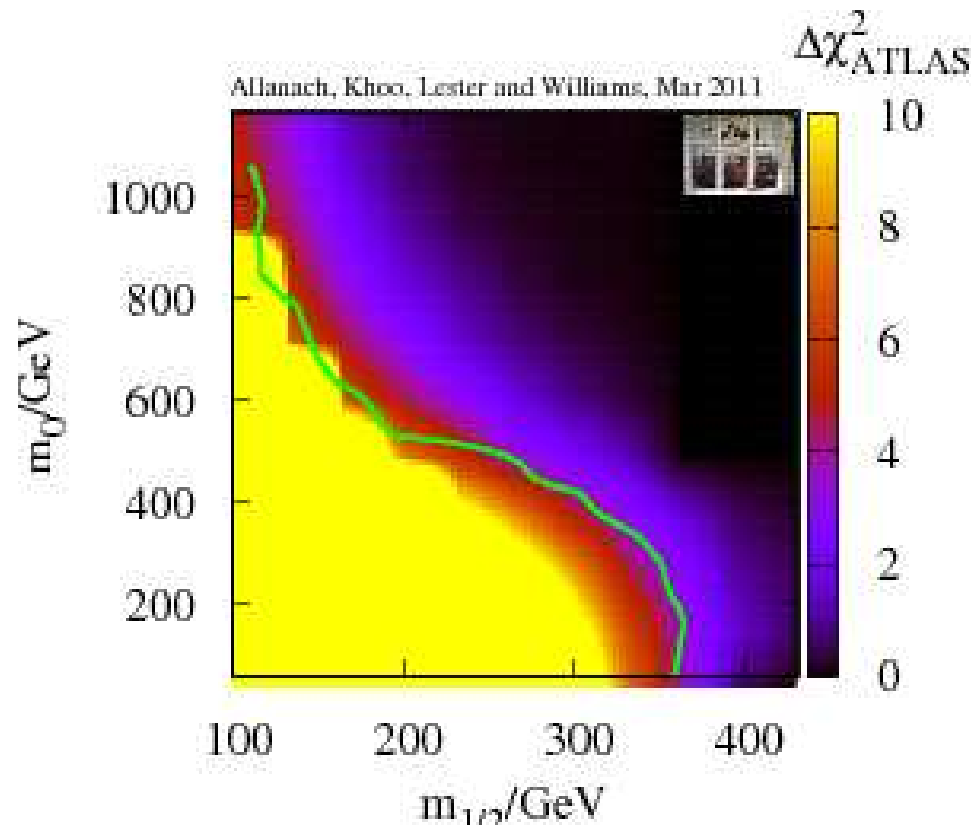
$$P_m(n_o^{(i)} | \vec{\Sigma}^{(i)}) = \int d\delta_s \int d\delta_b P_{sys}(n_o^{(i)}, \delta_s, \delta_b).$$

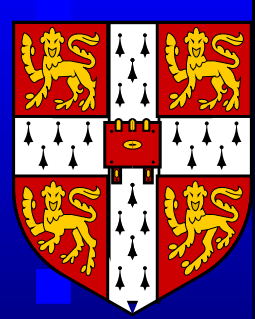


Validation of ATLAS Analysis III

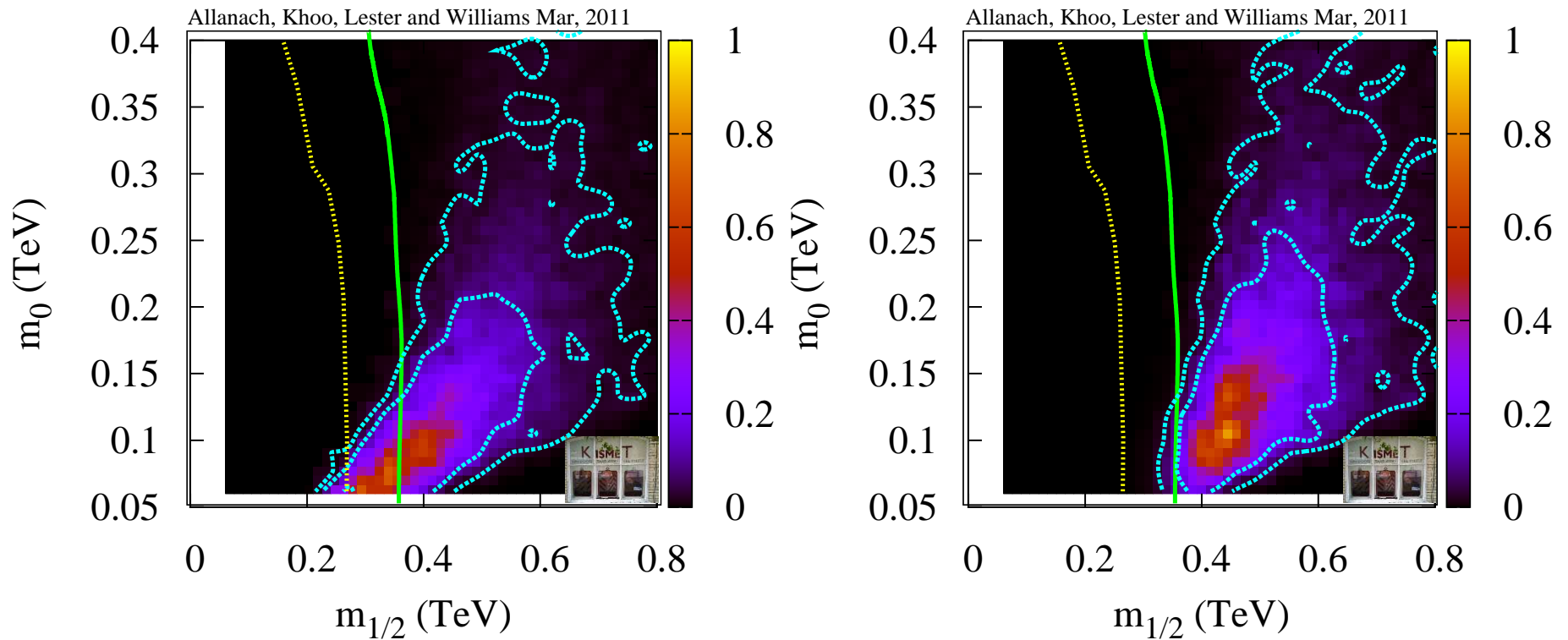
Found $\sigma_C = 0.6$, $\sigma_D = 0.3$ provide a reasonably good fit - by hand. $\vec{n} = (n_o^{(C)}, n_o^{(D)})$, $\vec{\lambda} = (\lambda_C, \lambda_D)$

$$(3) P(\vec{n}|\vec{\lambda}) = p(n_o^{(D)}|\lambda_D) p(n_o^{(D)} - n_o^{(C)}|\lambda_C - \lambda_D).$$



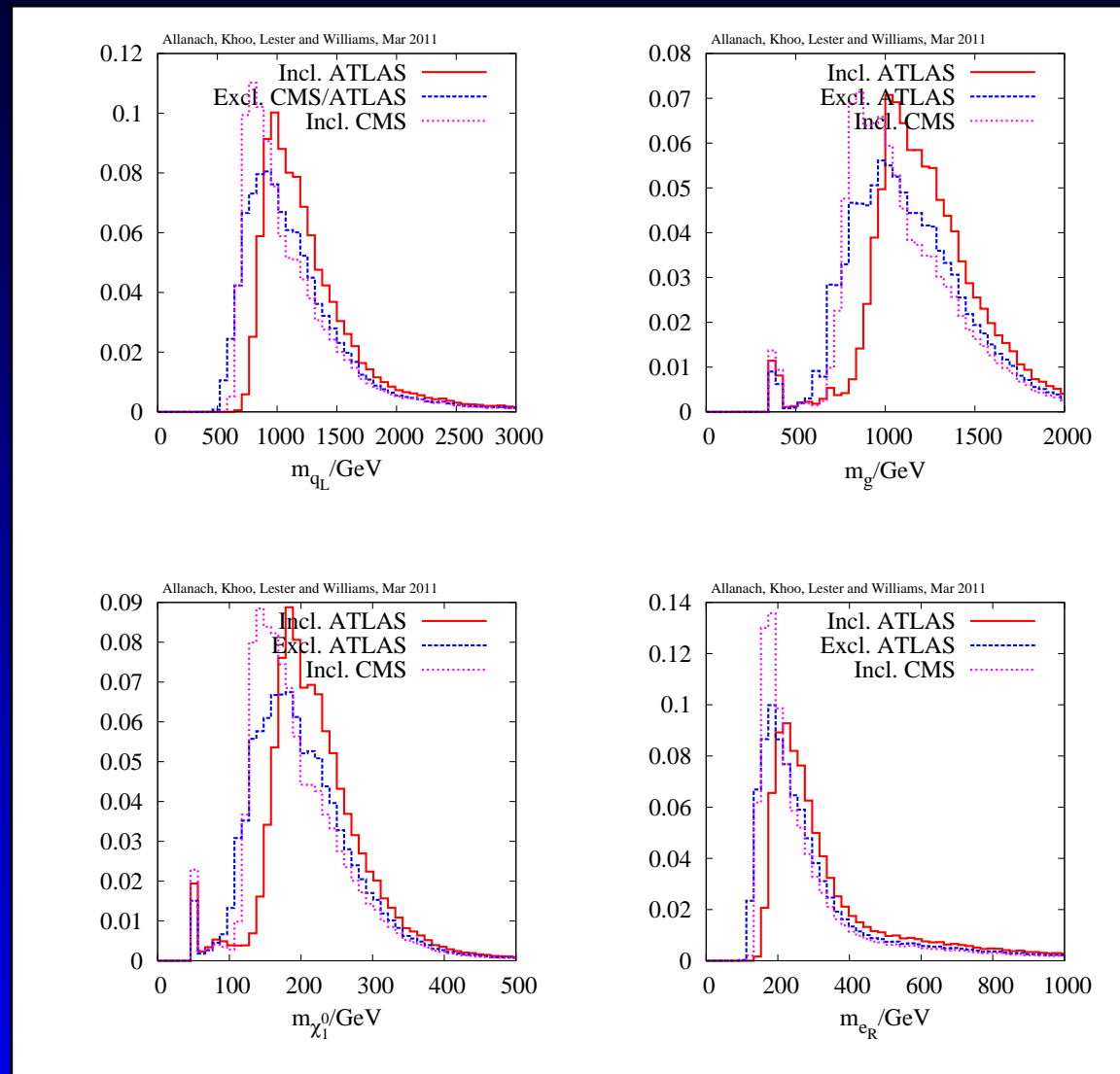
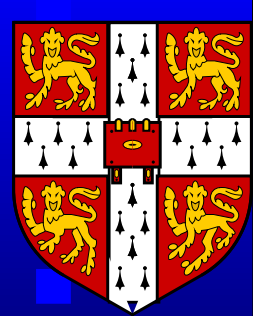


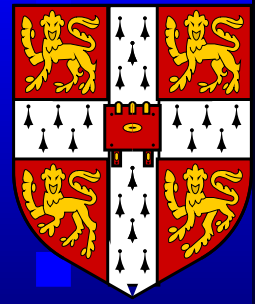
ATLAS Weighted Fits



Again, we assume A_0 - $\tan \beta$ independence and interpolate across m_0 and $m_{1/2}$.

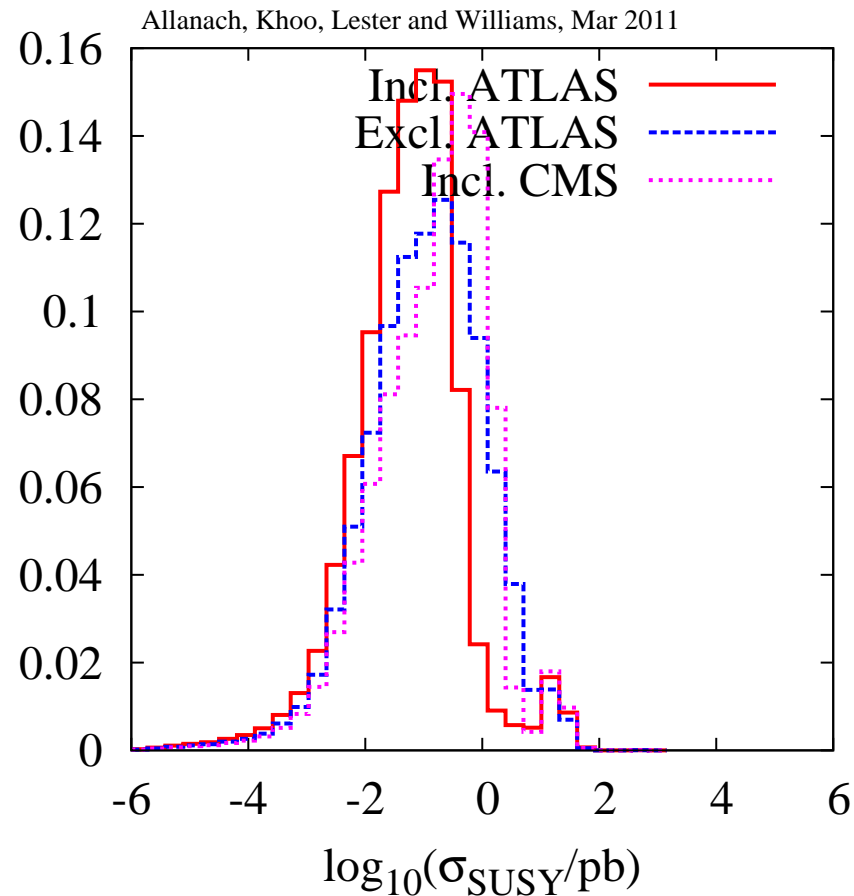
CMS/ATLAS Weighted Fits





Prospects for SUSY

Look good! 5fb^{-1} expected over the next couple of years



Summary

- LHC analyses providing a nice amount of information for interpretation of data. There's always room for improvement...
- Validation step very important for us when we're interpreting experiments' results.
- CMSSM *could well be discovered this year*
- Current searches reach squark and gluino masses of 775 GeV. This will be extended to 1000 GeV, covering much of the good-fit region.



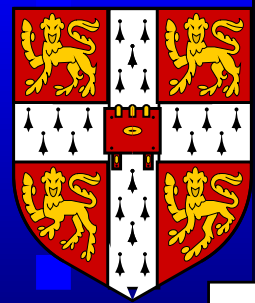
Other work

- Shortly after my first, the Mastercode collaboration^a performed fits of CMSSM, NUHM, VCMSSM, mSUGRA to CMS and ATLAS 11 data based on an **informed guess** of the likelihood function, fitted to the exclusion contours. Validated against one point. CMSSM results very similar to our analysis.
- More recently Akula et al^b examined ATLAS 0 and 1-lepton analyses at varying A_0 , $\tan \beta$ in a scan, showing where the indirect constraints apply.

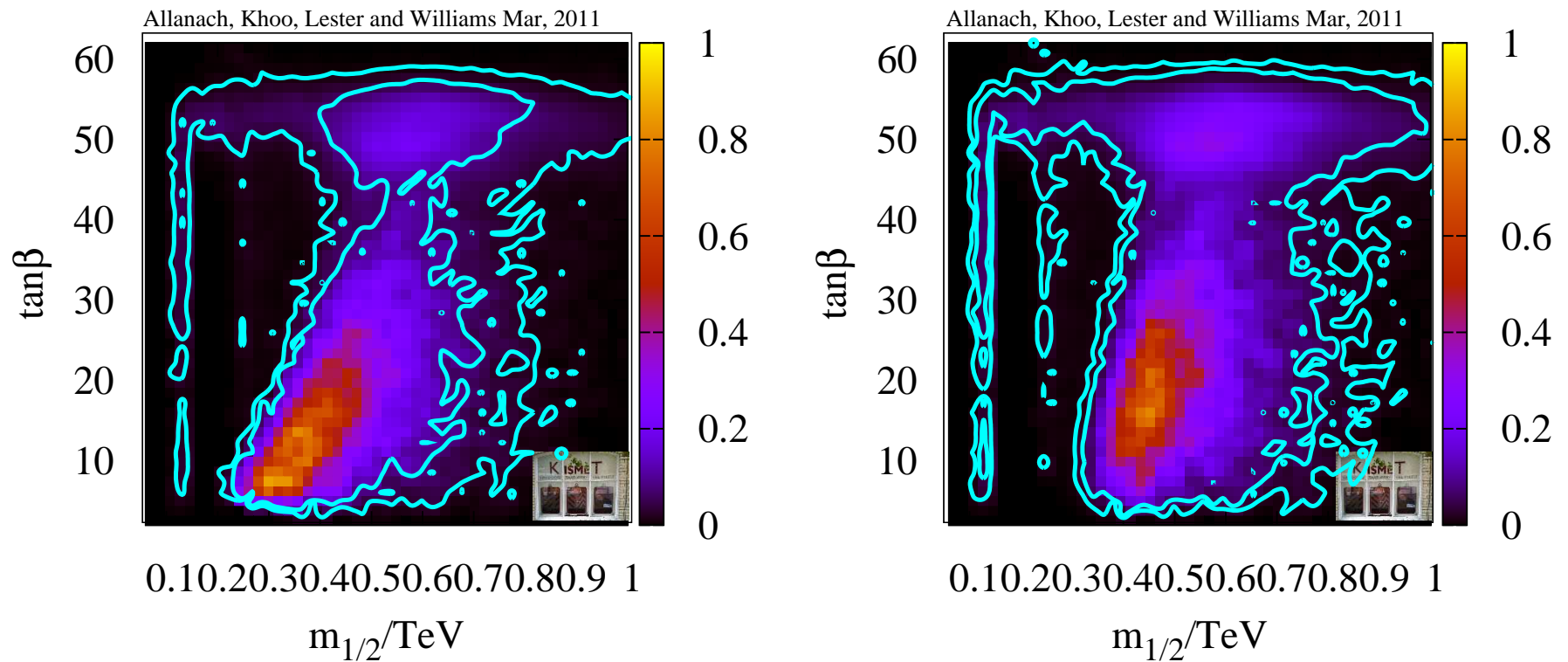
^a [arXiv://1102.4585](https://arxiv.org/abs/1102.4585)

^b [arXiv://1103.1197](https://arxiv.org/abs/1103.1197)



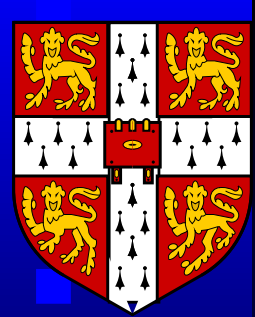


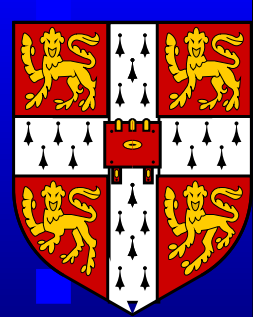
Log Fits



Before (left) and after (right) ATLAS 0-lepton exclusion limits.

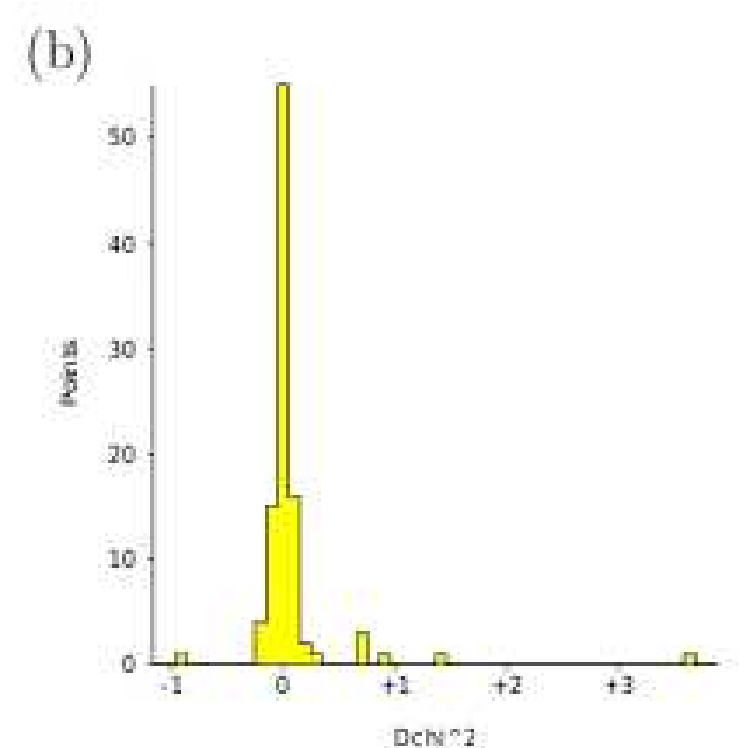
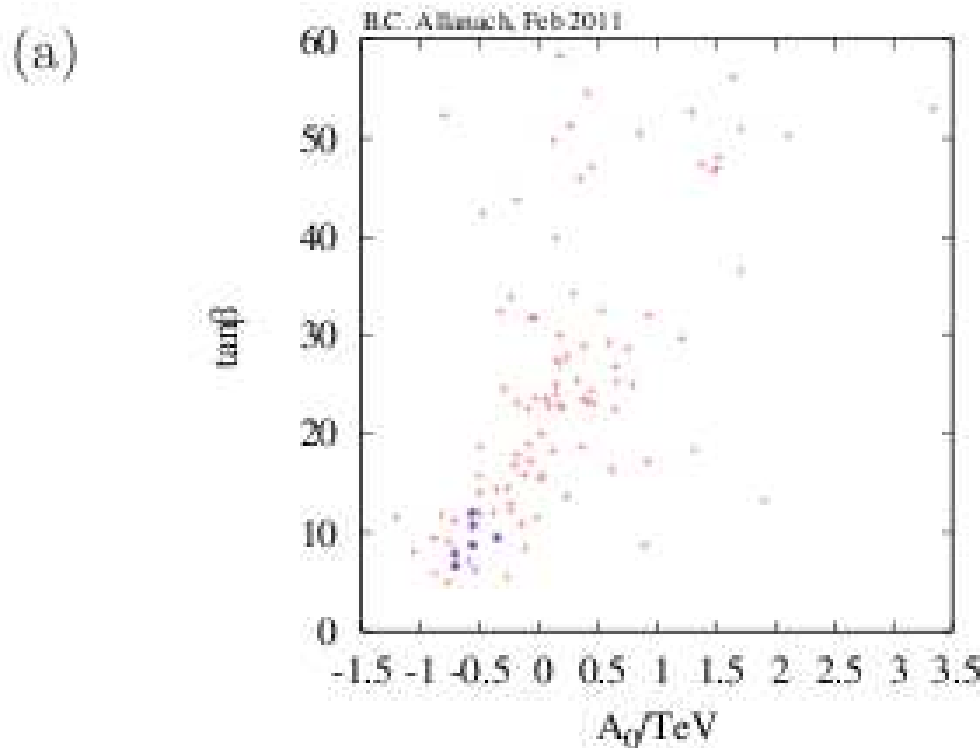
Supplementary Material

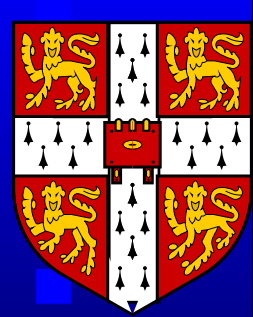




Checking A_0 - $\tan \beta$ Independence

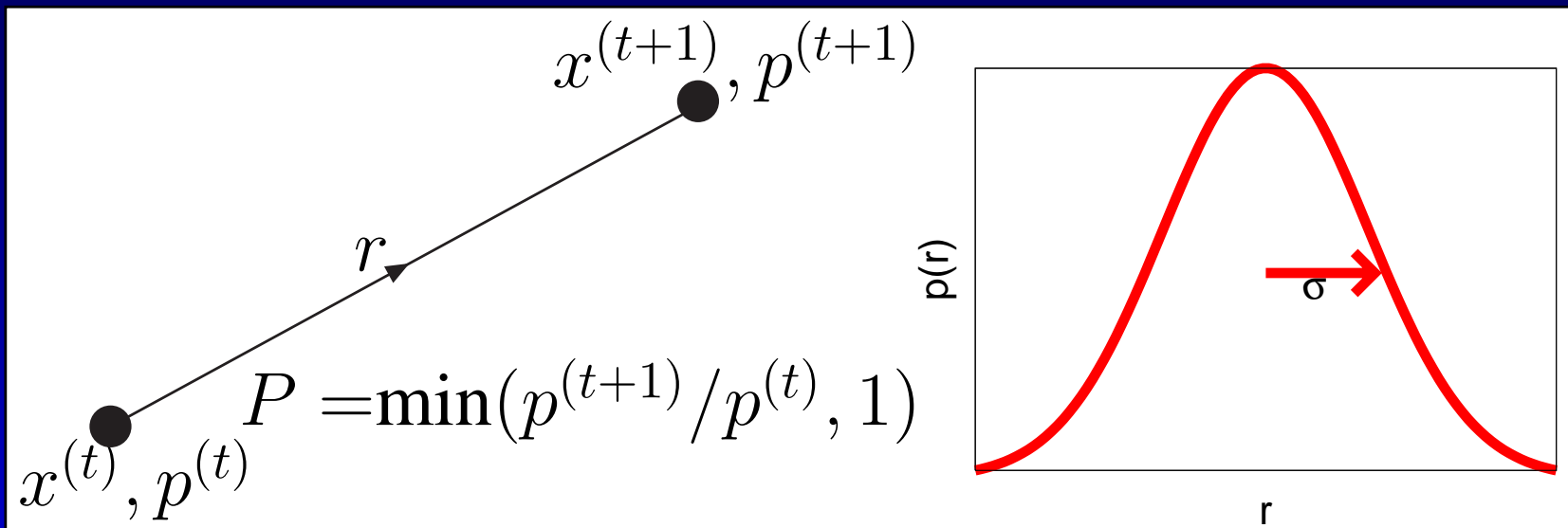
Choose samplings from the global fit at random and perform simulation on them. Then compare with $m_0 - m_{1/2}$ interpolation at $\tan \beta = 3$ and find how good the approximation is.



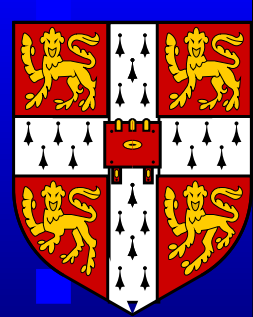


Markov-Chain Monte Carlo

Metropolis-Hastings Markov chain sampling consists of list of parameter points $x^{(t)}$ and associated posterior probabilities $p^{(t)}$.



Final density of x points $\propto p$. Required number of points goes *linearly* with number of dimensions.



Ice Cube

Neutralinos can become trapped in the sun $\tilde{h}^0 - Z$ coupling $\sigma_{\chi^0 p, SD} \propto [|N_{1d}|^2 - |N_{1u}|^2]^2$ dominates.
 $A^\odot \equiv \sigma v / V$:

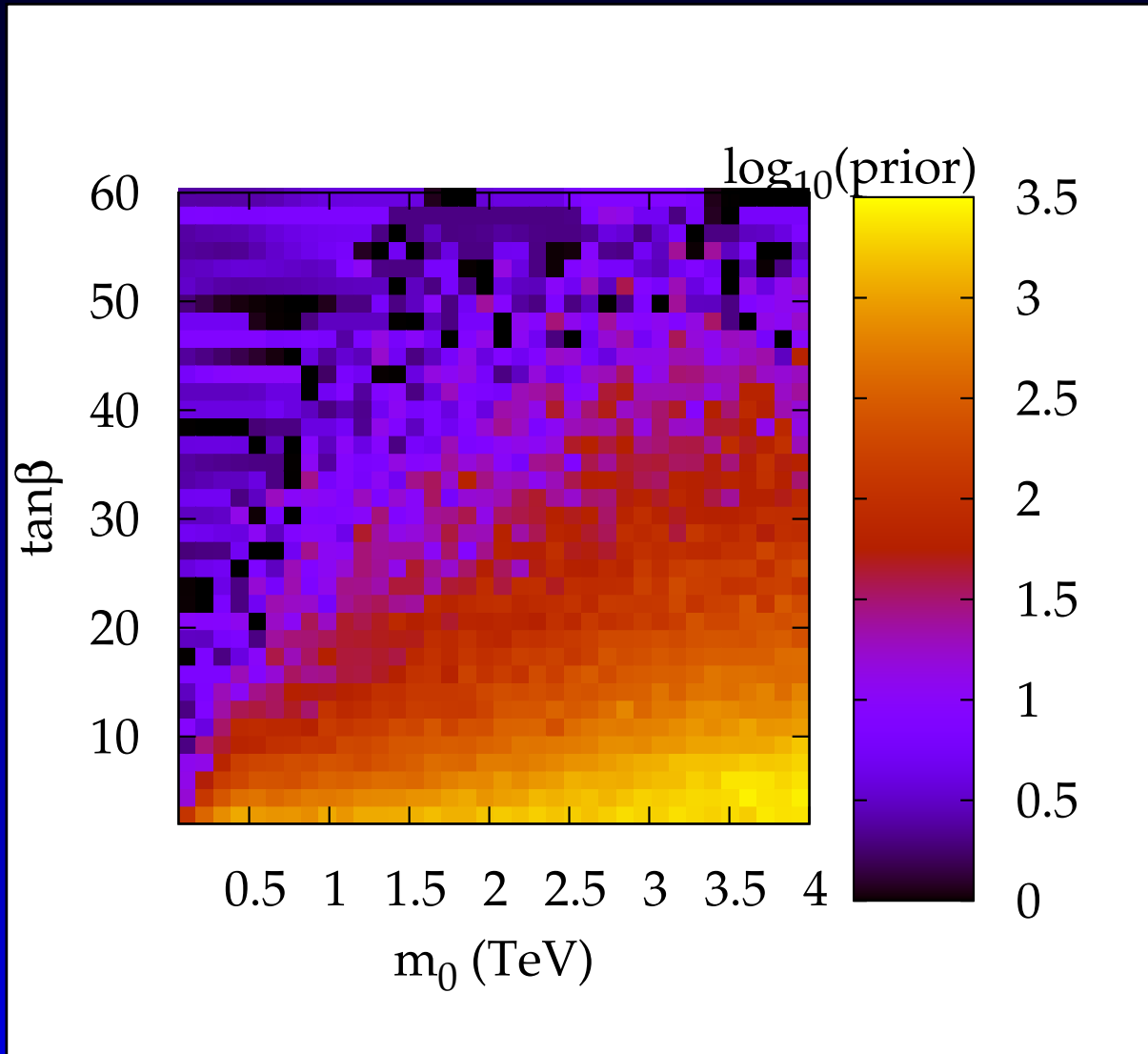
$$\dot{N} = C^\odot - A^\odot N^2,$$

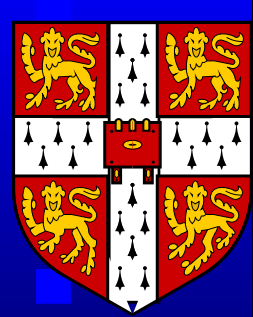
$$\Gamma = \frac{1}{2} A^\odot N^2 = \frac{1}{2} C^\odot \tanh^2 \left(\sqrt{C^\odot A^\odot} t_\odot \right)$$

$$\frac{dN_{\nu_\mu}}{dE_{\nu_\mu}} = \frac{C_\odot F_{\text{Eq}}}{4\pi D_{\text{ES}}^2} \left(\frac{dN_\nu}{dE_\nu} \right)^{\text{Inj}}$$

$$N_{\text{ev}} \approx \int \int \frac{dN_{\nu_\mu}}{dE_{\nu_\mu}} \frac{d\sigma_\nu}{dy} R_\mu((1-y) E_\nu) A_{\text{eff}} dE_{\nu_\mu} dy$$

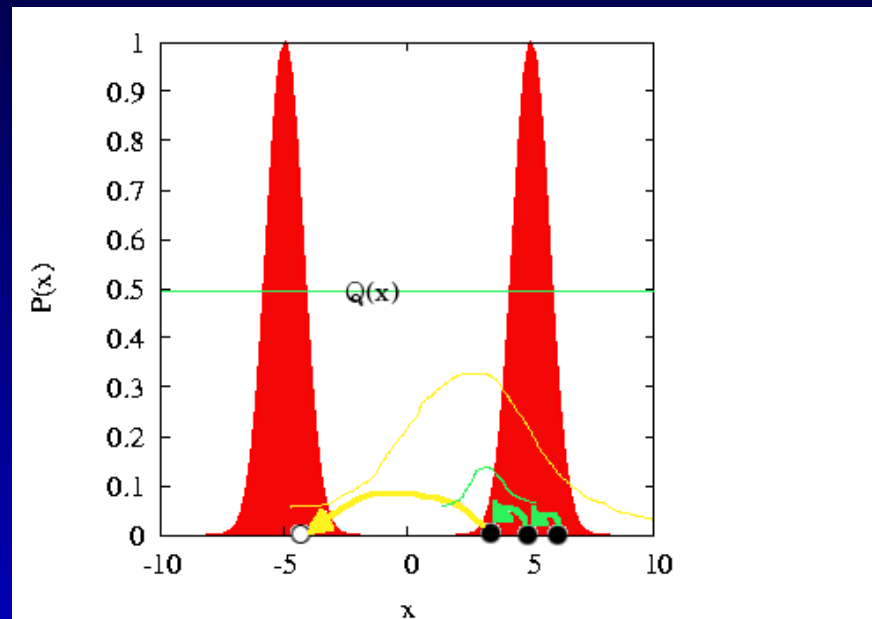
Naturalness priors



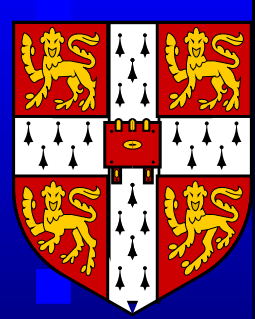


Potential Problem

Often, people use a **flat** $Q(x)$. The trouble with this “*blind drunk*” sampling is the following situation:



Either **large** or **small** proposal widths σ lead to low efficiencies of sampling. Our proposal is to determine a $Q(x)$ closer to $P(x)$ *semi-automatically*.



Bank Sampling

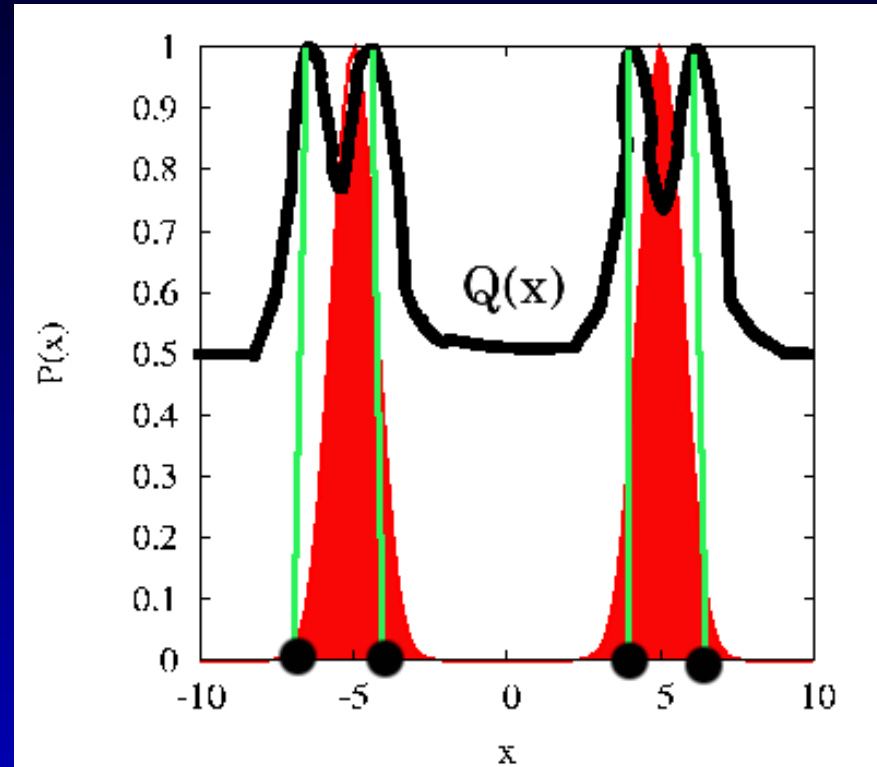


Figure 1: Bank points determined from previous runs:
want to have at least one point in each maximum.

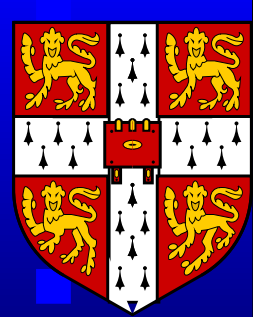
Knowledgeable drunk

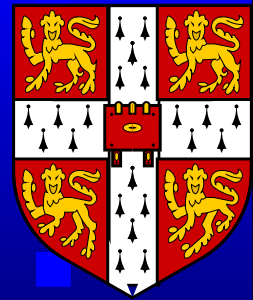
Proposal Distribution

$$Q_{bank}(\mathbf{x}; \mathbf{x}^{(t)}) = (1-\lambda)K(\mathbf{x}; \mathbf{x}^{(t)}) + \lambda \sum_{i=1}^N w_i K(\mathbf{x}; \mathbf{y}^{(i)})$$

w_i are a set of N weights: $\sum_{i=1}^N w_i = 1$, $0 < \lambda < 1$, while K is the proposal distribution.

With probability $(1 - \lambda)$ propose a local Metropolis step of the usual kind, i.e. “close” to the last point in the chain. With probability λ , teleport to the vicinity of one of the number of “banked” points, chosen with weight w_i from within the bank.



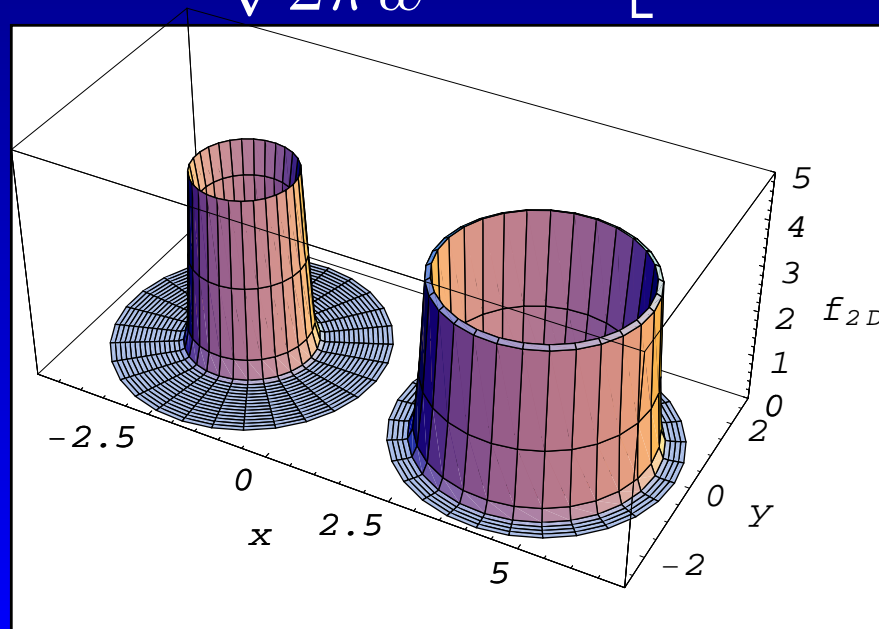


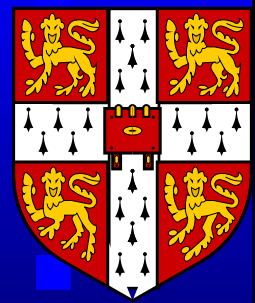
Example Distribution

$$f_{2D}(\mathbf{x}) = \text{circ}(\mathbf{x}; c_1, r_1, w_1) + \text{circ}(\mathbf{x}; c_2, r_2, w_2)$$

where $c_1 = (-2, 0)$, $r_1 = 1$, $w_1 = 0.1$, $c_2 = (+4, 0)$,
 $r_2 = 2$, $w_2 = 0.1$ and

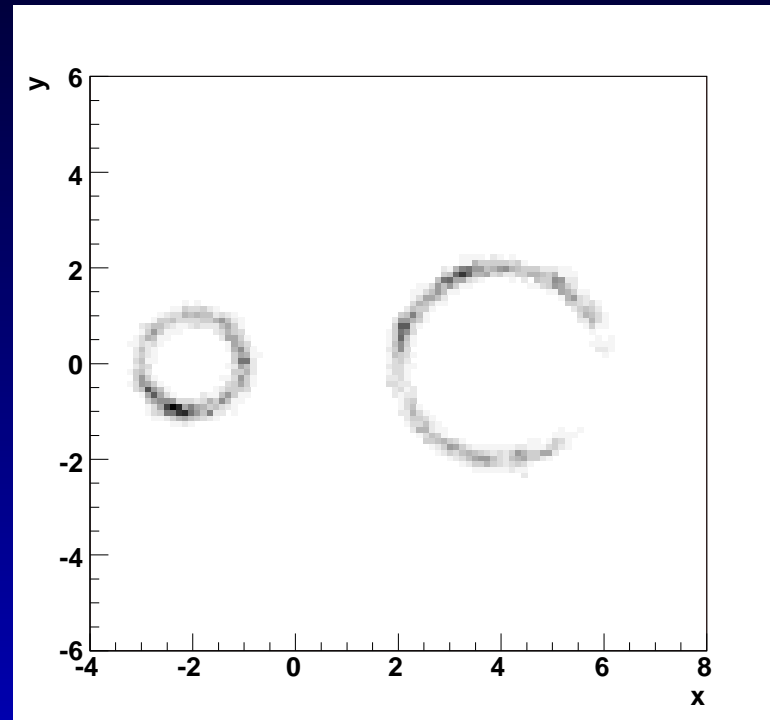
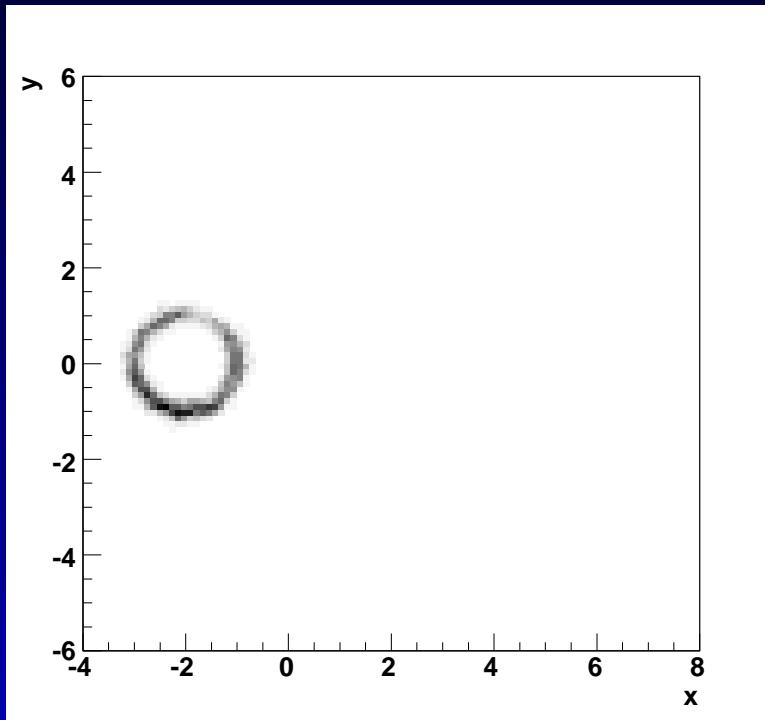
$$\text{circ}(\mathbf{x}; \mathbf{c}, r, w) = \frac{1}{\sqrt{2\pi w^2}} \exp \left[-\frac{(|\mathbf{x} - \mathbf{c}| - r)^2}{2w^2} \right].$$

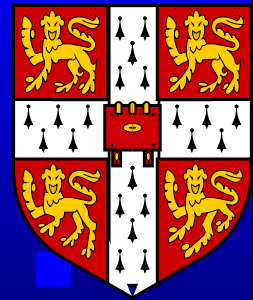




Bank vs Metropolis

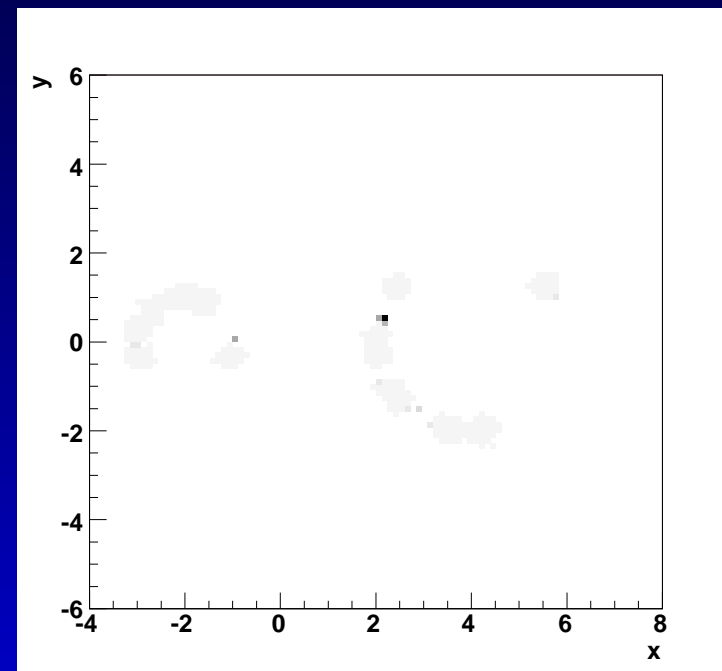
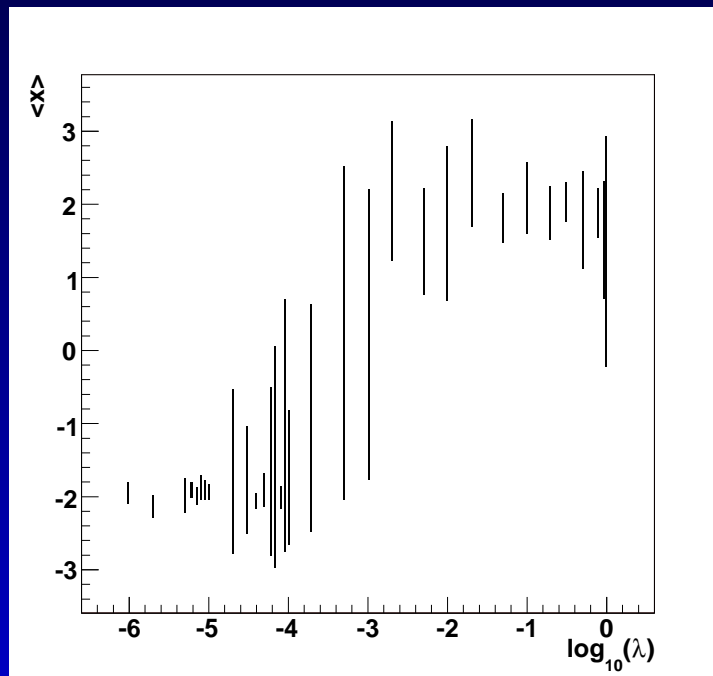
10 000 samples for MCMC and bank sampling:





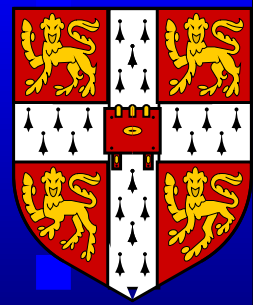
Safety with respect to λ

10 bank samplers, with 10 bank points generated in each circle: 10 000 samples. All started from $x = -2$. Correct $\langle x \rangle = 2$. $\lambda \approx 1$ is importance sampling limit.



Q: What values of λ are “safe”?

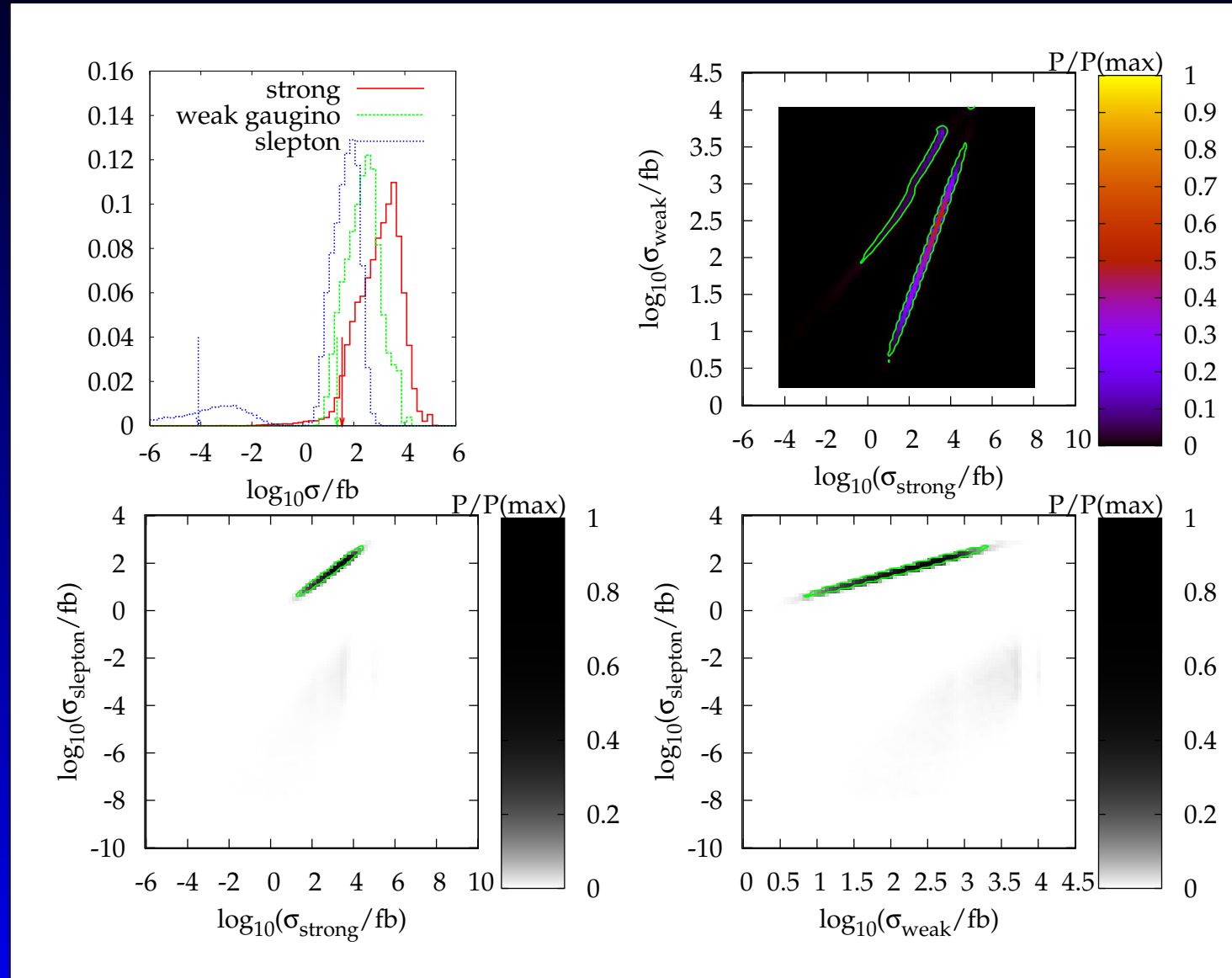
A: [0.001, 0.9]



LHC Cross-sections

Science & Technology
Facilities Council

Supersymmetry
Cambridge
Working group





Collider Check

Need corroboration with *direct detection*.

If we can pin particle physics down, a comparison between the predicted relic density and that observed is a test of the cosmological assumptions used in the prediction.^a

Thus, if it doesn't fit, you change the cosmology until it does.

^aBCA, G. Belanger, F. Boudjema, A. Pukhov, JHEP 0412 (2004) 020.; M. Nojiri, D. Tovey, JHEP 0603 (2006) 063

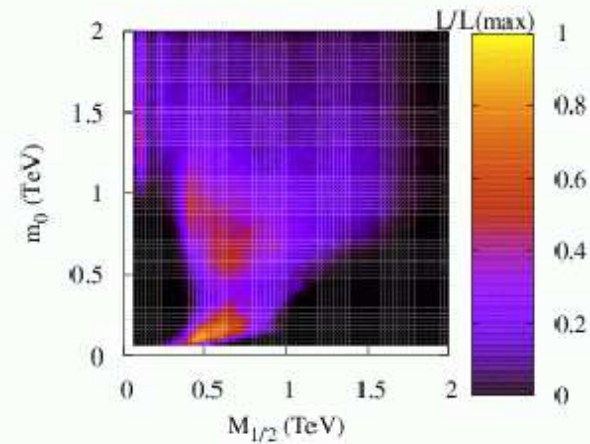
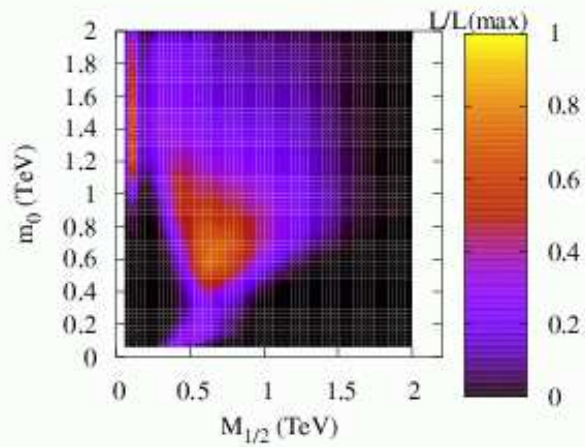
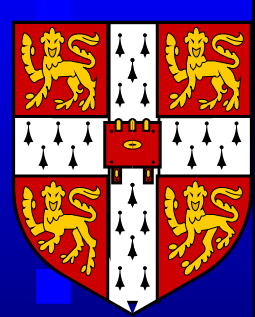


CMSSM Regions

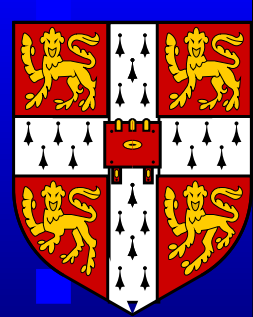
After WMAP+LEP2, **bulk region** diminished. Need specific mechanism to reduce overabundance:

- **$\tilde{\tau}$ coannihilation**: small m_0 , $m_{\tilde{\tau}_1} \approx m_{\chi_1^0}$. Boltzmann factor $\exp(-\Delta M/T_f)$ controls ratio of species. $\tilde{\tau}_1 \chi_1^0 \rightarrow \tau \gamma$, $\tilde{\tau}_1 \tilde{\tau}_1 \rightarrow \tau \bar{\tau}$.
- **Higgs Funnel**: $\chi_1^0 \chi_1^0 \rightarrow A \rightarrow b\bar{b}/\tau\bar{\tau}$ at large $\tan \beta$. Also via^a h at large m_0 small $M_{1/2}$.
- **Focus region**: Higgsino LSP at large m_0 : $\chi_1^0 \chi_1^0 \rightarrow WW/ZZ/Zh/t\bar{t}$.
- **\tilde{t} coannihilation**: high $-A_0$, $m_{\tilde{t}_1} \approx m_{\chi_1^0}$. $\tilde{t}_1 \chi_1^0 \rightarrow gt$, $\tilde{t}\tilde{t} \rightarrow tt$

Comparison



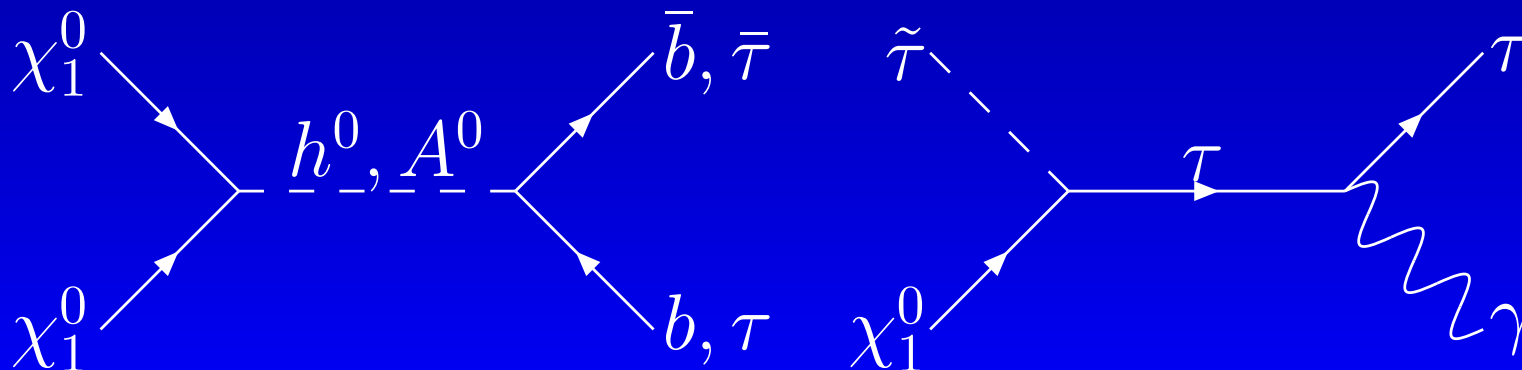
- LHS: allowing non thermal- χ_1^0 contribution
- RHS: only χ_1^0 dark matter
- *(flat priors)*



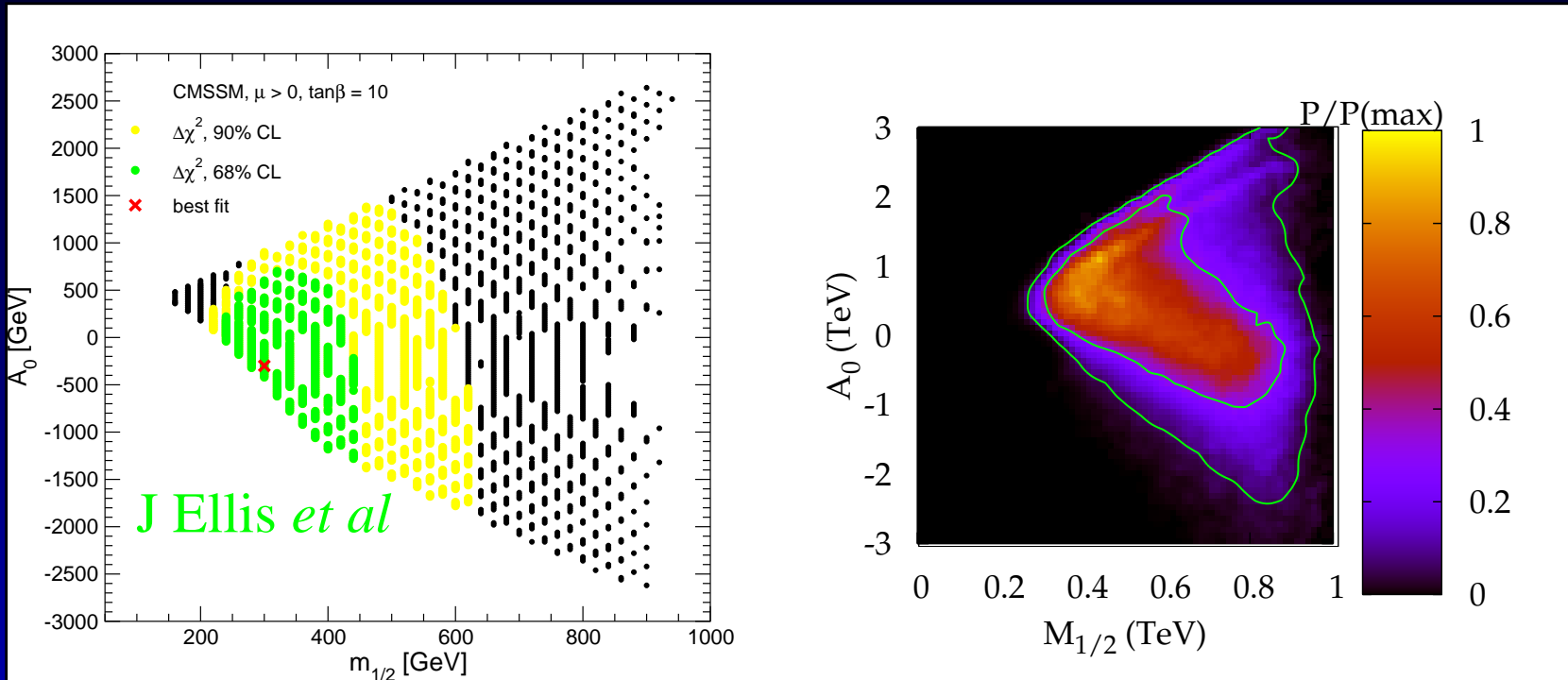
Annihilation Mechanism

Define stau co-annihilation when $m_{\tilde{\tau}}$ is within 10% of $m_{\chi_1^0}$ and Higgs pole when $m_{h,A}$ is within 10% of $2m_{\chi_1^0}$.

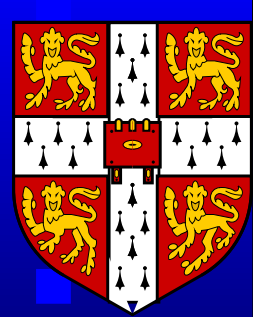
mechanism	flat prior	natural prior
h^0 –pole	0.025	0.07
A^0 –pole	0.41	0.14
$\tilde{\tau}$ –co-annihilation	0.26	0.18
rest	0.31	0.61



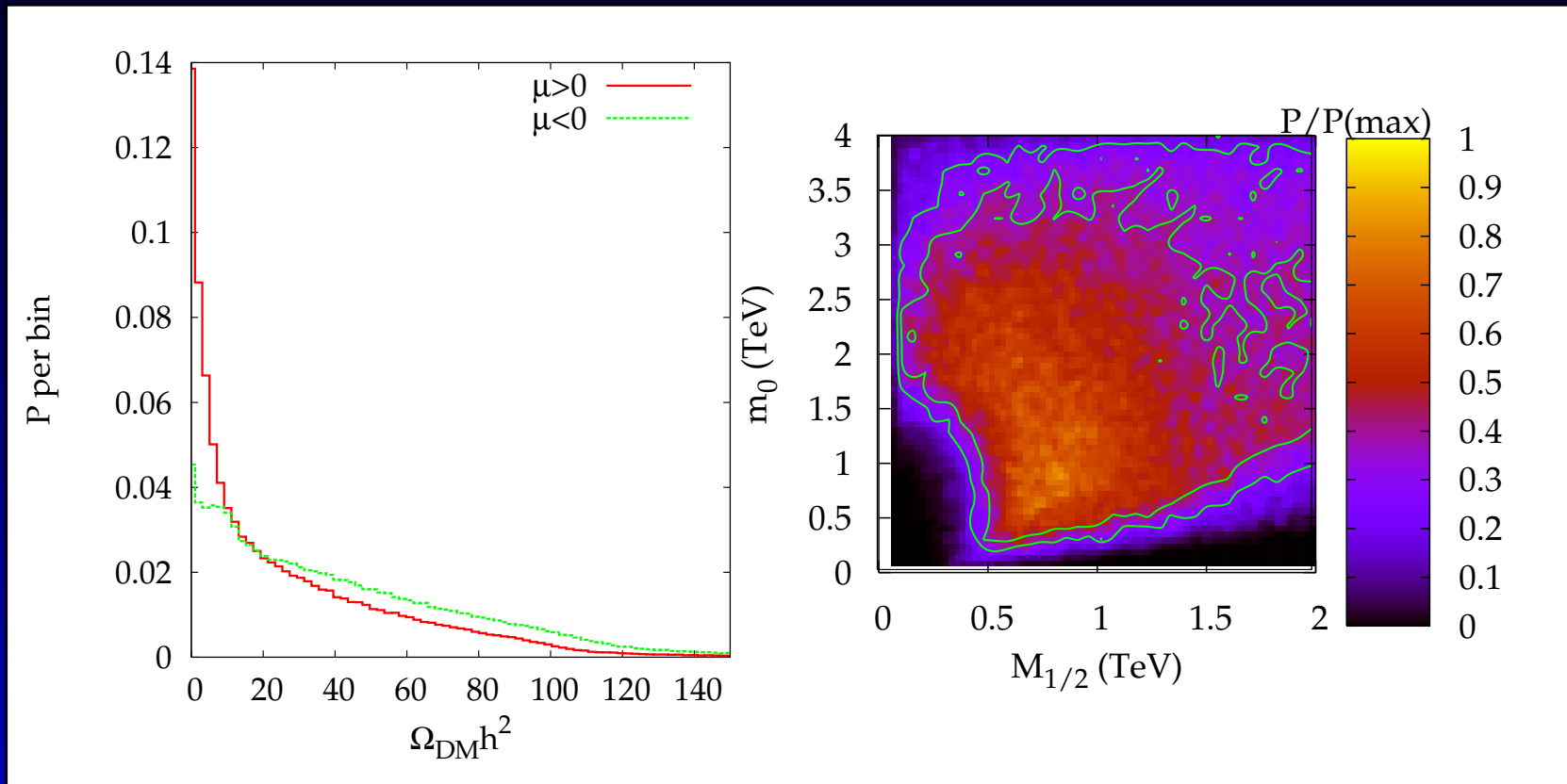
Comparison



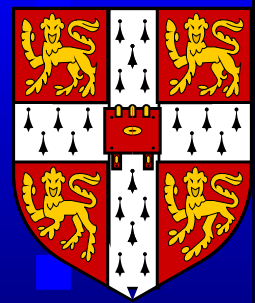
- Fix $\tan\beta = 10$ and all SM inputs
- Restrict $m_0, M_{1/2} < 1$ TeV.
- *Same* fits!



No Dark Matter Fits

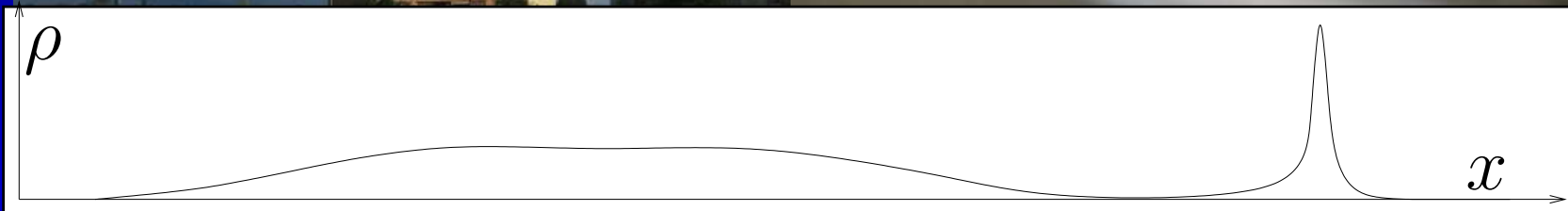


Huge χ^2 from the dark matter relic density.



Volume Effects

Can't rely on a good χ^2 in non-Gaussian situation



Likelihood and Posterior

Q: What's the chance of observing someone to be pregnant, given that they are female?

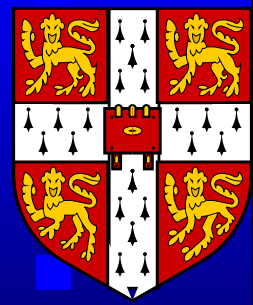


Likelihood

$$p(\text{pregnant} \mid \text{female, human}) = 0.01$$

Posterior

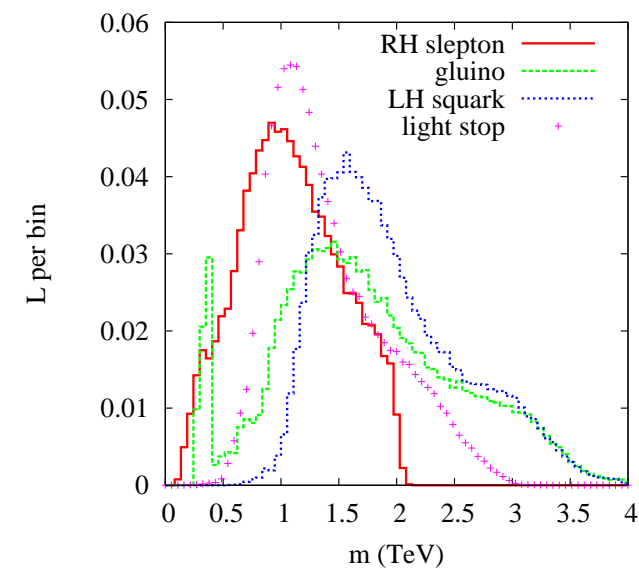
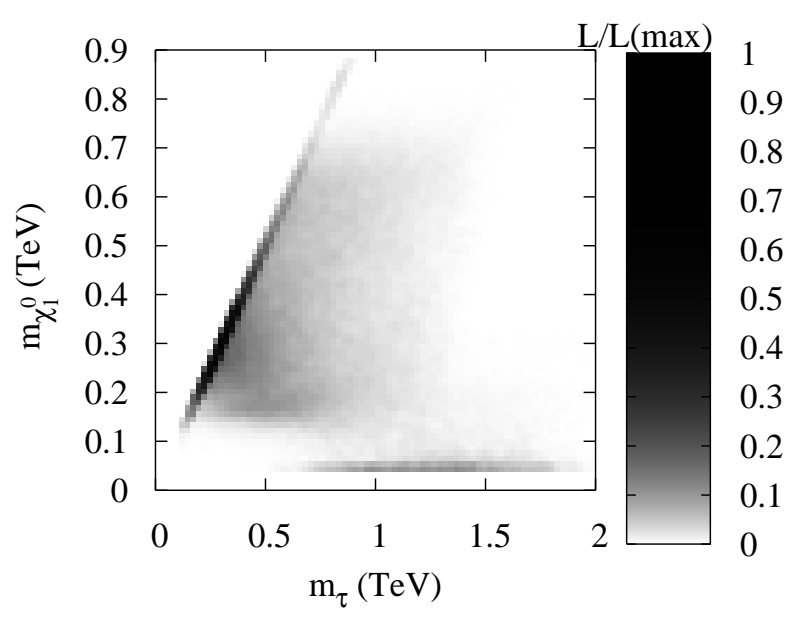
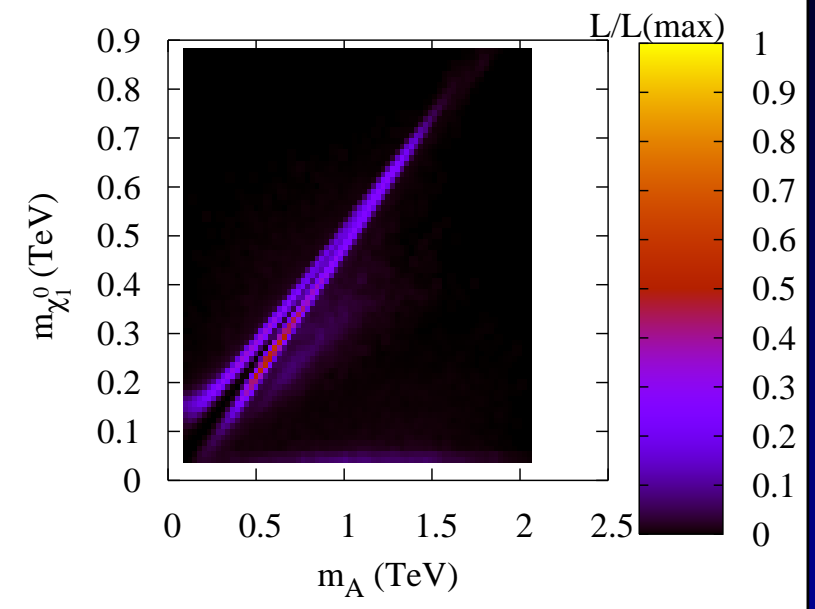
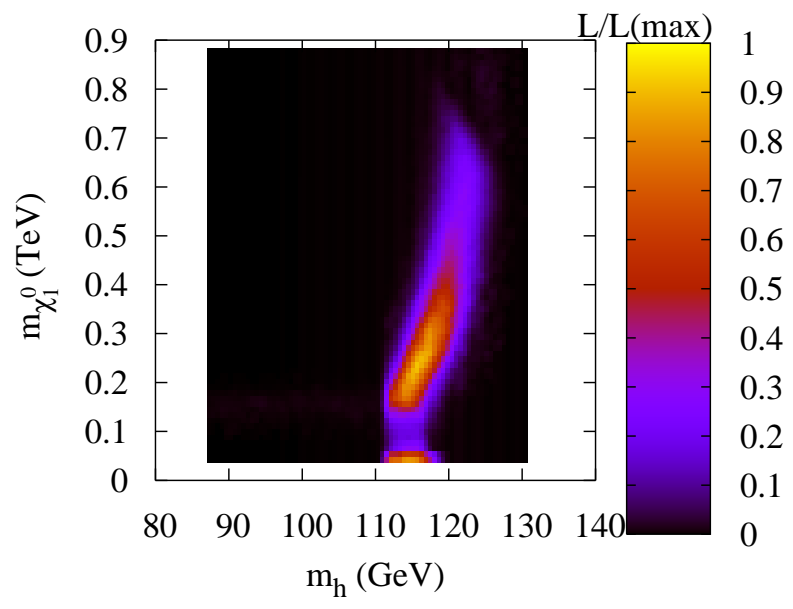
$$p(\text{female} \mid \text{pregnant, human}) = 1.00$$



Sanity Check

Science & Technology
Facilities Council

Supersymmetry
Cambridge
Working group



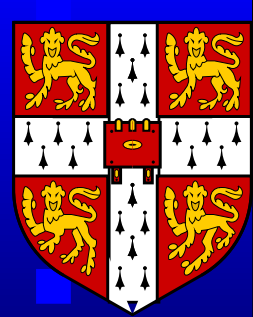
LHC vs LC in SUSY Measurement

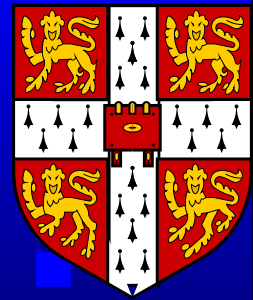
- **LHC** (start date 2007) produces strongly interacting particles up to a few TeV. Precision measurements of mass *differences* possible if the decay chains exist: possibly per mille for leptons, several percent for jets.
- **ILC** has several energy options: 500-1000 GeV, CLIC up to 3 TeV. Linear colliders produce less strong particles but much easier to make precision measurements of masses/couplings.

Q: What energy for LC?

Q: What do we get from LHC^a?

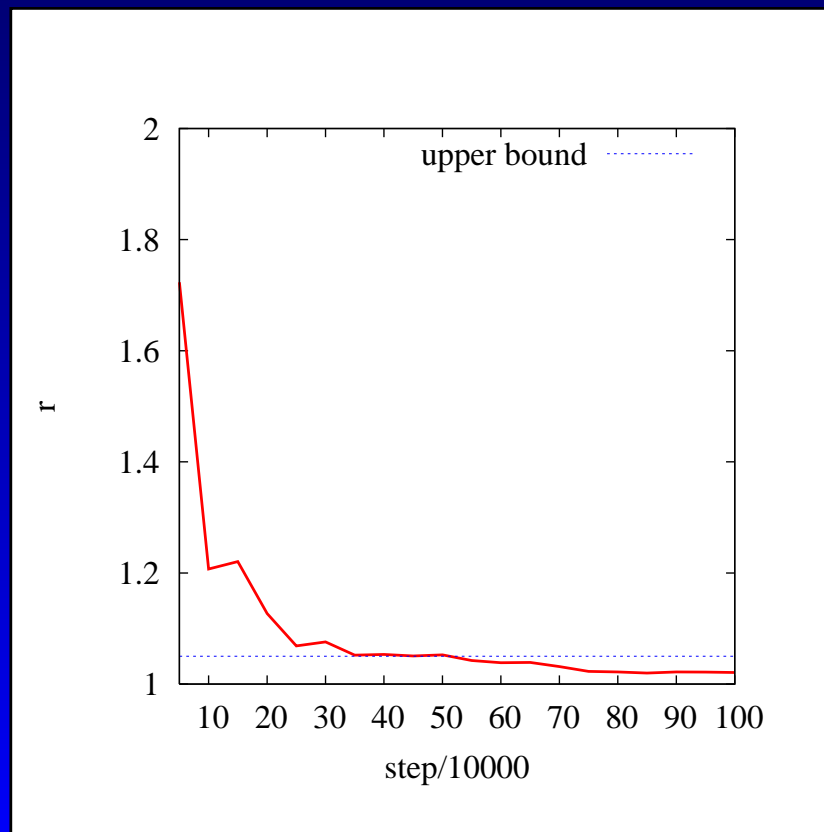
^aLHC/ILC Working Group Report: hep-ph/0410364

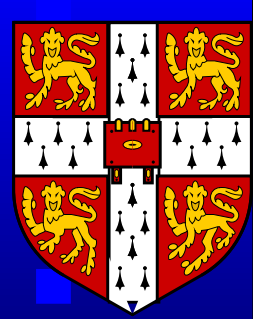




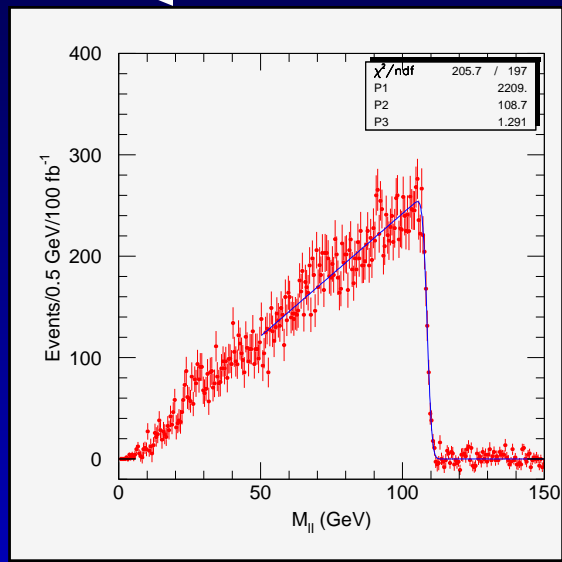
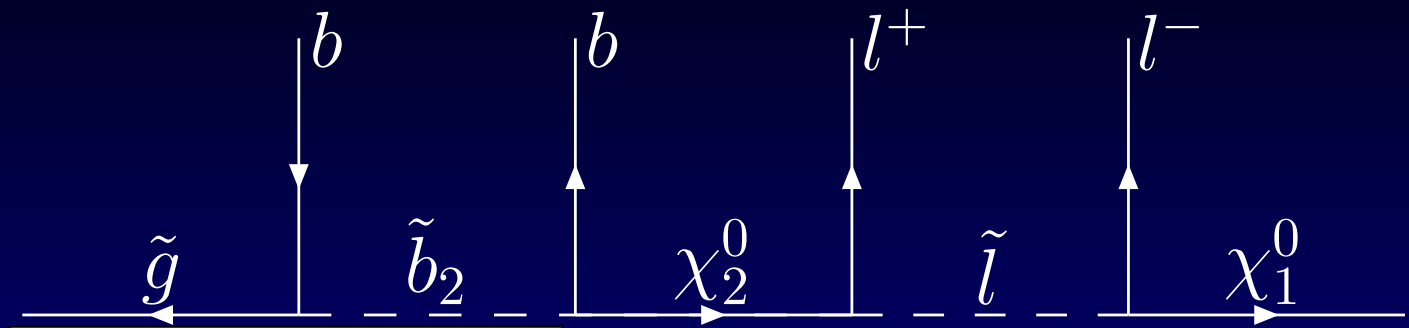
Convergence

We run $9 \times 1\,000\,000$ points. By comparing the 9 independent chains with random starting points, we can provide a statistical measure of convergence: an upper bound r on the expected variance decrease for infinite statistics.





LHC SUSY Measurements



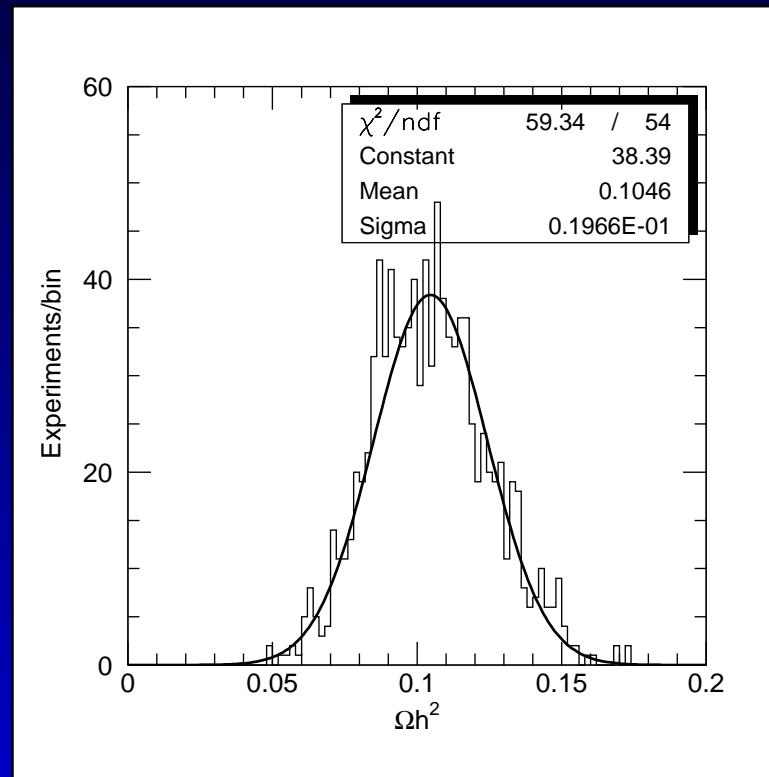
$$m_{ll}^2 = \frac{(m_{\chi_2^0}^2 - m_{\tilde{l}}^2)(m_{\tilde{l}}^2 - m_{\chi_1^0}^2)}{m_{\tilde{l}}^2}$$

Q: Can we measure enough of these to pin SUSY^a down?

^aBCA, Lester, Parker, Webber, JHEP 0009 (2000) 004

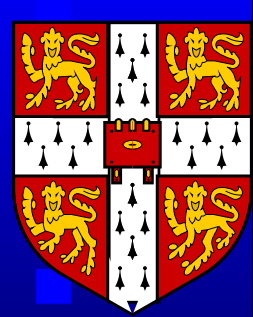
Predicting Ωh^2

Not much left that's allowed but edge measurements allow reasonable Ωh^2 error^a for 300 fb^{-1} .



Q: What about other bits of parameter space?

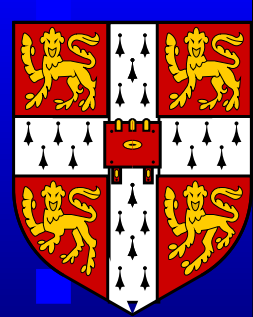
^aM Nojiri, G Polesello, D Tovey, JHEP 0603 (2006) 063,
[hep-ph/0512204](https://arxiv.org/abs/hep-ph/0512204).



Bulk Region

M Nojiri, G Polesello, D Tovey, JHEP 0603 (2006) 063, hep-ph/0512204. for 300 fb^{-1} . SPA point $m_0 = 70 \text{ GeV}$, $m_{1/2} = 250 \text{ GeV}$, $A_0 = -300 \text{ GeV}$, $\tan \beta = 10$, $\mu > 0$: $\Omega h^2 = 0.108$. Put in m_{ll}^{max} , m_{llq}^{max} , m_{lq}^{low} , m_{lq}^{high} , m_{llq}^{min} , $m_{lL} - m_{\chi_1^0}$, $m_{ll}^{max}(\chi_4^0)$, $m_{\tau\tau}^{max}$, m_h .

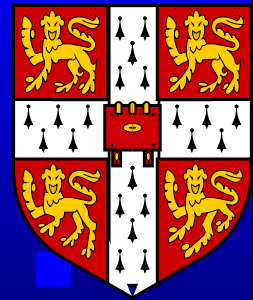
$\tilde{\chi}_1^0 \tilde{\chi}_1^0 \rightarrow l^+ l^-$	40%
$\tilde{\chi}_1^0 \tilde{\chi}_1^0 \rightarrow \tau^+ \tau^-$	28%
$\tilde{\chi}_1^0 \tilde{\chi}_1^0 \rightarrow \nu \bar{\nu}$	3%
$\tilde{\chi}_1^0 \tilde{\tau}_1 \rightarrow Z \tau$	4%
$\tilde{\chi}_1^0 \tilde{\tau}_1 \rightarrow A \tau$	18%
$\tilde{\tau}_1 \tilde{\tau}_1 \rightarrow \tau \tau$	2%



Neutralino mass matrix

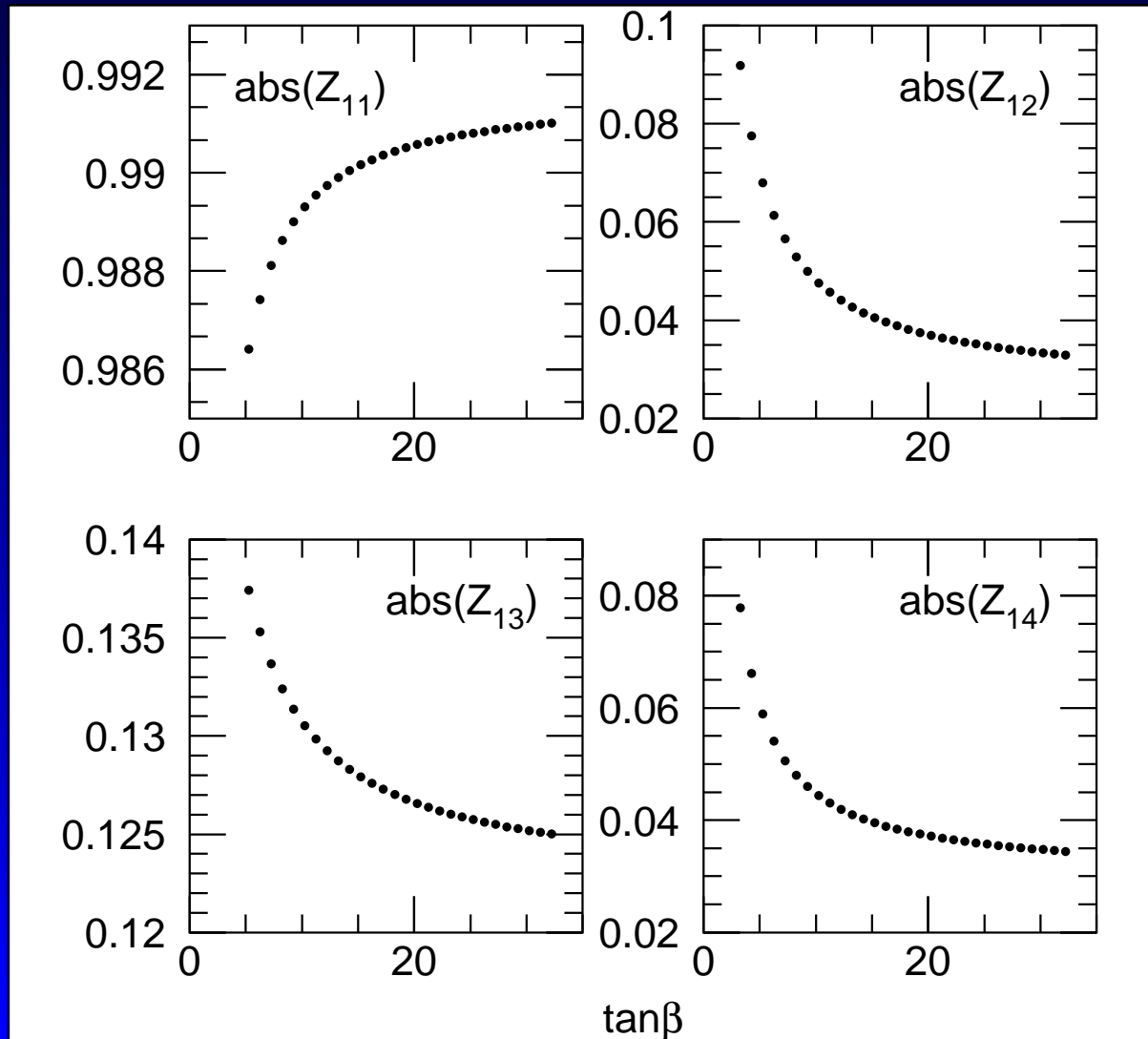
Neutralino masses measured: $\chi_{1,2,4}^0$ but need mixing matrix to determine couplings. Left with $\tan \beta$.

$$(4) \quad \begin{bmatrix} M_1 & 0 & -m_Z c_\beta s_W & m_Z s_\beta s_W \\ 0 & M_2 & m_Z c_\beta c_W & -m_Z s_\beta c_W \\ -m_Z c_\beta s_W & m_Z c_\beta c_W & 0 & -\mu \\ m_Z s_\beta s_W & -m_Z s_\beta c_W & -\mu & 0 \end{bmatrix}$$



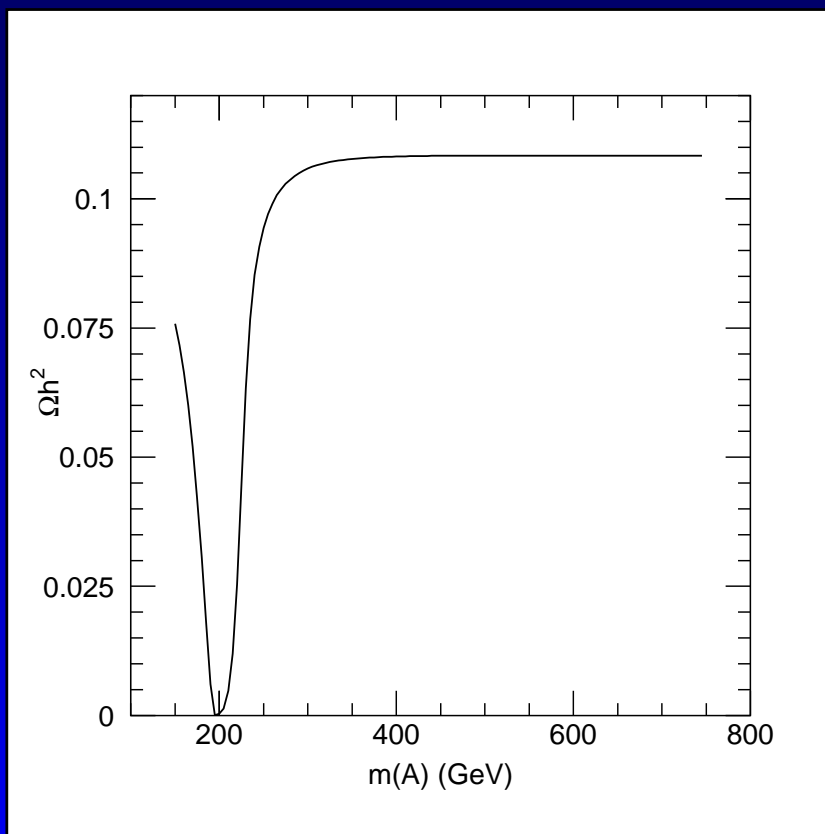
Neutralino mass matrix

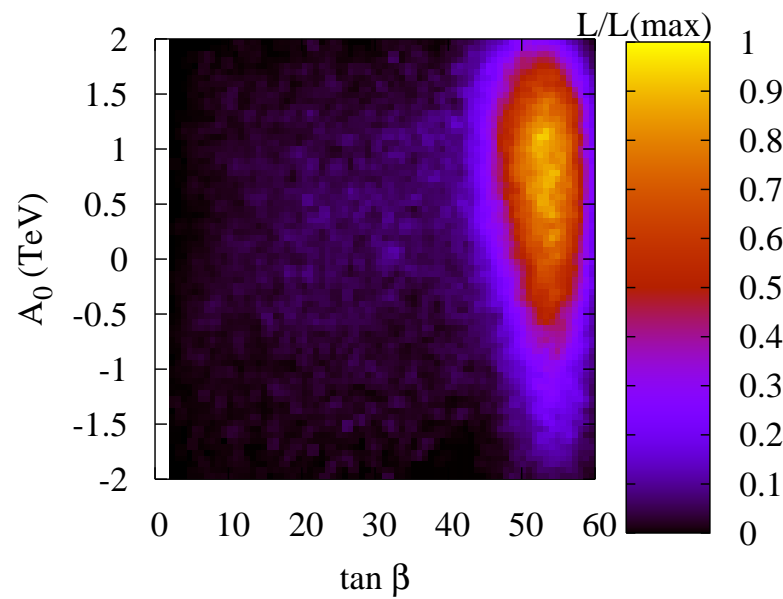
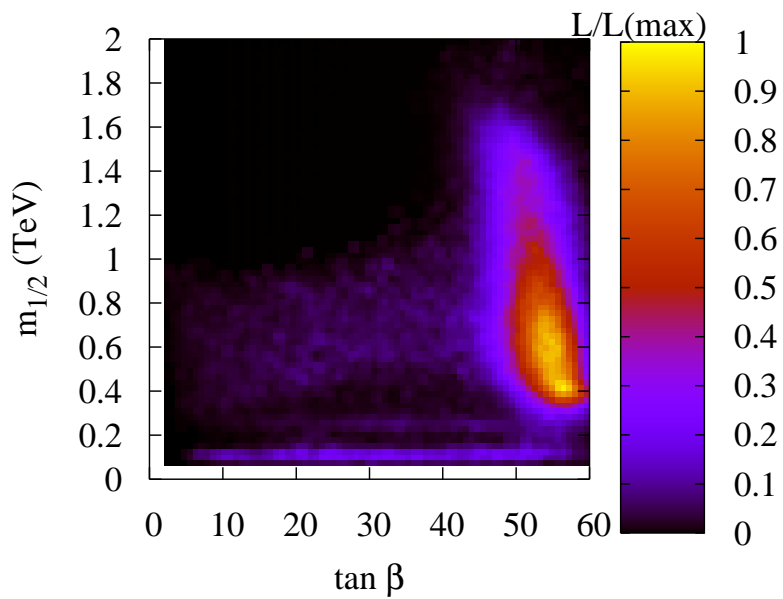
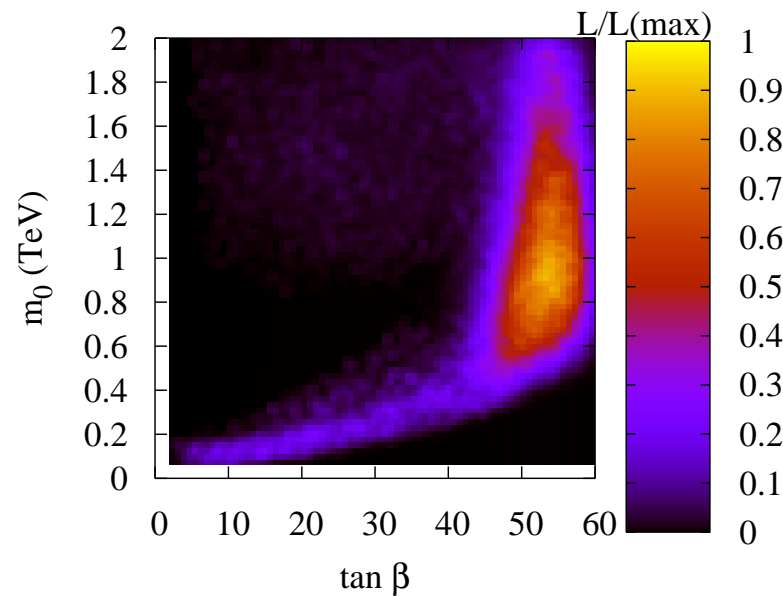
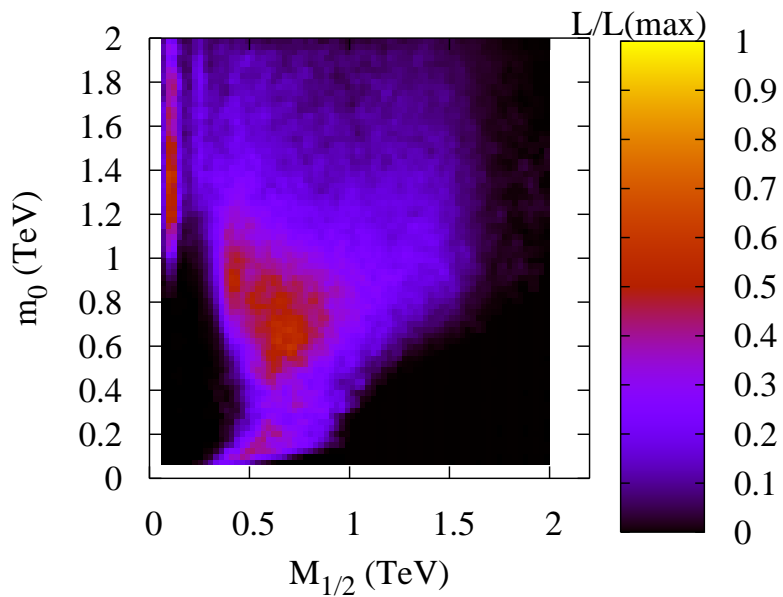
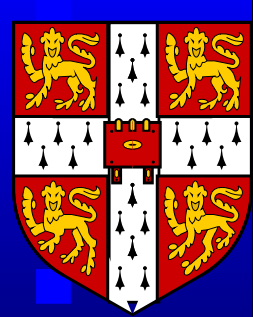
Neutralino masses measured: $\chi_{1,2,4}^0$ but need mixing matrix to determine couplings. Left with $\tan\beta$.



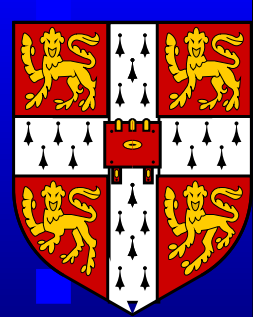
Slepton/ A^0 Higgs

$\Gamma(\chi_2^0 \rightarrow \tilde{l}_R l) / \Gamma(\chi_2^0 \rightarrow \tilde{\tau}_1 \tau)$ then helps determine θ_τ for a given $\tan \beta$. Exclusion of A^0 helps you to exclude A^0 appearing in cascade decays. Measurement of m_h provides constraints in $m_A - \tan \beta$ plane.





v



Uncertainties in Relic Density

Bulk region: $\tilde{B}\tilde{B} \rightarrow Z, h \rightarrow l\bar{l}$. Coannihilation: $\tilde{\tau}\chi_1^0 \rightarrow \tau + X$

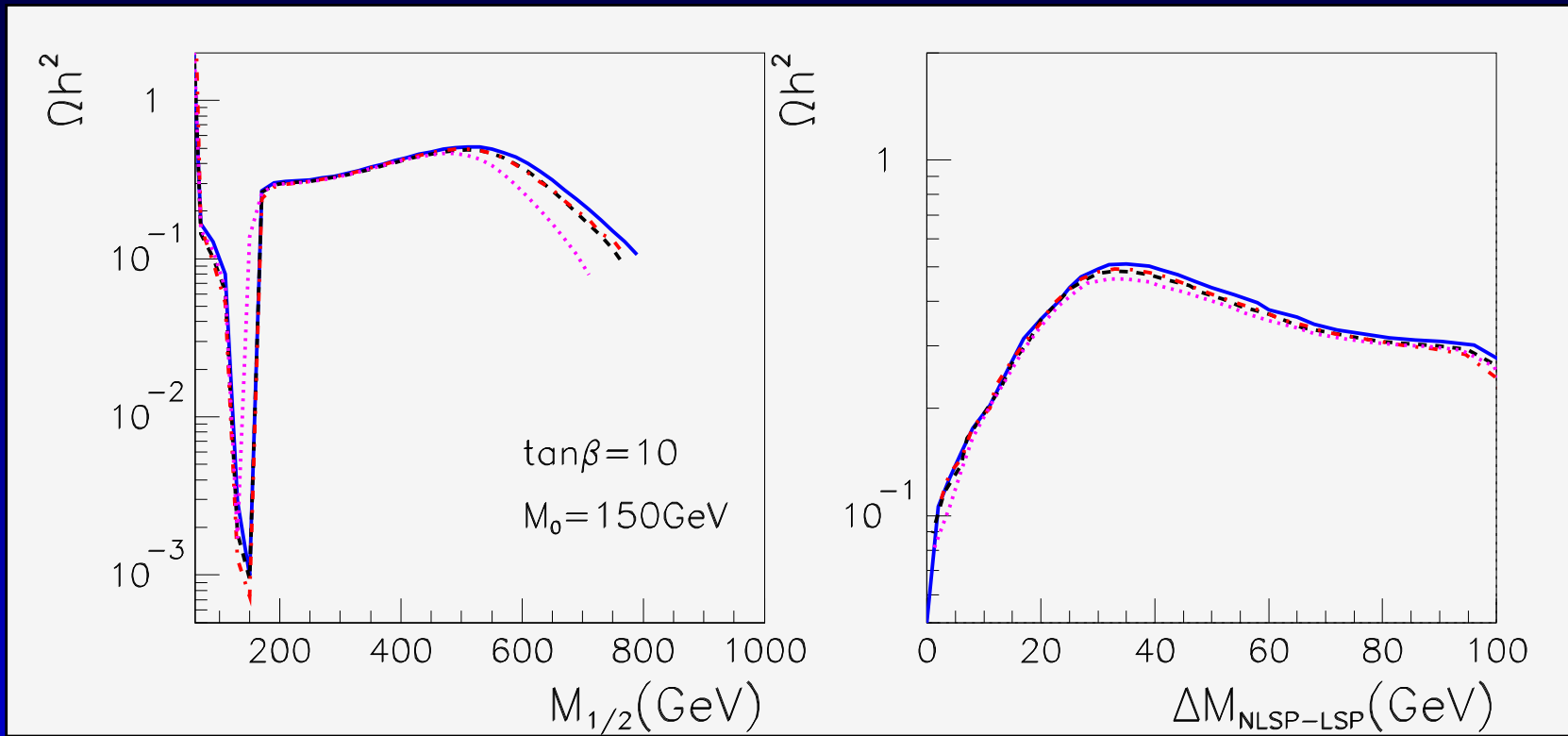
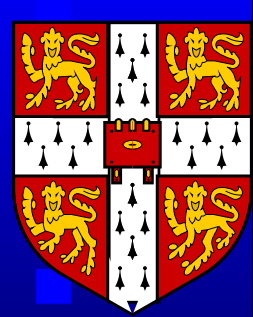


Figure 3: Bulk/coannihilation region. Full: SoftSusy, dotted: SPheno.



Focus Point

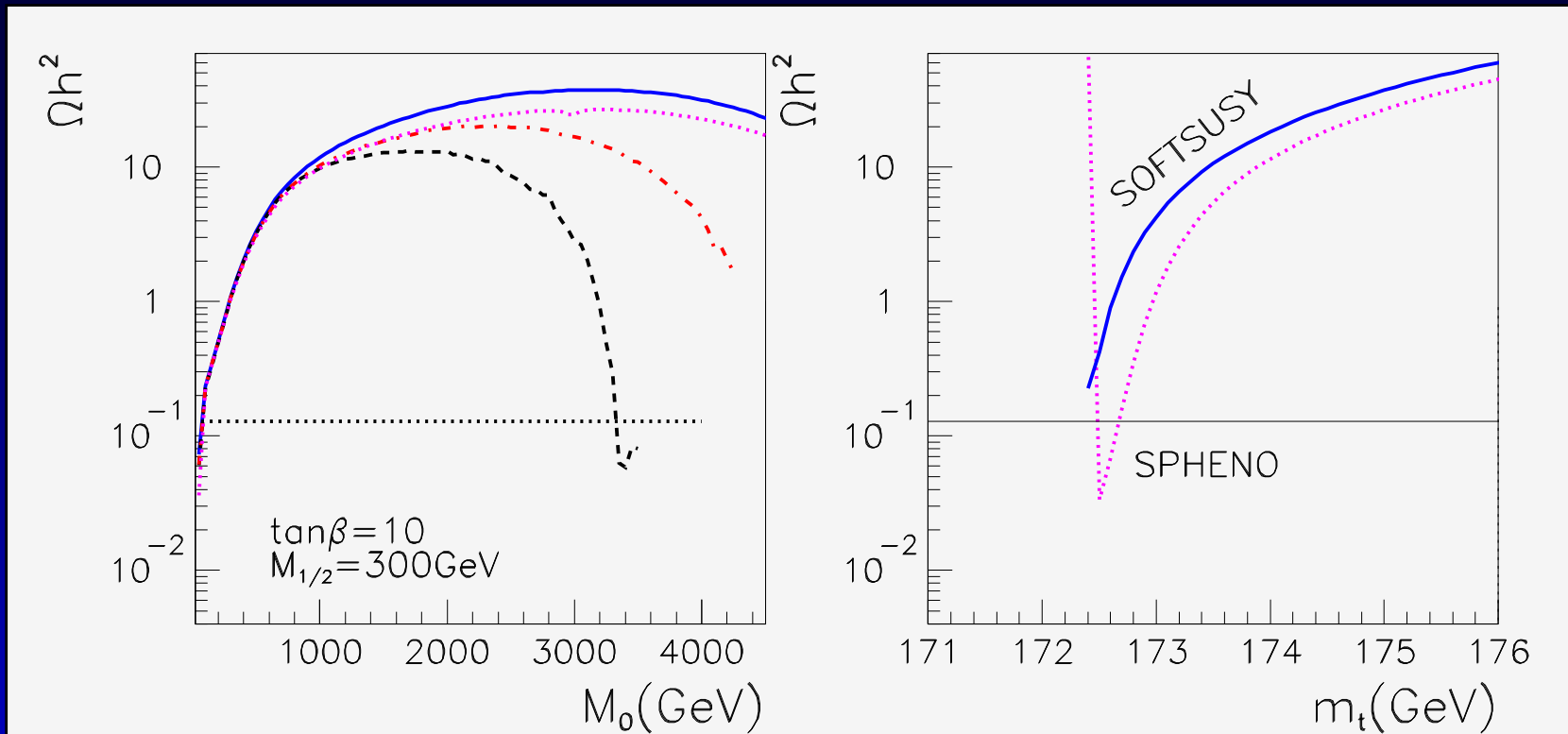
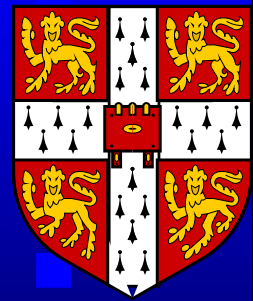


Figure 4: Focus point region. Full: SoftSusy, dotted: SPheno, dashed: SuSpect. Higgsino LSP annihilates into ZZ/WW



High $\tan \beta$

BCA, Belanger, Boudjema, Pukhov, Porod, hep-ph/0402161. Baer *et al*

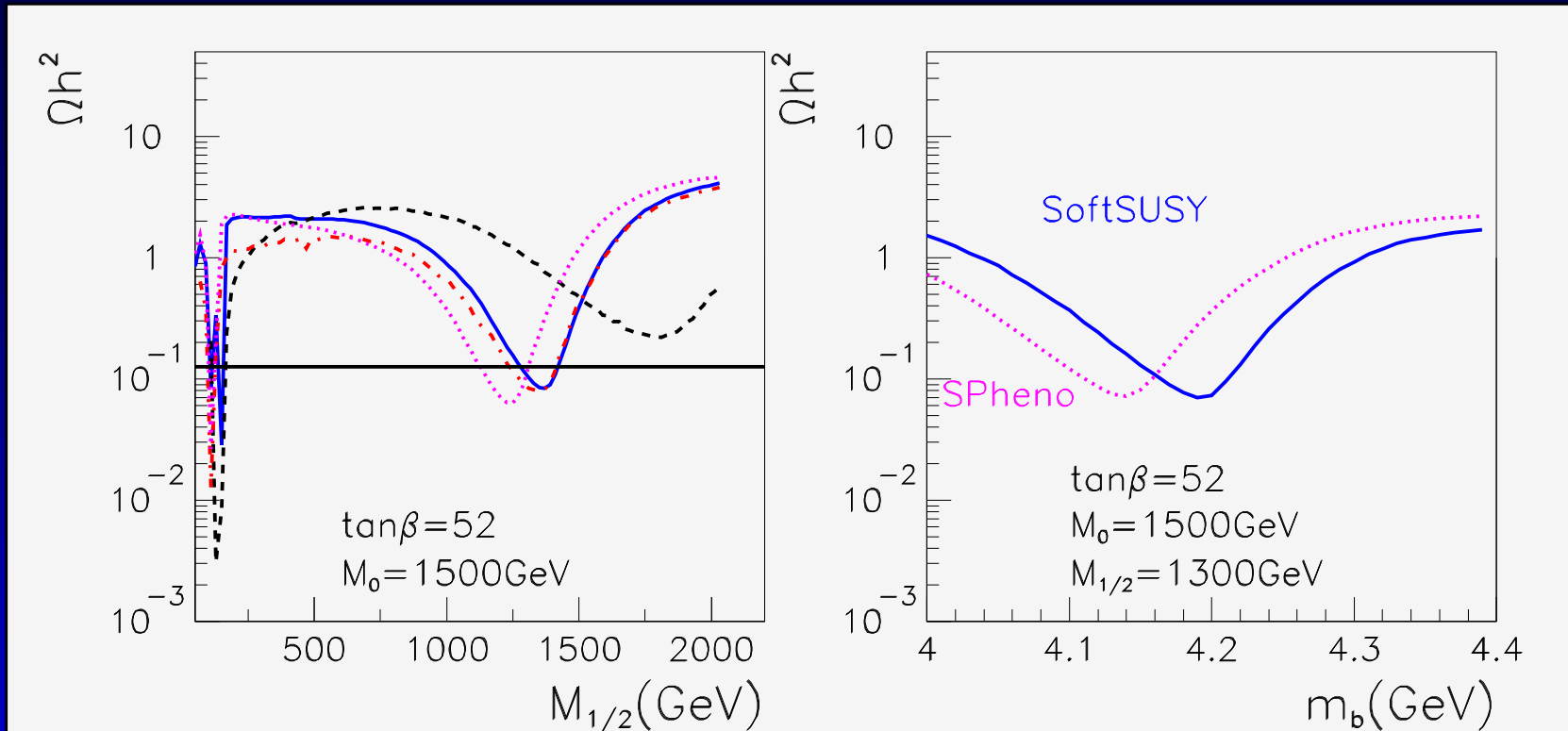
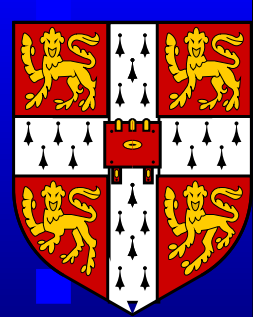
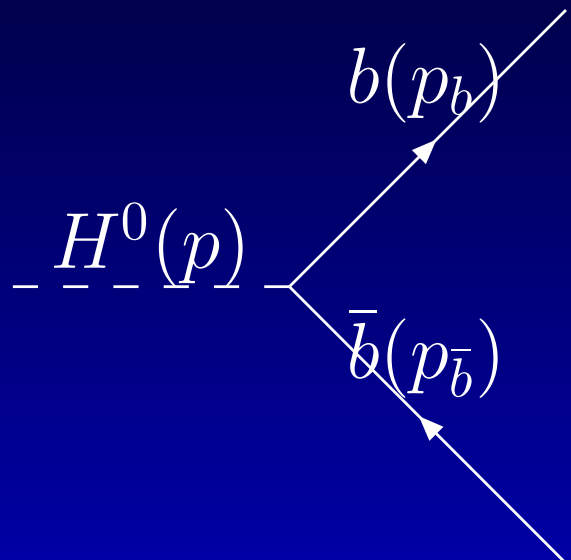


Figure 5: High $\tan \beta$ region. Full: SoftSUSY, dotted: SPheno, dashed: SuSpect. Get annihilation into A .



SUSY Kinematics: a Reminder

Take a particle decaying into 2 particles, eg $H^0 \rightarrow b\bar{b}$.
We define the **invariant mass** of the $b\bar{b}$ pair such that:

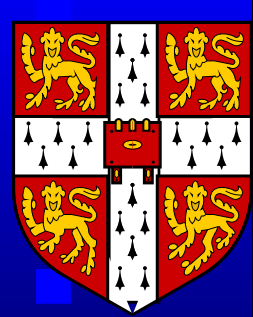


The diagram shows a horizontal dashed line on the left labeled $H^0(p)$. Two arrows branch out from this line: one pointing up and to the right labeled $b(p_b)$, and one pointing down and to the right labeled $\bar{b}(p_{\bar{b}})$.

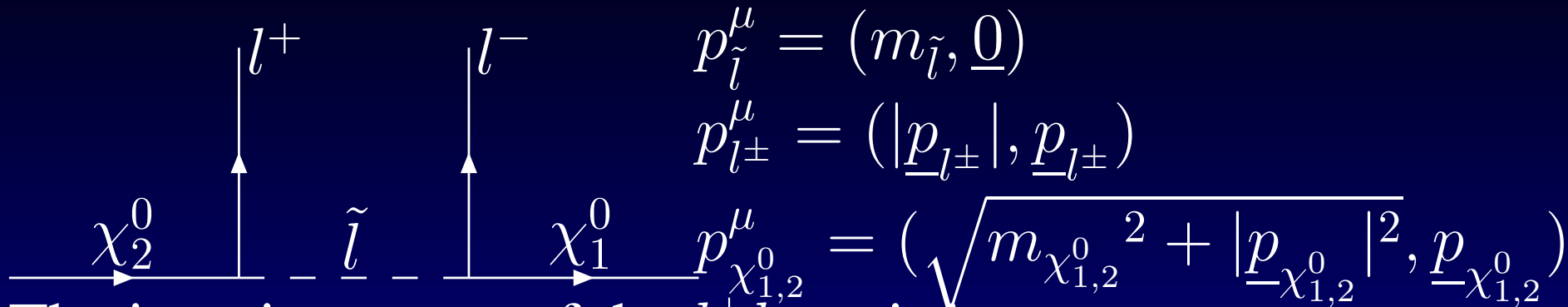
$$p^\mu = (\sqrt{m_H^2 + p^2}, \underline{p}) = p_b^\mu + p_{\bar{b}}^\mu$$
$$\Rightarrow p^2 = m_H^2 = (p_b + p_{\bar{b}})^2$$

Is *invariant* in boosted frames

Question: What happens to invariant mass in SUSY cascade decays, where we miss the final particle?



Cascade Decay



The invariant mass of the l^+l^- pair is

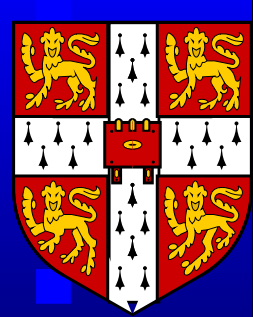
$$m_{ll}^2 = (p_{l^+} + p_{l^-})^\mu (p_{l^+} + p_{l^-})_\mu = p_{l^+}^2 + p_{l^-}^2 + 2p_{l^+} \cdot p_{l^-} \\ = 2|\underline{p}_{l^+}||\underline{p}_{l^-}|(1 - \cos \theta) \leq 4|\underline{p}_{l^+}||\underline{p}_{l^-}|.$$

Momentum conservation:

$$\Rightarrow \underline{p}_{\chi_2^0} + \underline{p}_{l^+} = \underline{0}, \quad \underline{p}_{l^-} + \underline{p}_{\chi_1^0} = \underline{0}.$$

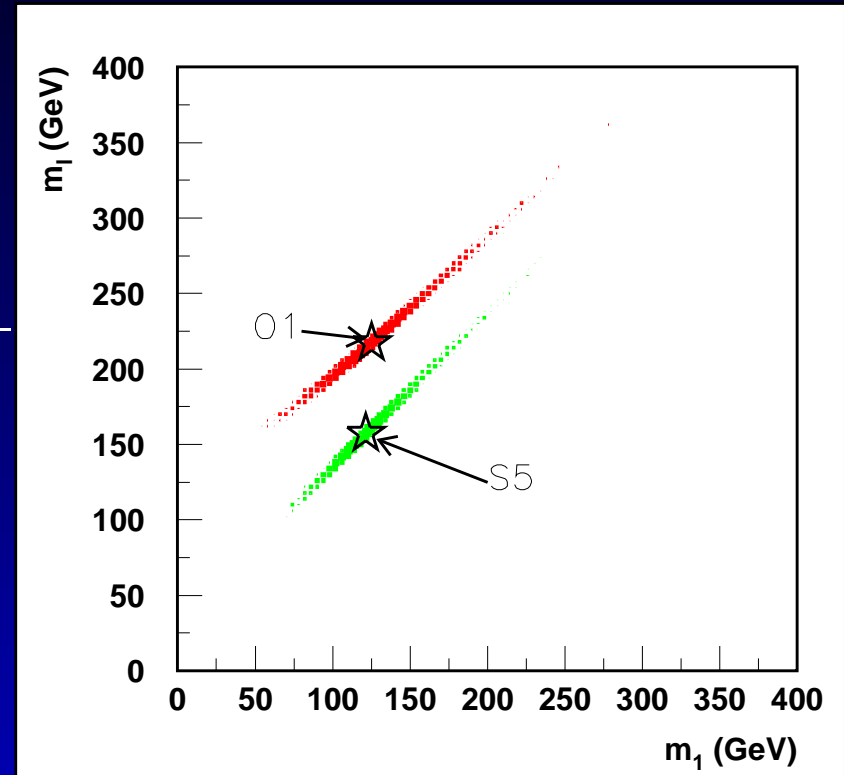
Energy conservation: $\sqrt{m_{\chi_2^0}^2 + |\underline{p}_{l^+}|^2} = m_{\tilde{l}} + |\underline{p}_{l^+}|,$

$$\Rightarrow |\underline{p}_{l^+}| = \frac{m_{\chi_2^0}^2 - m_{\tilde{l}}^2}{2m_{\tilde{l}}}. \quad \text{Similarly } |\underline{p}_{l^-}| = \frac{m_{\tilde{l}}^2 - m_{\chi_1^0}^2}{2m_{\tilde{l}}}.$$



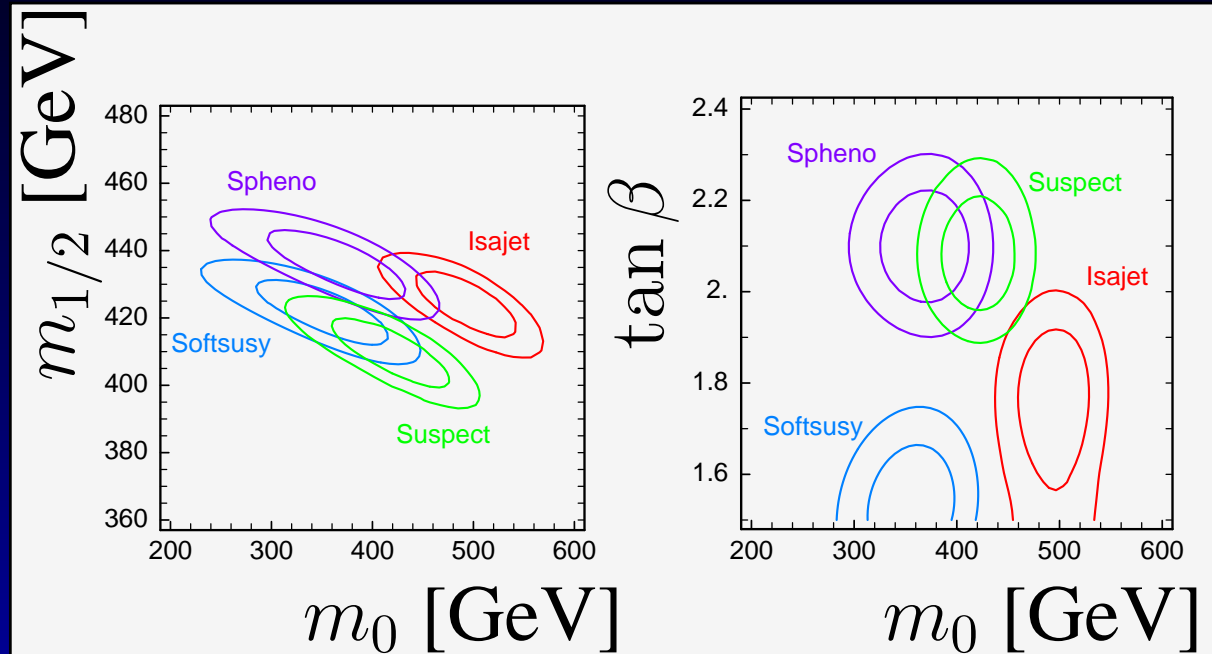
Edge to Mass Measurements

	width S5	width O1
χ_1^0	17	22
\tilde{l}_R	17	20
χ_2^0	17	20
\tilde{q}	22	20



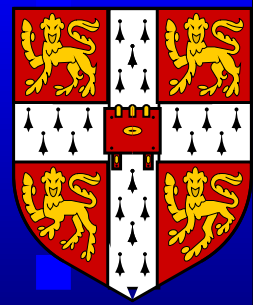
Mass differences well constrained, but overall mass scale not so well constrained by LHC

Fitting to SUSY Breaking Model

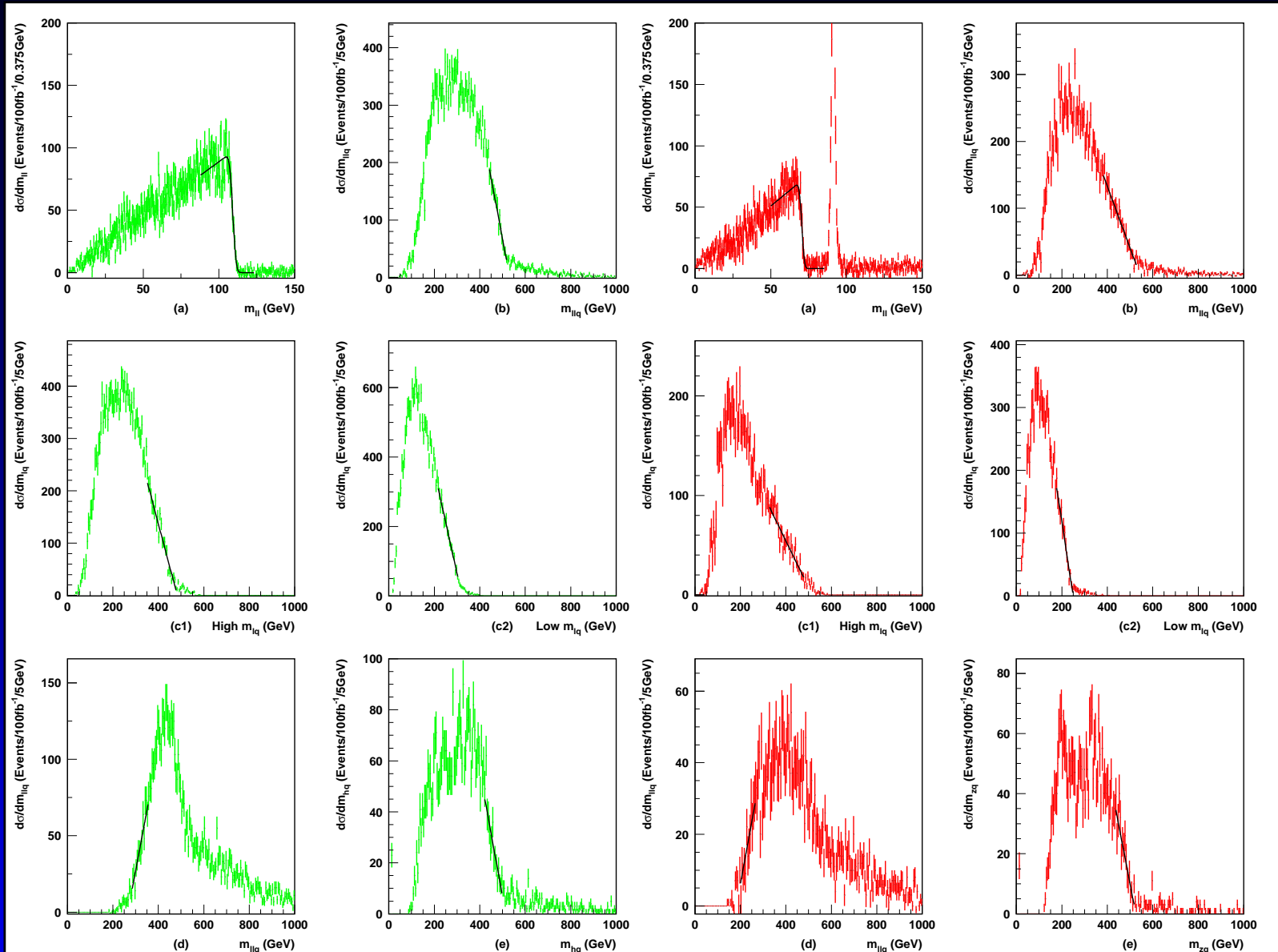


- Experimenters pick a SUSY breaking point
- They derive observables and errors after detector simulation
- We fit^a this “data” with our codes

^aBCA, S Kraml, W Porod, JHEP 0303 (2003) 016



Edge Fitting at S5 and O1



Edge Positions

endpoint	S5 fit	O1 fit
m_{ll}	109.10 ± 0.13	70.47 ± 0.15
m_{llq} edge	532.1 ± 3.2	544.1 ± 4.0
lq high	483.5 ± 1.8	515.8 ± 7.0
lq low	321.5 ± 2.3	249.8 ± 1.5
llq thresh	266.0 ± 6.4	182.2 ± 13.5

Best case lepton mass measurements can be as accurate as 1 per mille, but jets are a few percent

SOFTSUSY

Get $g_i(M_Z), h_{t,b,\tau}(M_Z)$.

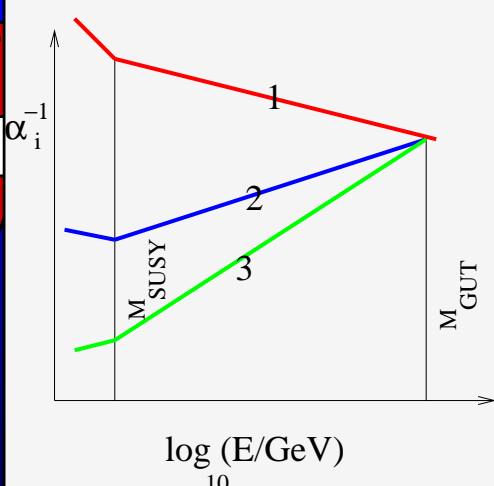
Run to M_S .

REWSB, iterative solution of μ

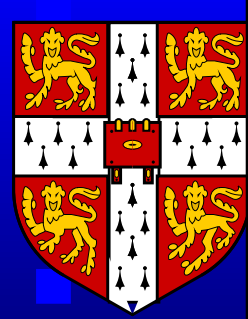
M_X . Soft SUSY breaking BC.

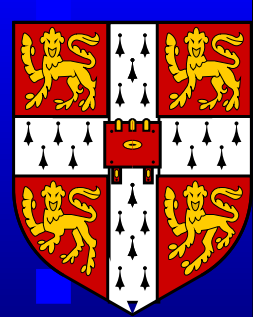
Run to M_S . Calculate^a sparticle pole masses.

Run to M_Z



^aBCA, Comp. Phys. Comm. 143 (2002) 305.





Other Observables

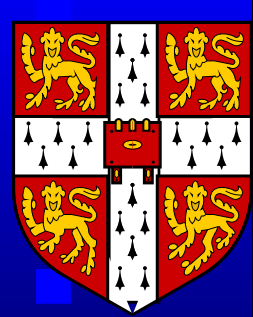
Often more complicated, eg m_{llq} edge:

$$\max \left[\frac{(m_{\tilde{q}}^2 - m_{\chi_2^0}^2)(m_{\chi_2^0}^2 - m_{\chi_1^0}^2)}{m_{\chi_2^0}^2}, \frac{(m_{\tilde{q}}^2 - m_{\tilde{l}}^2)(m_{\tilde{l}}^2 - m_{\chi_1^0}^2)}{m_{\tilde{l}}^2}, \frac{(m_{\tilde{q}}m_{\tilde{l}} - m_{\chi_2^0}m_{\chi_1^0})(m_{\chi_2^0}^2 - m_{\tilde{l}}^2)}{m_{\chi_2^0}m_{\tilde{l}}} \right]$$

Also m_{lq}^{high} , m_{lq}^{low} , llq *threshold*^a, $M_{T_2}^2(m) =$

$$\min_{\not{p}_1 + \not{p}_2 = \not{p}_T} \left[\max \left\{ m_T^2(p_T^{l_1}, \not{p}_1, m), m_T^2(p_T^{l_2}, \not{p}_2, m) \right\} \right],$$

$\max[M_{T_2}(m_{\chi_1^0})] = m_{\tilde{l}}$ for dilepton production.



Same order prior

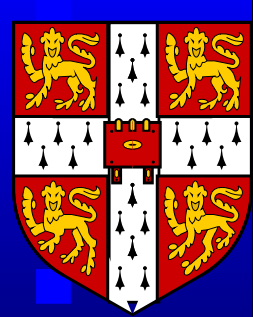
We wish to encode the idea that “**SUSY breaking terms should be of the same order of magnitude**”

$$p(m_0|M_S) = \frac{1}{\sqrt{2\pi w^2 m_0}} \exp\left(-\frac{1}{2w^2} \log^2\left(\frac{m_0}{M_S}\right)\right),$$

$$p(A_0|M_S) = \frac{1}{\sqrt{2\pi e^{2w} M_S}} \exp\left(-\frac{1}{2e^{2w}} \frac{A_0^2}{M_S^2}\right),$$

We don't know SUSY breaking scale M_S :

$$p(m_0, M_{1/2}, A_0, \mu, B) = \int_0^\infty dM_S p(m_0, M_{1/2}, A_0, \mu, B|M_S) p(M_S)$$



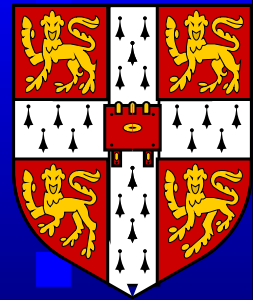
Naturalness

$$M_Z^2 = \tan 2\beta [m_{H_2}^2 \tan \beta - m_{H_1}^2 \cot \beta] - 2\mu^2$$

Cancellation implied by sparticle mass bounds.
Quantify by

$$f = \max_x \left\{ \left\| \frac{d \ln M_Z^2}{d \ln x} \right\| \right\}$$

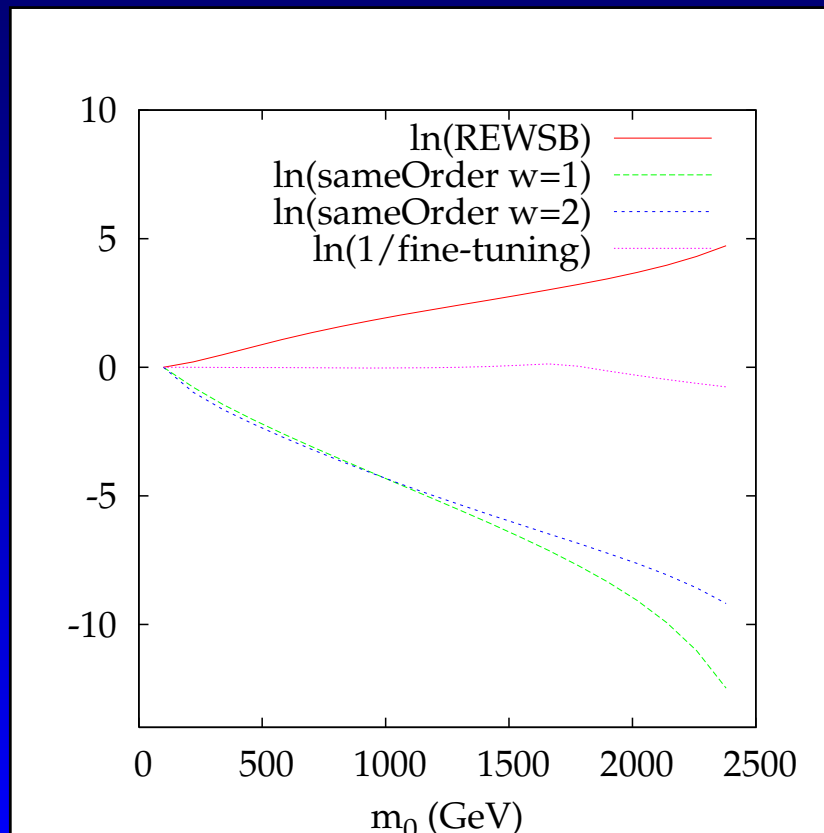
where $x \in \{M_{1/2}, m_0, A_0, \mu, B\}$. We will choose the prior to be $1/f$.



Fine Tuning

Compare with usual definition of *fine-tuning*:

$$f = \max_p \frac{d \ln M_Z}{d \ln p}$$



SPS1a Point

$$M_{1/2} = 250 \text{ GeV}$$

$$\tan \beta = 10 \text{ GeV}$$

$$A_0 = -100 \text{ GeV}$$