



Muon $g-2$

Precision Precession

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OUTLINE

- **What is it?**
- **Why measure it (again)?**
- **How?**
 - **Goals and how to achieve them:**
 - Brief recap of technique
 - Upgrades!
 - Beam, detectors, field
- **Status and Conclusions**

† The material for this talk has been shamelessly stolen from many including: B. Lee Roberts, Leah Welty-Reiger, Mark Lancaster, Thomas Teubner, Chris Polly, Andreas Kronfeld, Ruth Van de Water.....

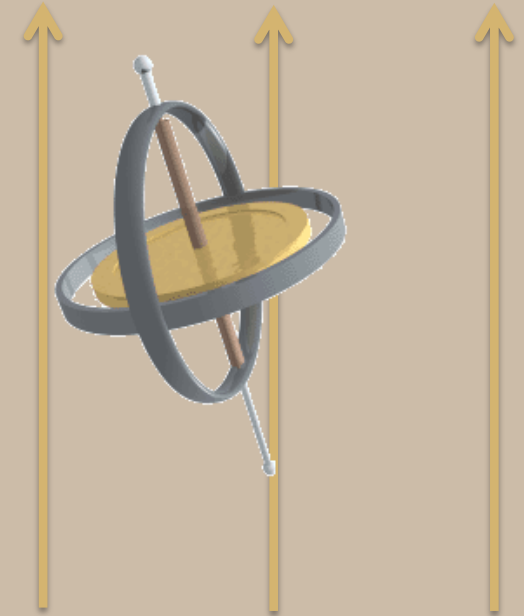
Magnetic Moments

- Magnetic moment of elementary particles related to their spin by the “g-factor”

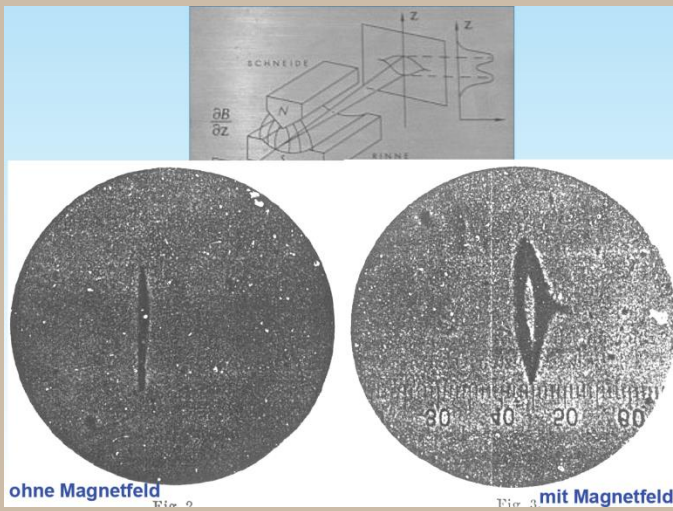
$$\vec{\mu}_s = g \frac{Qe}{2m} \vec{S}$$

$$\vec{\tau} = \vec{\mu} \times \vec{B}$$

Larmor frequency



A little history...



1924 Stern-Gerlach
Magnetic moment of silver atom in its ground state is 1 Bohr magneton. (10%)

...but not understood as spin 1/2

Spin $\frac{1}{2}$?

1925/26 Uhlenbeck And Goldschmidt proposed electron spin to explain fine structure....

...but prediction off by factor of 2!

Rescued by Thomas precession (1926)

- relativistic kinematics effect

(successive non-collinear boosts give rotation).



nucleus. On account of its magnetic moment, the electron will be acted on by a couple just as if it were placed at rest in a magnetic field of magnitude equal to the vector product of the nuclear electric field and the velocity of the electron relative to the nucleus divided by the velocity of light. This couple will cause a slow precession of the spin axis, the conservation

g=2

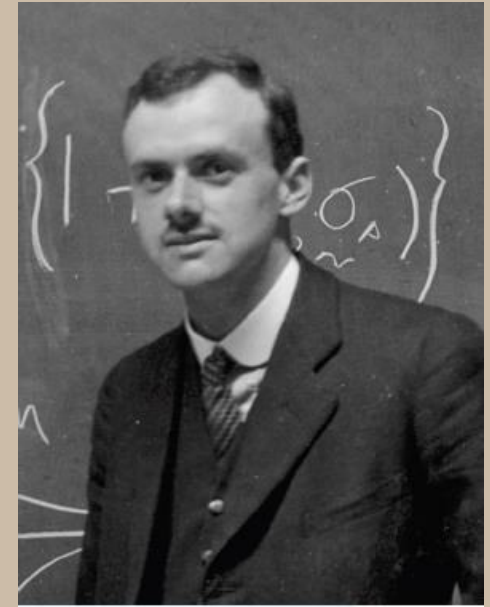
1928

$$\left(i\gamma^\mu \left(\partial_\mu + ieA_\mu \right) - m \right) \psi = 0$$

Non-relativistic reduction \Rightarrow

$$i\hbar \frac{\partial \psi}{\partial t} = \left\{ \frac{p^2}{2m} - \frac{e}{2m} \left[\mathbf{1}\vec{L} + \mathbf{2}\vec{S} \right] \cdot \vec{B} \right\} \psi$$

$$g_L = 1$$



$$g_S = 2$$

The Quantum Theory of the Electron.

By P. A. M. DIRAC, St. John's College, Cambridge.

Greater Experimental Precision...

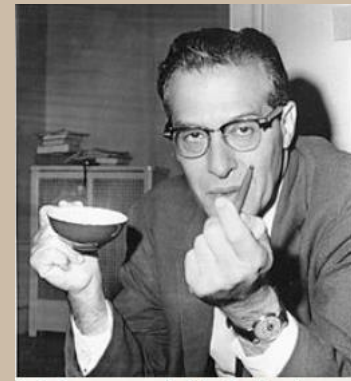
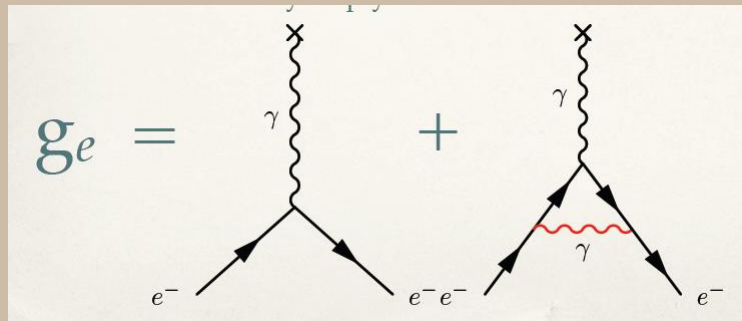
...1947 (Nafe, Nelson, Rabi) Hyperfine structure of H and D did not fit $g=2$... (It was a 5 sigma effect)

1948 Kusch and Foley : A precision measurement: $g_e = 2(1.00119 \pm 0.00005)$

An anomaly! Define $a = \frac{g - 2}{2}$

It takes QED to begin to explain the anomaly...

$$a_e = \frac{\alpha}{2\pi} = 0.001161$$



More QED...

$$\frac{g}{2} = 1 + C_1 \left(\frac{\alpha}{\pi}\right) + C_2 \left(\frac{\alpha}{\pi}\right)^2 + C_3 \left(\frac{\alpha}{\pi}\right)^3 + C_4 \left(\frac{\alpha}{\pi}\right)^4 + C_5 \left(\frac{\alpha}{\pi}\right)^5 +$$

Laporta,
Rameddi

Kinoshita et al.

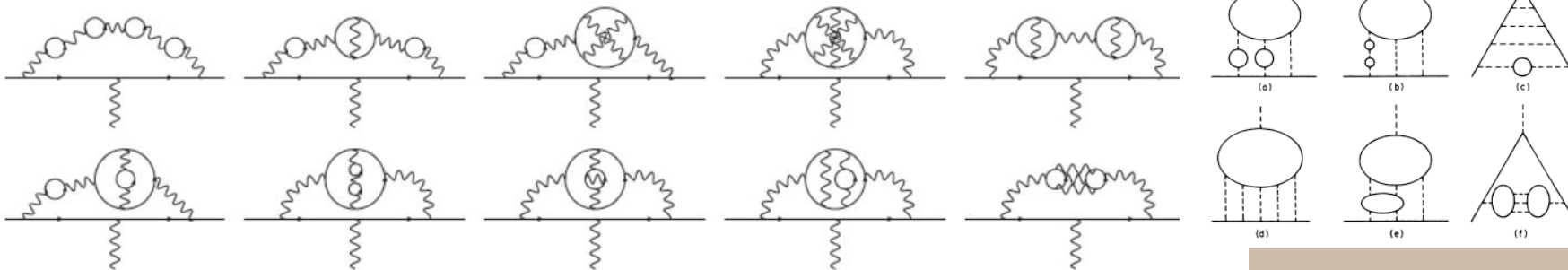
Even analytically...

4 loops: [Baikov,Broadhurst'95]

5 loops: [Baikov,Chetyrkin,Kühn,Sturm'13; Baikov,Marquard,Maier'13]

(see also [Aguilar,Greynat,De Rafael'08])

Some very
weird
diagrams!



Together with a succession of experiments \Rightarrow

Status of electron g-2

PRL 100, 120801 (2008)

PHYSICAL REVIEW LETTERS

week ending
28 MARCH 2008



New Measurement of the Electron Magnetic Moment and the Fine Structure Constant

D. Hanneke, S. Fogwell, and G. Gabrielse*

Department of Physics, Harvard University, Cambridge, Massachusetts 02138, USA

(Received 4 January 2008; published 26 March 2008)

$$a_e^{\text{exp}} = 1159652180.73(28) \times 10^{-12}$$

Ultra-precise agreement

$$\Delta a_e = (1.05 \pm 0.82) \times 10^{-12}$$

Gives best value of α_E

$$a_e^{\text{thy}} = 1159652181.78(77) \times 10^{-12}$$

Tenth-Order QED Contribution to the Electron $g - 2$ and an Improved Value of the Fine Structure Constant

Tatsumi Aoyama,^{1,2} Masashi Hayakawa,^{3,2} Toichiro Kinoshita,^{4,2} and Makiko Nio²

¹*Kobayashi-Maskawa Institute for the Origin of Particles and the Universe (KMI), Nagoya University, Nagoya, 464-8602, Japan*

²*Nishina Center, RIKEN, Wako, Japan 351-0198*

³*Department of Physics, Nagoya University, Nagoya, Japan 464-8602*

⁴*Laboratory for Elementary Particle Physics, Cornell University, Ithaca, New York, 14853, USA*

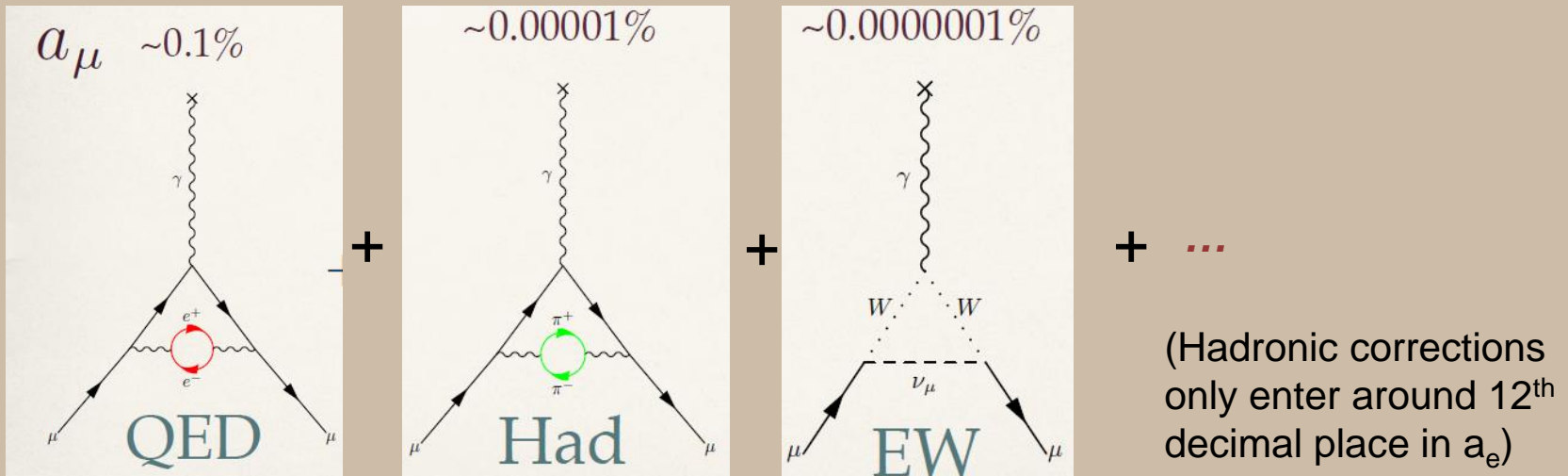
(Received 24 May 2012; published 13 September 2012)

a_μ

Standard Model Physics predicts electron magnetic moment anomaly at ppt level!

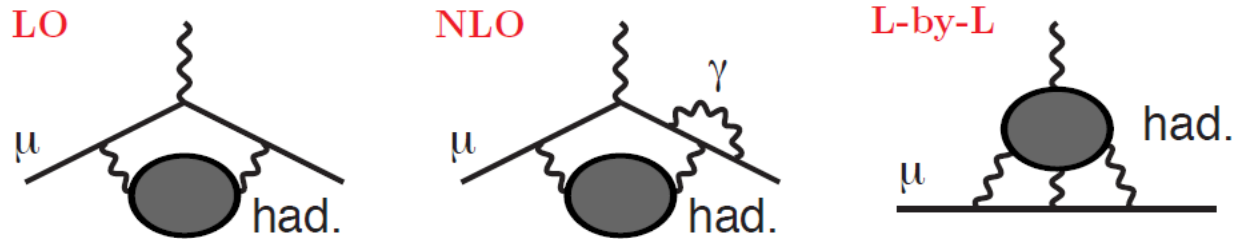
But the story is different for the muon... It's heavier

More sensitive to more contributions...



- QED well known
- EW contributions also understood (only couple of loop accuracy needed)
- **Hadronic contributions are significant and the biggest source of uncertainty.**

$$a_{\mu}^{\text{had}} = a_{\mu}^{\text{had,VP LO}} + a_{\mu}^{\text{had,VP NLO}} + a_{\mu}^{\text{had,Light-by-Light}}$$

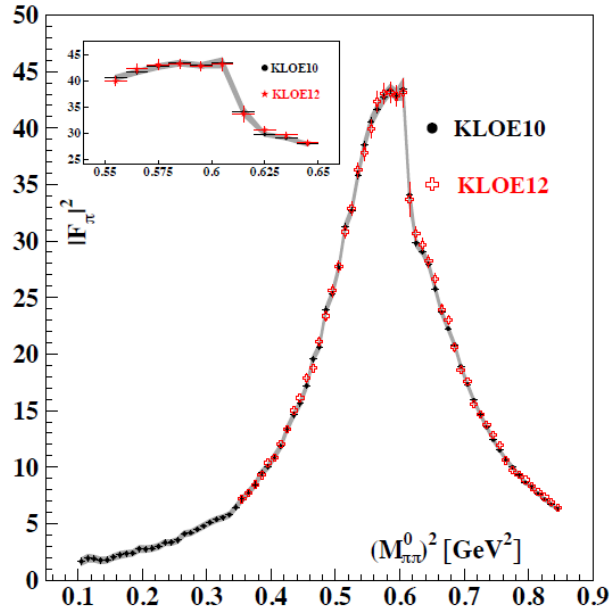


Non-perturbative - cannot be calculated.
 Determined from experiment
 Low energy $e^+e^- \rightarrow$ hadrons.
 + some lattice QCD for L-by-L contribution

$$\text{had.} = \int \frac{ds}{\pi(s-q^2)} \text{Im} \text{had.}$$

$$2 \text{Im} \text{had.} = \sum_{\text{had.}} \int d\Phi \left| \text{had.} \right|^2$$

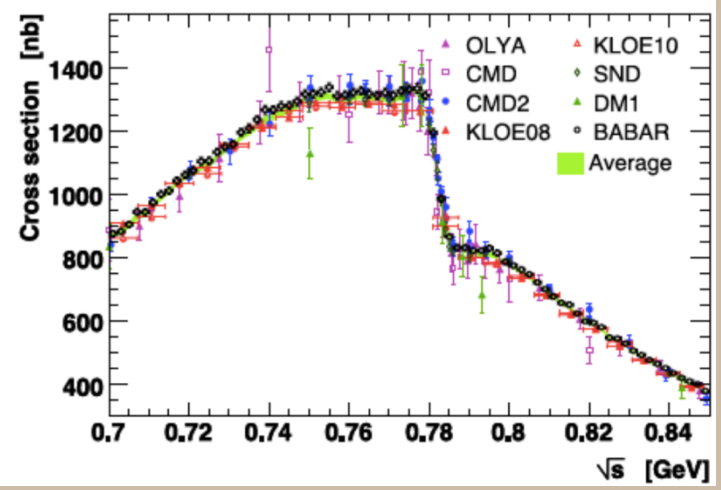
$$a_{\mu}^{\text{had,LO}} = \frac{m_{\mu}^2}{12\pi^3} \int_{s_{\text{th}}}^{\infty} ds \frac{1}{s} \hat{K}(s) \sigma_{\text{had}}(s)$$



new

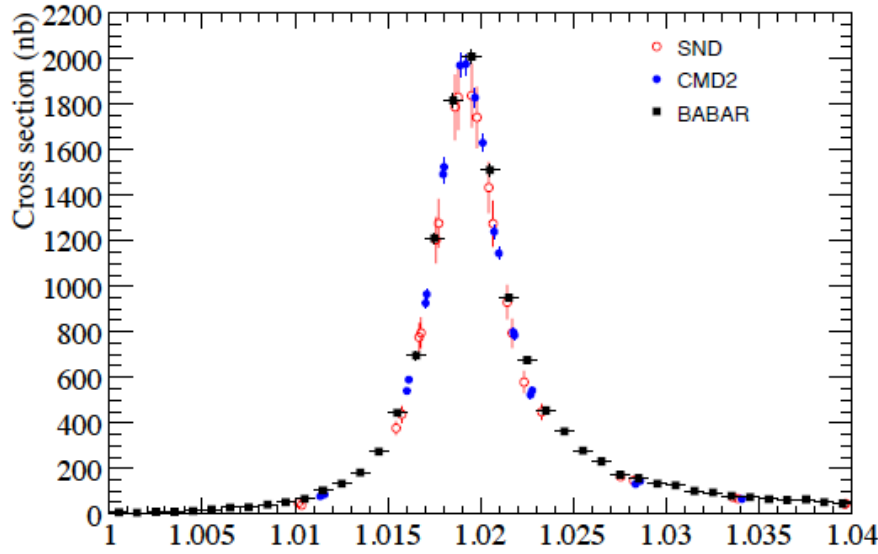
$\pi^+\pi^-$

Older e^+e^- data



PLB720(2013)336, :

new
 $K^+K^- \gamma$



PHIPSI13
Rome, 2013

How the contributions stack up:

Determination of hadronic contribution to muon g-2 has become an industry

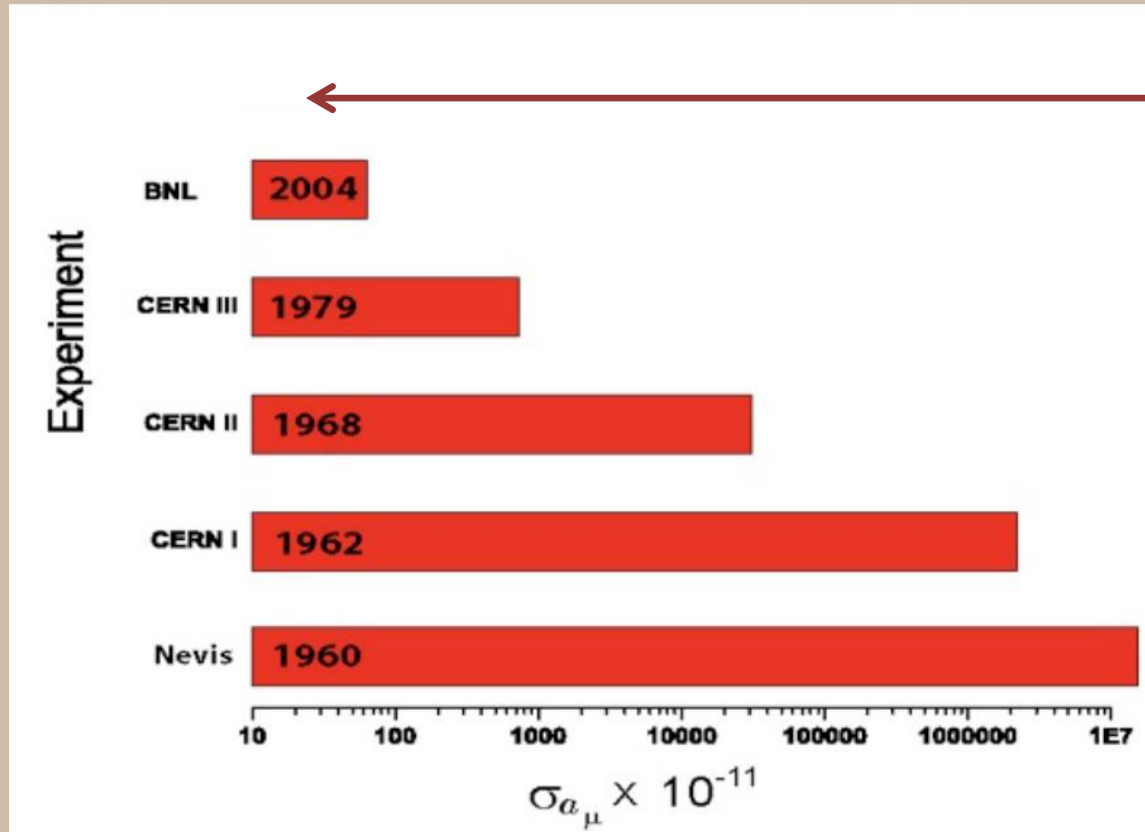
	VALUE ($\times 10^{-11}$) UNITS
QED ($\gamma + \ell$)	$116\,584\,718.951 \pm 0.009 \pm 0.019 \pm 0.007 \pm 0.077_\alpha$
HVP(lo) [47]	$6\,923 \pm 42$
HVP(lo) [48]	$6\,949 \pm 43$
HVP(ho) [48]	-98.4 ± 0.7
HLbL [61]	105 ± 26
EW [54]	153.6 ± 1.0
Total SM [47]	$116\,591\,802 \pm 42_{\text{H-LO}} \pm 26_{\text{H-HO}} \pm 2_{\text{other}} (\pm 49_{\text{tot}})$
Total SM [48]	$116\,591\,828 \pm 43_{\text{H-LO}} \pm 26_{\text{H-HO}} \pm 2_{\text{other}} (\pm 45_{\text{tot}})$

DHMZ
HLMNT

Paralleled by g-2 measurements...

Experimental Determination of a_μ .

A succession of improving measurements



FNAL GOES HERE

...Details to follow!

The current state of the art:

$$a_{\mu}^{exp} = 116\,592\,089\,(63) \times 10^{-11}$$

$$a_{\mu}^{SM} = 116\,591\,802\,(49) \times 10^{-11}$$

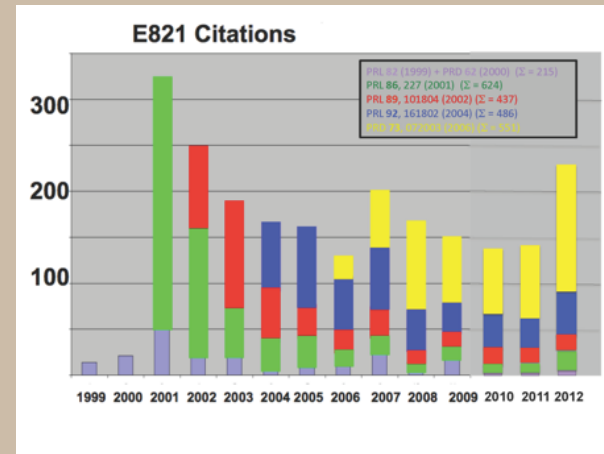
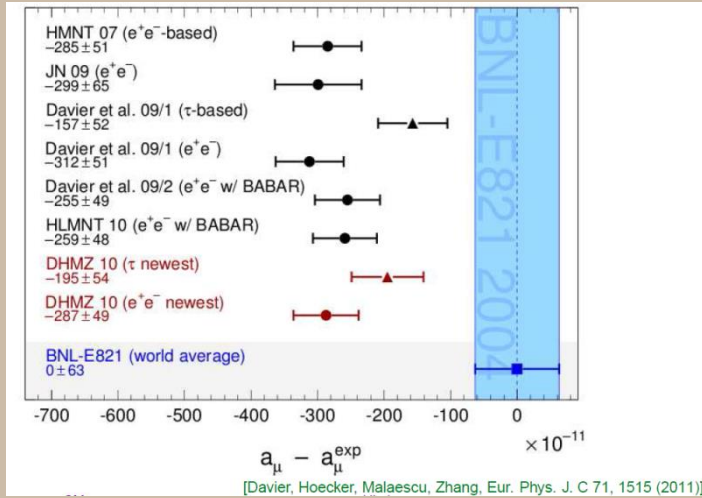
$$a_{\mu}^{exp} - a_{\mu}^{SM} = 287\,(80) \times 10^{-11}$$

Not same precision as the electron but compensated by higher mass.

Muon anomalous magnetic moment is sensitive to most of the standard model... and to new physics.

A tantalising but inconclusive
3.3-3.6 σ discrepancy

There is no shortage of interest in this intriguing result!



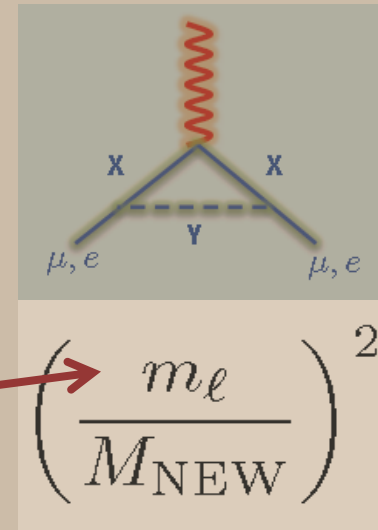
Were it to persist...

- Strong indicator of BSM physics...

Loop contributions sensitive to new particles running round loop...

μ better than e

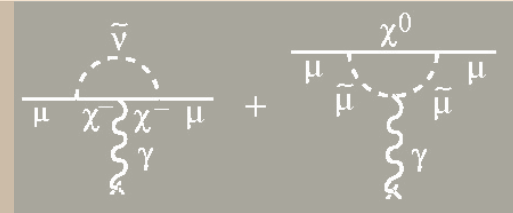
$$\left(\frac{m_\mu}{m_e} \right)^2 \approx 40,000$$



e.g. SUSY

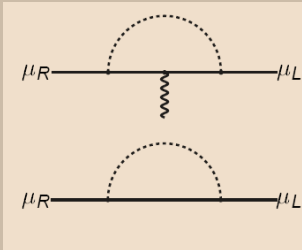
$$a_\mu(\text{SUSY}) \simeq \text{sgn}(\mu) 130 \times 10^{-11} \tan \beta \left(\frac{100 \text{ GeV}}{\Lambda} \right)^2$$

NP



But broad spectrum of sensitivity in TeV mass range...

a_μ related to m_μ



highly model dependent

Generically:

$$a_\mu^{NP} = \mathcal{O}(1) \times \left(\frac{m_\mu}{M_{NEW}} \right)^2 \times \left(\frac{\delta m_\mu^{NP}}{m_\mu} \right)^2$$

❖ flavour-conserving, CP-conserving, chirality flipping, loop-induced

Large +ve anomaly wrt SM



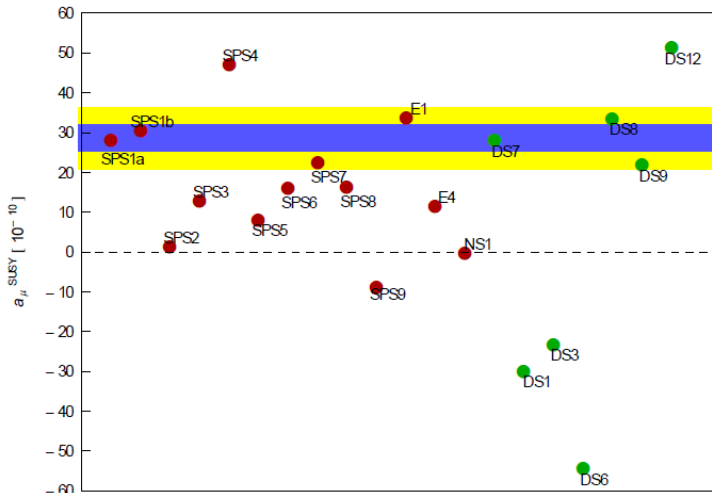
Extended technicolor (fermion masses)

SUSY (natural, gauge-mediated, compressed), RS ED

Z', W', Little Higgs, Universal ED

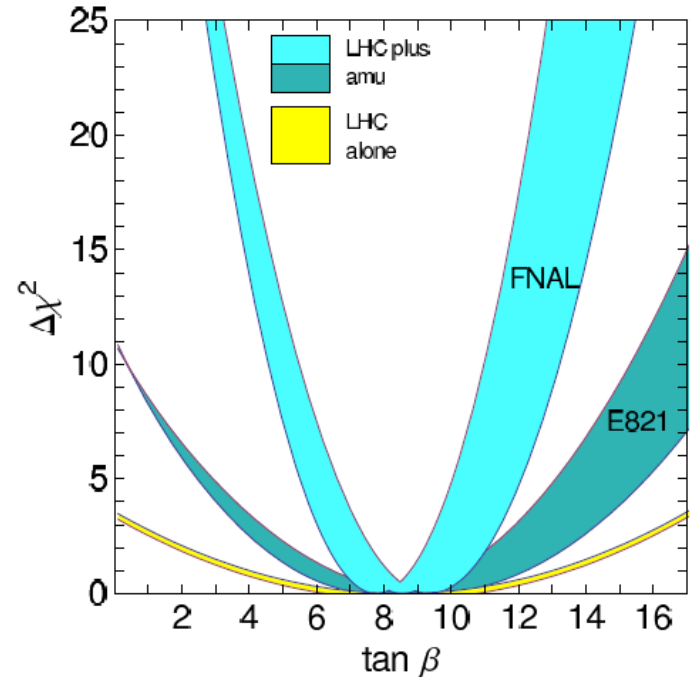
Value consistent with SM

a_μ provides discriminating power...



SPS benchmark points

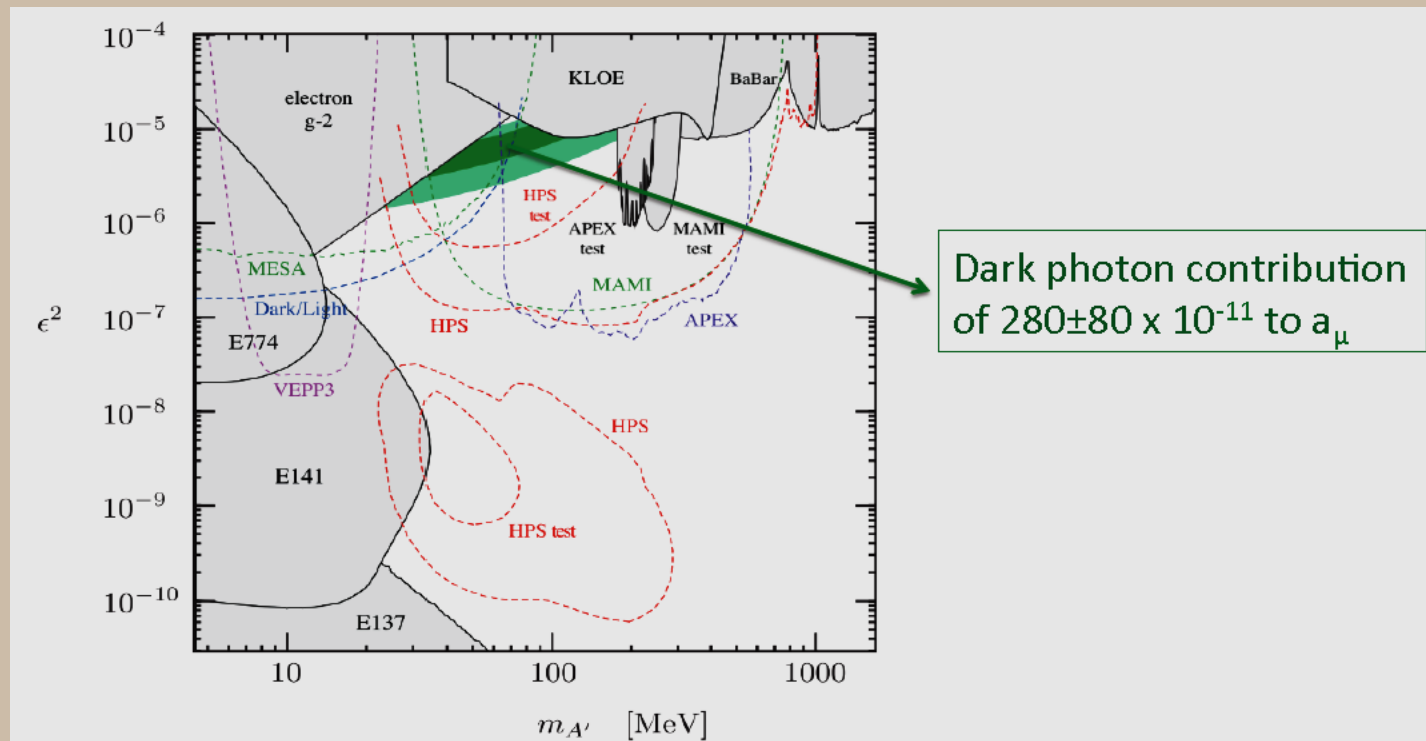
LHC Inverse Problem (300fb^{-1})
 can't be distinguished at LHC
 [Sfitter: Adam, Kneur, Lafaye,
 Plehn, Rauch, Zerwas '10]



[Hertzog, Miller, de Rafael, Roberts, DS '07]

...also can inform, low mass, below LHC reach...

e.g. Dark photons:

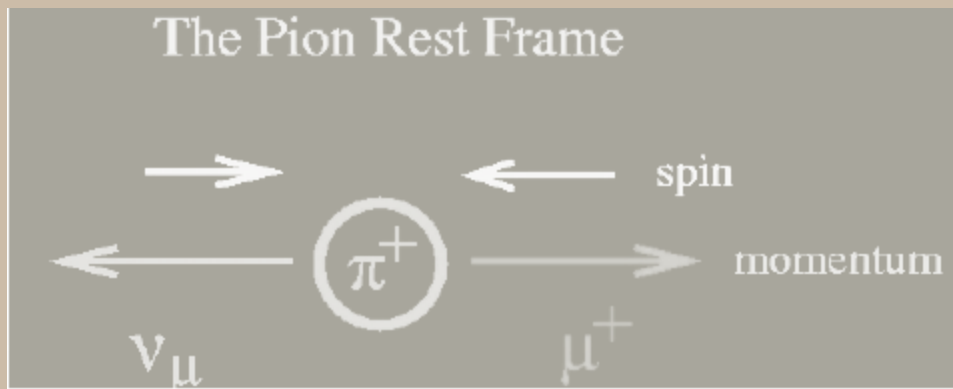


How do we measure $g-2$?

First make your muons... ...from pions.

Fortune of nature number 1

Parity violation delivers conveniently polarised muons:



\Rightarrow beam of polarised muons

inject into a (very) uniform magnetic field...

Muon momentum turns with cyclotron frequency $\omega_C = -\frac{QeB}{m\gamma}$

Spin turns with frequency $\omega_S = -g\frac{QeB}{2m} - (1-\gamma)\frac{QeB}{\gamma m}$

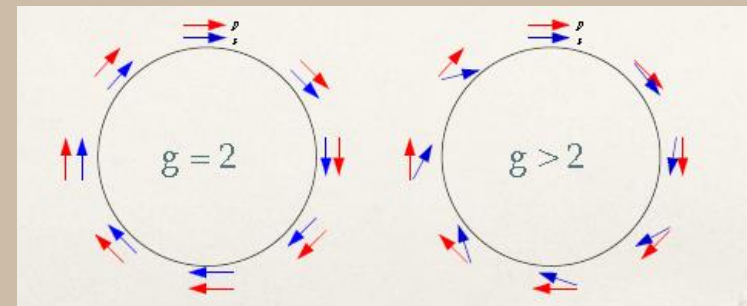
Fortune of nature number 2:

$$\omega_a \equiv \omega_S - \omega_C = -\left(\frac{g-2}{2}\right)\frac{QeB}{m_\mu} = -a_\mu \frac{QeB}{m_\mu}$$

Direct dependence on the anomaly: an immediate 3 orders of magnitude gain over measuring μ in at-rest experiments!

We need to measure ω_a and B

...and know m_μ very accurately?



Actually measure:

Normalise magnetic field to Larmor frequency of proton

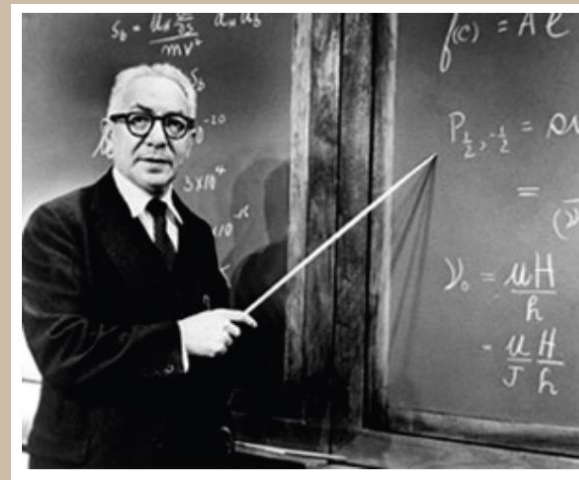
$$a_{\mu} = \frac{\frac{\omega_a}{\omega_p}}{\frac{\mu_{\mu}}{\mu_p} - \frac{\omega_a}{\omega_p}}$$

$$\lambda \equiv \frac{\mu_{\mu}}{\mu_p}$$

Measured from hyperfine structure of muonium:
currently known to 120ppb[†]

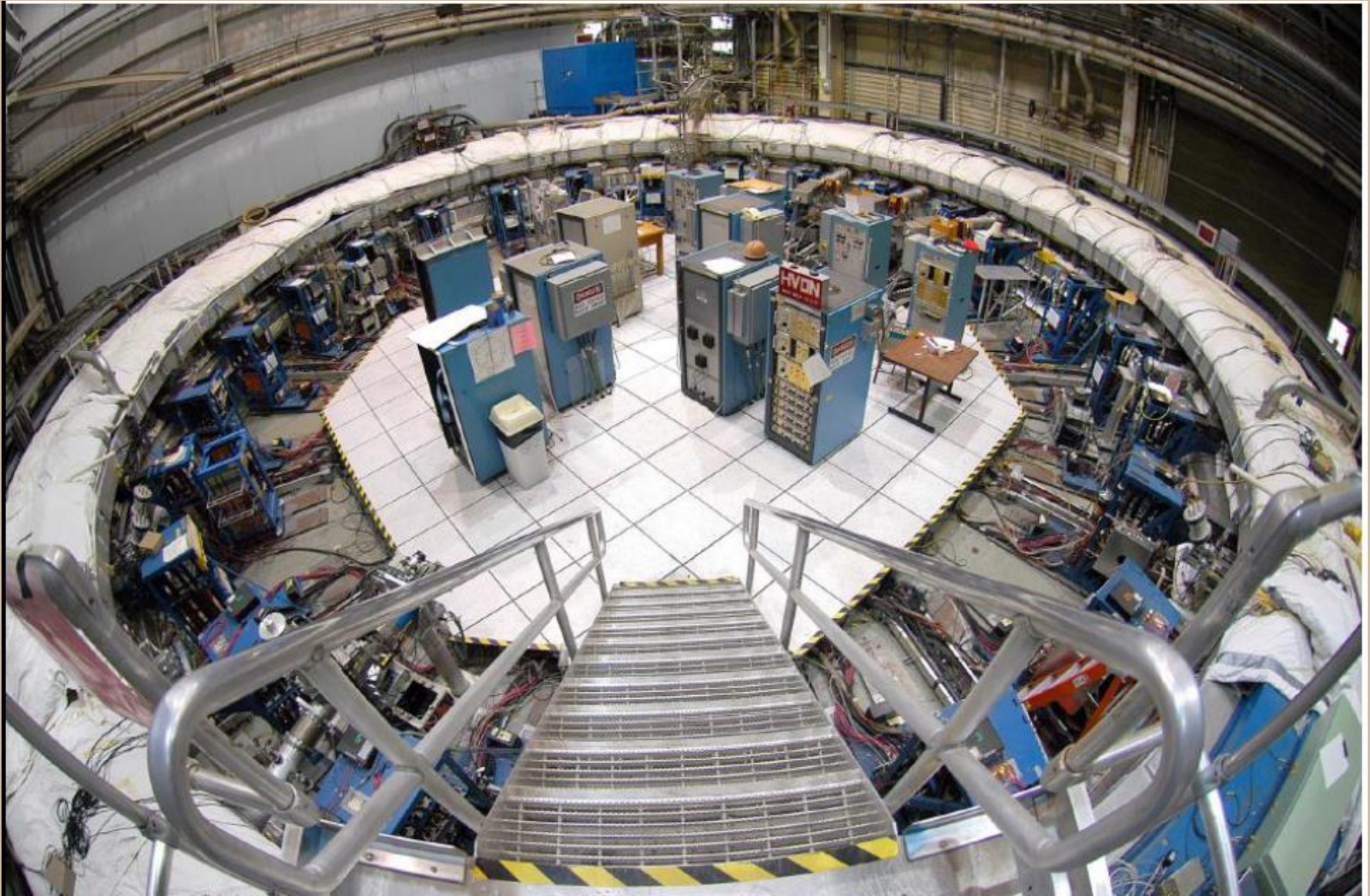
“Never measure anything but frequency”

I. Rabi



[†]JPARC expt. to reduce this to ppb level

E821 at BROOKHAVEN



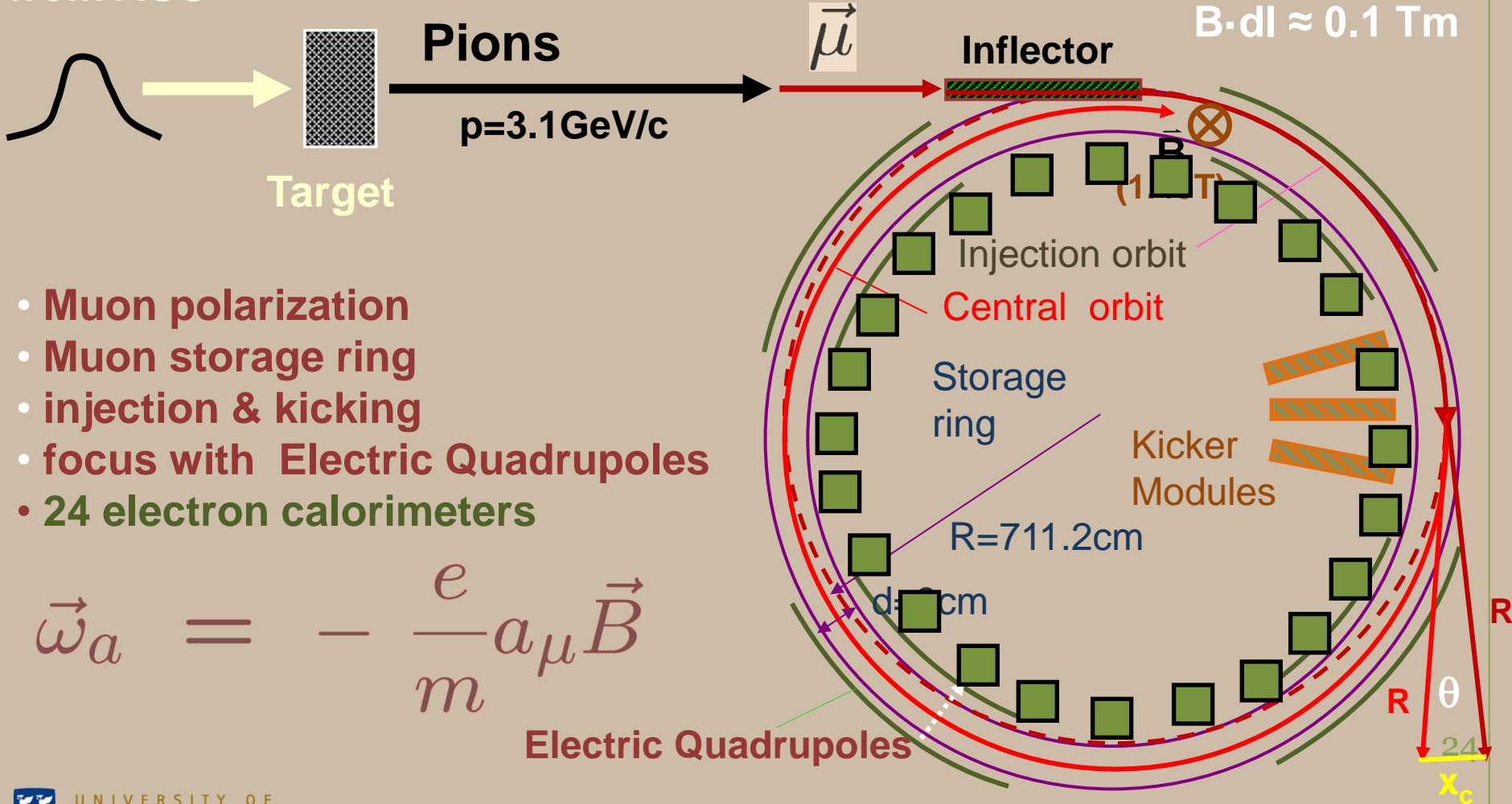
E821 Experimental Technique

25ns bunch of
 5×10^{12} protons
 from AGS

$x_c \approx 77$ mm

$\theta \approx 10$ mrad

$B \cdot dl \approx 0.1$ Tm



- Muon polarization
- Muon storage ring
- injection & kicking
- focus with Electric Quadrupoles
- 24 electron calorimeters

$$\vec{\omega}_a = - \frac{e}{m} a_\mu \vec{B}$$

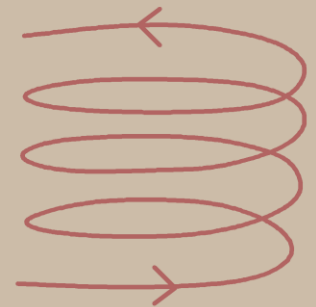
Electric Quadrupoles

Magic γ



- Vertical magnetic field – need vertical focussing to stop muons spiralling out of ring
- Achieve using electrostatic dipoles
- The E-field modifies the precession frequency:

$$\vec{\omega}_a = \frac{e}{mc} \left[a\vec{B} - \left(a - \frac{1}{\gamma^2 - 1} \right) \vec{\beta} \times \vec{E} \right]$$



- Unwelcome source of additional systematics
- Can be made to vanish for 'magic' γ . Extremely lucky that size of a_μ makes this possible!

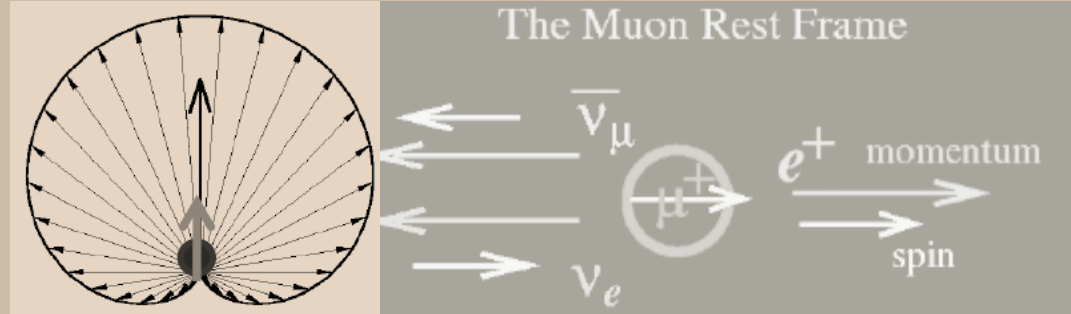
$$\gamma_{\text{magic}} = 29.3 \quad \Rightarrow \quad p_\mu = 3.09 \text{ GeV}$$

Method pioneered by 3rd CERN g-2

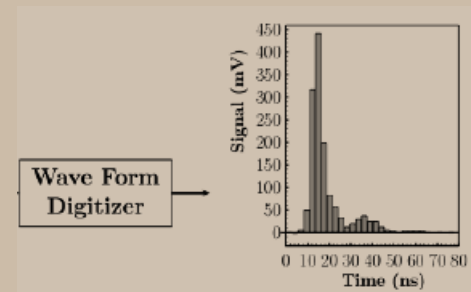
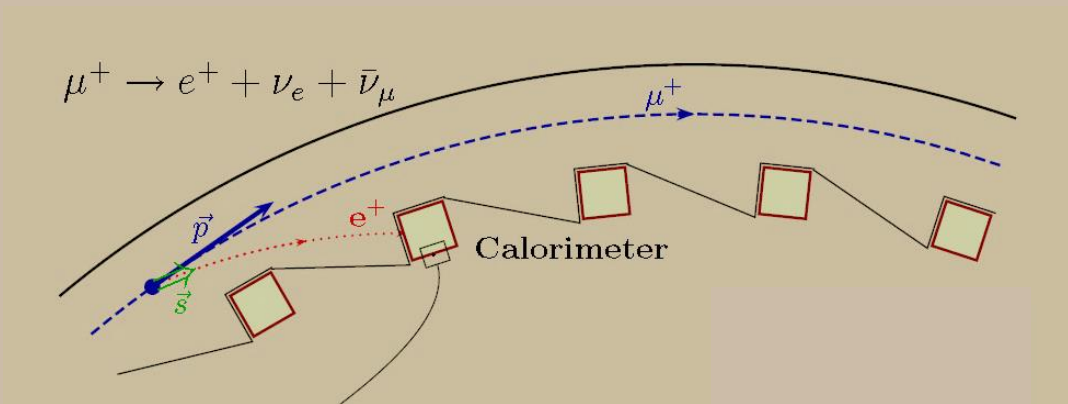
...but sadly, not every μ will be magic!

But how to measure ω_a ?

Parity violation again!



- Highest energy e^+ emitted along direction of μ^+ spin
- Use calorimeters to count e^+ above an energy threshold vs. t

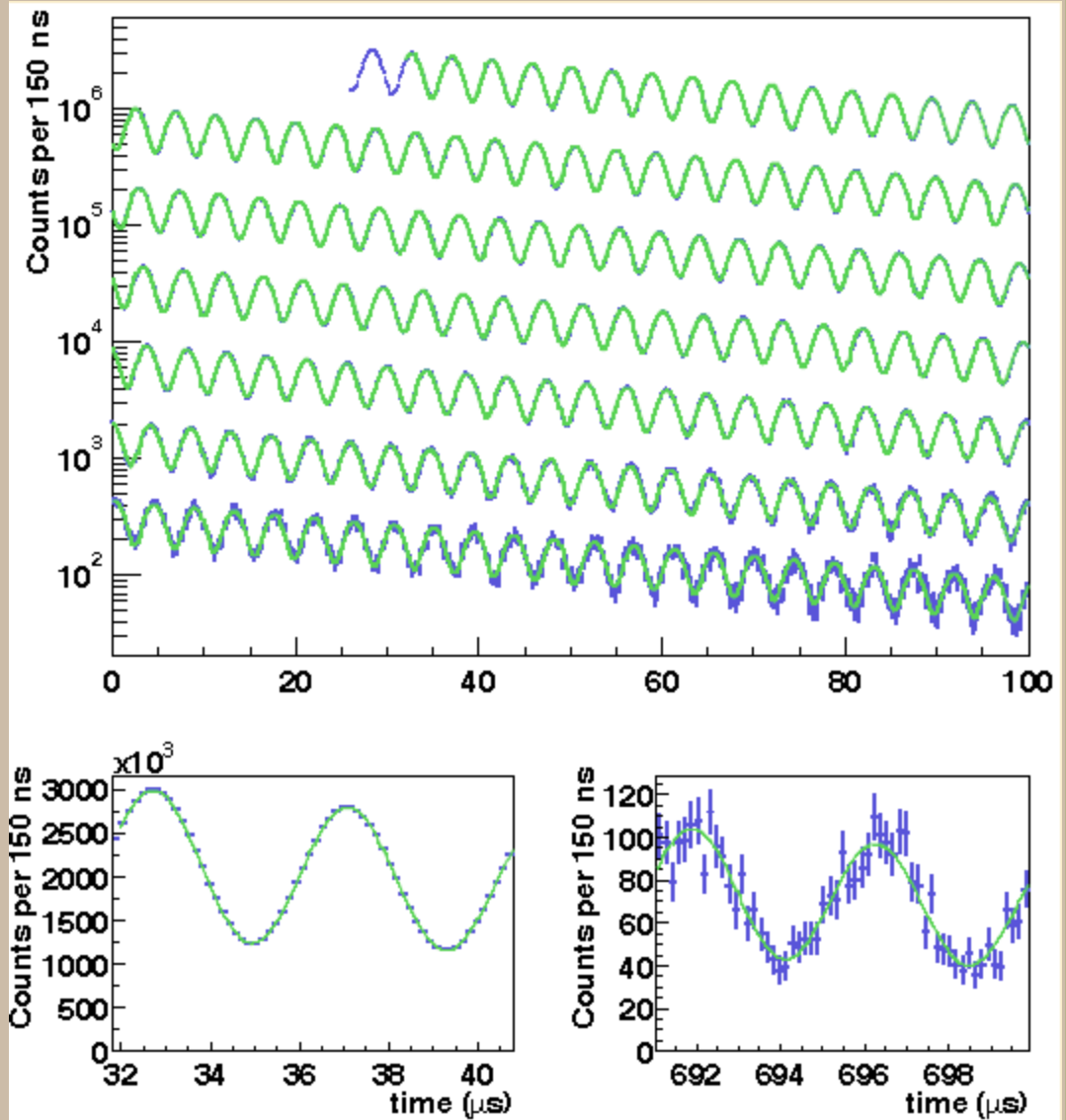


...an iconic plot

E 821

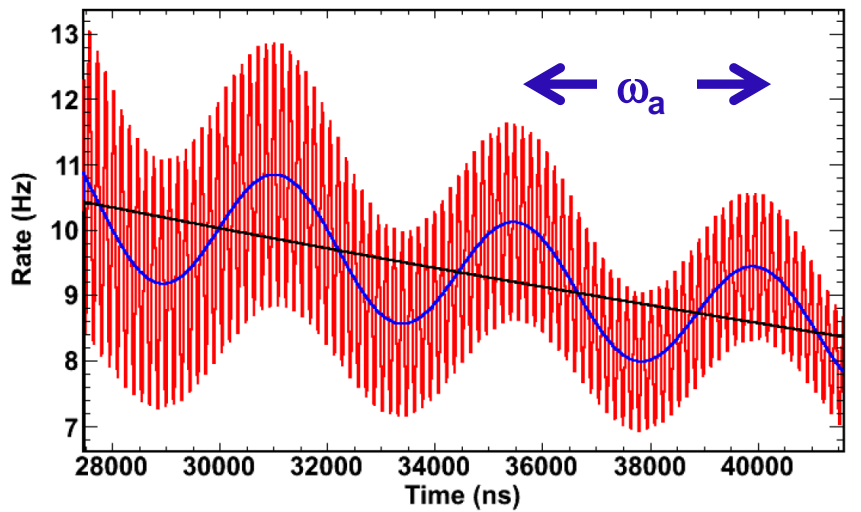
$$N(t) = N_0 e^{-t/\gamma\tau_\mu} \left[1 - A \cos(\omega_a t + \phi) \right]$$

“5-parameter fit”



Measuring ω_a ...some reality

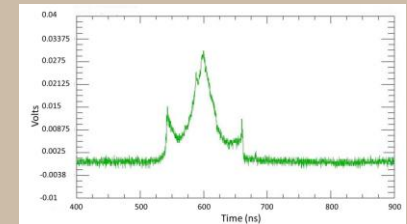
Simulated for E989



High frequency modulation because muon bunch initially doesn't fill ring...decays as bunch spreads. This is good – can get p distribution of muons

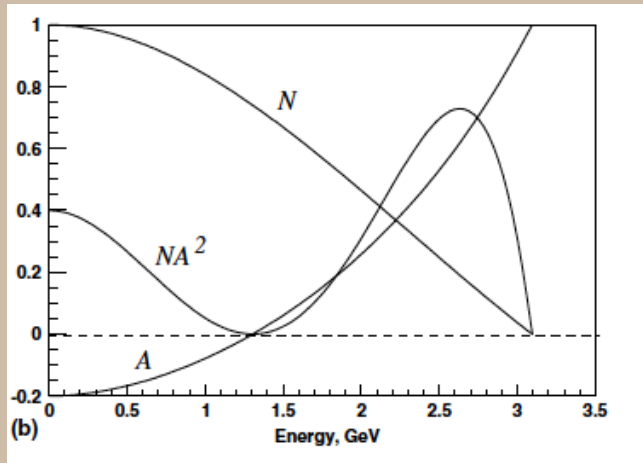
Expected for E989: ω_c 149ns

Bunch length 120ns at injection



$$N(t) = N_0 e^{-t/\gamma\tau_\mu} \left[1 - A \cos(\omega_a t + \phi) \right]$$

N, A depend on energy



Many sources of systematic error.

Particularly insidious are 'early-to-late' errors

Example: Effect of pile up.

$$(\omega_a t + \phi)$$

Time dependence in phase: $\phi(t) = \phi_0 + \alpha t + \beta t^2 + \dots \equiv \phi_0 + \alpha t$

$$\cos(\omega_a t + \phi(t)) \approx \cos((\omega_a + \alpha)t + \phi_0)$$

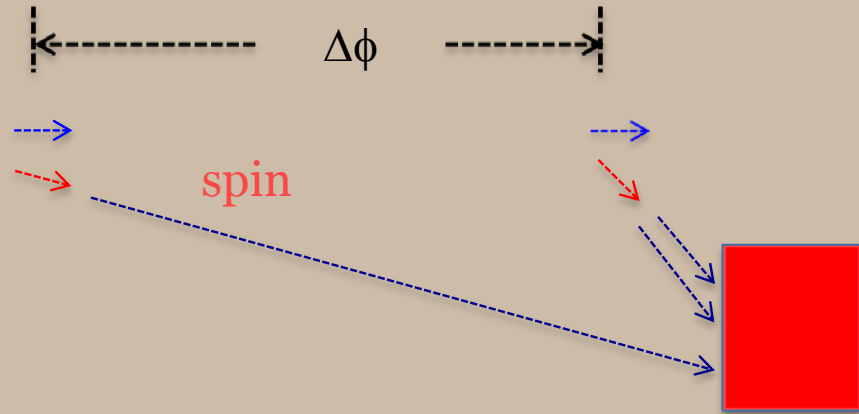
...but why should ϕ change? Things which change early to late in the fill can lead to a phase change in the accepted events \rightarrow direct bias to extracted ω_a .

Higher energy positrons come from further away.

If we get the energy wrong, we get the phase wrong.

If we have pile-up, two low energy positrons fake a high energy positron.

More pile-up early in the fill.



$$N(t) = \frac{N_0}{\tau} \Lambda(t) V(t) B(t) C(t) [1 - A'(t) \cos(\omega_a t + \phi'(t))], \text{ where}$$

$$B(t) = 1 - A_{br} e^{-t/\tau_{br}} \text{ with } \tau_{br} = 5\mu\text{s.}$$

Beam relaxation

$$V(t) = (1 - e^{-t/\tau_{vw}} [1 - A_{vw} \cos(\omega_{vw} t + \phi_{vw})]),$$

Vertical breathing

$$A'(t) = A(1 - e^{-t/\tau_{cbo}} [1 - A_2 \cos(\omega_{cbo} t + \phi_2)]), \text{ and}$$

$$\phi'(t) = \phi(1 - e^{-t/\tau_{cbo}} [1 - A_3 \cos(\omega_{cbo} t + \phi_3)]).$$

3 CBO terms

$$C(t) = 1 - e^{-t/\tau_{cbo}} [1 - A_1 \cos(\omega_{cbo} t + \phi_1)].$$

$$\Lambda(t) = 1 - C e^{-t_0/\tau} \int_{t_0}^t L(t') e^{t'/\tau} dt',$$

Muons lost from ring

...strong incentive to repeat the measurement with increased precision:

E989 Muon $g-2$ at FNAL aims to:

Build on BNL result and:

increase number of muons by factor ~21
reduce total systematics by factor ~3

$$10^{11} \times a_{\mu}^{E821} = 116592089 (54)_{stat} (33)_{sys}$$

$$(54)_{stat} \oplus (33)_{sys} \rightarrow (11)_{stat} \oplus (11)_{sys}$$

$$0.54\text{ppm} \rightarrow 0.14\text{ppm}$$

$$\delta a_{\mu} \leq \pm 16 \times 10^{-11}$$

$$\text{i.e. } \sim 3.5\sigma \rightarrow \gtrsim 5\sigma$$

How these goals will be achieved:

Move the entire E821 storage ring to FNAL!

Use the same experimental technique as E821 but:

- exploit the unique FNAL facilities to deliver more muons

Reduce systematics by improving and refining

- the detectors
- the stored beam dynamics
- Uniformity and measurement of magnetic field

Fermilab Muon Campus

Multipurpose Building designed for future experiments as well



At FNAL...

Get more and cleaner beam:

~21 times statistics of E821 and

Use beam transfer and delivery as
1900m decay line \Rightarrow no pion
background, **no hadronic flash**

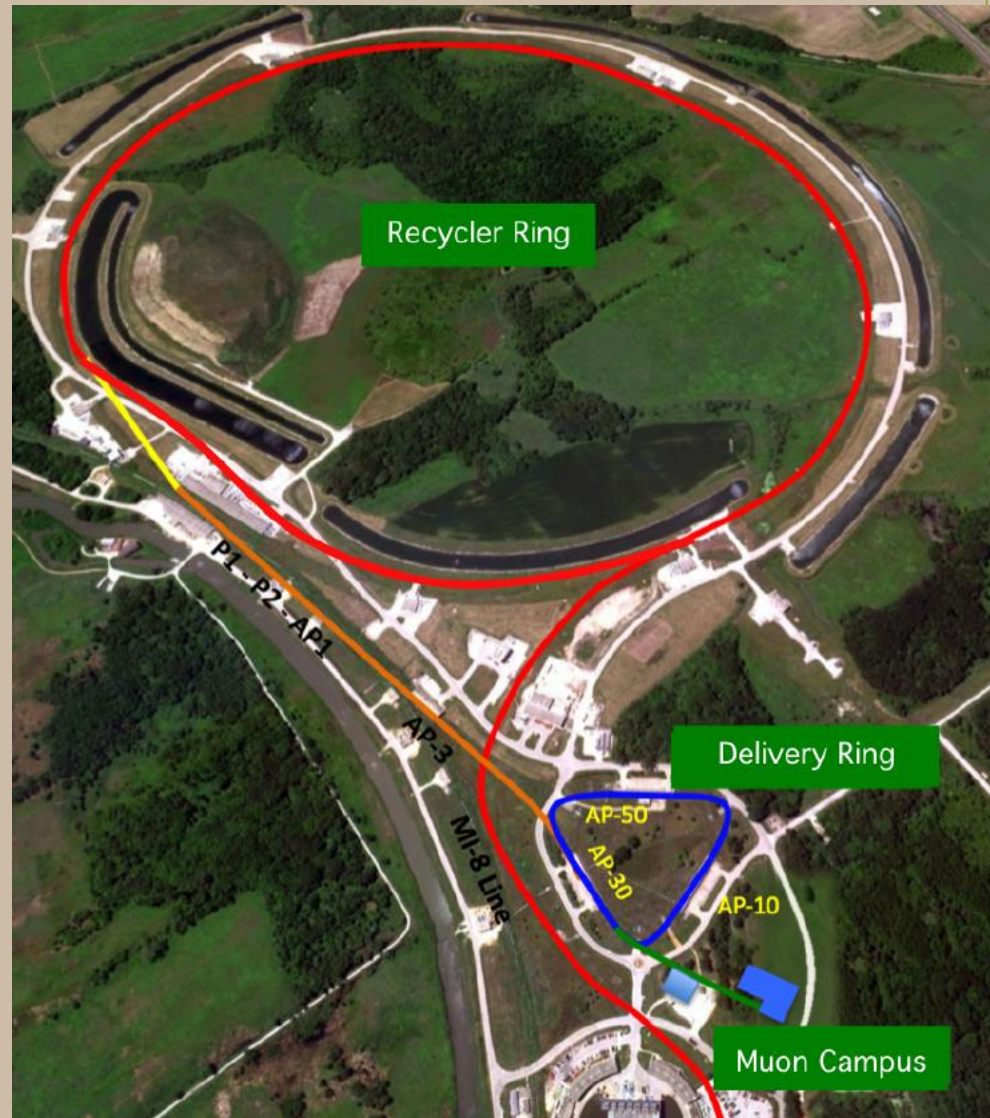
Goal is

1.8×10^{11} Detected decays

Systematic errors of better than:

± 0.07 ppm on ω_a

± 0.07 ppm on ω_p



Factor ~3 reduction in systematics built from large number of individual improvements:

ω_a

calorimeter

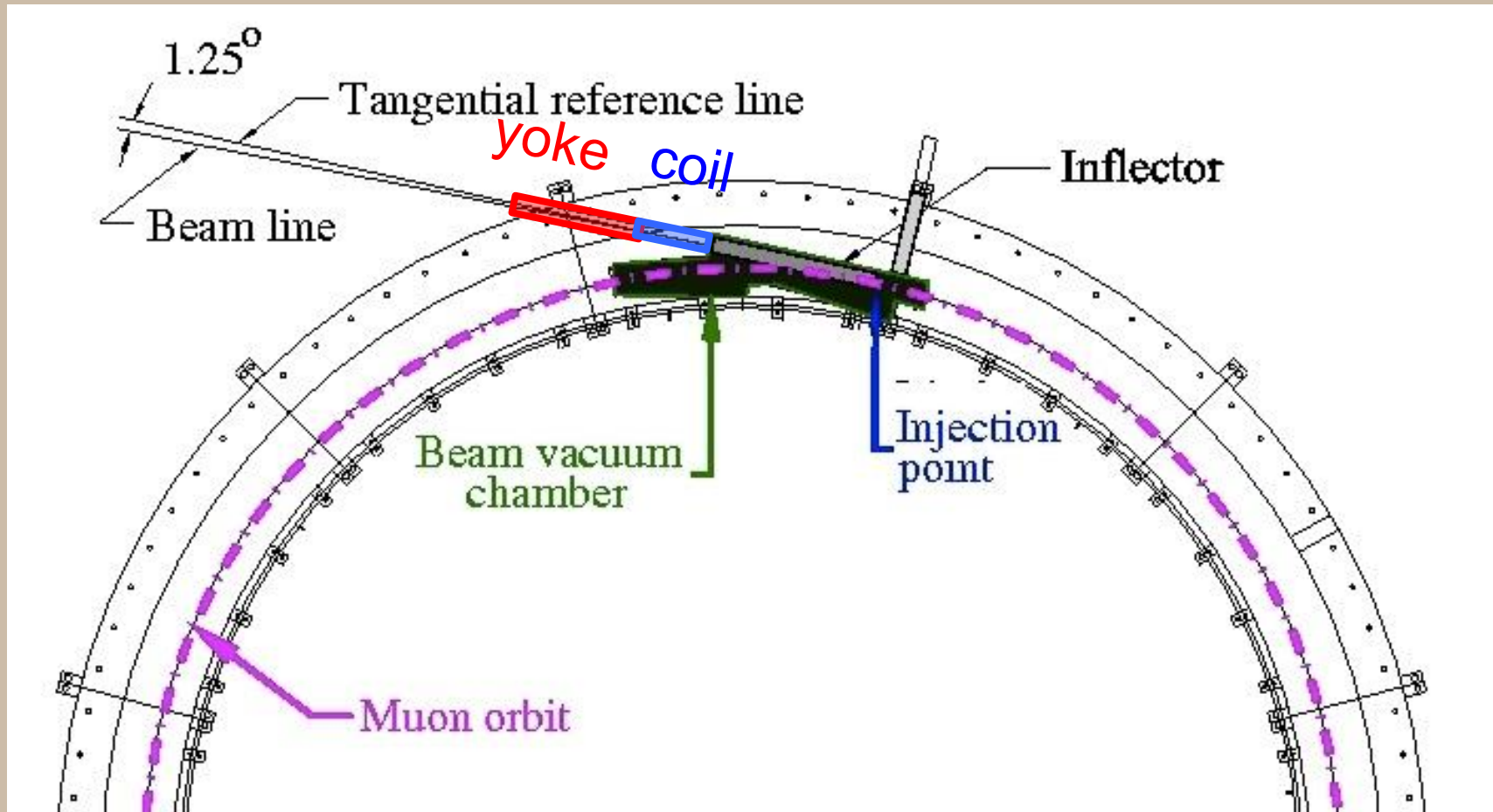
beam

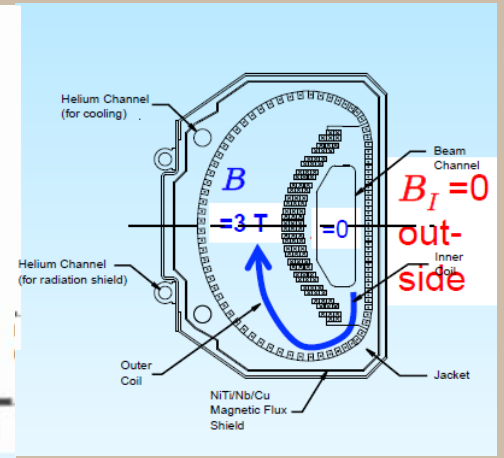
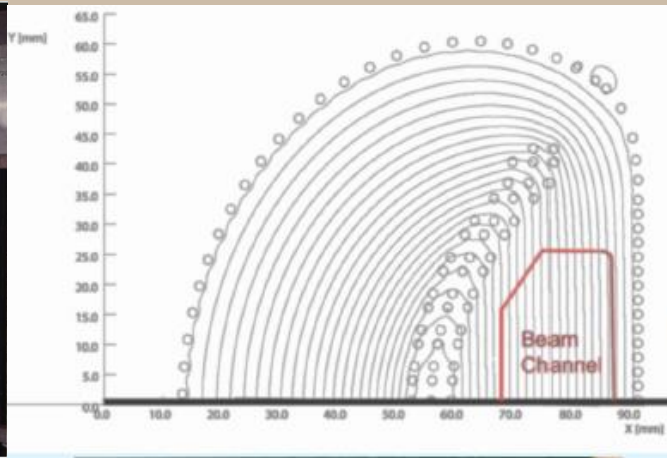
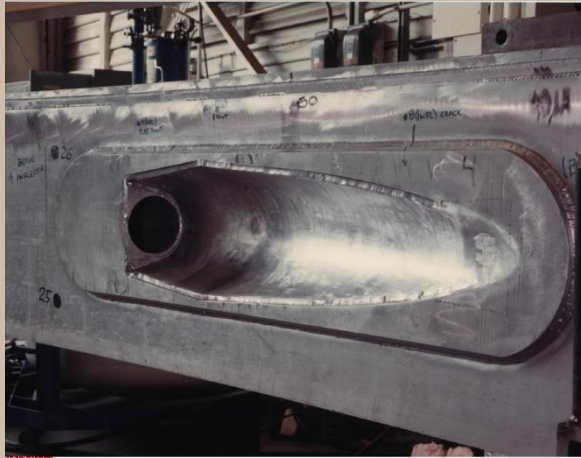
tracker

Category	E821 [ppm]	E989 Improvement Plans	Goal [ppm]
Gain changes	0.12	Better laser calibration low-energy threshold	0.02
Pileup	0.08	Low-energy samples recorded calorimeter segmentation	0.04
Lost muons	0.09	Better collimation in ring	0.02
CBO	0.07	Higher n value (frequency) Better match of beamline to ring	< 0.03
E and pitch	0.05	Improved tracker Precise storage ring simulations	0.03
Total	0.18	Quadrature sum	0.07

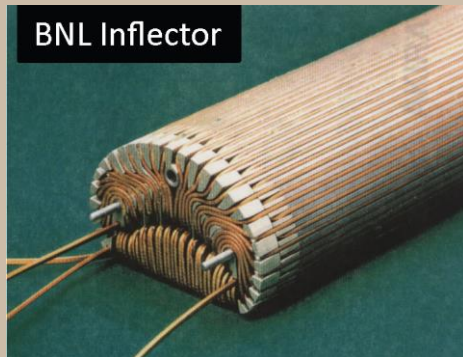
Beam injection, beam dynamics

INFLECTOR





Possible redesign of inflector



Replace with open ended design and larger aperture. Shielding challenging.

Improved acceptance, improved matching between delivery and ring leading to more muons and reduced beam oscillations
 → possible factor ~4 in storage efficiency.

Beam Injection and Ring

Numerous Improvements in collimation, beam tune etc. New inflector, New kickers.

- Reduction in muon loss
- Better control of coherent betatron oscillations and their impact on ω_a .

vertical and horizontal oscillations

Avoid this E821 feature!

Muons not all magic, not all perpendicular to B,E

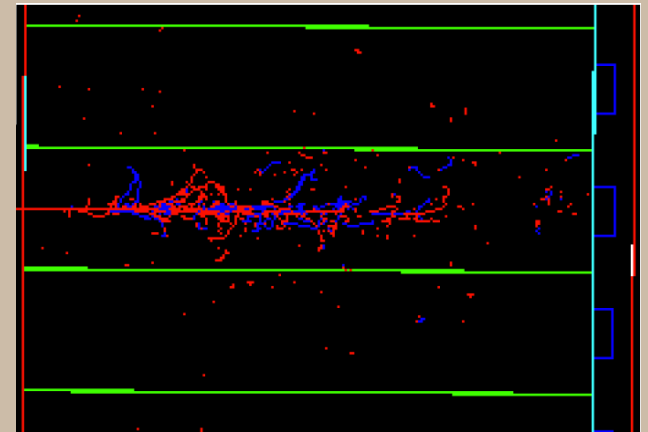
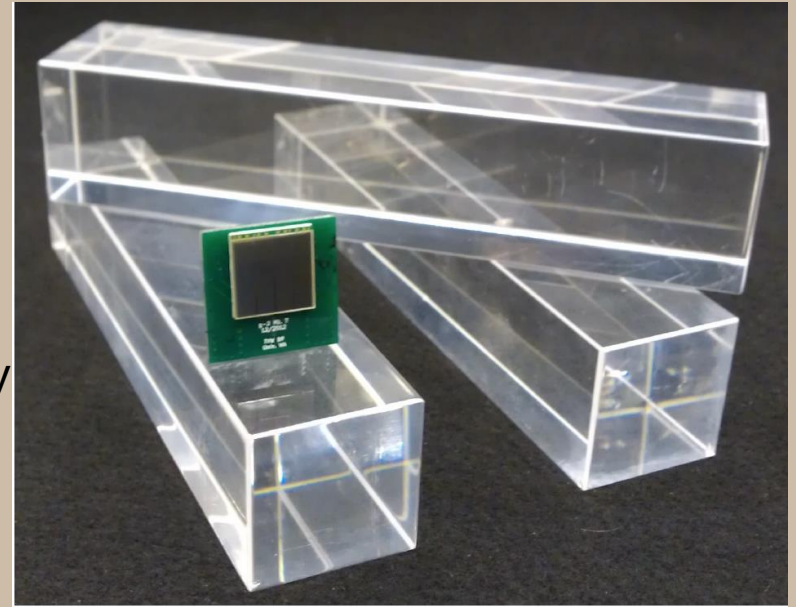
Pitch corrections needed.

Quantity	Expression	Frequency	Period
f_a	$\frac{e}{2\pi mc} a_\mu B$	0.23 MHz	4.37 μ s
f_c	$\frac{v}{2\pi R_0}$	6.7 MHz	149 ns
f_x	$\sqrt{1-n} f_c$	6.23 MHz	160 ns
f_y	$\sqrt{n} f_c$	2.48 MHz	402 ns
f_{CBO}	$f_c - f_x$	0.477 MHz	2.10 μ s
f_{VW}	$f_c - 2f_y$	1.74 MHz	0.574 μ s

Calorimeters

New for E989:

- Segmented: 6x9 PbF₂ crystals with SiPM readout. **Attack pileup systematic.**
- Pileup: muon phase correlated with e⁺ energy
 - overlapping pulses → wrong phase shifts
 - varying fraction of pileup within fill produces early → late shift in average phase → direct impact on ω_a .
- one pulse should not affect gain of subsequent pulse on same channel. Should be able to separate at 5ns level
- Fills ~ 700μs long
- Gain variations and time shifts over this period feed into systematics. $\Delta G(t) < 0.1\%$

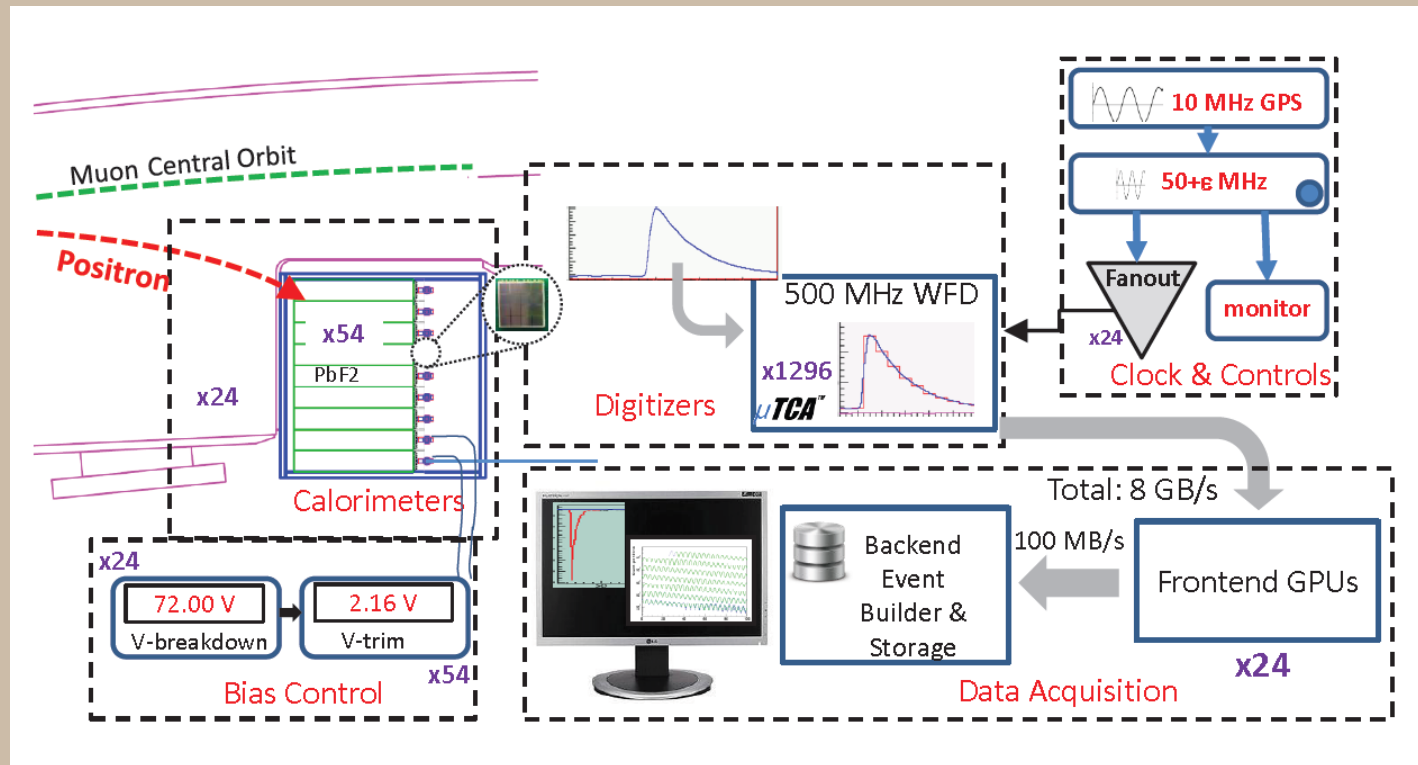


0.05 ppm systematic budget for ω_a

Calorimeters II

Continuous distribution of muons with random decay probability

- Determine arrival time
- Determine energy
- Pileup separation \Rightarrow Wave Form Digitisation
- Laser calibration system



Improvements: Gain changes 0.12 \rightarrow 0.02ppm

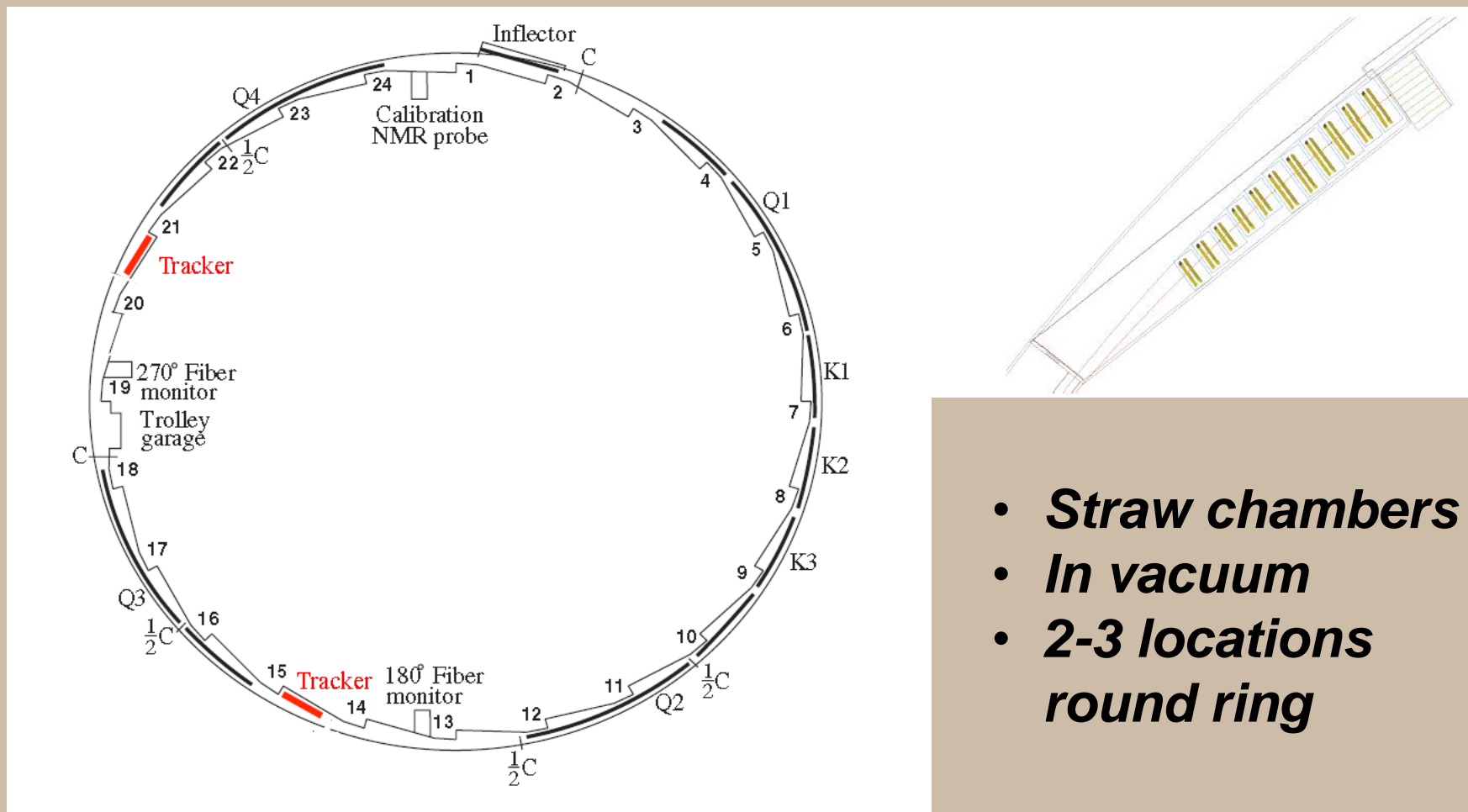
Pileup: 0.08 \rightarrow 0.04ppm

Tracking detectors

- Measure beam profile at several locations as function of time in fill.
 - Convolute μ spatial distribution with field to determine effective field seen by μ 's **0.03 \rightarrow 0.01ppm**
 - Momentum spread and betatron oscillations lead to ppm corrections to ω_α from non-magic muons - E-field and pitch corrections...
Beam dynamics corrections **0.05 \rightarrow 0.03ppm**
 - Pileup identification. **0.08 \rightarrow 0.04ppm**
 - Independent momentum measurement. Verify calorimeter gain
 - Correct for acceptance changes in calorimeters from betatron oscillations. Validate calorimeter based determinations of pileup corrections, gain, muon loss. **0.12 \rightarrow 0.02ppm**
- **EDM** measure positron vertical angle – asymmetry.
Design will allow a factor ~ 200 /month increase in statistics over Brookhaven measurement \Rightarrow factor of 10 on EDM very quickly and ~ 100 eventually.

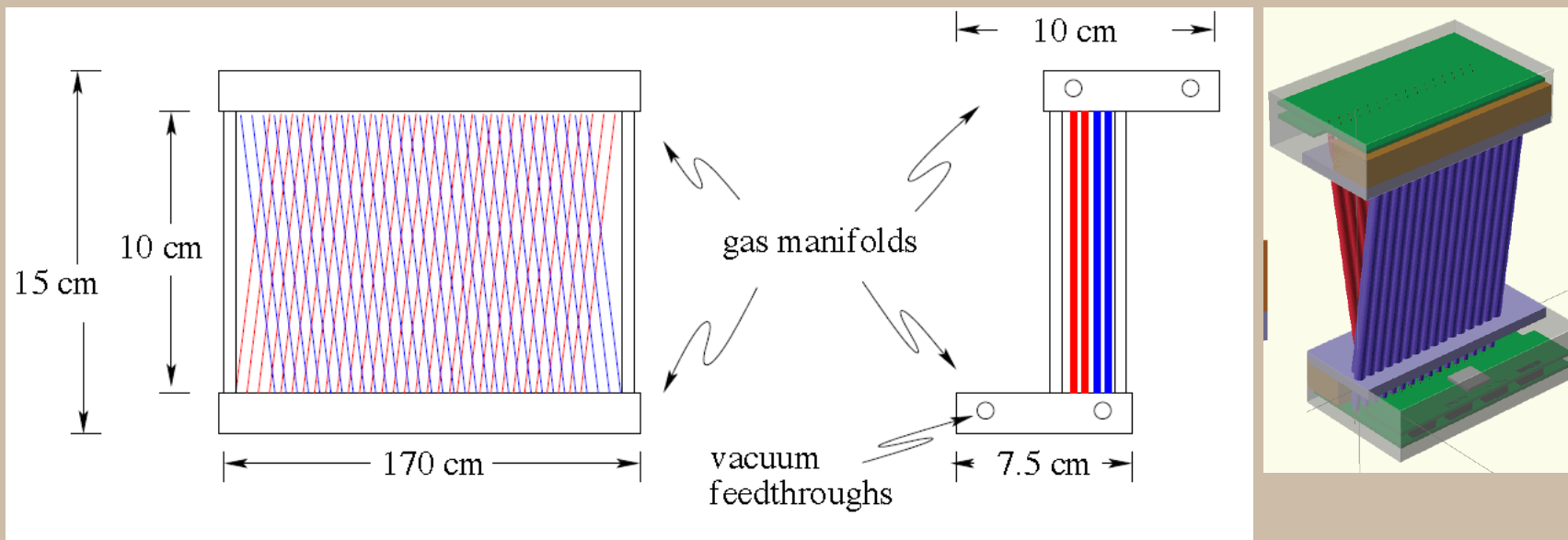
New Tracking detectors

- Need: long lever arm $\sim 1\text{mm}$ determination of muon decay up to 10m away
- Continuously distributed decay points, muon momenta \Rightarrow distributed detector



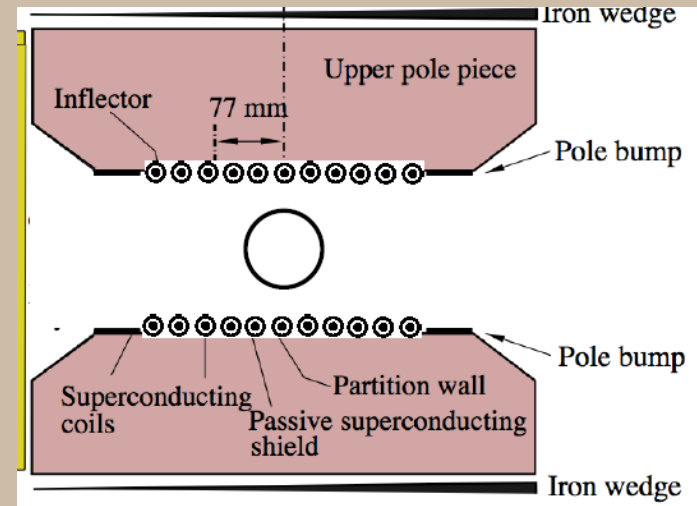
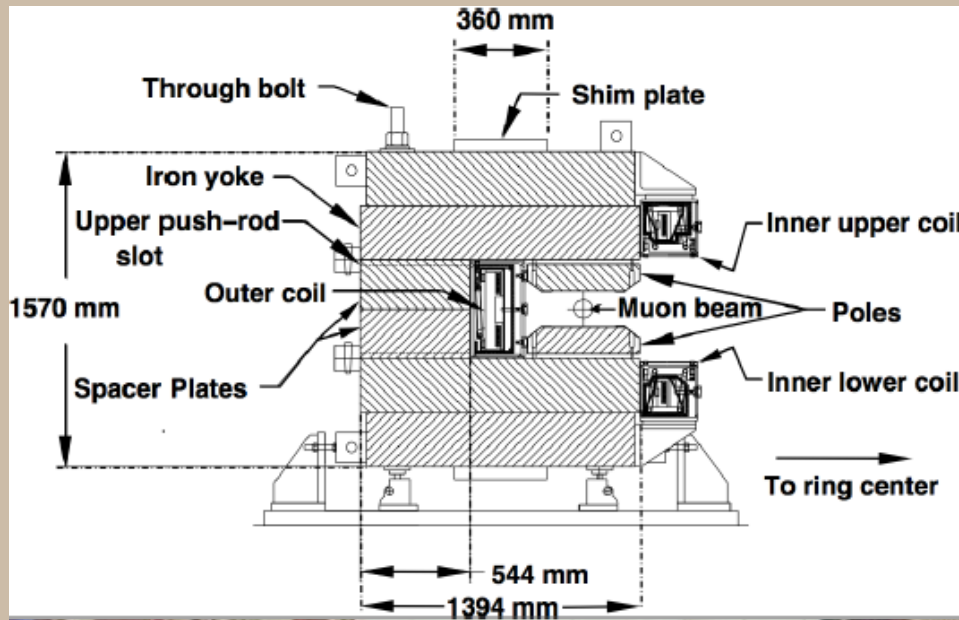
- **Straw chambers**
- **In vacuum**
- **2-3 locations round ring**

Straws do the trick:



- Low mass $\sim 0.1X_0$ per station, non-magnetic, OK in vacuum
- 5mm diameter, 12cm long straws. Mylar coated with Al+Au.
- 25μ gold-plated tungsten wires
- Based on Mu2e straws.
- 80:20 Argon:CO₂ gas
- $\pm 7.5^\circ$ UV layers to give vertical resolution

What about ω_p ?



Goal is to get this to 0.07ppm accuracy...

A lot of shimming

A lot of measuring and monitoring

ω_p Field measured with set of NMR probes

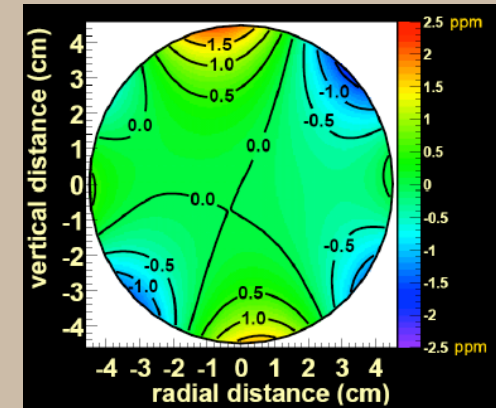
Average muon orbits
~400 times sampling
toroidal region

- fixed probes
- plunging probes
- trolley probes

Again, requires concerted attack to beat down systematics.

Improvements from reduced position uncertainties, more frequent measurements, better electronics etc.

Source of errors	R01 [ppm]	E989 [ppm]
Absolute calibration of standard probe	0.05	0.035
Calibration of trolley probes	0.09	0.03
Trolley measurements of B_0	0.05	0.03
Interpolation with fixed probes	0.07	0.03
Uncertainty from muon distribution	0.03	0.01
Inflector fringe field uncertainty	–	–
Time dependent external B fields	–	0.005
Others †	0.10	0.03
Total systematic error on ω_p	0.17	0.070

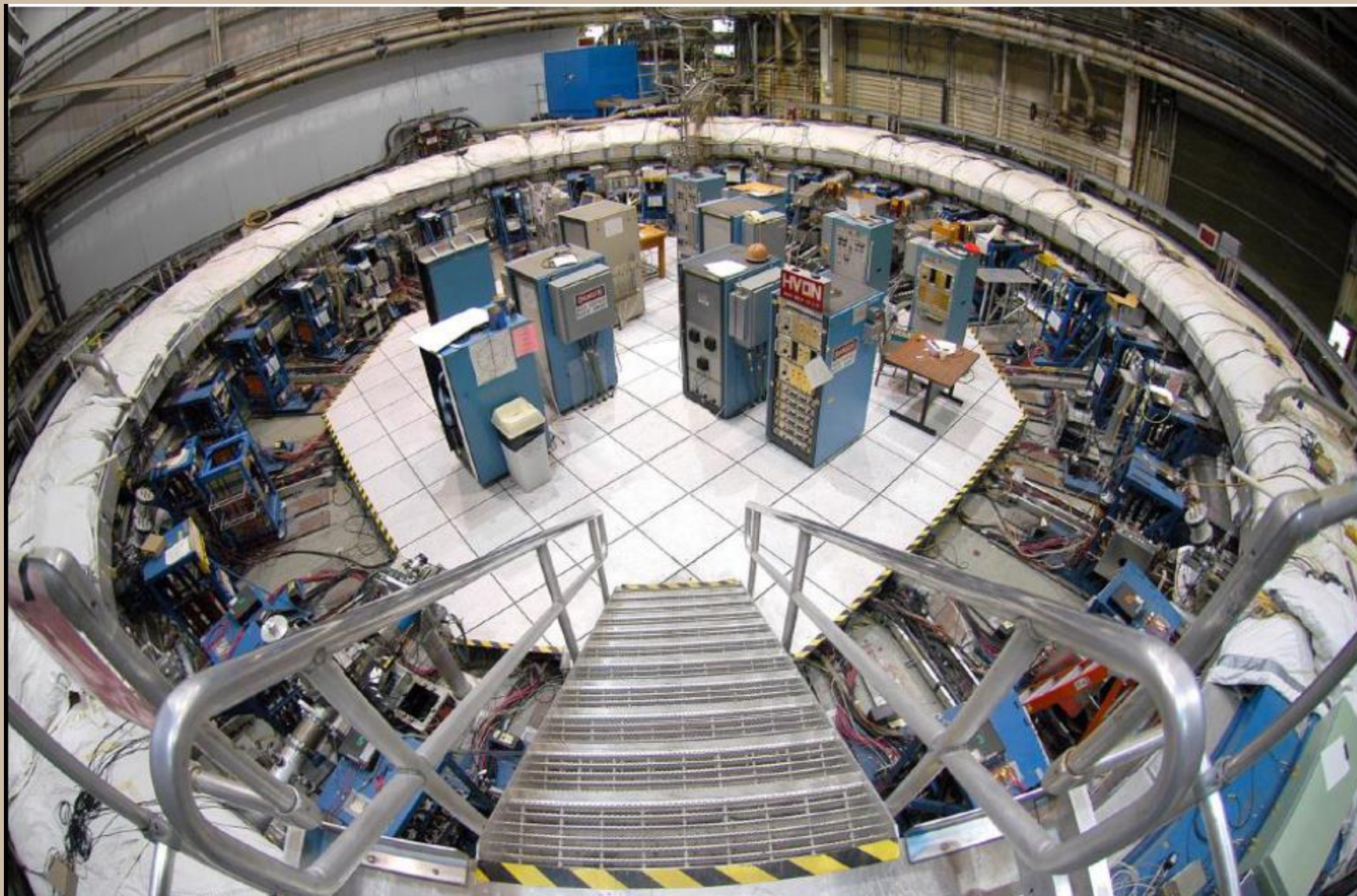


e.g. **Absolute Calibration**
Dedicated 1.45T calibration
magnet, more probes.

Possibility of using ^3He as
well as water-based NMR
probes.

But first... Vital bits of E821
have to get from BNL to
FNAL

The move...



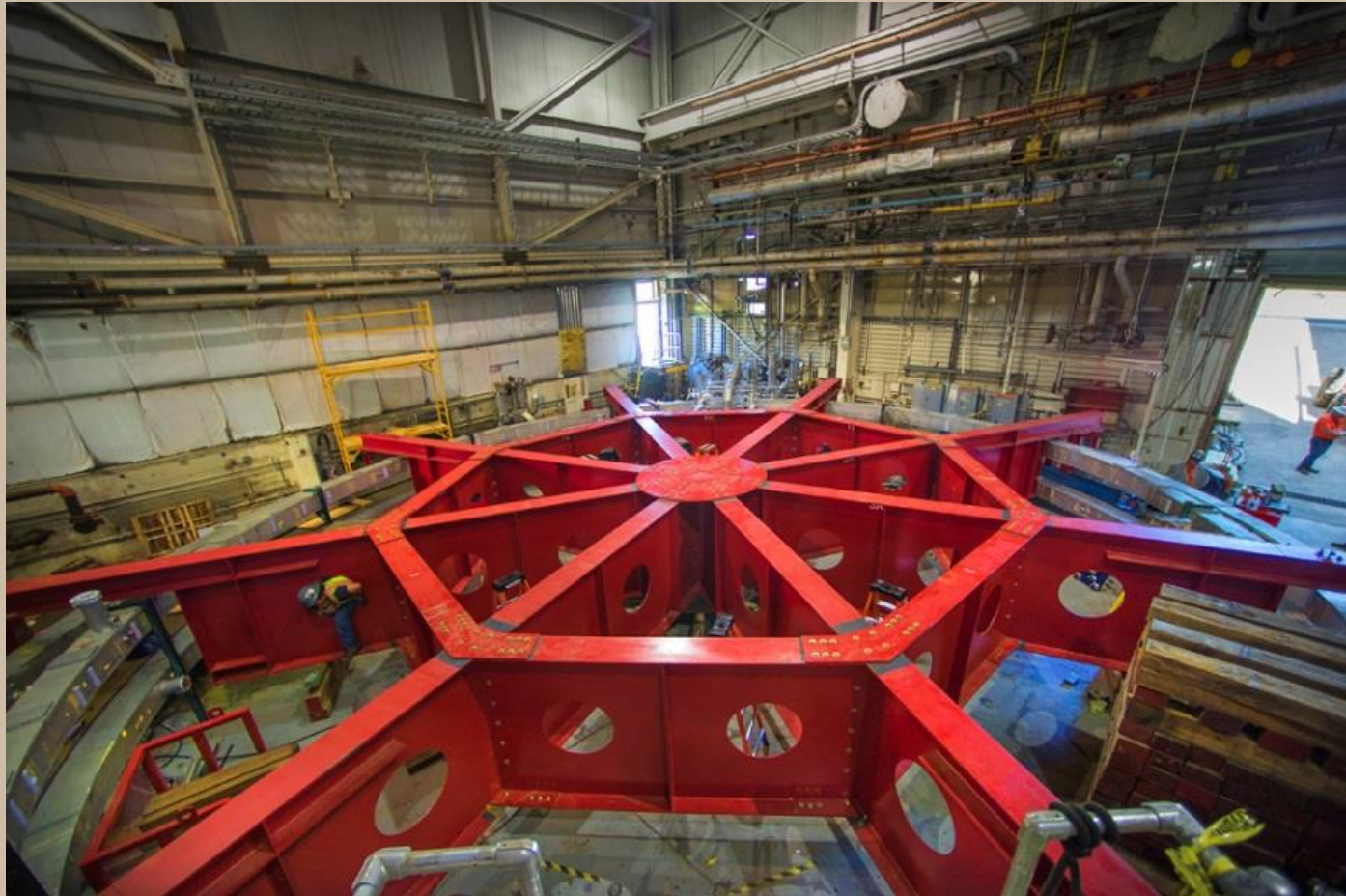
Disassembly September 2012



May 2013 : Yoke pieces arriving at FNAL



Building support structure...







*"Yeah,
We can move that "*

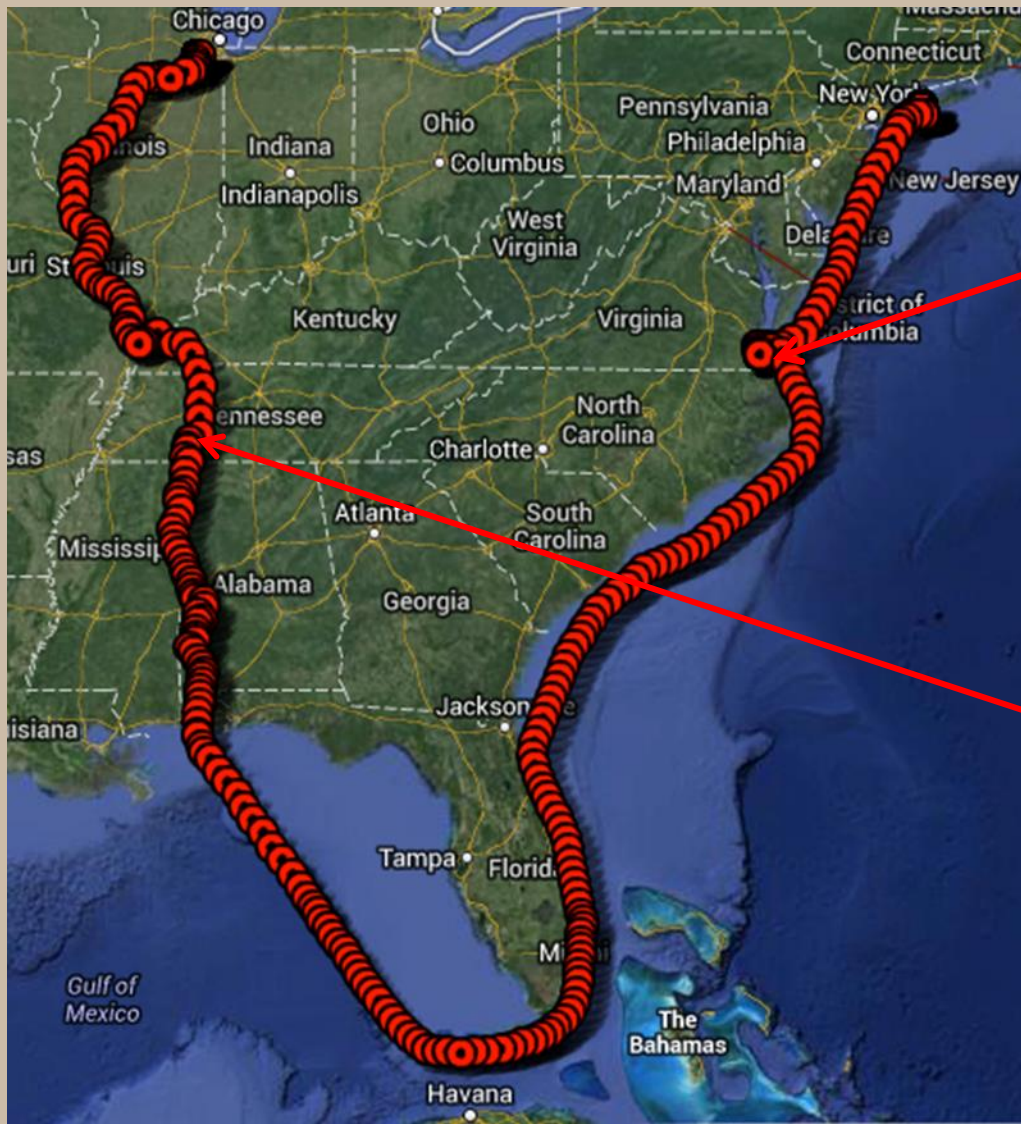






Big barge to limit pitch, roll and heave...





5 nights sheltering from Storm near Norfolk Virginia.

Tennessee-Tombigbee Waterway
Mississippi, Illinois and
Des Plaines rivers.

25th June - July 20th

Miss Katie







Lemont, IL. ...safely ashore



It fits...





**‘Overdaying’
at Costco’s
supermarket!**





...after 3200 mile journey.

CONCLUSIONS/Timeline

- Important bits of E821 now at Fermilab
 - coils, ~2/3 steel
- Building under construction
 - expect beneficial occupancy February 2014
- Arduous series of CD-1 reviews this year nearly over
 - (then CD-2,... next year!)
- Many upgrades well-underway
- On-schedule for:
- Magnet powered 2015
- Beam in 2016 or 2017



Thank You