Challenges and future in three-body heavy meson decays

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#BlackLifesMatter

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Motivation

- Standard Model works quite well but... some gaps!

  → baryogenesis !

- 1967, the Russian physicist Andrey Sakharov proposed three conditions for generating the observed matter/anti-matter asymmetry of the Universe:

  1) baryon number violation
  2) C and CP violation
  3) departure from thermal equilibrium

- CP-Violation on Hadronic decays
  - SM predicts CPV in B sector but ….. lot to be understood

    ↳ massive phase-space localized Asymmetry in $B^{\pm} \to h^{\pm} h^{-} h^{+}$

  - 2019 1st observation in charm $D^0(\bar{D}^0) \to \pi^+\pi^- - K^+K^-$

    ↳ CPV on three-body?

→ can lead to new physics

Context

- D and B three-body HADRONIC decays are dominated by low E resonances

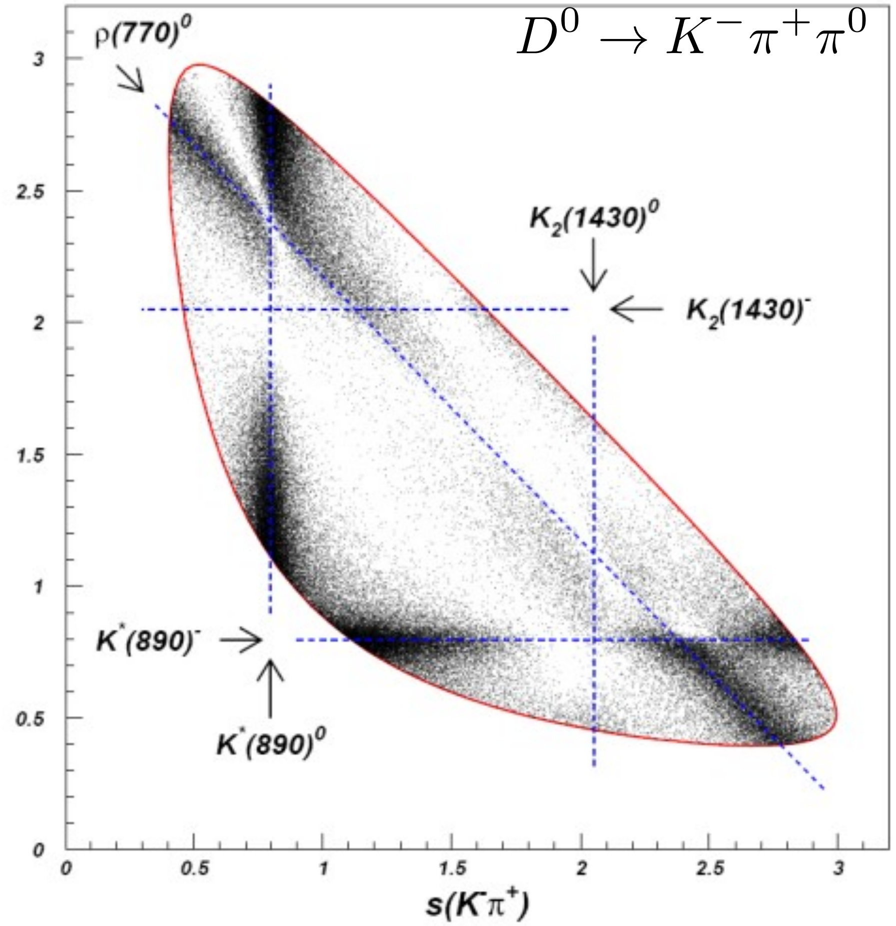


image credit:Brian Meadows

- spectroscopy: new resonances, their properties…
- information of MM interactions

1st observation of
σ [$f_0(600)$] and κ [$K_0^*(700)$]
in D decays

- build up the idea that the main dynamic in 3-body is driven by 2-body resonances

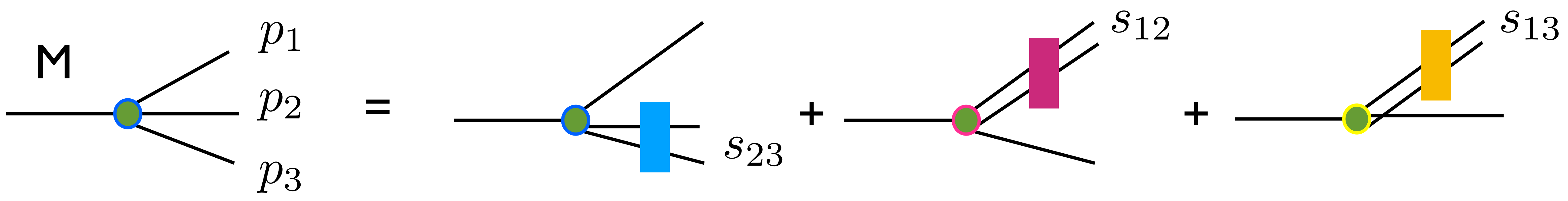
- new high data sample from LHCb ⟶ more to come from LHCb, BelleII, BESIII

↳ simple models (only focus on two-body resonances) are not enough to explain data anymore

theoretical challenge !

Three-body kinematics : DALITZ plot

- How to describe the kinematics of three-body HADRONIC decays?



- Mandelstam variables for 3-body

$$s_{12} = (p_1 + p_2)^2 = m_{12}^2$$

$$s_{13} = (p_1 + p_3)^2 = m_{13}^2 \quad \rightarrow \quad s_{12} + s_{13} + s_{12} = M^2 + m_1^2 + m_2^2 + m_3^2$$

$$s_{23} = (p_2 + p_3)^2 = m_{23}^2$$

- In the rest frame of M (**P**=0): final particle are in the same plane

↳ final particle distribution in the phase-space will depend on: - average of spin
- Euler angles

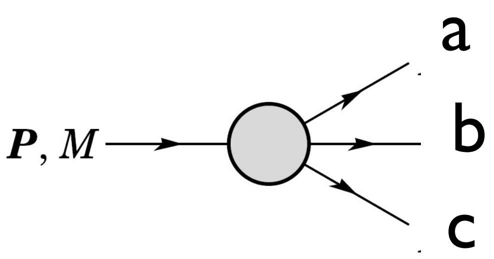
→ decay rate can be written as: $d\Gamma = \frac{1}{(2\pi)^3}\frac{1}{32M^2}|\bar{\mathcal{M}}|^2 s_{12} s_{23}$

↳ Amplitude, dynamic!

Three-body kinematics : DALITZ plot

- The phase-space is NOT one-dimension!

$$\frac{d\Gamma}{ds_{12}ds_{23}} = \frac{1}{(2\pi)^3}\frac{1}{32M^3}\boxed{|\mathcal{A}(s_{12},s_{23})|^2}$$

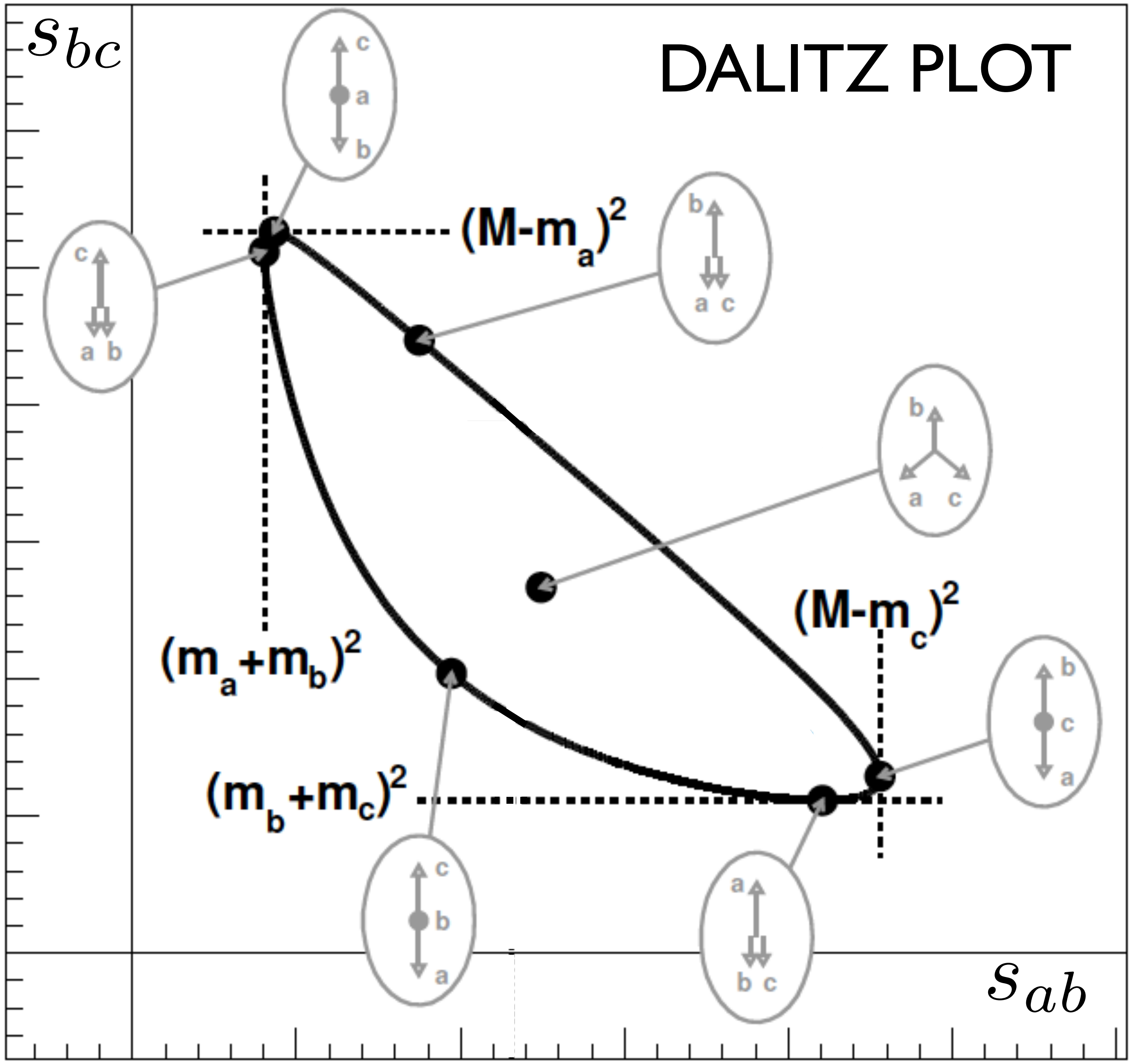


- DP proposed by Richard Dalitz (1925-2006) in 1953

- the perimeter depends on the masses
  min: $s_{ij} > (m_i + m_j)^2$
  max: in s_{ij} , $(M - m_k)^2$

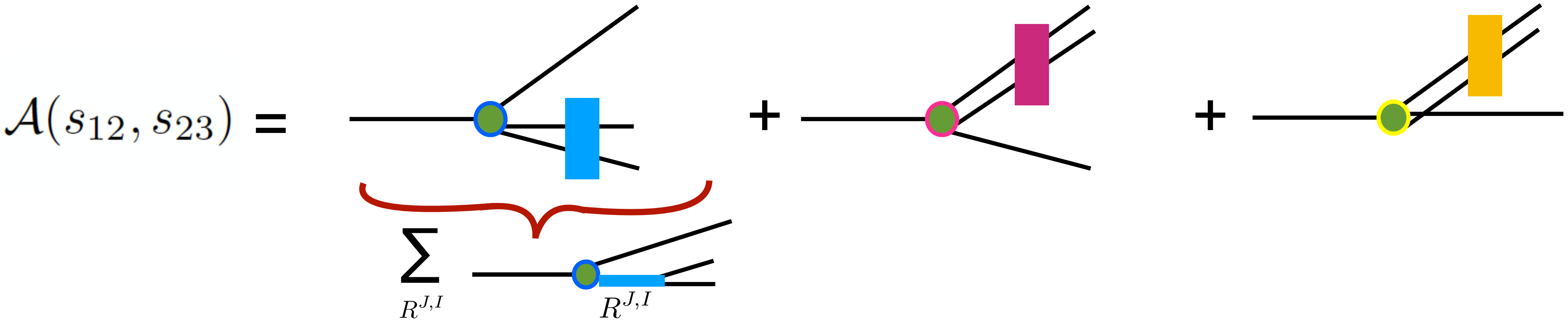
- inside this contour there are all combinations of momenta distribution

- The probability of each point inside is given by the dynamic amplitude A



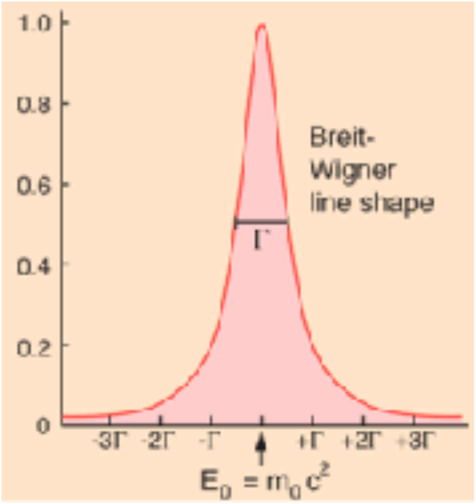
→ tool for analyse data

Two-body resonances signature in DP



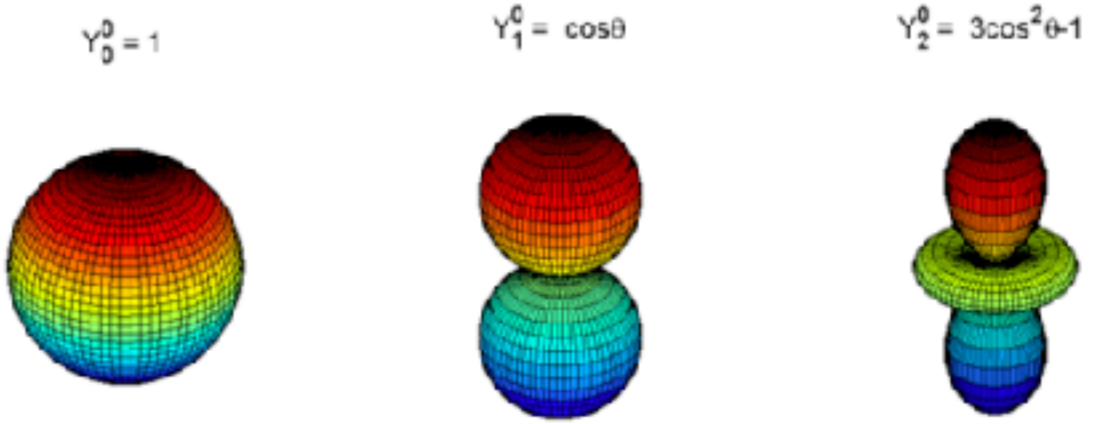
- 2-body resonances have spin and isospin well defined: $R^{J,I}$
  - typically amplitudes are bumps (like the Breit-Wigner)
  - contribute to a specific partial wave

$$\mathcal{M}_{ba}(s,t) = \sum_{j=0}^{\infty} (2j+1)\mathcal{M}^{j}_{ba}(s) P_j(\cos(\theta))$$



credit:hyperphysics.phy

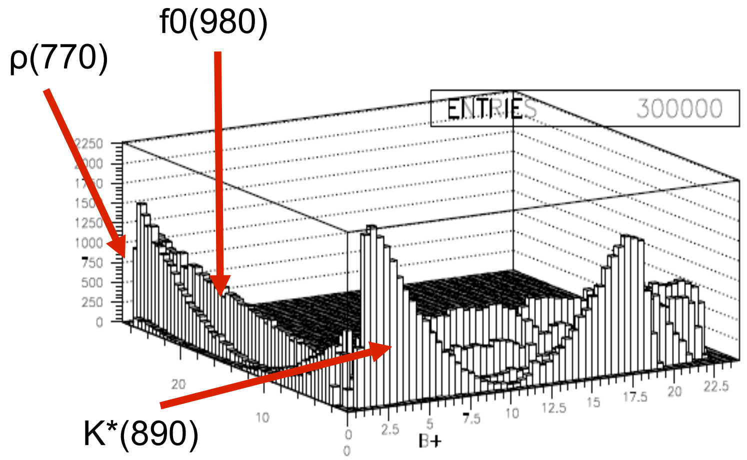
↳ besides the amplitude bump (intensity/probability) the resonance will have a spin signature in DP: $P_j(\cos(\theta))$ (same as spherical harmonics)…



scalar vector tensor

credit: knowino.org

⟶ this pattern in Dalitz Plot

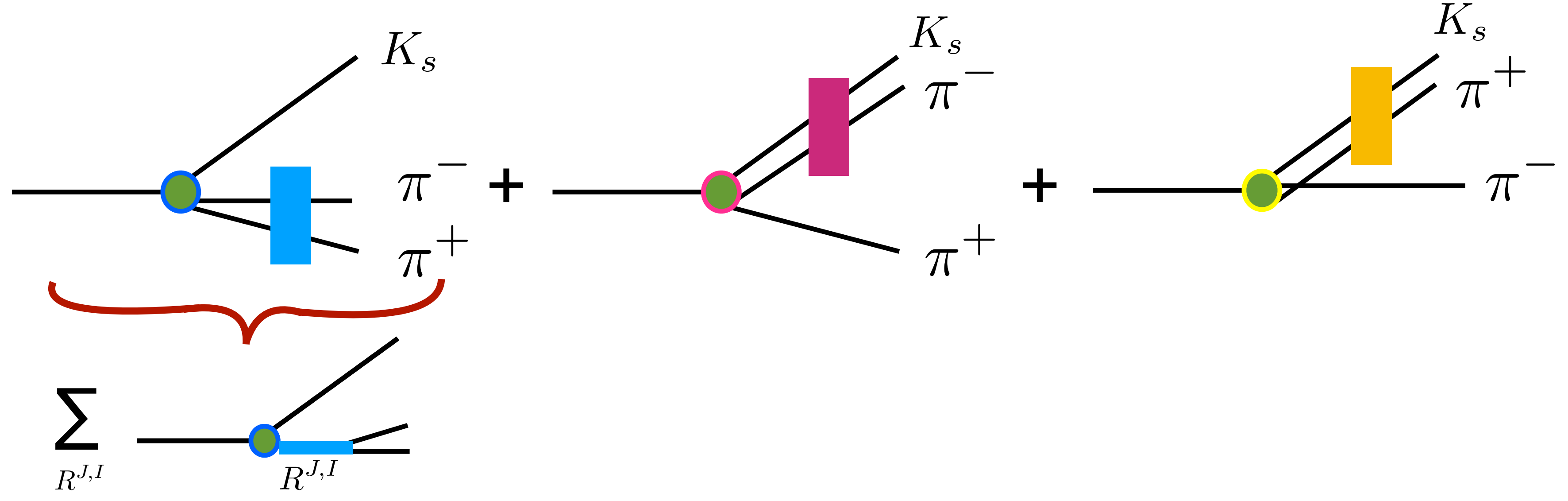


Two-body resonances signature in DP

- common cartoon to described 3-body decay

$D^0 \rightarrow K_s \pi^- \pi^+$

$\mathcal{A}(s_{12}, s_{23}) =$



- one expect to see all 3 channels res:

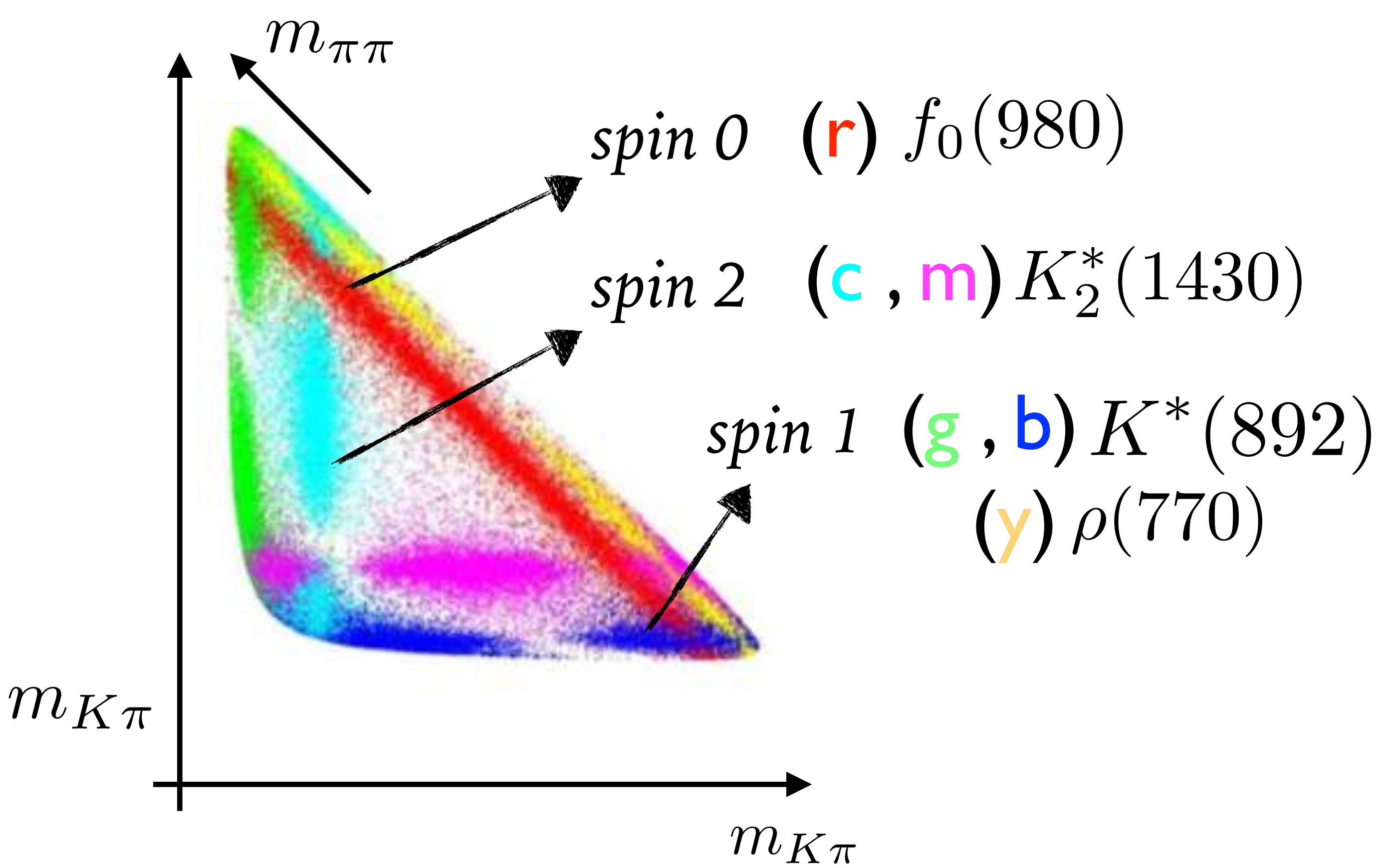
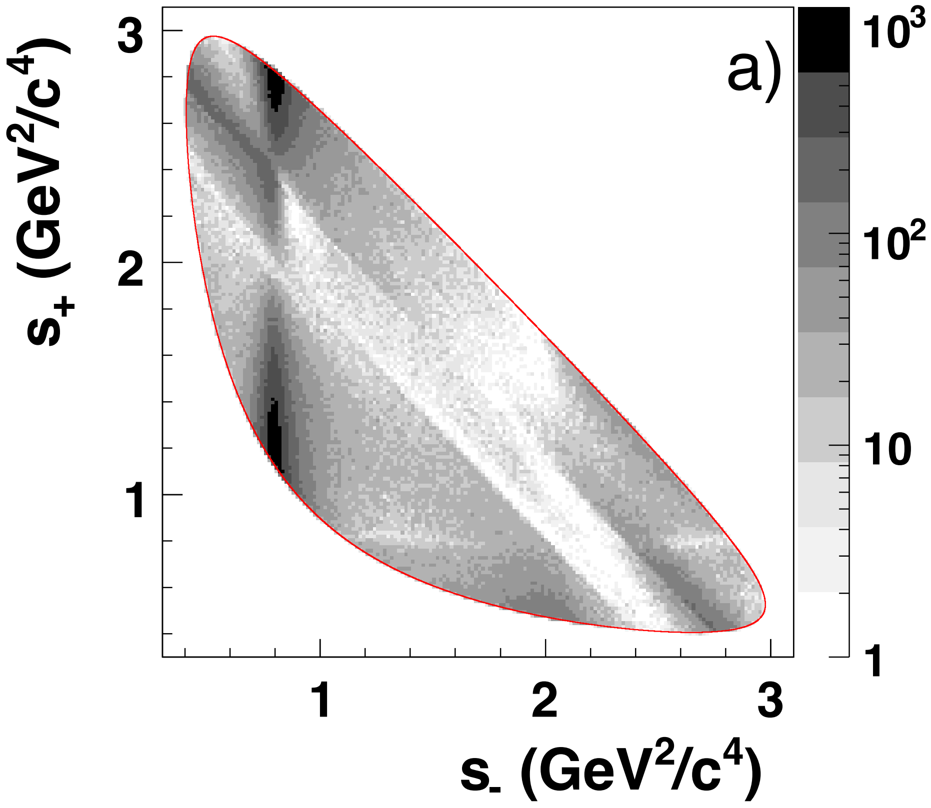


image credit:Tom Latham

→ But in reality…….

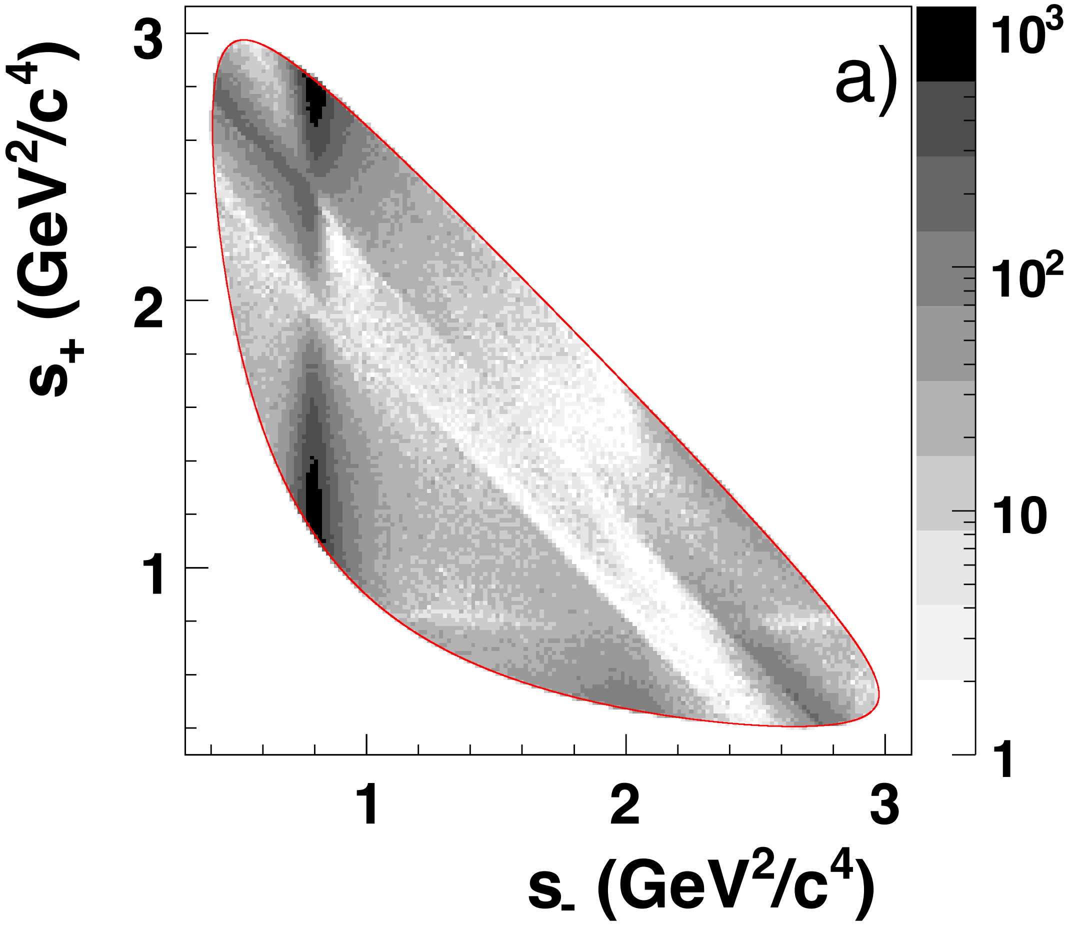
not all of them are clearly present



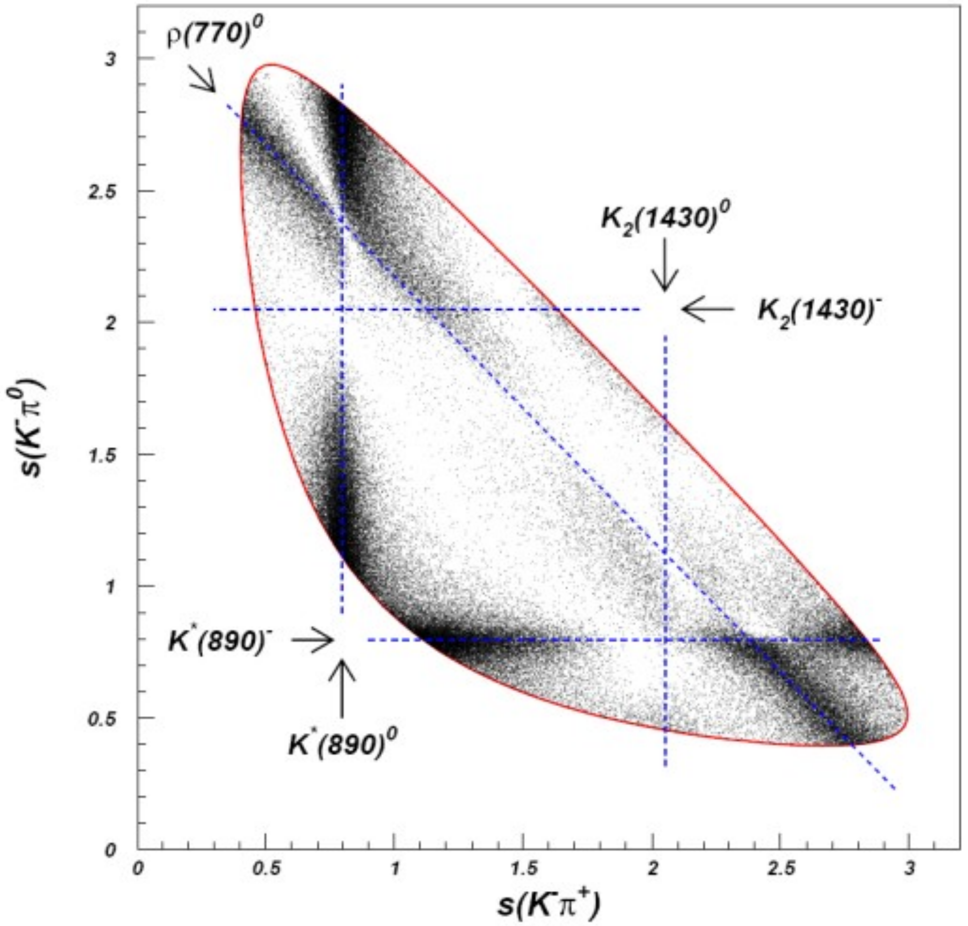
BABAR Phys.Rev. Lett. 105 (2010) 081803

Three-body kinematics : DALITZ plot

- $D^0 \to K_s \pi^- \pi^+$



- $D^0 \to K^- \pi^+ \pi^0$



credit:Brian Meadows

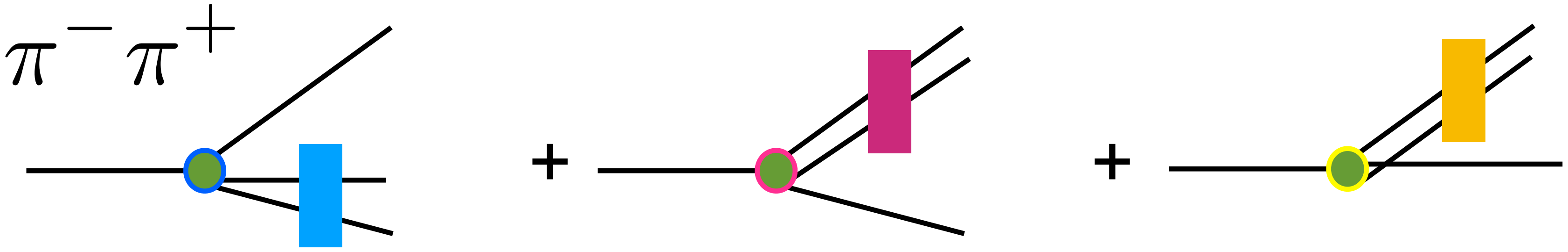
⟶ Similar final state but different interference pattern

↳ different dynamics to be understood

⟶ to disentangle the interference we need amplitude analysis

2-body x 3-body phases

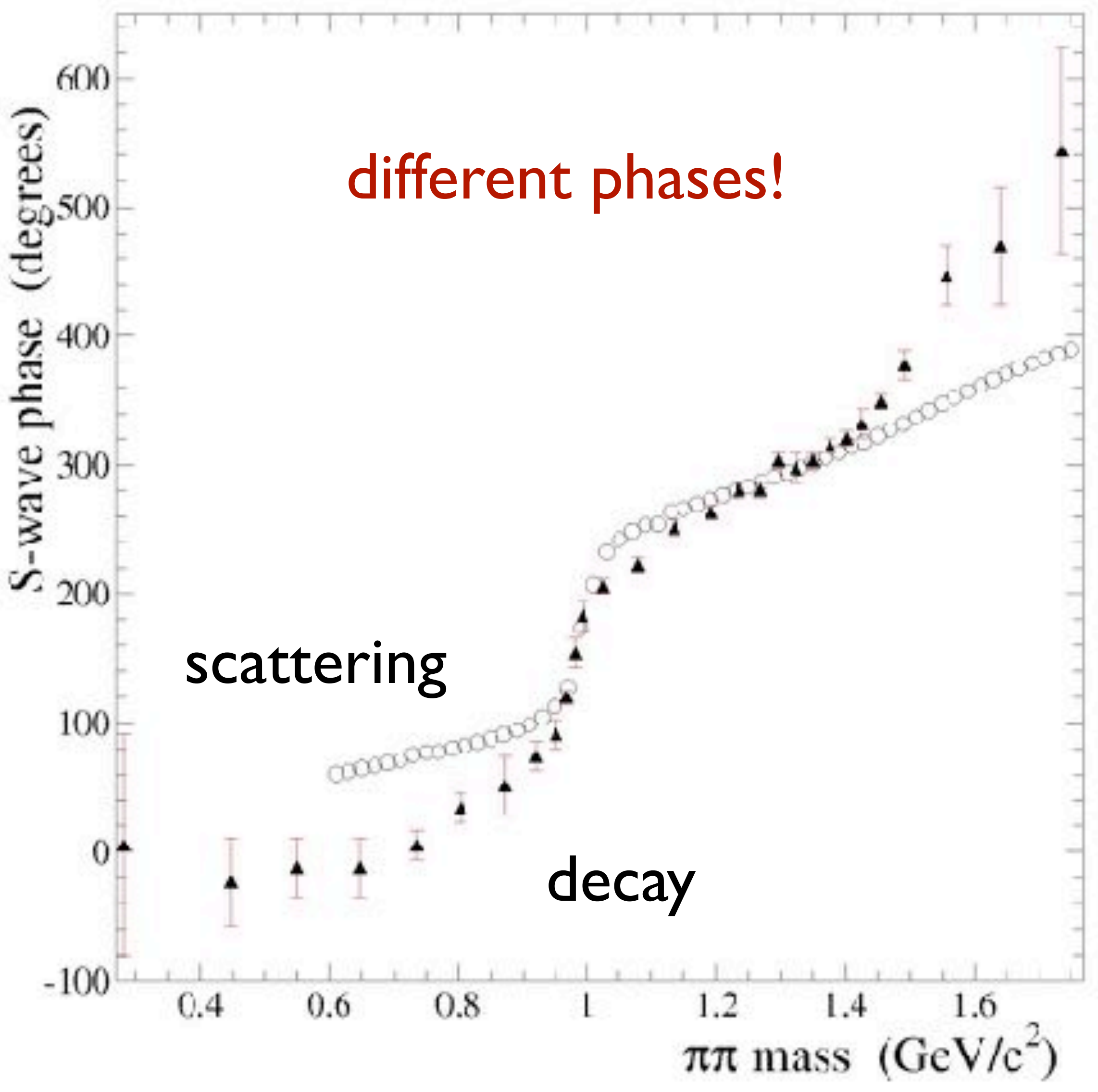
- $D_s^+ \to \pi^+\pi^-\pi^+$



  - If this is the “nature” picture ⟶ once it only contain 2-body information, decay phase should be the same as scattering

→ Is not as simple as it look like!

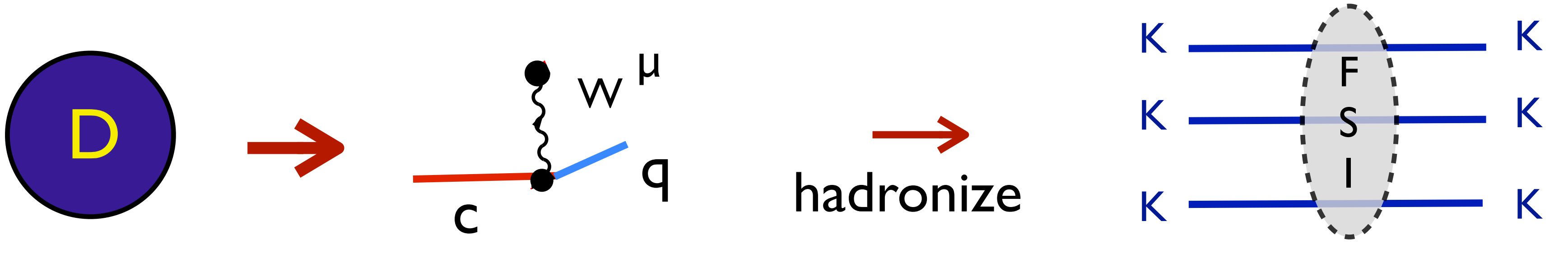
- Quantum numbers:
  - 2-body amplitude: spin and isospin well defined!
  - 3-body data: only spin! and $\neq$ dynamics



Phys.Rev. D 79 (2009) 032003

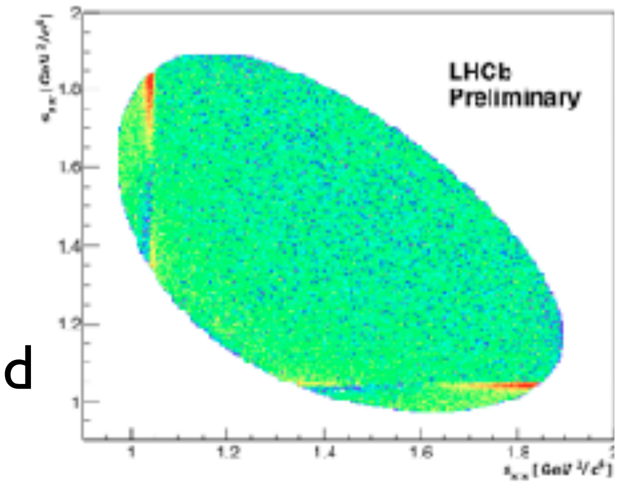
Three-body heavy meson decay

- dynamics $D^+ \to K^-K^+K^-$



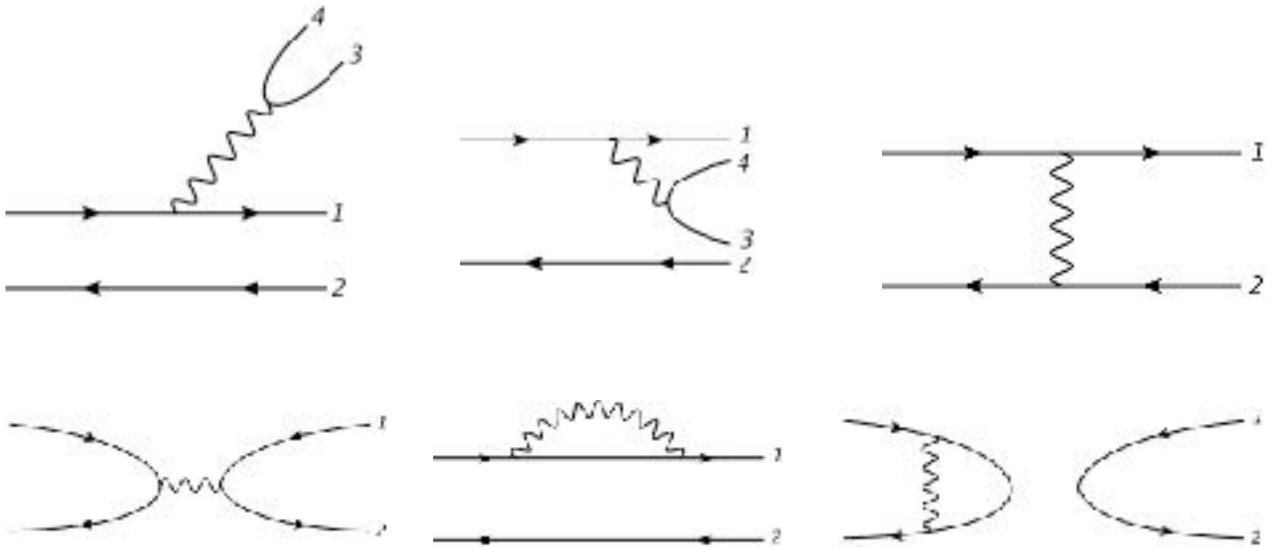
hadronize

observed

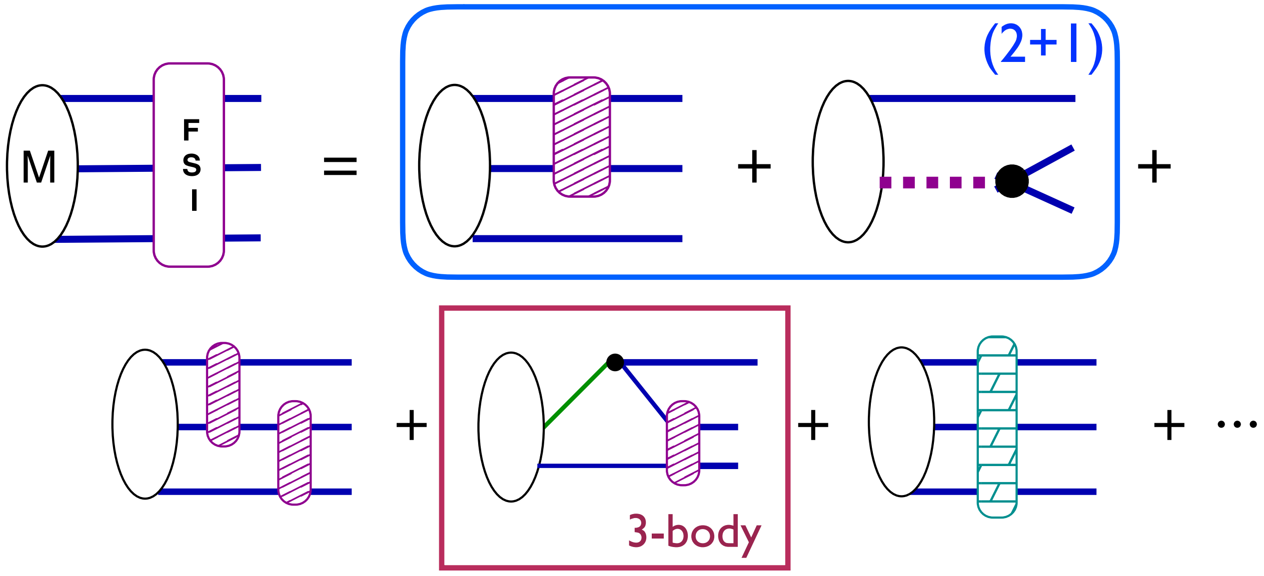


primary vertex - weak -

Final State Interactions - strong -

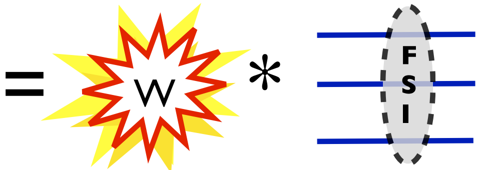


QCD, CKM coupling and phase



2-body is crucial!!!!

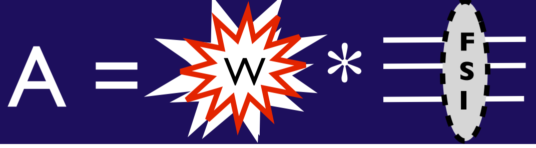
To extract information from data we need an **amplitude MODEL**

A = 

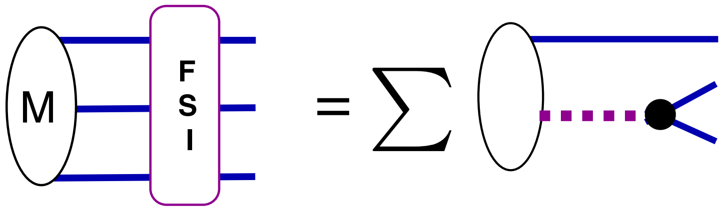
$$\frac{d\Gamma}{ds_{12}ds_{23}} = \frac{1}{(2\pi)^3}\frac{1}{32M^3}|\mathcal{A}(s_{12},s_{23})|^2$$

dynamics

Models available



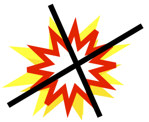
- isobar model: widely used by experimentalists
  - (2+1) approximation:



→ ignore the 3rd particle (bachelor)

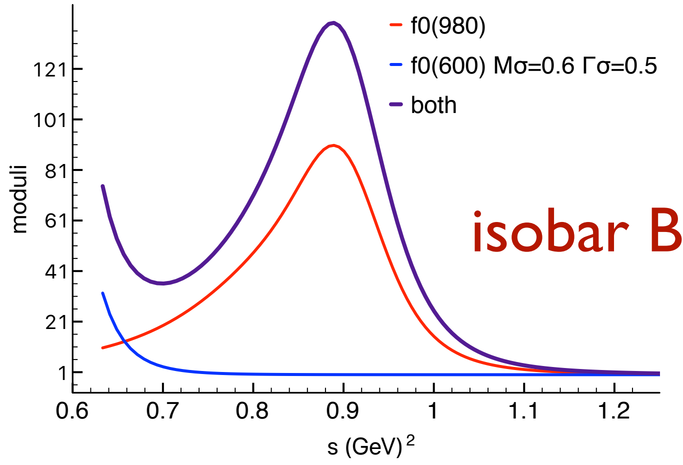
$A = \sum c_k A_{k;}$ + NR { non-resonant as constant or exponential!

each resonance as Breit-Wigner $\mathrm{BW}(s_{12}) = \dfrac{1}{m_R^2 - s_{12} - i m_R \Gamma(s_{12})},$



weak vertex is not considered explicitly

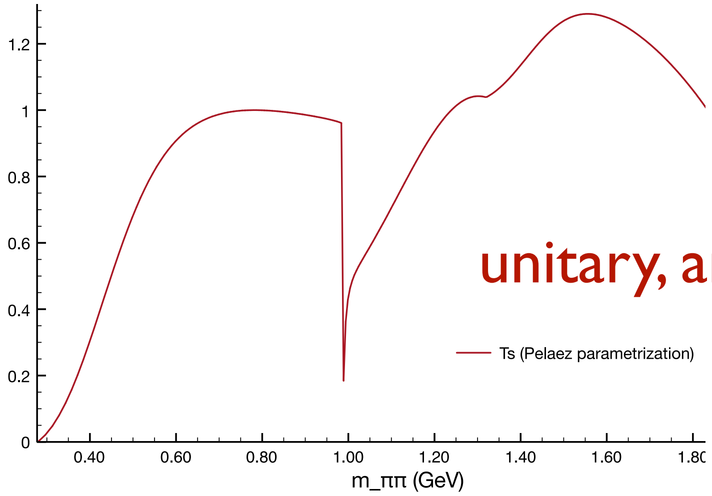
- worst problems: ππ S-wave



isobar BW

→ fit could change this interference

more than 2 scalars ←

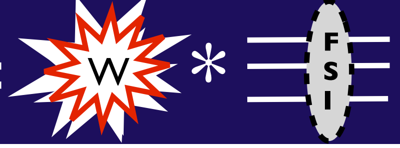


unitary, analytic,..

Pelaez, Yndurain PRD71(2005) 074016

- sum of BW violates two-body unitarity (2 res in the same channel);
- do NOT include rescattering and coupled-channels;
- free parameters are not connected with theory !

!

Models available A = 

- movement to use better 2-body (unitarity) inputs in data analysis

  - “K-matrix" : ππ S-wave 5 coupled-channel modulated by a production amplitude

    ↳ used by Babar, LHCb, BES II

    Anisovich PLB653(2007)

  - rescattering $\pi\pi \rightarrow KK$ contribution in LHCb $\begin{cases} B^{\pm} \rightarrow \pi^{+}\pi^{-}\pi^{\pm} \\ B^{\pm} \rightarrow K^{-}K^{+}\pi^{\pm} \end{cases}$ [arXiv:1909.05212; 1909.05211] [arXiv:1905.09244]

    Pelaez, Yndurain PRD71(2005) 074016

    ↳ new parametrization Pelaez, and Rodas EPJ. C78 (2018) 11, 897

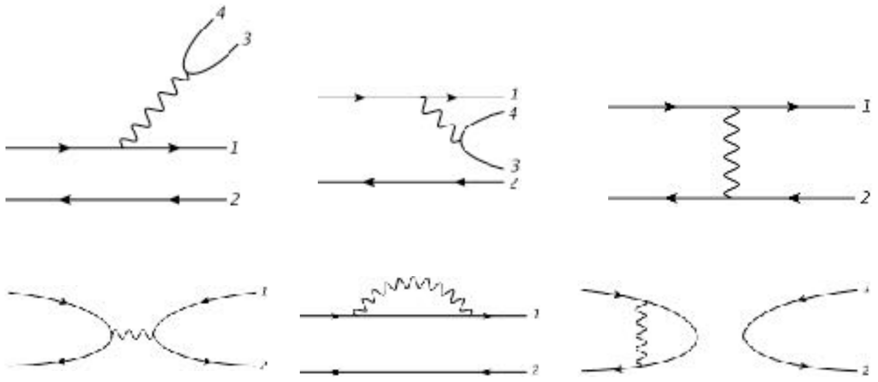
→ other scalar and vector form factors available

Limited to low E (2 GeV)!

$<\pi\pi\|0>$	scalar	Moussallam EPJ C 14, 111 (2000); Daub, Hanhart, and B. Kubis JHEP 02 (2016) 009.
	vector	Hanhart, PL B715, 170 (2012); Dumm and Roig EPJ C 73, 2528 (2013).
$<K\pi\|0>$	scalar	Moussallam EPJ C 53, 401 (2008); Jamin, Oller and Pich, PRD 74, 074009 (2006)
	vector	Boito, Escribano, and Jamin EPJ C 59, 821 (2009).
$<KK\|0>$ (no data)	Fit from 3-body data	PCM, Robilotta + LHCb JHEP 1904 (2019) 063
	extrapolate from unitarity model	Albaladejo and Moussallam EPJ C 75, 488 (2015).
	quark model with isospin symmetry	Bruch,Khodjamirian, and Kühn , EPJ C 39, 41 (2005)

Models available A = W * FSI

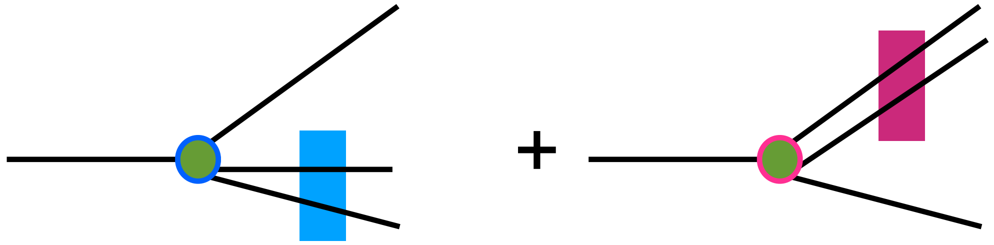
- QCD factorization approach → factorize the quark currents



Chau [Phys. Rep. 95,1(1983)]

- challenging for 3-body
- not all FSI and 3-body NR
- scale issue with charm

!



$$\mathcal{H}_{\rm eff}^{\Delta B=1} = \frac{G_F}{\sqrt{2}} \sum_{p=u,c} V^*_{pq} V_{pb} \Big[C_1(\mu) O_1^p(\mu) + C_2(\mu) O_2^p(\mu) + \sum_{i=3}^{10} C_i(\mu) O_i(\mu) + C_{7\gamma}(\mu) O_{7\gamma}(\mu) + C_{8g}(\mu) O_{8g}(\mu) \Big] + \text{h.c.} \,,$$

→ ex: $B^+ \to \pi^+\pi^-\pi^+$

how to describe it?

$$A \sim \underbrace{\langle[\pi^+(p_2)\pi^-(p_3)]}_{R}\ |(\bar{u}b)_{V-A}|B^-\rangle\ \langle\pi^-(p_1)|(\bar{d}u)_{V-A}|0\rangle + \langle\pi^-(p_1)|(\bar{d}b)_{sc-ps}|B^-\rangle\ \underbrace{\langle[\pi^+(p_2)\pi^-(p_3)]\ |(\bar{d}d)_{sc+ps}|0\rangle}_{FF}$$

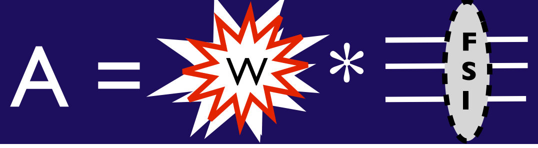
- naive factorization
  - intermediate by a resonance R;
  - FSI with scalar and vector form factors FF

↳ parametrizations for B and D→3h Boito et al. PRD96 113003 (2017)

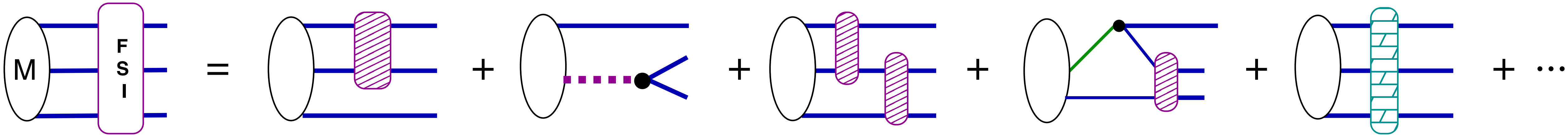
- modern QDC factorization: improvement to include “long distance” still developing

Klein, Mannel, Virto, Keri Vos JHEP10 117 (2017)

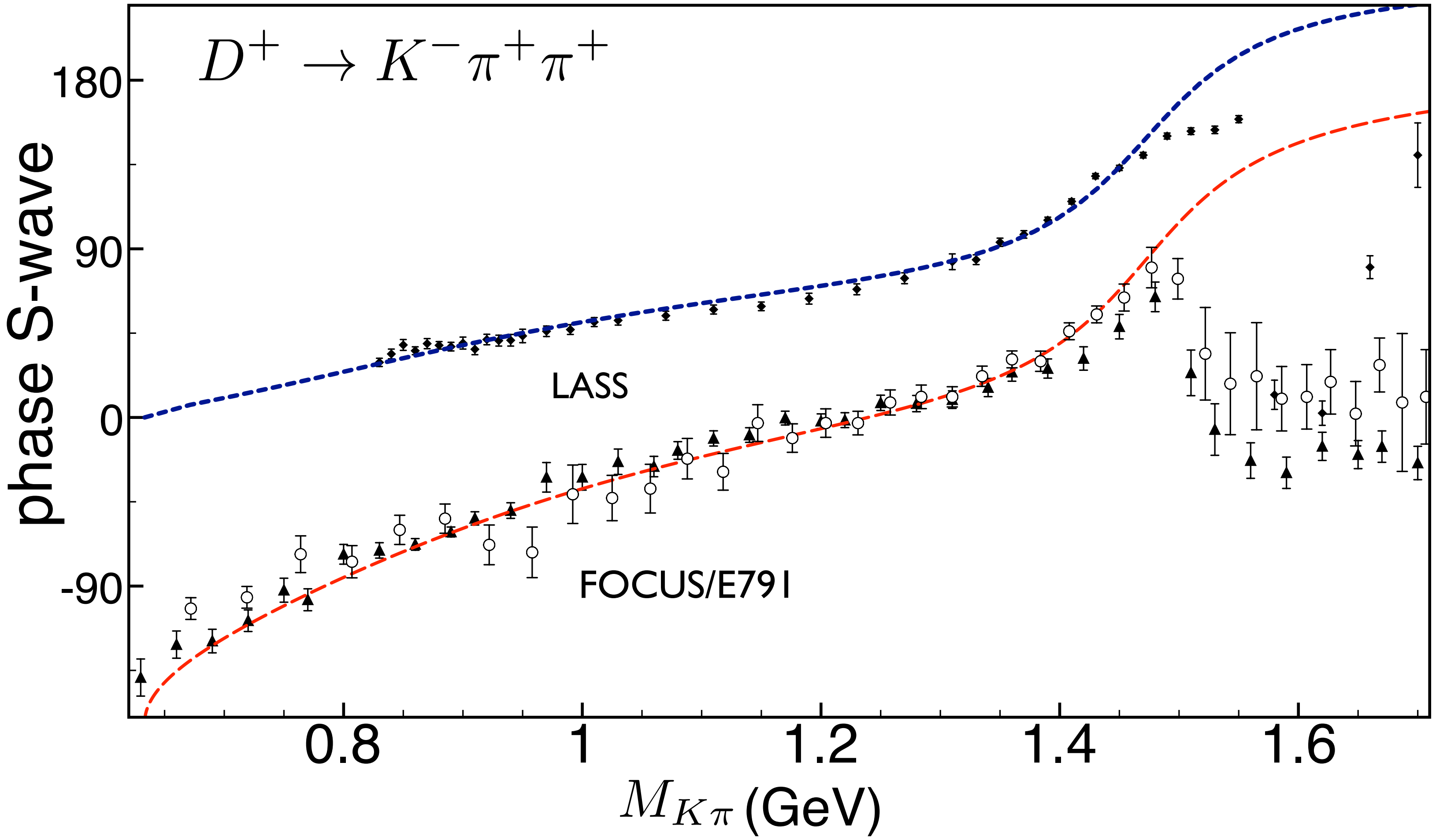
Models available

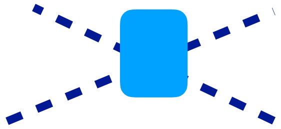


- Three-body FSI (beyond 2+1)



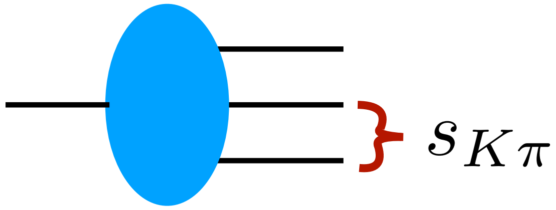
  - shown to be relevant on charm sector



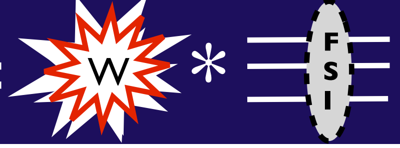


Scattering

Decay projected in one pair mass



Models available

A = 

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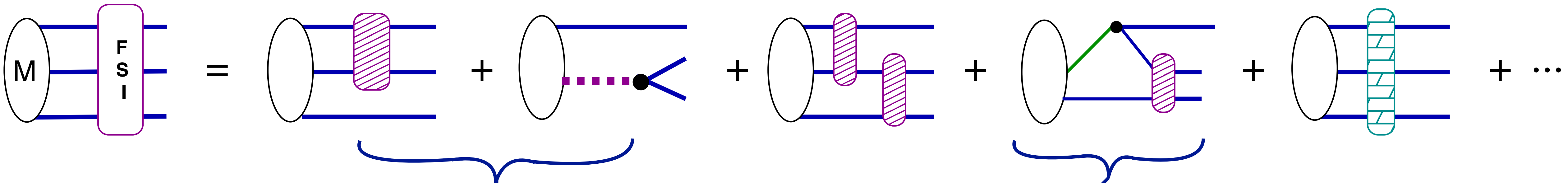
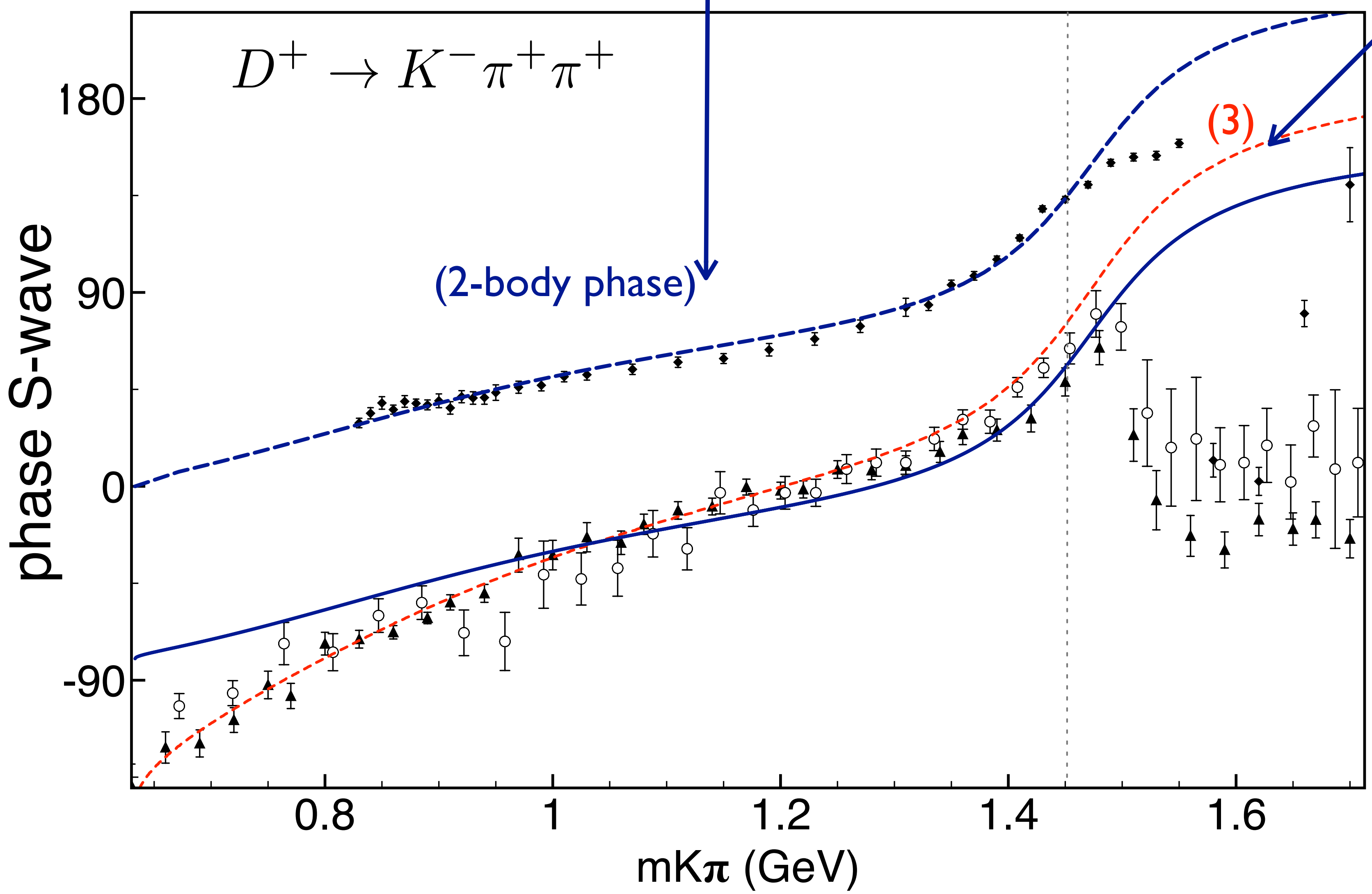


PRD92 094005 (2015)

- shown to be relevant on charm sector



- 3-body approaches

PCM et.al: PRD84 094001 (2011),
S.Nakamura PRD93 014005 (2016)
Niecknig, Kubis, JHEP10 142 (2015)

↳ 3-body FSI play a role

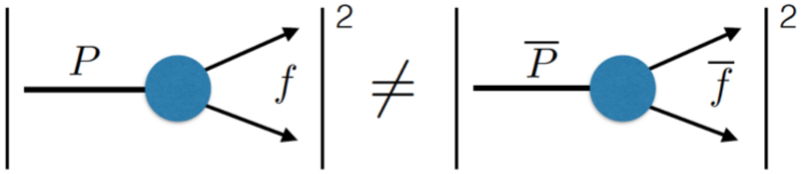
↳ data analysis…

Final State Interaction in B decays as a source of CP violation

CPV on data: Puzzle!

- Charge Parity Violation

$$\Gamma(M \to f) \neq \Gamma(\bar{M} \to \bar{f})$$



  - condition to CPV

    → $2 \neq$ amplitudes, SAME final state with strong (δ_i) and weak (ϕ_i) phase

$$\langle f \,|\, T \,|\, M \rangle = A_1\, e^{i(\delta_1+\phi_1)} + A_2\, e^{i(\delta_2+\phi_2)}$$

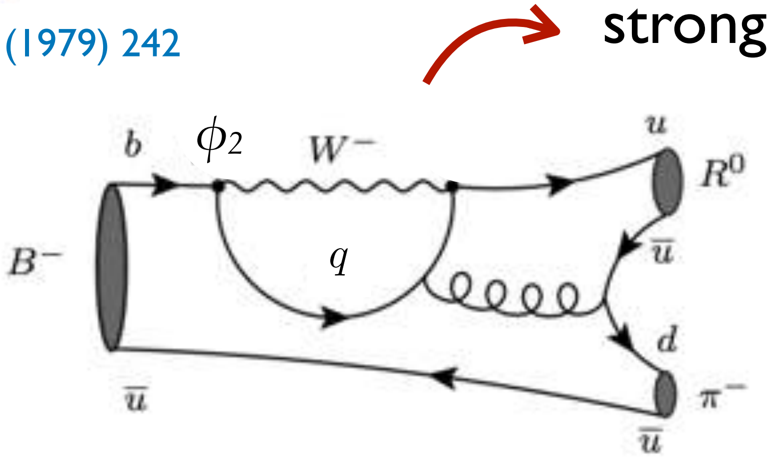
$$\Big\downarrow \text{CP}$$

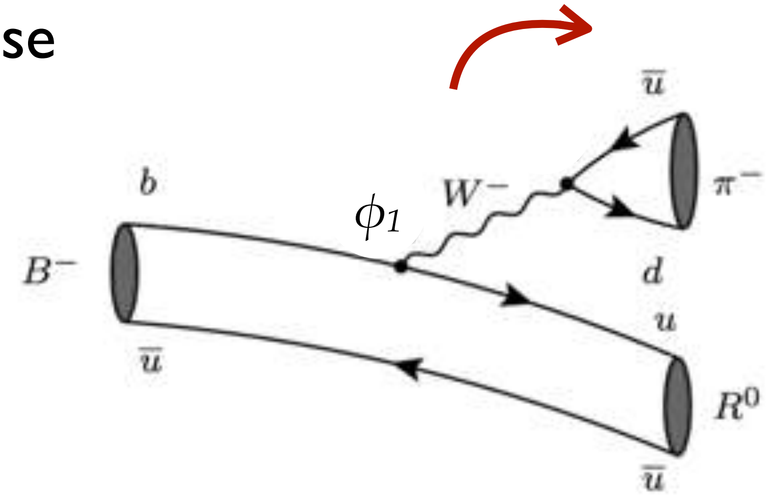
$$\langle \bar{f} \,|\, T \,|\, \bar{M} \rangle = A_1\, e^{i(\delta_1-\phi_1)} + A_2\, e^{i(\delta_2-\phi_2)}$$

$$\therefore\ \Gamma(M \to f) - \Gamma(\bar{M} \to \bar{f}) = |\langle f \,|\, T \,|\, M \rangle|^2 - |\langle \bar{f} \,|\, T \,|\, \bar{M} \rangle|^2 = -4A_1A_2 \sin(\delta_1 - \delta_2)\, \sin(\phi_1 - \phi_2)$$

- BSS model  + 

Bander Silverman & Soni PRL 43 (1979) 242

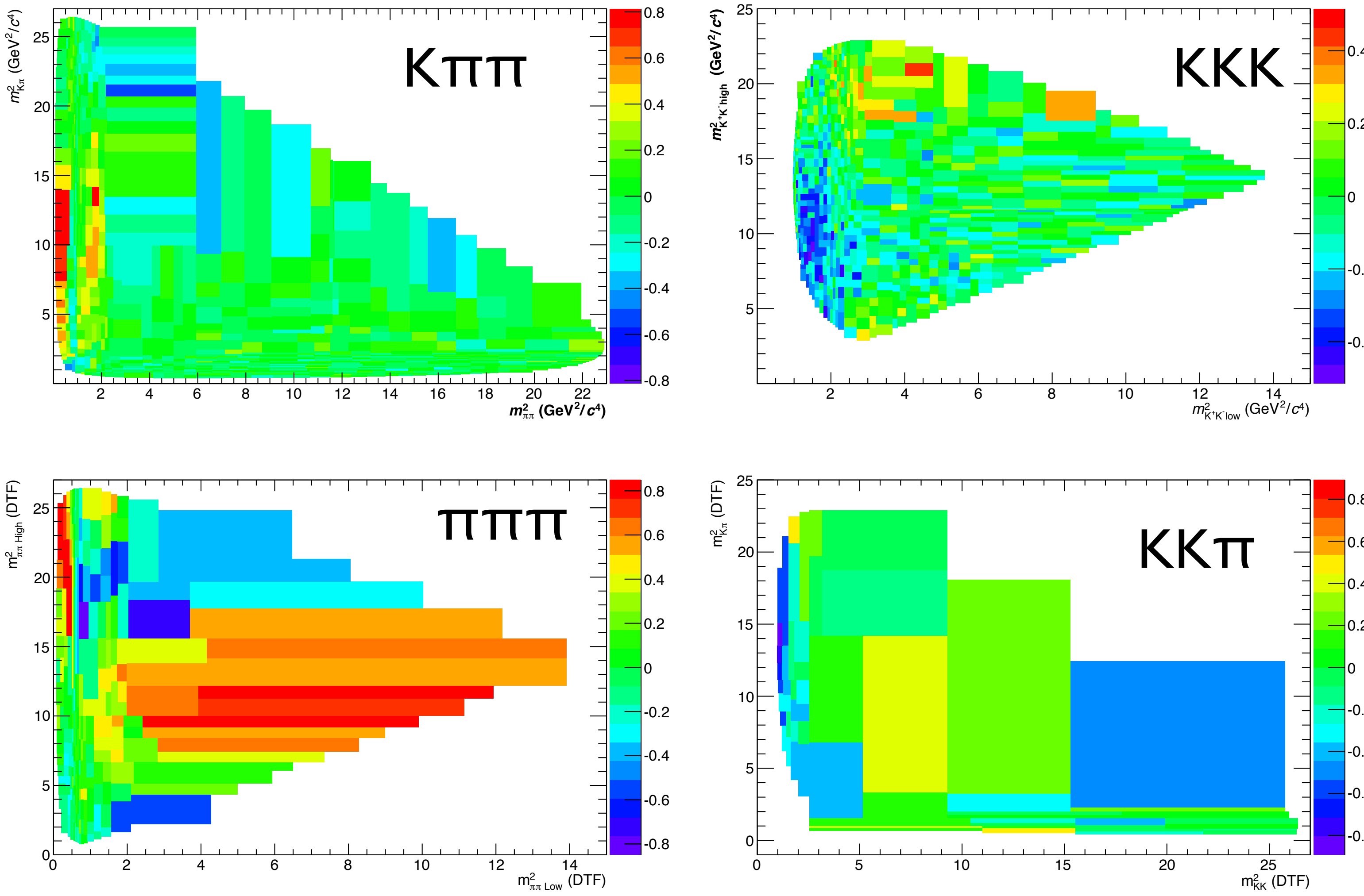

strong phase


weak phase: CKM
Vub

CPV on data: Puzzle!

- $B^{\pm} \rightarrow h^{\pm}h^{-}h^{+}$ massive localized Acp
  - suggest dynamic effect
  - middle looks “empty" → CPV

$$A_{CP} = \frac{\Gamma(M \rightarrow f) - \Gamma(\bar{M} \rightarrow \bar{f})}{\Gamma(M \rightarrow f) + \Gamma(\bar{M} \rightarrow \bar{f})}$$



LHCb PRD90 (2014) 112004

- BSS model  + 

not enough!!

- hadronic interactions → strong phase

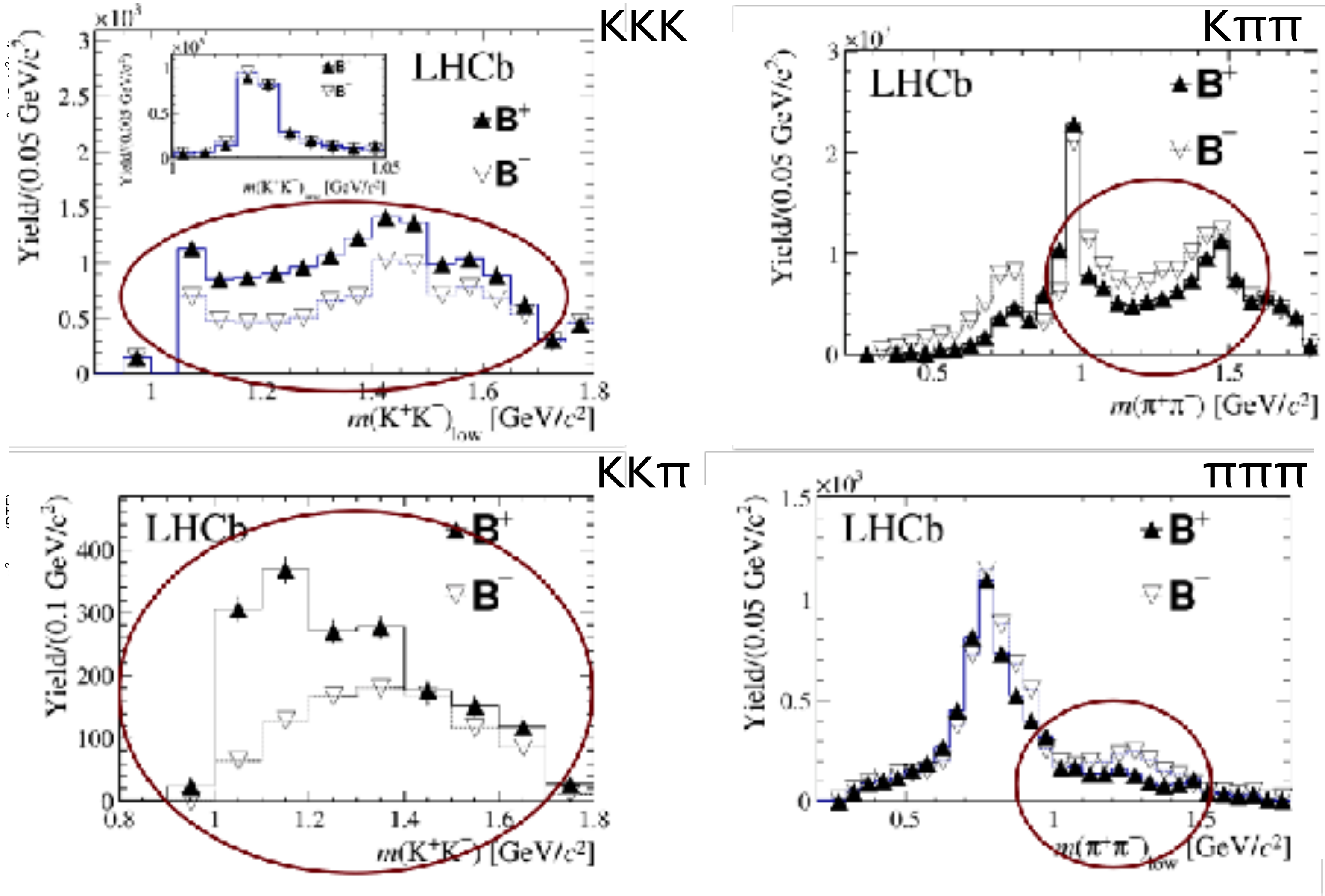
- FSI → strong phase

Wolfenstein PRD43 (1991) 151

CPV on data

- $B^{\pm} \to h^{\pm}\pi^{-}\pi^{+}$ and $B^{\pm} \to h^{\pm}K^{-}K^{+}$

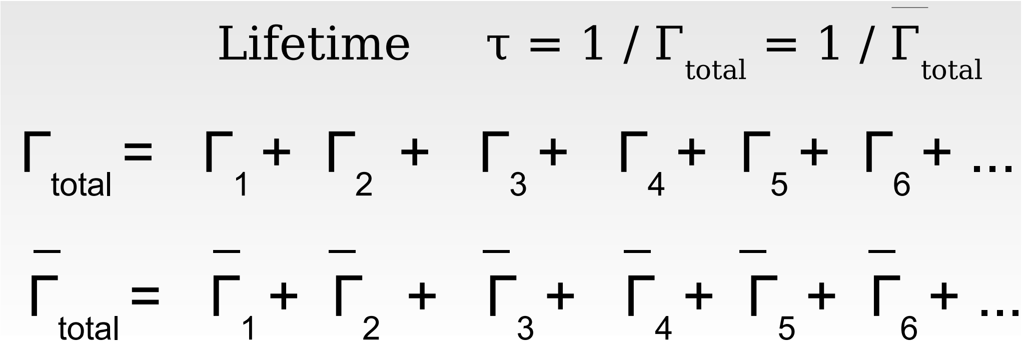
$$\rightarrow \boxed{\pi\pi \to KK}$$



- low-energy CPV
  with opposite signs

Frederico PRD89(2014)094013

- CPT:



Lifetime $\tau = 1 / \Gamma_{\text{total}} = 1 / \overline{\Gamma}_{\text{total}}$

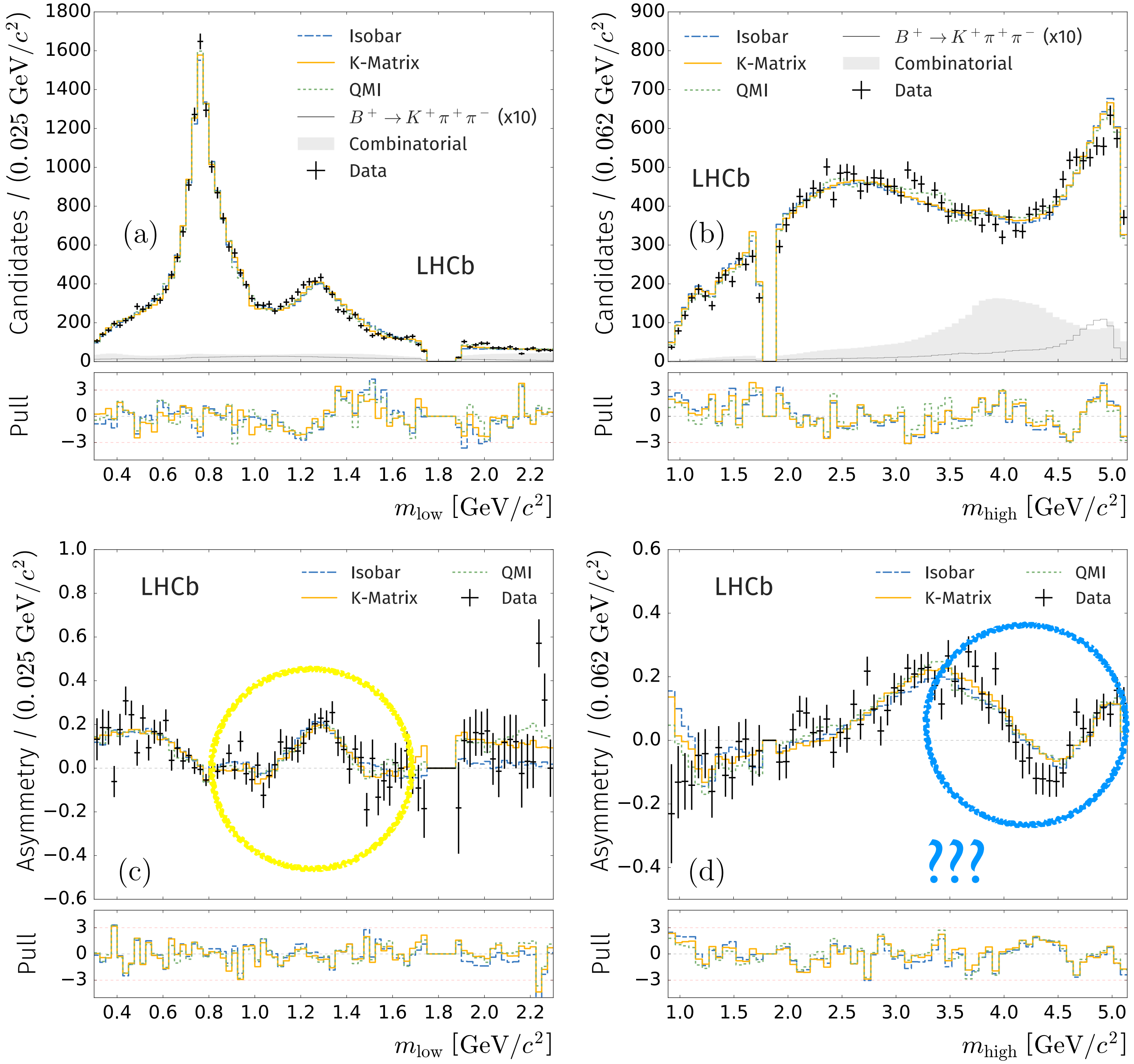
$$\Gamma_{\text{total}} = \Gamma_1 + \Gamma_2 + \Gamma_3 + \Gamma_4 + \Gamma_5 + \Gamma_6 + \dots$$

$$\overline{\Gamma}_{\text{total}} = \overline{\Gamma}_1 + \overline{\Gamma}_2 + \overline{\Gamma}_3 + \overline{\Gamma}_4 + \overline{\Gamma}_5 + \overline{\Gamma}_6 + \dots$$

→ CPV in one channel should be compensated by another one with opposite sign

CPV: amplitude analysis $B^{\pm} \to \pi^{-}\pi^{+}\pi^{\pm}$

- LHCb recent Amplitude analysis $B^{\pm} \to \pi^{-}\pi^{+}\pi^{\pm}$ [arXiv:1909.05212(PRD); 1909.05211(PRL)]
    - $(\pi^{-}\pi^{+})_{S-Wave}$ 3 different model:
        - ↪ σ as BW (!) + rescattering;
        - ↪ P-vector K-Matrix;
        - ↪ binned freed lineshape (QMI);



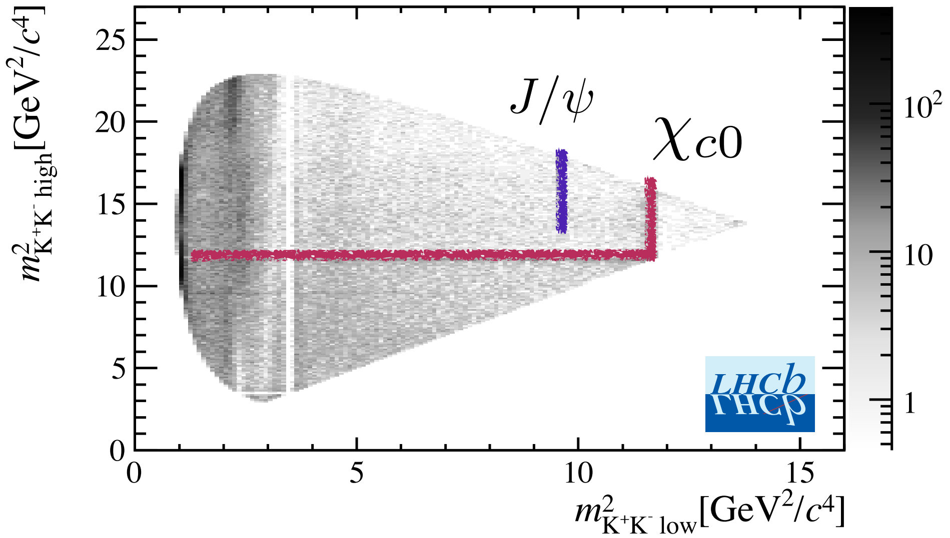
Contribution	Fit fraction (10^{-2})	A_{CP} (10^{-2})	B^+ phase (°)	B^- phase (°)
Isobar model				
$\rho(770)^0$	$55.5 \pm 0.6 \pm 2.5$	$+0.7 \pm 1.1 \pm 1.6$	—	—
$\omega(782)$	$0.50 \pm 0.03 \pm 0.05$	$-4.8 \pm 6.5 \pm 3.8$	$-19 \pm 6 \pm 1$	$+8 \pm 6 \pm 1$
$f_2(1270)$	$9.0 \pm 0.3 \pm 1.5$	$+46.8 \pm 6.1 \pm 4.7$	$+5 \pm 3 \pm 12$	$+53 \pm 2 \pm 12$
$\rho(1450)^0$	$5.2 \pm 0.3 \pm 1.9$	$-12.9 \pm 3.3 \pm 35.9$	$+127 \pm 4 \pm 21$	$+154 \pm 4 \pm 6$
$\rho_3(1690)^0$	$0.5 \pm 0.1 \pm 0.3$	$-80.1 \pm 11.4 \pm 25.3$	$-26 \pm 7 \pm 14$	$-47 \pm 18 \pm 25$
S-wave	$25.4 \pm 0.5 \pm 3.6$	$+14.4 \pm 1.8 \pm 2.1$	—	—
Rescattering	$1.4 \pm 0.1 \pm 0.5$	$+44.7 \pm 8.6 \pm 17.3$	$-35 \pm 6 \pm 10$	$-4 \pm 4 \pm 25$
σ	$25.2 \pm 0.5 \pm 5.0$	$+16.0 \pm 1.7 \pm 2.2$	$+115 \pm 2 \pm 14$	$+179 \pm 1 \pm 95$
K-matrix				
$\rho(770)^0$	$56.5 \pm 0.7 \pm 3.4$	$+4.2 \pm 1.5 \pm 6.4$	—	—
$\omega(782)$	$0.47 \pm 0.04 \pm 0.03$	$-6.2 \pm 8.4 \pm 9.8$	$-15 \pm 6 \pm 4$	$+8 \pm 7 \pm 4$
$f_2(1270)$	$9.3 \pm 0.4 \pm 2.5$	$+42.8 \pm 4.1 \pm 9.1$	$+19 \pm 4 \pm 18$	$+80 \pm 3 \pm 17$
$\rho(1450)^0$	$10.5 \pm 0.7 \pm 4.6$	$+9.0 \pm 6.0 \pm 47.0$	$+155 \pm 5 \pm 29$	$-166 \pm 4 \pm 51$
$\rho_3(1690)^0$	$1.5 \pm 0.1 \pm 0.4$	$-35.7 \pm 10.8 \pm 36.9$	$+19 \pm 8 \pm 34$	$+5 \pm 8 \pm 46$
S-wave	$25.7 \pm 0.6 \pm 3.0$	$+15.8 \pm 2.6 \pm 7.2$	—	—
QMI				
$\rho(770)^0$	$54.8 \pm 1.0 \pm 2.2$	$+4.4 \pm 1.7 \pm 2.8$	—	—
$\omega(782)$	$0.57 \pm 0.10 \pm 0.17$	$-7.9 \pm 16.5 \pm 15.8$	$-25 \pm 6 \pm 27$	$-2 \pm 7 \pm 11$
$f_2(1270)$	$9.6 \pm 0.4 \pm 4.0$	$+37.6 \pm 4.4 \pm 8.0$	$+13 \pm 5 \pm 21$	$+68 \pm 3 \pm 66$
$\rho(1450)^0$	$7.4 \pm 0.5 \pm 4.0$	$-15.5 \pm 7.3 \pm 35.2$	$+147 \pm 7 \pm 152$	$-175 \pm 5 \pm 171$
$\rho_3(1690)^0$	$1.0 \pm 0.1 \pm 0.5$	$-93.2 \pm 6.8 \pm 38.9$	$+8 \pm 10 \pm 24$	$+36 \pm 26 \pm 46$
S-wave	$26.8 \pm 0.7 \pm 2.2$	$+15.0 \pm 2.7 \pm 8.1$	—	—

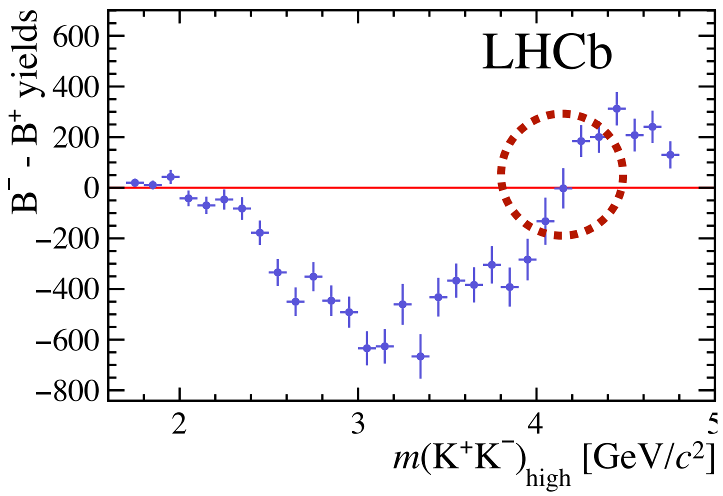
- ANA for $B^{\pm} \to \pi^{\pm}K^{-}K^{+}$ [arXiv:1905.09244]

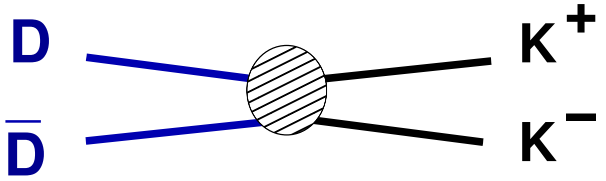
Contribution	Fit Fraction(%)	A_{CP}(%)	Magnitude (B^+/B^-)	Phase[^{o}] (B^+/B^-)
$K^*(892)^0$	$7.5 \pm 0.6 \pm 0.5$	$+12.3 \pm 8.7 \pm 4.5$	$0.94 \pm 0.04 \pm 0.02$ $1.06 \pm 0.04 \pm 0.02$	0 (fixed) 0 (fixed)
$K_0^*(1430)^0$	$4.5 \pm 0.7 \pm 1.2$	$+10.4 \pm 14.9 \pm 8.8$	$0.74 \pm 0.09 \pm 0.09$ $0.82 \pm 0.09 \pm 0.10$	$-176 \pm 10 \pm 16$ $136 \pm 11 \pm 21$
Single pole	$32.3 \pm 1.5 \pm 4.1$	$-10.7 \pm 5.3 \pm 3.5$	$2.19 \pm 0.13 \pm 0.17$ $1.97 \pm 0.12 \pm 0.20$	$-138 \pm 7 \pm 5$ $166 \pm 6 \pm 5$
$\rho(1450)^0$	$30.7 \pm 1.2 \pm 0.9$	$-10.9 \pm 4.4 \pm 2.4$	$2.14 \pm 0.11 \pm 0.07$ $1.92 \pm 0.10 \pm 0.07$	$-175 \pm 10 \pm 15$ $140 \pm 13 \pm 20$
$f_2(1270)$	$7.5 \pm 0.8 \pm 0.7$	$+26.7 \pm 10.2 \pm 4.8$	$0.86 \pm 0.09 \pm 0.07$ $1.13 \pm 0.08 \pm 0.05$	$-106 \pm 11 \pm 10$ $-128 \pm 11 \pm 14$
Rescattering	$16.4 \pm 0.8 \pm 1.0$	$-66.4 \pm 3.8 \pm 1.9$	$1.91 \pm 0.09 \pm 0.06$ $0.86 \pm 0.07 \pm 0.04$	$-56 \pm 12 \pm 18$ $-81 \pm 14 \pm 15$
$\phi(1020)$	$0.3 \pm 0.1 \pm 0.1$	$+9.8 \pm 43.6 \pm 26.6$	$0.20 \pm 0.07 \pm 0.02$ $0.22 \pm 0.06 \pm 0.04$	$-52 \pm 23 \pm 32$ $107 \pm 33 \pm 41$

CPV high energy

- $B^+ \rightarrow K^- K^+ K^+$
  - $\mathcal{A}_{cp}$ change sign ~ $D\bar{D}$ open channel

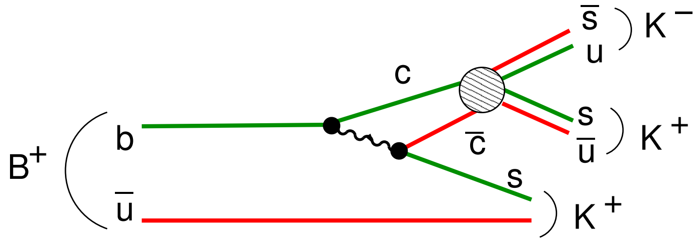


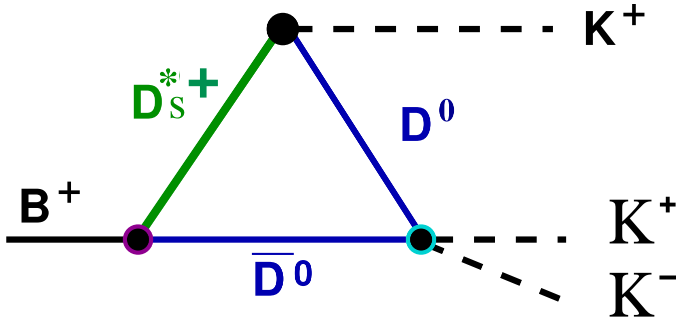




- charm intermediate processes as source of strong phase

I. Bediaga, PCM, T Frederico
PLB 780 (2018) 357





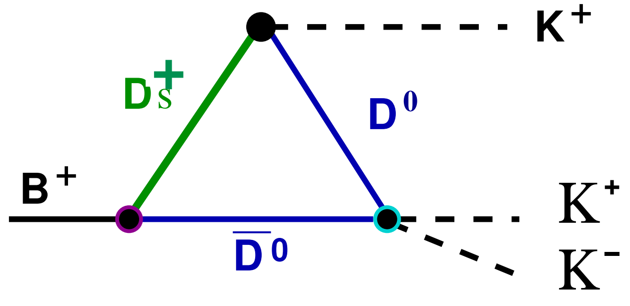
- even dynamically suppressed $Br\left[B \rightarrow D D_s^*\right]$ ~1% → 1000 x $Br\left[B \rightarrow KKK\right]$

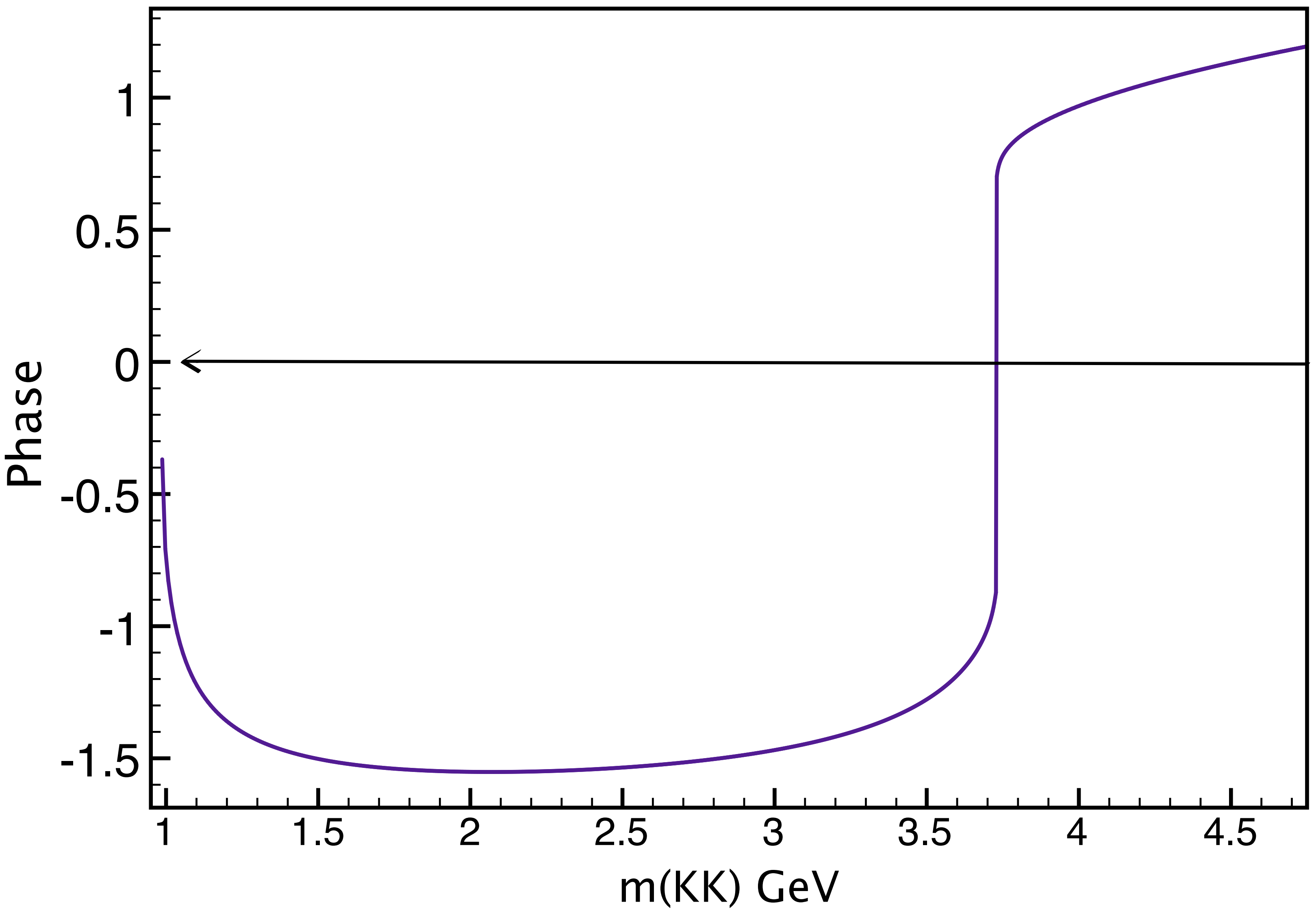
- hadronic loop technique $D^+ \rightarrow \pi^+ K^- \pi^+$

PCM & M Robilotta PRD 92 094005 (2015)
PCM et al PRD 84 094001 (2011)

hadronic loop results for $B^{\pm} \to K^{\pm} K^{-} K^{+}$

- Triangle hadronic loop with charm rescattering can generate a phase that change signal near DD threshold





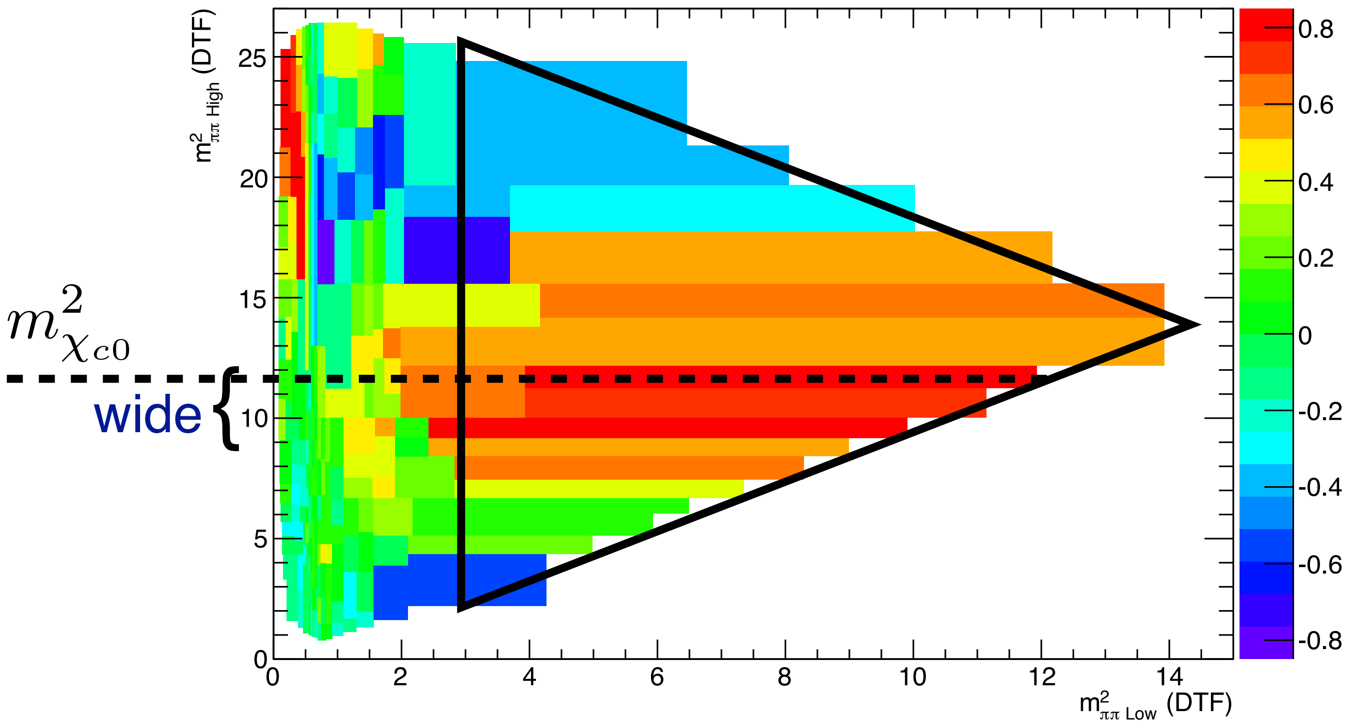
- how this can be translated to the observable CPV?

we need inference with weak-phase!

charm rescattering in $B^{\pm} \to \pi^{\pm}\pi^{-}\pi^{+}$

Bediaga, Frederico, PCM - PLBX (2020)[arXiv:2003.10019]

- high mass CPV study in $B^{\pm} \to \pi^{\pm}\pi^{-}\pi^{+}$
  - Focus on $m^2_{\pi\pi} > 3\, GeV^2$
    ↳ avoid low energy resonances
  - include χ_{c0} (expected in Run II)



- Important data features
  - data shows a huge CP asymmetry around $m^2_{\chi c0} = 11.65\, GeV^2$
  - wide CP asymmetry: same source for a nonresonant amplitude and χ_{c0}
    ↳ charm loop and χ_{c0}

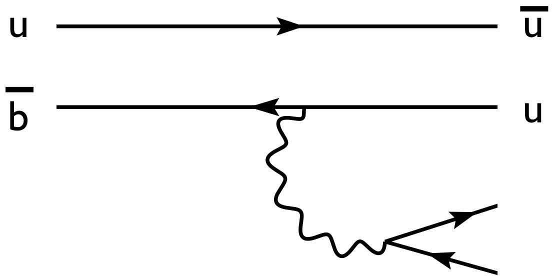
Amplitude model

NEW!

- Amplitude Model for $B^{\pm} \to \pi^{\pm}\pi^{-}\pi^{+}$ high mass $m^2_{\pi\pi} > 3\,GeV^2$

$$A_{B^{\pm}\to\pi^-\pi^+\pi^{\pm}}(s_{12}, s_{23}) = A^{\pm}_{tree}(s_{12}, s_{23}) + A_{D\bar{D}}(s_{12}, s_{23})$$

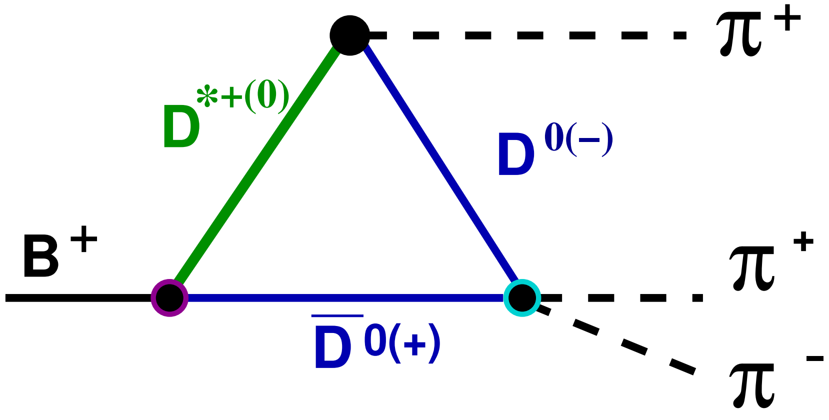
- $A^{\pm}_{tree} = a_0\, e^{\pm i\gamma}$: weak phase γ from the dominant $b \to u$ tree diagram



→ Nonresonant (only resonances tails)

→ a_0 is complex (strong phase)

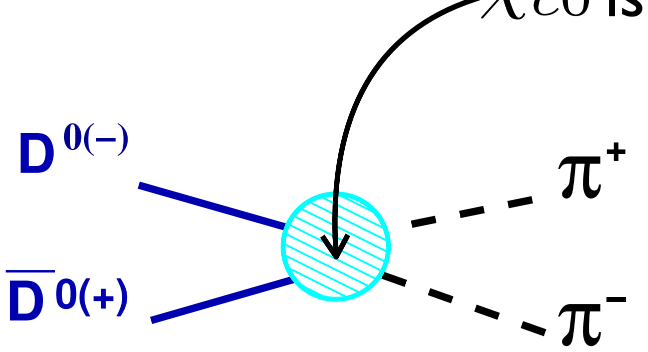
- $A_{D\bar{D}}$ charm rescattering with χ_{c0} : source of strong phase variation



→ similar triangle loop

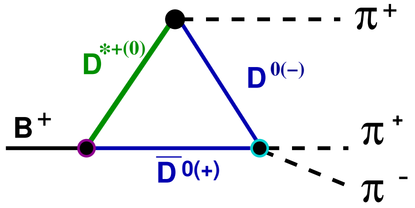
→

χ_{c0} is a pole bellow threshold



Results

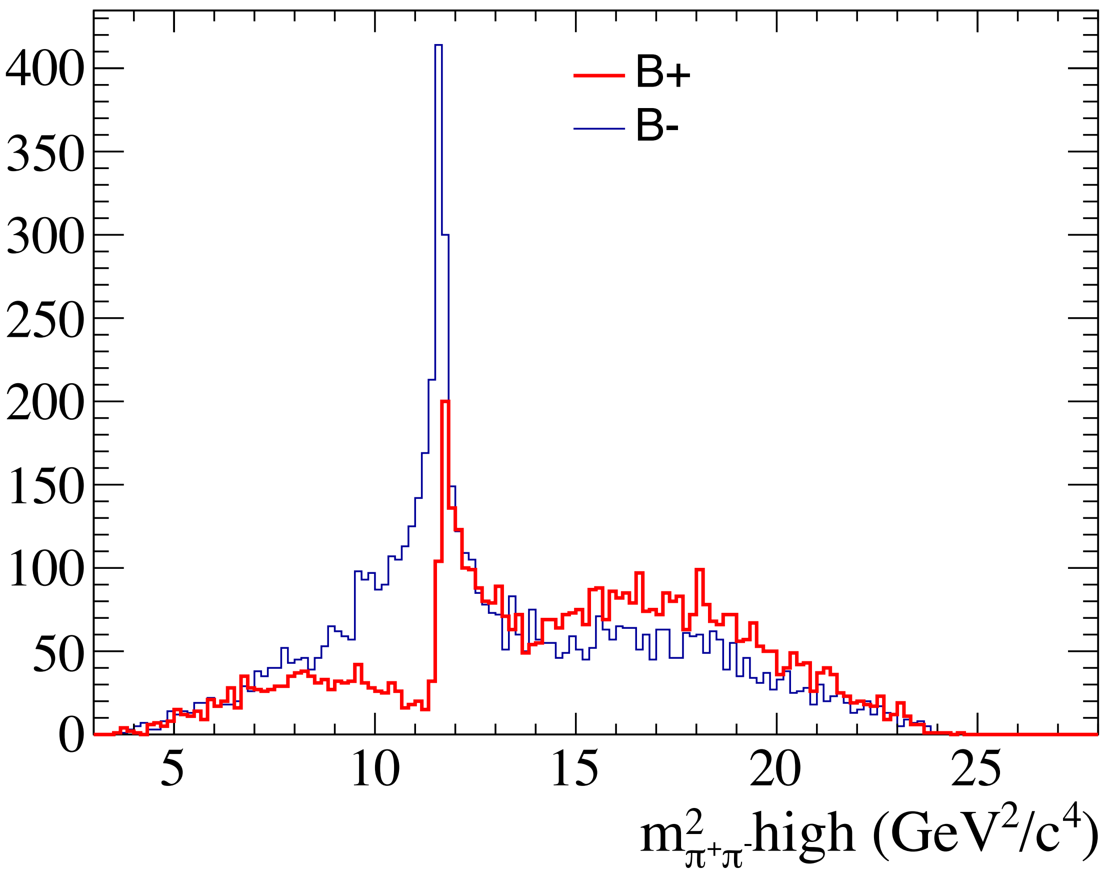
- $A_{B^\pm\to\pi^-\pi^+\pi^\pm}(s_{12},s_{23}) =$

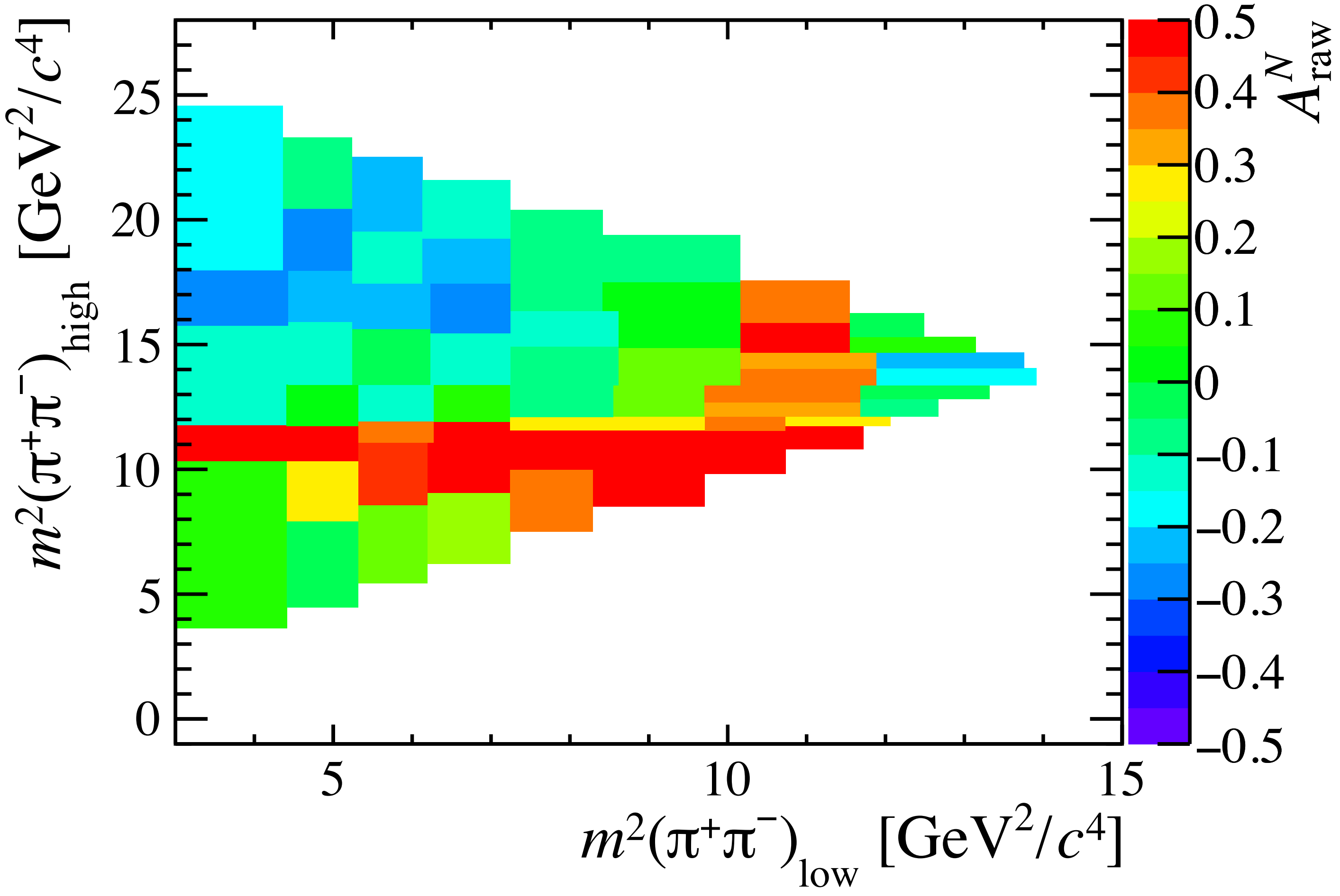


$+ \quad a_0\, e^{\pm i\gamma}$

$$\boxed{\begin{aligned}\gamma &= 70^o\\ a_0 &= 2\, e^{i(\delta_s=45^o)}\end{aligned}}$$

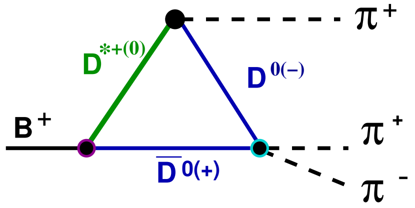
- the goal was to reproduce the main observed CPV characteristics


Amplitude projection


Acp signature

Results

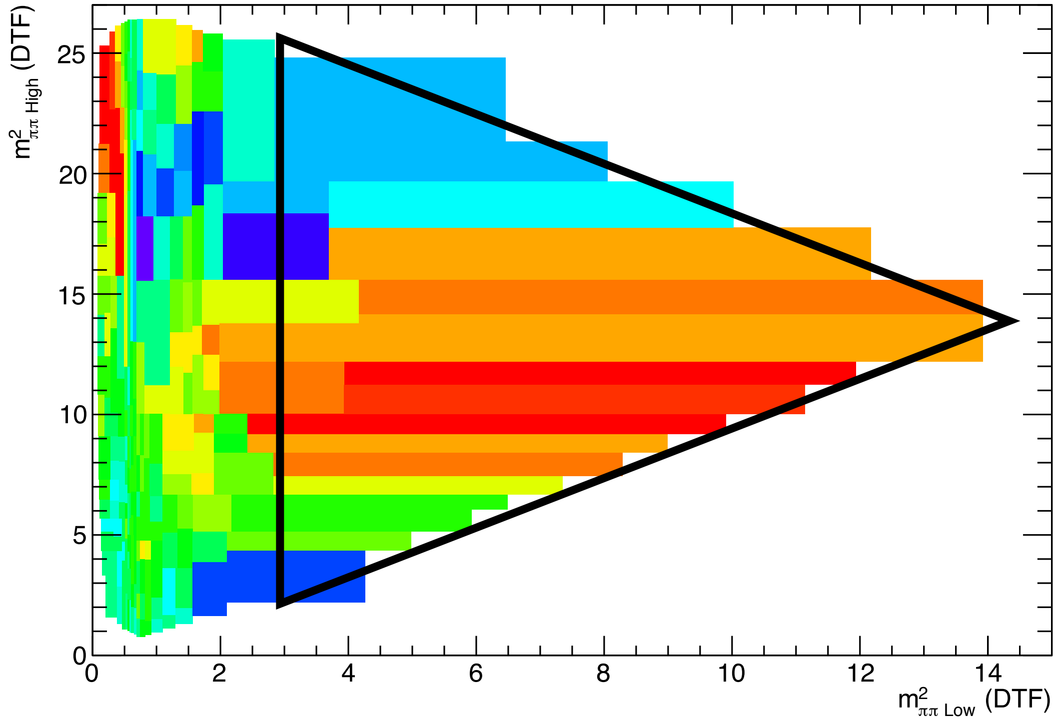
- $A_{B^\pm\to\pi^-\pi^+\pi^\pm}(s_{12},s_{23}) =$



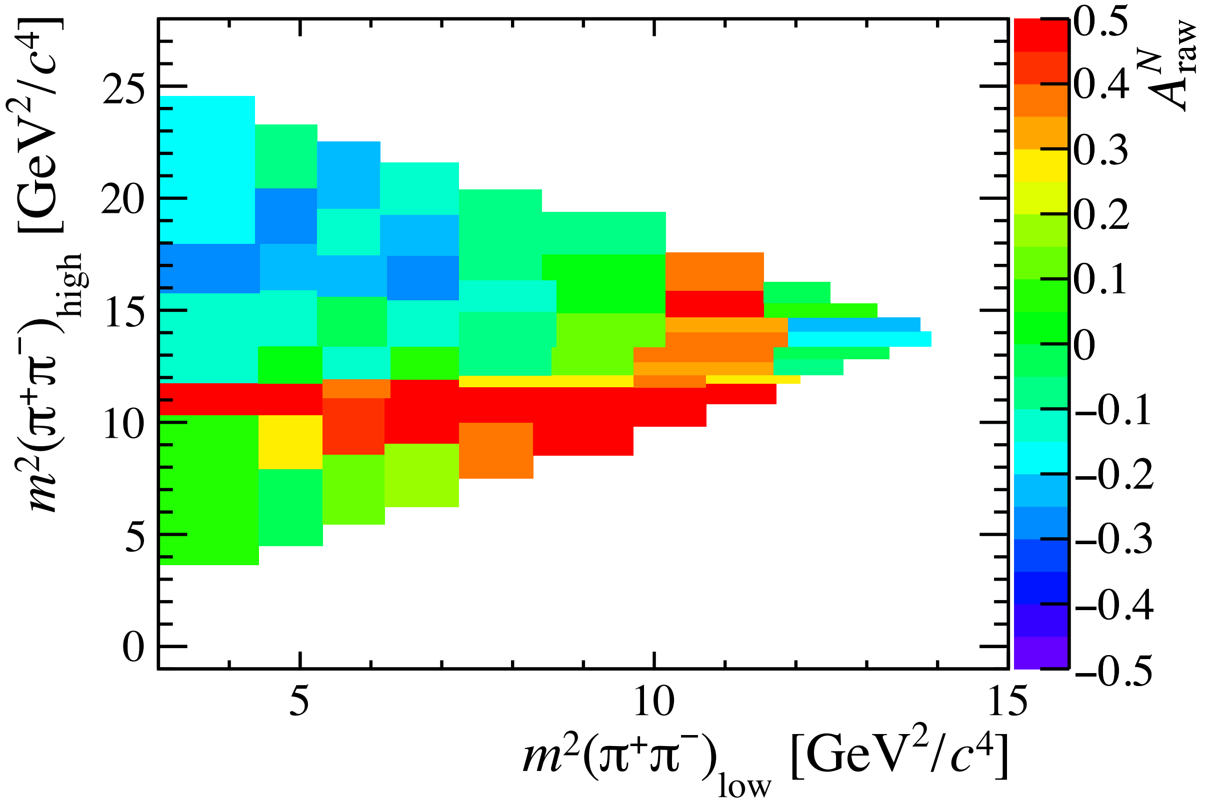
$+ \quad a_0\, e^{\pm i\gamma}$

$$\boxed{\begin{aligned}\gamma &= 70^o\\ a_0 &= 2\, e^{(\delta_s=45^o)}\end{aligned}}$$

- Run I



- our model



→ not the same binning and scale

→ mimic some of the CPV pattern at high mass

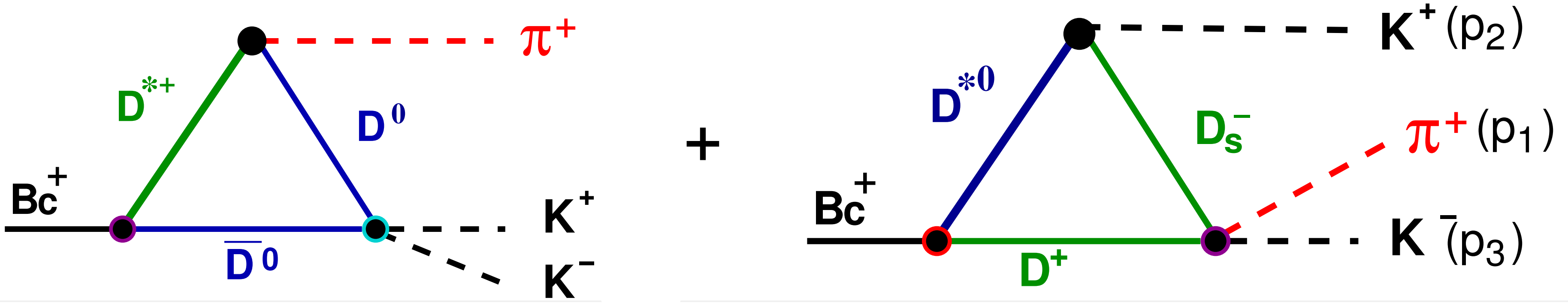
→ **should be included in amplitude analysis of Run II data to confirm or disprove**

final remarks

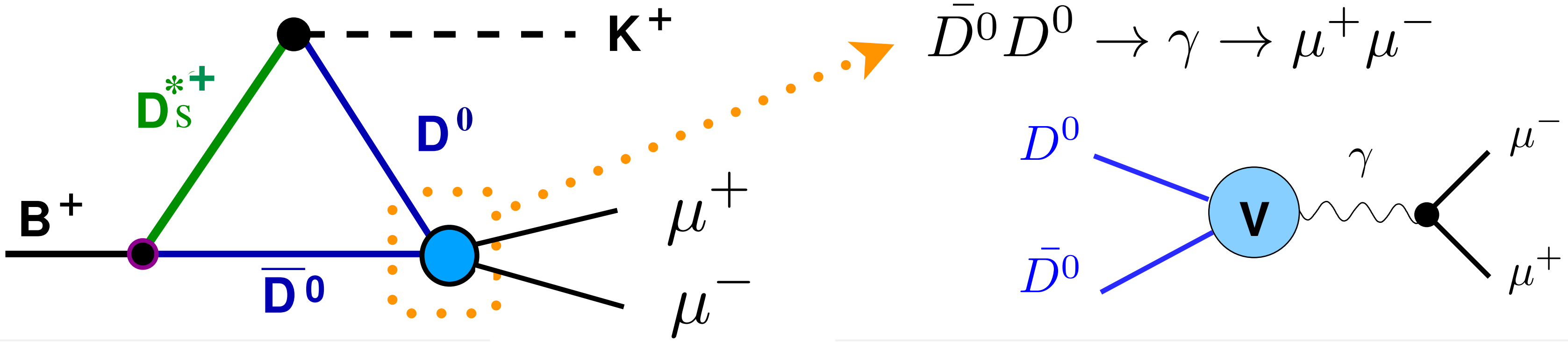
- interference between charm loop and nonresonant can give a similar CPV signature observed in data
- Charm rescattering triangles is an important FSI mechanism

charm rescattering to $B_c^+ \to K^- K^+ \pi^+$

I. Bediaga, PCM, T Frederico
PLB 785 (2018) 581



Hadronic effect in $B \to K\mu\mu$



$\bar{D}^0 D^0 \to \gamma \to \mu^+\mu^-$

final remarks

FSI are important and play a major role in hadronic 3-body decays!

→ superposition of resonant and non-resonant at low and high energy

- Lots of theoretical limitations to be developed:
  - need to merge the short and long distance descriptions!
  - extend the meson-meson interaction to high E, …
- Successful examples of cooperation between theory and experiment !!!

→ Important tool !

Thank you!

obrigada!!

FSI in three-body decay :

references

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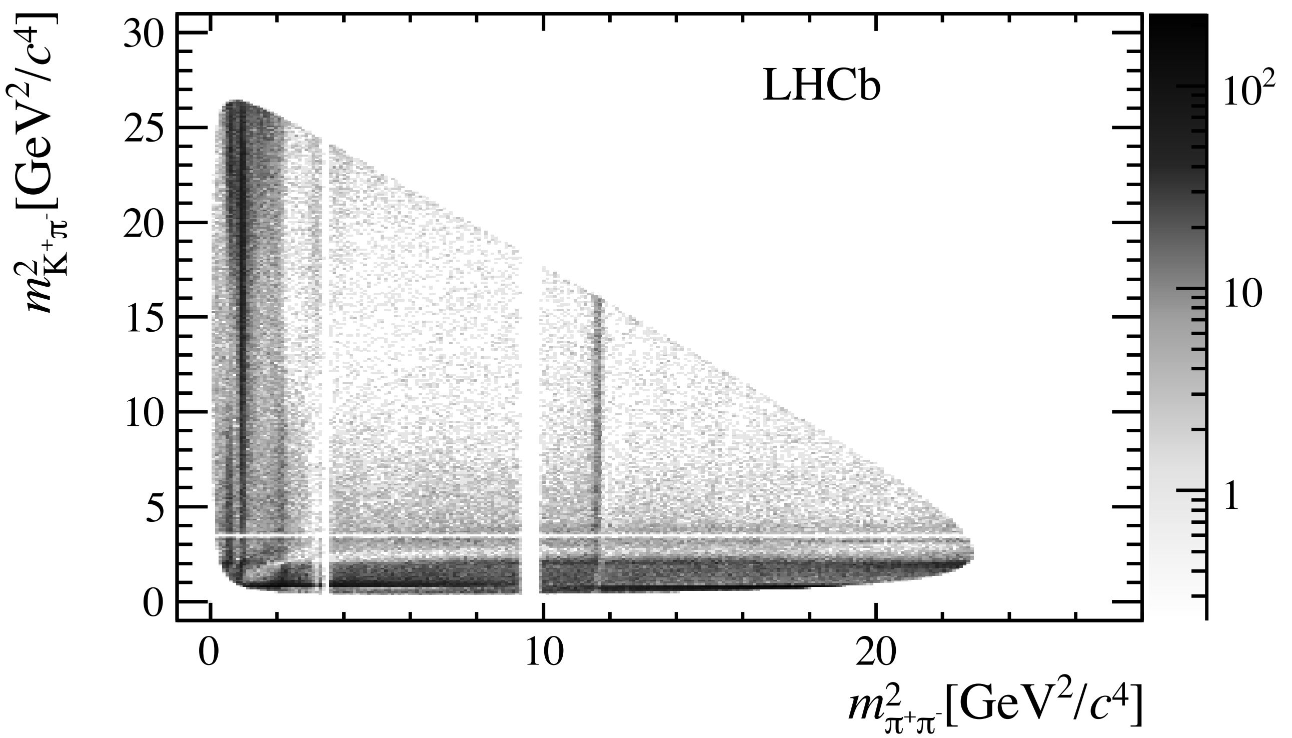
many more ...

if needed

Kpp



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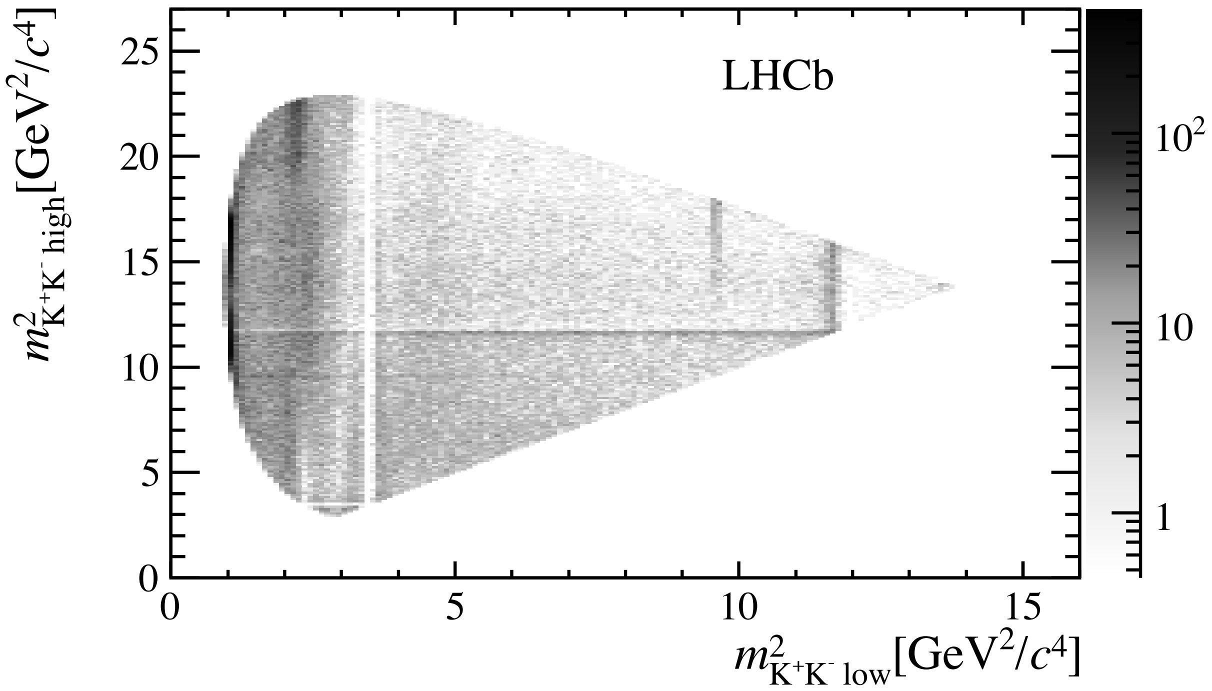

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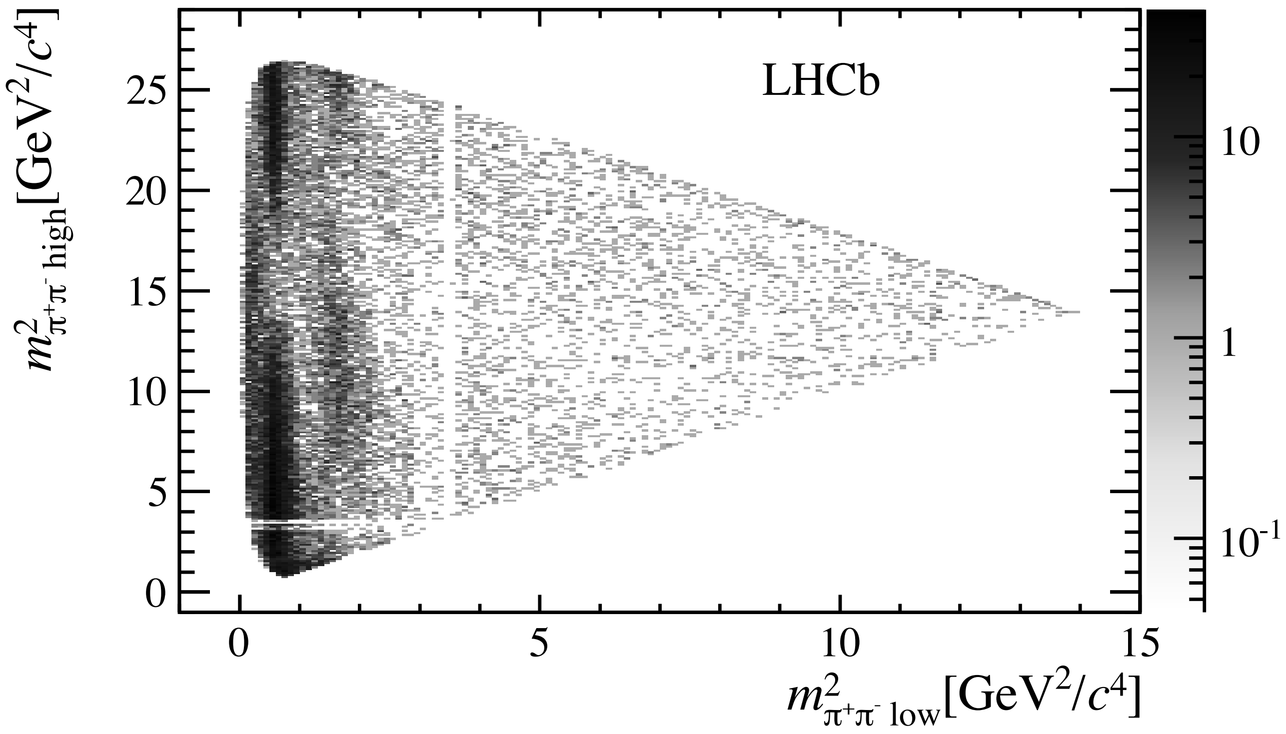

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