

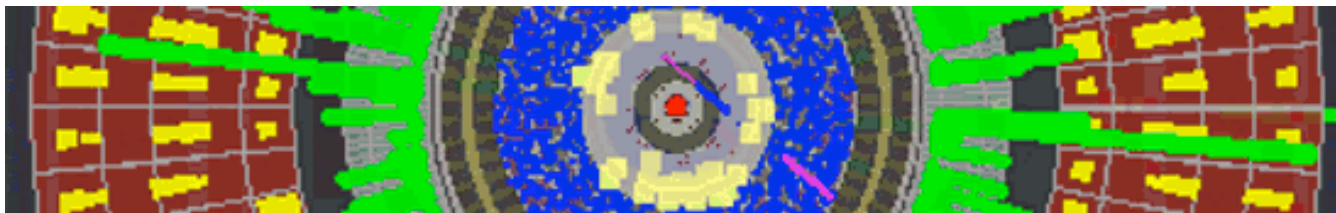
Neutrinos and the Universe

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College
LONDON

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**London Centre
for Terauniverse
Studies (LCTS)
AdV 267352**



**Seminar,
February 20 2013**

OUTLINE

Neutrinos & Baryon Asymmetry in Universe

Neutrinos & the Dark Sector of Universe:

CPT Violation in the Early Universe & Neutrinos

Conclusions - Outlook

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Role of (heavy) Majorana
Right-handed Neutrinos
In Leptogenesis/Baryogenesis

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anomalous generation of neutrino mass
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CPT Violation in Early Universe
Geometries & particle/antiparticle
asymmetries already
in thermal equilibrium

Conclusions - Outlook

PART I
NEUTRINOS,
BARYOGENESIS
LEPTOGENESIS

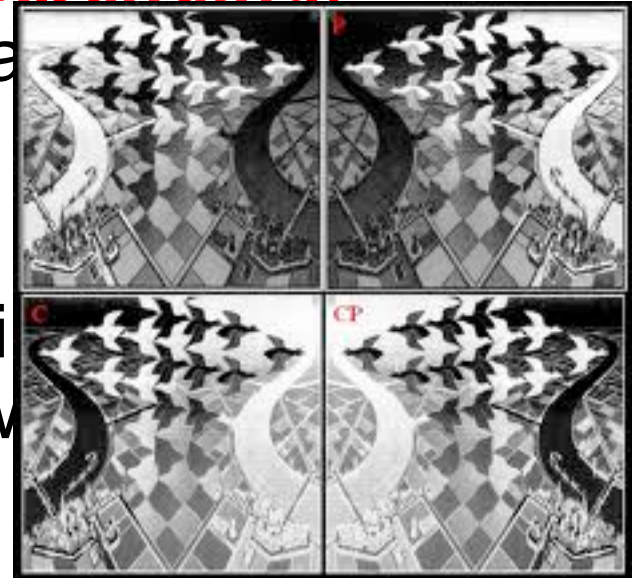
Generic Concepts

- ***Leptogenesis***: physical ***out of thermal equilibrium*** processes in the (***expanding***) Early Universe that produce an asymmetry between leptons & antileptons
- ***Baryogenesis***: The corresponding processes that produce an asymmetry between baryons and antibaryons
- ***Ultimate question: why is the Universe made only of matter?***

Generic Concepts

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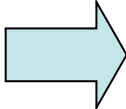
- **Baryogenesis**: The corresponding processes that produce an asymmetry between baryons and antibaryons



escher

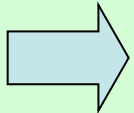
- **Ultimate question: why is the Universe made only of matter?**

NEUTRINOS & LEPTOGENESIS

- Matter-Antimatter asymmetry in the Universe  Violation of Baryon # (B), C & CP
- Tiny CP violation ($O(10^{-3})$) in Labs: e.g. $K^0 \bar{K}^0$
- But Universe consists only of matter

$$\frac{n_B - \bar{n}_B}{n_B + \bar{n}_B} \sim \frac{n_B - \bar{n}_B}{s} = (8.4 - 8.9) \times 10^{-11} \quad T > 1 \text{ GeV}$$

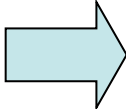
Sakharov : Non-equilibrium physics of early Universe, **B, C, CP violation**



$$n_B - \bar{n}_B$$

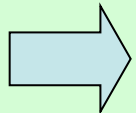
but **not quantitatively in SM**, still a mystery

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Assume CPT

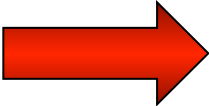
ELECTROWEAK THEORY & FERMION # NON-CONSERVATION

Classical conservations of EW theory: B, L_e, L_μ, L_τ

Quantum Anomalies in Standard Model (SM):

$$\partial_\mu J_\mu^B = \partial_\mu J_\mu^L = \frac{n_f}{32\pi^2} \text{Tr} F_{\mu\nu} \tilde{F}_{\mu\nu} + U(1) \text{ part}$$

Allowed Processes (change of B by multiples of 3)

bosons \leftrightarrow bosons + $9q + 3l$  $L_i - B/3$ Conserved
(three quantities)

BUT:
**OBSERVED NEUTRINO
FLAVOUR OSCILLATIONS**



L-B conserved (one quantity)
L = total Lepton #

If neutrinos Majorana



L violated, No conserved numbers

OBSERVED CP VIOLATION UNLIKELY TO EXPLAIN BARYON ASYMMETRY IN THE UNIVERSE

In SM: bosons \leftrightarrow bosons + 9q + 3l

Kuzmin, Rubakov, Shaposhnikov

Rate of B violation in Early Universe

$$\Gamma \sim \begin{cases} (\alpha_W T)^4 \left(\frac{M_{\text{sph}}}{T}\right)^7 \exp\left(-\frac{M_{\text{sph}}}{T}\right), & T \lesssim M_{\text{sph}}, \\ \alpha_W (\alpha_W T)^4 \log(1/\alpha_W), & T \gtrsim M_{\text{sph}}, \end{cases}$$

$\alpha_W = \text{SU}(2)$ fine structure ``constant''

Sphaleron Mass Scale
 $(M_W/\alpha_W) = \text{height of energy}$
Barrier separating SU(2) vacua
with different topologies

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Thermal Equilibrium (i.e. $\Gamma > H$ (Hubble)) for B non conserv. occurs only for:

$$T_{\text{sph}}(m_H) < T < (\alpha_W)^5 M_{Pl} \sim 10^{12} \text{ GeV}$$

$$T_{\text{sph}}(m_H) \in [130, 190] \text{ GeV}$$

$$m_H \in [100, 300] \text{ GeV}$$

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BAU could be produced this way only when sphaleron interactions freeze out, i.e.

$$T \simeq T_{\text{sph}}$$

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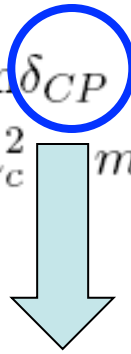
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Use CKM Matrix for $T > T_{\text{sph}}$

Within the Standard Model, lowest CP Violating structures

$$d_{CP} = \sin(\theta_{12})\sin(\theta_{23})\sin(\theta_{13})\sin\delta_{CP} \cdot (m_t^2 - m_c^2)(m_t^2 - m_u^2)(m_c^2 - m_u^2)(m_b^2 - m_s^2)(m_b^2 - m_d^2)(m_s^2 - m_d^2)$$



Cabbibo-Kobayashi-Maskawa CP Violating phase

Shaposhnikov

$$D = \text{Im Tr} [\mathcal{M}_u^2 \mathcal{M}_d^2 \mathcal{M}_u \mathcal{M}_d]$$

$$\delta_{KM}^{CP} \sim \frac{D}{T_{12}} \sim 10^{-20}$$

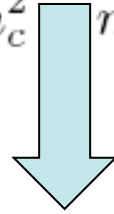
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**This CP Violation
Cannot be the
Source of Baryon
Asymmetry in
The Universe**

Role of Neutrinos?

- Several Ideas to go beyond the SM (e.g. GUT models, Supersymmetry, extra dimensional models etc.)

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- Several Ideas to go beyond the SM (e.g. GUT models, Supersymmetry, extra dimensional models etc.)
- Massive ν are **simplest** extension of SM
- Right-handed massive ν may provide extensions of SM with:
 - extra CP Violation** and thus Origin of Universe's **matter-antimatter asymmetry** due to neutrino masses, Dark Matter

BASICS: TYPES OF NEUTRINO MASS TERMS

THE MOST GENERAL, LORENTZ-INVARIANT NEUTRINO MASS TERM

$$\mathcal{L}^{\text{D+M}} = -\frac{1}{2} \bar{\nu}_L M_L^{\text{M}} (\nu_L)^c - \bar{\nu}_L M^{\text{D}} \nu_R - \frac{1}{2} \overline{(\nu_R)^c} M_R^{\text{M}} \nu_R + \text{h.c.}$$

$$\nu_L = \begin{pmatrix} \nu_{eL} \\ \nu_{\mu L} \\ \nu_{\tau L} \end{pmatrix}$$

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$$\nu_{lL}(x) = \sum_{i=1} U_{li} \nu_{iL}(x) \quad (l = e, \mu, \tau)$$

MIXING

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DIRAC

CONSERVE
TOTAL LEPTON
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$$(\nu_{lL})^c = C \bar{\nu}_{lL}^T$$

C = Charge Conjugation



**VIOLATE
LEPTON
(L) NUMBER**

**MAJORANA
LEFT-HANDED**

$$\nu_L = \begin{pmatrix} \nu_{eL} \\ \nu_{\mu L} \\ \nu_{\tau L} \end{pmatrix}$$



$$(\nu_{lR})^c = C \bar{\nu}_{lR}^T$$

**VIOLATE
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**MAJORANA
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VIOLATE
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**MAJORANA FIELDS
ARE MASS EIGENSTATES
PARTICLE=ANTIPARTICLE**

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$$(\nu^M(x))^c = \nu^M(x)$$

$$\nu^M = U^\dagger \nu_L + (U^\dagger \nu_L)^c = \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}, \quad m = \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix}$$



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FOR SEESAW: NO LEFT-HANDED MAJORANA

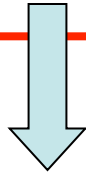
SM Extension with N extra right-handed neutrinos

$$L = L_{SM} + \bar{N}_I i \partial_\mu \gamma^\mu N_I - F_{\alpha I} \bar{L}_\alpha N_I \tilde{\phi} - \frac{M_I}{2} \bar{N}_I^c N_I + \text{h.c.}$$

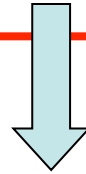
Paschos, Hill, Luty, Minkowski,
Yanagida, Mohapatra, Senjanovic,
de Gouvea..., Liao, Nelson,
Buchmuller, Anisimov, di Bari...
Akhmedov, Rubakov, Smirnov,
Davidson, Giudice, Notari, Raidal,
Riotto, Strumia, **Pilaftsis**, Underwood,
Shaposhnikov ... Hernandez, Giunti...

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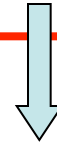
Right-handed
Massive **Majorana**
neutrinos



Leptons

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Higgs scalar SU(2)

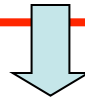
Dual: $\tilde{\phi}_i = \epsilon_{ij} \phi_j^*$

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ν MSM

Boyarski, Ruchayskiy, Shaposhnikov

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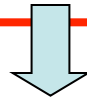
**Yukawa couplings
Matrix (N=2 or 3)**

$$F = \tilde{K}_L f_d \tilde{K}_R^\dagger$$

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Majorana masses
to (2 or 3) active
neutrinos via **seesaw**

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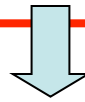
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NB: Upon Symmetry Breaking
 $\langle \phi \rangle = v \neq 0 \rightarrow$ Dirac mass term



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Light Neutrino Masses through see saw

$$m_\nu = -M^D \frac{1}{M_I} [M^D]^T$$

$$M_D = F_{\alpha I} v$$

$$v = \langle \phi \rangle \sim 175 \text{ GeV}$$

$$M_D \ll M_I$$

Minkowski, Yanagida,
Mohapatra, Senjanovic
Sechter, Valle ...



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From Constraints
(compiled ν oscillation data)
on (light) sterile neutrinos:

Giunti, Hernandez ...

N=1 excluded by data

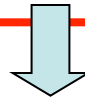
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Model with 2 or 3 singlet fermions works well in reproducing Baryon Asymmetry and is consistent with Experimental Data on neutrino oscillations

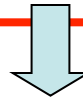
Model with N=3 also works fine, and in fact it allows **one** of the Majorana fermions to almost **decouple** from the rest of the SM fields, thus providing candidates for **light** (keV region of mass) sterile neutrino **Dark Matter**.

SM Extension with N extra right-handed neutrinos

ν MSM

Boyarski, Ruchayskiy, Shaposhnikov

$$L = L_{SM} + \bar{N}_I i \partial_\mu \gamma^\mu N_I - F_{\alpha I} \bar{L}_\alpha N_I \tilde{\phi} - \frac{M_I}{2} \bar{N}_I^c N_I + \text{h.c.}$$



**Yukawa couplings
Matrix (N=3)**

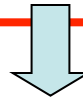
$$F = \tilde{K}_L f_d \tilde{K}_R^\dagger$$

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Yukawa couplings
Matrix (N=3)

$$F = \tilde{K}_L f_d \tilde{K}_R^\dagger$$

$$f_d = \text{diag}(f_1, f_2, f_3), \quad \tilde{K}_L = K_L P_\alpha, \quad \tilde{K}_R^\dagger = K_R^\dagger P_\beta$$

$$P_\alpha = \text{diag}(e^{i\alpha_1}, e^{i\alpha_2}, 1), \quad P_\beta = \text{diag}(e^{i\beta_1}, e^{i\beta_2}, 1)$$

Majorana
phases

Mixing

$$K_L = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{L23} & s_{L23} \\ 0 & -s_{L23} & c_{L23} \end{pmatrix} \begin{pmatrix} c_{L13} & 0 & s_{L13} e^{-i\delta_L} \\ 0 & 1 & 0 \\ -s_{L13} e^{i\delta_L} & 0 & c_{L13} \end{pmatrix} \begin{pmatrix} c_{L12} & s_{L12} & 0 \\ -s_{L12} & c_{L12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$c_{Lij} = \cos(\theta_{Lij}) \text{ and } s_{Lij} = \sin(\theta_{Lij}).$$

$$L = L_{SM} + \bar{N}_I i \partial_\mu \gamma^\mu N_I - F_{\alpha I} \bar{L}_\alpha N_I \tilde{\phi} - \frac{M_I}{2} \bar{N}_I^c N_I + \text{h.c.}$$

$$|F|^2 \approx \frac{m_{\text{atm}} M_I}{v^2} \sim 2 \times 10^{-15} \frac{M_I}{\text{GeV}} \quad |\Delta m_{\text{atm}}^2| \equiv m_{\text{atm}}^2 = 2.40_{-0.11}^{+0.12} \times 10^{-3} \text{eV}^2$$

(Decay) processes in Early Universe

$$N t \leftrightarrow \nu t, \quad H \leftrightarrow N \nu \quad \text{or} \quad N \leftrightarrow H \nu$$

Rate: $9F^2 f_t^2 T / (64\pi^3)$

Akhmedov, Rubakov, Smirnov

f_t = top quark Yukawa coupling

Thermal equilibrium at temperatures

$$M_0 = M_P / (1.66 \sqrt{g_{\text{eff}}})$$

time = $M_0^2 / 2T^2$ (radiation era)

$$T_{\text{eq}} \simeq \frac{9f_t^2 m_{\text{atm}} M_0}{64\pi^3 v^2} M_I \simeq 5M_I$$

(for $T_{\text{eq}} > 100 \text{ GeV}$)

Thermal Properties

Two distinct physics cases

($M_W = \text{electroweak scale} = O(100) \text{ GeV}$):

(i) $M_I > M_W$

(ii) $M_I < M_W$

Thermal Properties

Shaposhnikov...

Two distinct physics cases: $M_I > M_W$ & $M_I < M_W$

(i) $M_I > M_W$ (electroweak scale)



Decay of
Right-handed
fermions

$$T_{\text{decay}} \simeq \left(\frac{m_{\text{atm}} M_0}{24\pi v^2} \right)^{\frac{1}{3}} M_I \simeq 3M_I$$

Out of equilibrium for:

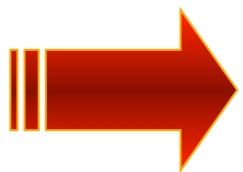
$$T > T_{\text{eq}} \text{ or for } T < T_{\text{decay}}$$

If $T_{\text{eq}} > T_{\text{sph}}$

Decays of Right-handed Majorana fermions occur for period of active Sphaleron processes



$$T_{\text{decay}} > T_{\text{sph}}$$



Thermal Leptogenesis

Fukugita,
Yanagida,

Thermal Leptogenesis

Heavy Right-handed Majorana neutrinos enter *equilibrium at $T = T_{eq}$*

Thermal Leptogenesis



***Independent of
Initial Conditions
@ $T \gg T_{eq}$***

Heavy Right-handed Majorana neutrinos enter ***equilibrium at $T = T_{eq}$***

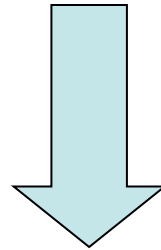
Thermal Leptogenesis

Independent of
Initial Conditions
@ $T \gg T_{eq}$

Heavy Right-handed Majorana neutrinos enter *equilibrium at $T = T_{eq} > T_{decay}$*

enhanced CP V. Lepton number
Violation

$$N_I \rightarrow H\nu, \bar{H}\bar{\nu}$$



Out of Equilibrium Decays

$$T \simeq T_{decay} > T_{sph}$$



Produce Lepton asymmetry

Fukugita, Yanagida,

Kuzmin, Rubakov,
Shaposhnikov

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Equilibrated electroweak
B, L violating sphaleron
interactions (B-L conserv)

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Produce Lepton asymm

Equilibrated electroweak
B, L violating sphaleron
interactions (B-L conserv)

$$L = \frac{2}{M} l_L l_L \phi \phi + \text{H.c.}$$

where

$$l_L = \begin{bmatrix} \nu_e \\ e \end{bmatrix}_L, \begin{bmatrix} \nu_\mu \\ \mu \end{bmatrix}_L, \begin{bmatrix} \nu_\tau \\ \tau \end{bmatrix}_L$$

**Observed Baryon Asymmetry
In the Universe (BAU)**

Thermal Leptogenesis

Independent of
Initial Conditions
@ $T \gg T_{eq}$

Heavy Right-handed Majorana neutrinos enter *equilibrium at $T = T_{eq} > T_{decay}$*

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Fukugita, Yanagida,

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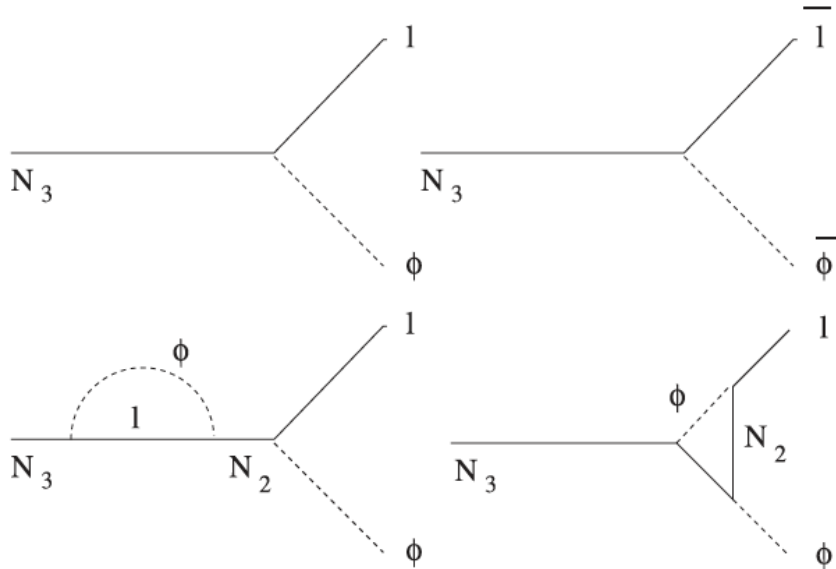
Kuzmin, Rubakov,
Shaposhnikov

**Estimate BAU by solving Boltzmann equations
for Heavy Neutrino Abundances**

Pilafsis, Riotto...
Buchmuller, di Bari *et al.*

Predicted BAU in such models is found to be of order:

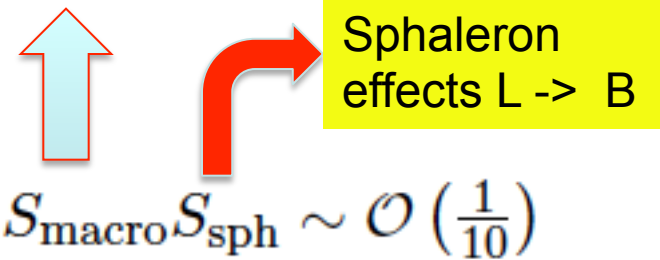
**Pilaftsis,
Shaposhnikov...**



$$\frac{n_B}{n_\gamma} = \Delta \sim \frac{1}{g_{\text{eff}}} \delta_{CP} \cdot S_{\text{macro}} S_{\text{sph}},$$

$$\delta_{CP} = \frac{\Gamma(N \rightarrow H\nu) - \Gamma(N \rightarrow \bar{H}\bar{\nu})}{\Gamma_{\text{tot}}}$$

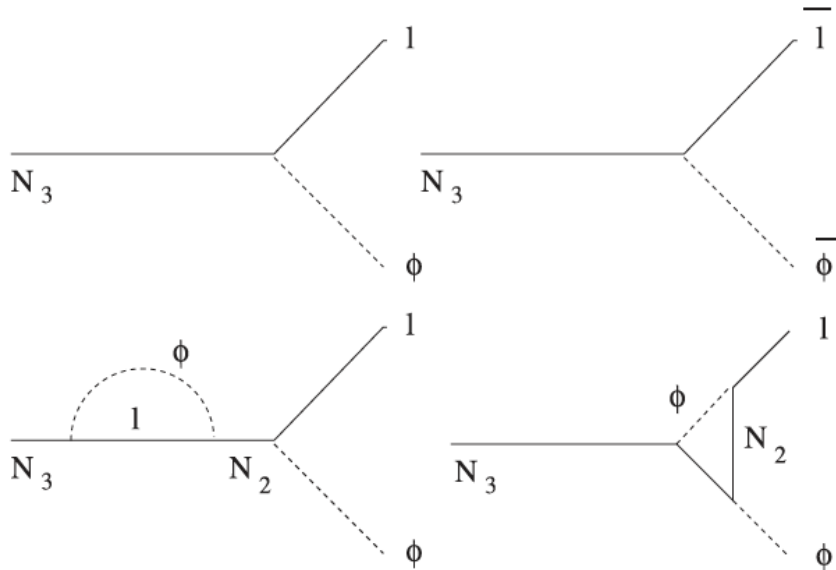
Majorana
fermion
Kinematics



$$S_{\text{macro}} S_{\text{sph}} \sim \mathcal{O}\left(\frac{1}{10}\right)$$

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**Pilaftsis,
Shaposhnikov...**



**Non mass degenerate
Majorana neutrinos**

$$|M_I - M_J| \sim M_K$$

$$\frac{n_B}{s} \sim 10^{-3} F^2 \simeq 10^{-10}$$



$$F^2 \sim 10^{-7}$$

reproduced observed BAU

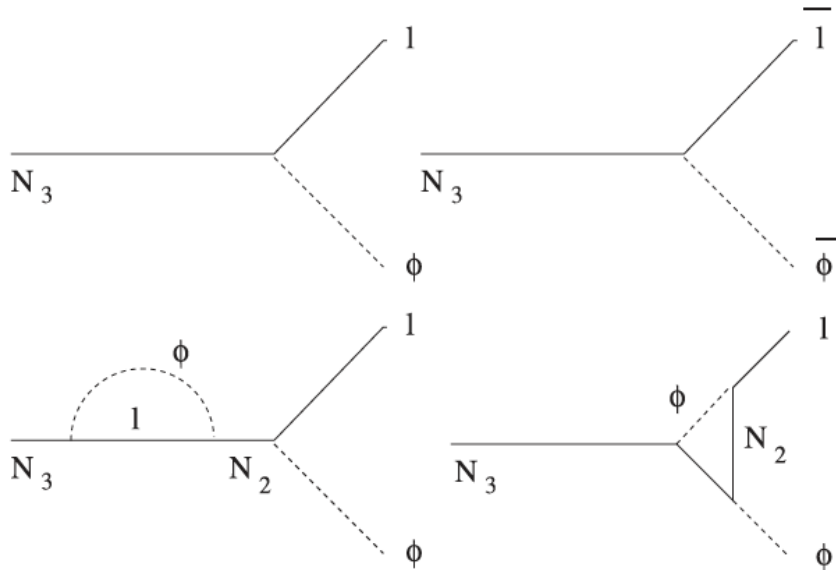
$$S_{\text{macro}} S_{\text{sph}} \sim \mathcal{O}\left(\frac{1}{10}\right)$$

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reproduced observed BAU

$$m_\nu = -M^D \frac{1}{M_I} [M^D]^T$$

$$M_D = F_{\alpha I} v$$

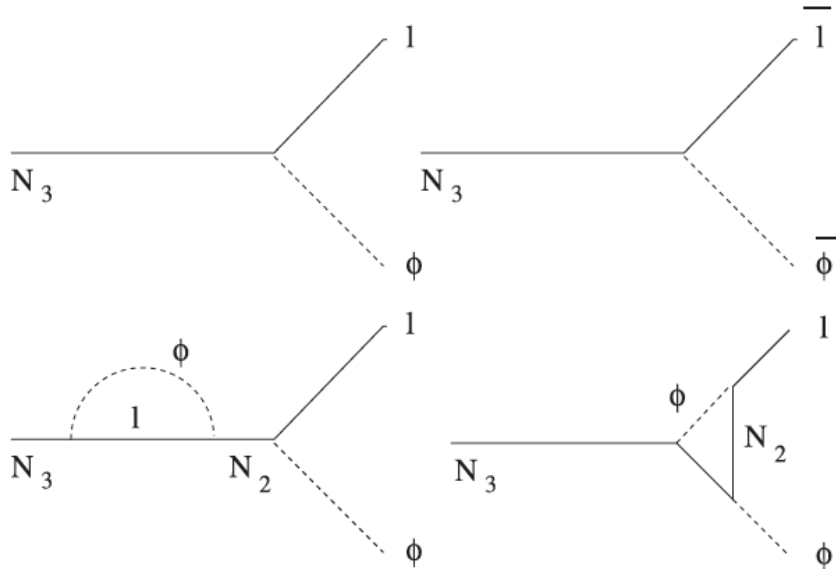
$$v = \langle \phi \rangle = 174 \text{ GeV}$$



$$M_N \sim 10^{11} \text{ GeV}$$

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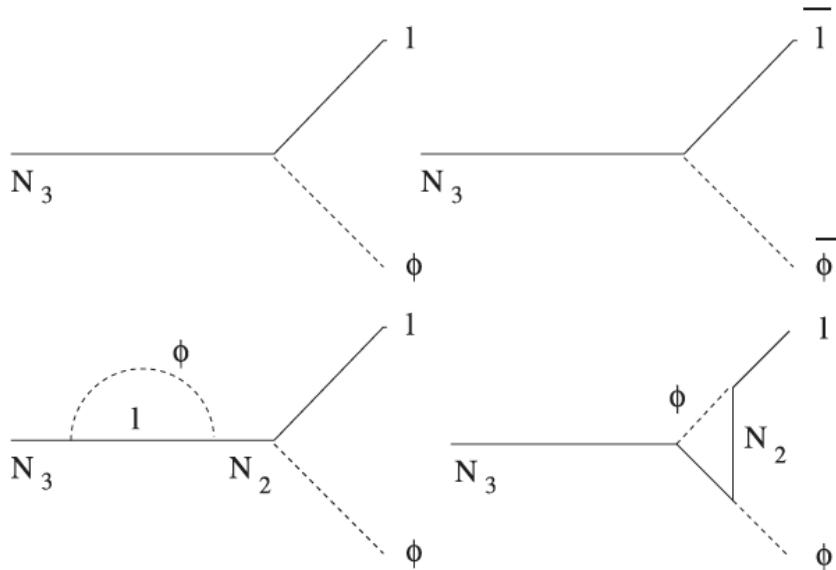


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**Pilaftsis,
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**Non mass degenerate
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**Stability
of Higgs
mass against
higher loops
in danger !**

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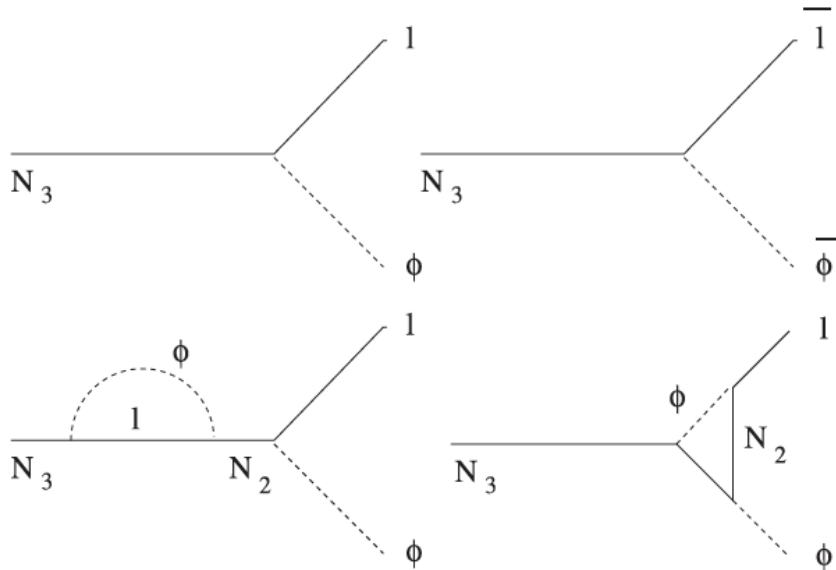
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**Non mass degenerate
Majorana neutrinos**



**Stability
of Higgs
mass against
higher loops
in danger !**

e.g. one loop

$$F^2 M_I^2 / (4\pi) \sim 10^{14} \text{ GeV}^2$$

reproduced observed BAU

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$$M_D = F_{\alpha I} v$$

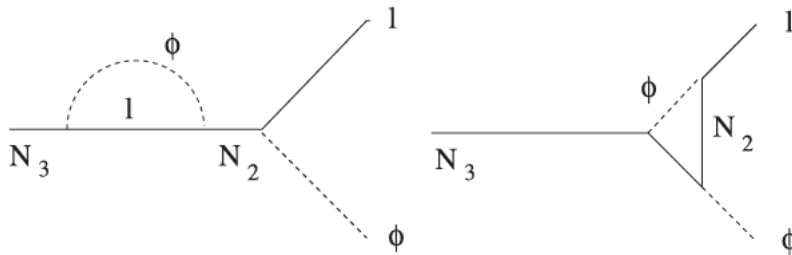
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POSSIBLE RESOLUTION: DEGENERATE RIGHT-HANDED NEUTRINOS

Pilaftsis
Shaposhnikov

If, say : N_2, N_3 *degenerate in mass*



enhanced CP violation contribution from mixing
(cf. neutral kaons)

but much smaller Yukawa couplings F allowed

BAU estimated in this case:

$$\frac{n_B}{s} \sim 10^{-3} f^2 \frac{M_2 \Gamma_{\text{tot}}}{(M_2 - M_3)^2 + \Gamma_{\text{tot}}^2}$$

$$|M_2 - M_3| \sim \Gamma_{\text{tot}}$$

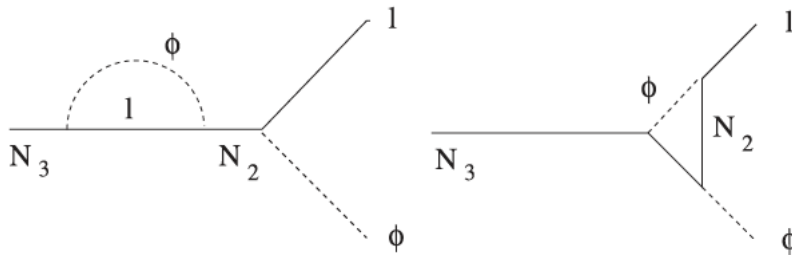
$$\frac{|M_2 - M_3|}{M_2} \sim f^2 \sim \frac{m_\nu M_W}{v^2} \sim 10^{-13}$$

$$M_I \sim M_W$$

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$$|M_2 - M_3| \sim \Gamma_{\text{tot}}$$


NB: For $M_I < 10^7$ GeV,

no Problem for Higgs mass stability

$$\frac{|M_2 - M_3|}{M_2} \sim f^2 \sim \frac{m_\nu M_W}{v^2} \sim 10^{-13}$$

$$M_I \sim M_W$$

A restricted Case : N_1 only **out of equilibrium** decay
 $N_{2,3}$ in thermal equilibrium

Resonant τ Leptogenesis

One lepton number (τ) resonantly produced by out-of-equilibrium decays


$$-\mathcal{L}_{Y,M} = \frac{1}{2}(\bar{\nu}_{iR})^C(M_S)_{ij}\nu_{jR} + \hat{h}_{ii}^l \bar{L}_i \Phi l_{iR} + h_{ij}^{\nu R} \bar{L}_i \tilde{\Phi} \nu_{jR} + \text{H.c.},$$

$$h^{\nu R} = \begin{pmatrix} \varepsilon_e & ae^{-i\pi/4} & ae^{i\pi/4} \\ \varepsilon_\mu & be^{-i\pi/4} & be^{i\pi/4} \\ \varepsilon_\tau & ce^{-i\pi/4} & ce^{i\pi/4} \end{pmatrix}.$$

Pilaftsis

**De Simone,
Riotto**

A restricted Case : N_1 only *out of equilibrium* decay
 $N_{2,3}$ in thermal equilibrium

Avoid L_τ excess  $N_{2,3}$ decay rates suppressed

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A restricted Case : N_1 only **out of equilibrium** decay
 $N_{2,3}$ in thermal equilibrium

Avoid L_τ excess



$N_{2,3}$ decay rates suppressed

Predicted BAU

$$\eta_B \sim -10^{-2} \frac{\delta_{N_1}^\tau}{K_{N_1}} \frac{\Gamma(N_1 \rightarrow L_\tau \Phi)}{\Gamma(N_{2,3} \rightarrow L_\tau \Phi)}$$

$$\sim -10^{-2} \frac{\delta_{N_1}^\tau}{K_{N_1}} \frac{\varepsilon_\tau^2}{c^2}$$



$$\bar{\Gamma}_{N_1}/H(z=1) \approx 10$$

$$z = m_{N_1}/T$$

$$-\mathcal{L}_{Y,M} = \frac{1}{2} (\bar{\nu}_{iR})^c (M_S)_{ij} \nu_{jR} + \hat{h}_{ii}^l \bar{L}_i \Phi l_{iR} \\ + h_{ij}^{\nu R} \bar{L}_i \tilde{\Phi} \nu_{jR} + \text{H.c.},$$

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agreement with observed BAU

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 $N_{2,3}$ in thermal equilibrium

Avoid L_τ excess



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Predicted BAU

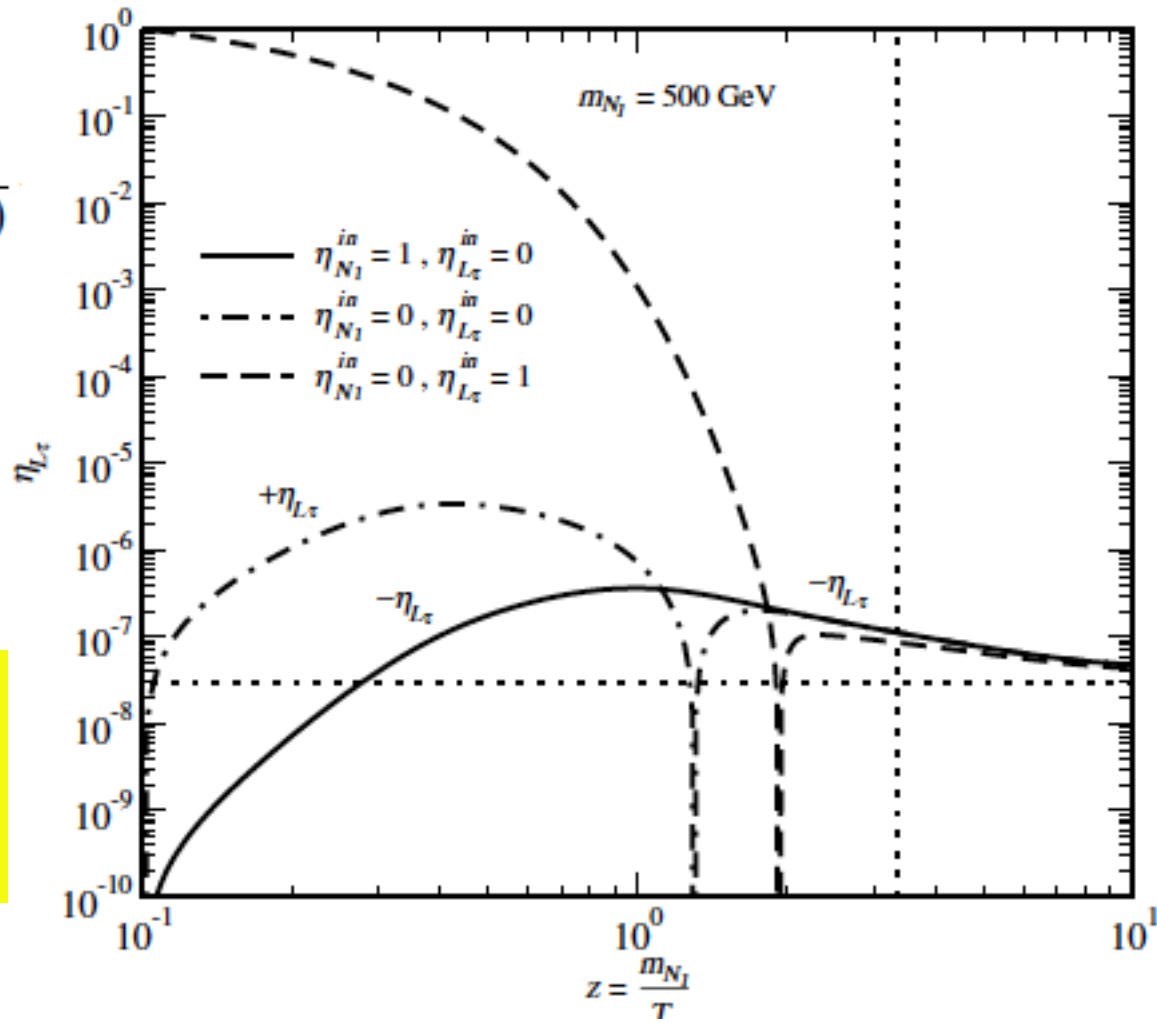
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$$|\delta_{N_1}^\tau| \sim 10^{-5} \text{ and } \varepsilon_\tau/c \sim 10^{-2}$$


Estimate agrees with:

Boltzmann eq calculated neutrino N_1 abundance



A restricted Case : N_1 only **out of equilibrium** decay
 $N_{2,3}$ in thermal equilibrium

Pilaftsis
 Underwood

Avoid L_τ excess  $N_{2,3}$ decay rates suppressed

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$$|\delta_{N_1}^\tau| \sim 10^{-5} \text{ and } \varepsilon_\tau/c \sim 10^{-2}$$

agreement with observed BAU

Leptogenesis possible for low-mass N_1 : $M_N = O(M_W - \text{TeV})$



A restricted Case : N_1 only **out of equilibrium** decay
 $N_{2,3}$ in thermal equilibrium

Pilaftsis
 Underwood

Predicted BAU



$$M_N = O(M_W - \text{TeV}) \quad \gg \quad \mu \rightarrow e\gamma$$

$$B(\mu \rightarrow e\gamma) \approx 6 \times 10^{-4} (a^2 b^2 v^4) / m_N^4$$

$$a, b \sim 3 \times 10^{-3}$$

$$B^{\text{exp}}(\mu \rightarrow e\gamma) \lesssim 1.2 \times 10^{-11} \quad \gg \quad \text{future sensitivity to } 10^{-13}$$

MEG experiment: $\mu^+ \rightarrow e^+ + \gamma$

1107.5547 $B(\mu^+ \rightarrow e^+ + \gamma) \leq 2.4 \times 10^{-12}$

A restricted Case : N_1 only **out of equilibrium** decay
 $N_{2,3}$ in thermal equilibrium

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Effects at e^+e^- linear collider? Study production of electroweak scale $N_{2,3}$ via their decays to e, μ (not τ**)**

Thermal Properties

Two distinct physics cases: $M_I > M_W$ & $M_I < M_W$

Thermal Properties

Ashaka, Shaposhnikov...

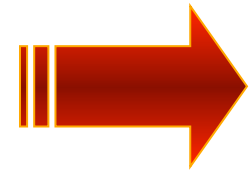
Two distinct physics cases: $M_I > M_W$ & $M_I < M_W$

(ii) $M_I < M_W$ (electroweak scale), e.g. $M_I = O(1)$ GeV



Keep light neutrino masses in right order , Yukawa couplings must be:

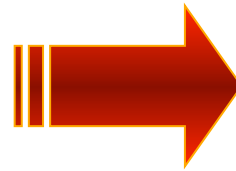
$$F_{\alpha I} \sim \frac{\sqrt{m_{\text{atm}} M_I}}{v} \sim 4 \times 10^{-8}$$



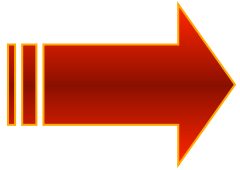
Baryogenesis through coherent oscillations right-handed singlet fermions

Akhmedov, Rubakov, Smirnov

$$F_{\alpha I} \sim \frac{\sqrt{m_{\text{atm}} M_I}}{v} \sim 4 \times 10^{-8}$$



Heavy Majorana fermions N_i
thermalize only for $T < M_W$



Out of Equilibrium decays of N_i for $T > M_W$

BAU depends in this case on initial conditions



But at the end of inflation we may reasonably assume that the **N_i populations** are washed out, hence set their end-of-inflation concentrations to **zero** value

Majorana masses small compared to Sphaleron freeze-out T

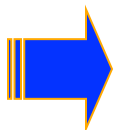


Total Lepton number conserved

Assume Mass generacy $N_{2,3}$ enhanced CP violation
Coherent Oscillations between these singlet fermions,



**Total Lepton zero
but unevenly
distributed between
active & sterile ν**



**Lepton number of active left-handed ν transferred to Baryons due to
equilibrated sphaleron processes**

BAU ESTIMATES



Assume *Mass degeneracy* $N_{2,3}$, hence enhanced CP violation
Coherent Oscillations between these singlet fermions

$$\omega \sim \frac{|M_2^2 - M_3^2|}{E_I} \sim \frac{M_2 \Delta M(T)}{T}$$

$$E_I \sim T$$
$$\Delta M(T) \ll M_2 \approx M_3$$



$N_2 - N_3$ MASS DIFF.

BAU ESTIMATES



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FOR CP VIOLATION TO OCCUR MUST HAVE: Oscillation rate > Hubble rate $H(T)$

Baryogenesis occurs @: $T_B \sim \left(M_I \Delta M(T) M_0 \right)^{1/3}$ eg O(100) GeV

$$\frac{n_B}{s} \simeq 1.7 \cdot 10^{-10} \delta_{CP} \left(\frac{10^{-5}}{\Delta M(T)/M_2} \right)^{2/3} \left(\frac{M_2}{10 \text{ GeV}} \right)^{5/3}$$

$$\delta_{CP} = 4s_{R23}c_{R23} \left[s_{L12}s_{L13}c_{L13} \left((c_{L23}^4 + s_{L23}^4)c_{L13}^2 - s_{L13}^2 \right) \cdot \sin(\delta_L + \alpha_2) \right. \\ \left. + c_{L12}c_{L13}^3 s_{L23}c_{L23} (c_{L23}^2 - s_{L23}^2) \cdot \sin \alpha_2 \right].$$

BAU ESTIMATES



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Coherent Oscillations between these singlet fermions

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Quite effective Mechanism: Maximal Baryon asymmetry $\Delta \equiv \frac{n_B - n_{\bar{B}}}{n_B + n_{\bar{B}}} \sim 1$

for $T_B = T_{\text{sph}} = T_{\text{eq}}$



Assumption: Interactions with plasma of SM particles
do not destroy quantum mechanical coherence of oscillations

BAU ESTIMATES



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Coherent Oscillations between these singlet fermions

$$\omega \sim \frac{|M_2^2 - M_3^2|}{E_I} \sim \frac{M_2 \Delta M(T)}{T}$$

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$$\text{Mass } N_2 (N_3) / (\text{Mass } N_1) = O(10^5)$$

N_1 Lightest Sterile neutrino is a natural DARK MATTER candidate

PART II
NEUTRINOS
&
THE DARK SECTOR
OF THE UNIVERSE

NEUTRINOS & THE DARK SECTOR OF THE UNIVERSE

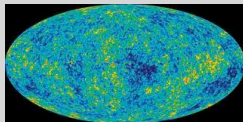
Current Energy Budget of the Cosmos

Observations from:

Supernovae Ia



CMB

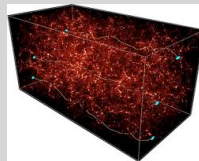


Baryon Acoustic Oscillations

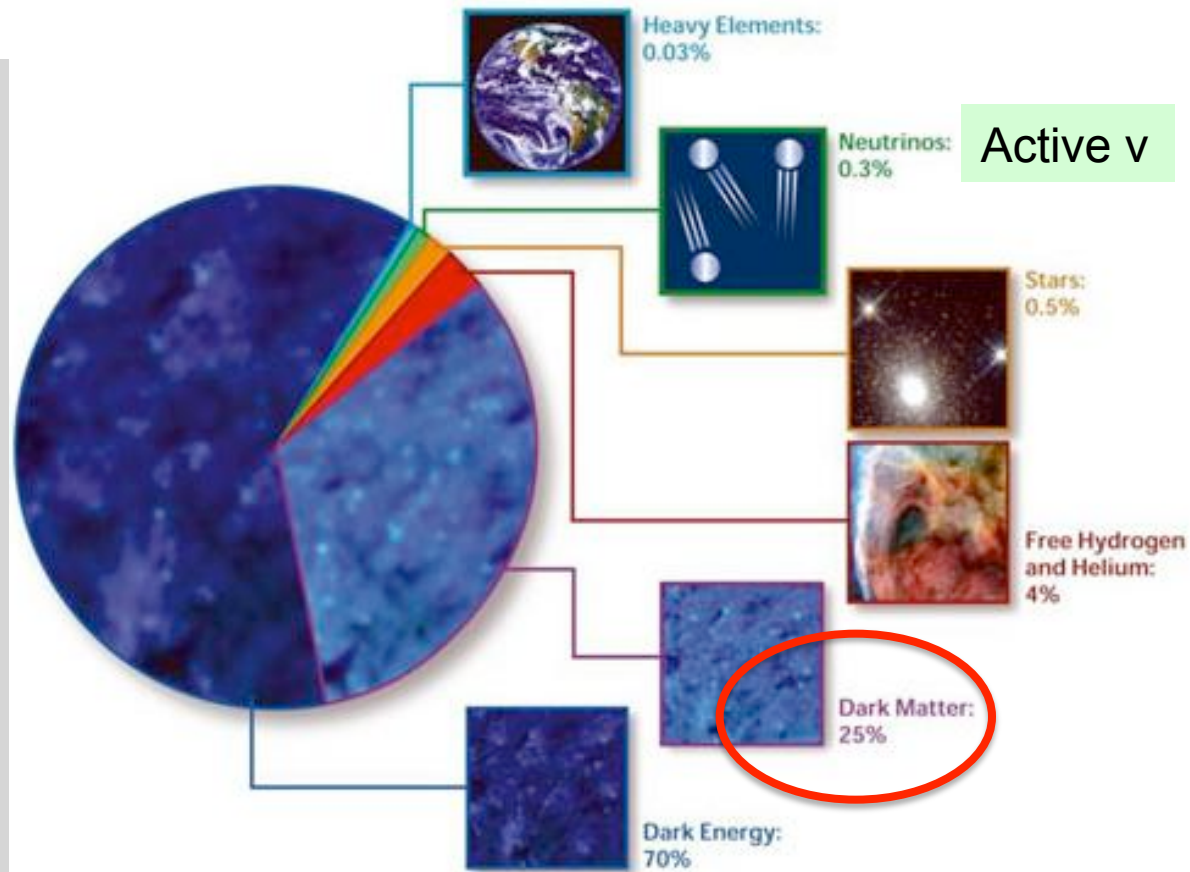


Galaxy Surveys

Structure Formation data



Strong & Weak lensing



TYPES OF DARK MATTER

- **HOT DARK MATTER (HDM)**: form of dark matter which consists of particles that travel with ultrarelativistic velocities: e.g. neutrinos
- **COLD DARK MATTER (CDM)**: form of dark matter consisting of slowly moving particles, hence cold,
 - e.g. WIMPS (stable supersymmetric particles (neutralinos etc.) or MACHOS.
- **WARM DARK MATTER (WDM)**: form of dark matter with properties between those of HDM and CDM, sterile neutrinos, light gravitinos-partner of gravitons in supergravity theories...)

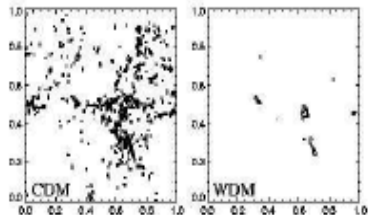
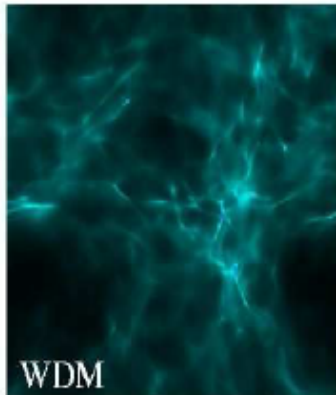
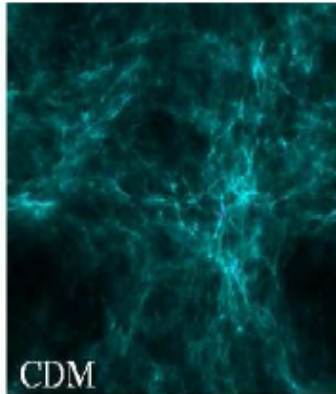
PHYSICS: WMAP and Dark Matter

WMAP results so far:

- Disfavor strongly hot dark matter (neutrinos), $\Omega_\nu h^2 < 0.0076$ ($\langle m_\nu \rangle_e < 0.23$ eV).
- Warm Dark Matter (gravitino) disfavoured by evidence for re-ionization at redshift $z \sim 20$.
- Cold Dark Matter (CDM) remains: axions, supersymmetric dark matter (lightest SUSY particle (LSP)), superheavy (masses $\sim 10^{14 \pm 5}$ GeV)

WMAP results: $\Omega_m h^2 = 0.135_{-0.009}^{+0.008}$ (matter), $\Omega_b h^2 = 0.0224 \pm 0.0009$ (baryons), hence, assuming CDM is the difference, $\Omega_{CDM} h^2 = 0.1126_{-0.0181}^{+0.0161}$, (2σ level).

WMAP excludes WARM Dark Matter



Numerical simulations for structure formation in Cold Dark Matter (CDM) (top) and Warm Dark Matter (WDM) (middle) with mass $m_X = 10$ KeV at $z = 20$. Bottom: Dark halos with mass $> 10^5 M_\odot$ for CDM (left) and for WDM (right).

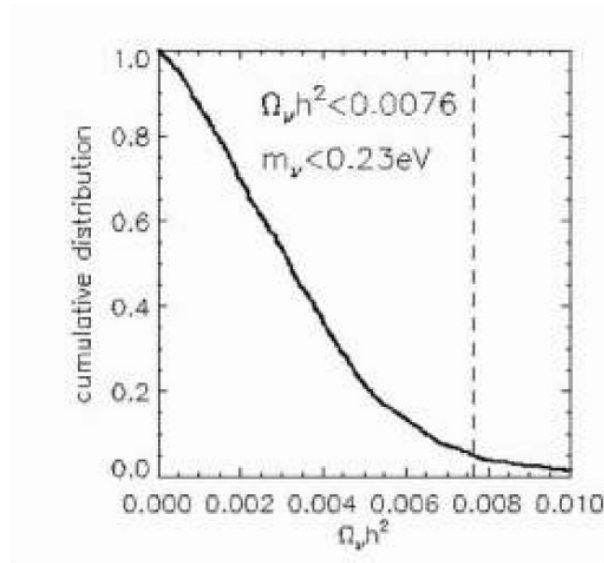
IMPORTANT COMMENTS:

Such structure formation arguments can only place a **lower bound** on mass of the WDM candidate: $m_X > 10$ KeV.

Above results exclude Light Gravitino Models ($m_X < 0.5$ KeV) of Particle Physics as DM candidates.

NB! WDM with $m_X \geq 100$ KeV becomes **indistinguishable** from Cold Dark Matter, as far as structure formation is concerned.

WMAP excludes HOT Dark Matter



Contribution of neutrinos to energy density of Universe: $\Omega_\nu h^2 = \frac{\sum_i m_i}{94.0 \text{ eV}}$

(sum over light neutrino species (decouple while still relativistic)).

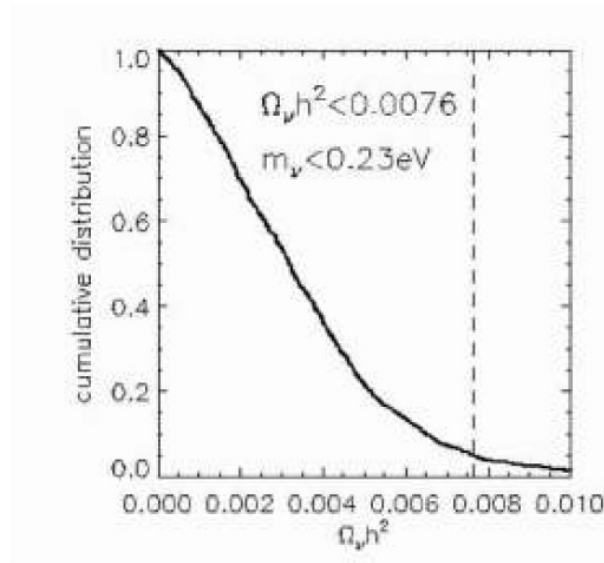
WMAP and other experiments (the Lyman α data etc) $\Omega_\nu h^2 < 0.0076 \Rightarrow \langle m_\nu \rangle_e < 0.23 \text{ eV}$:

Excludes HOT DM.

NB: WMAP still consistent with Majorana neutrinos, and also marginally with $\beta\beta$ -decay

(Heidelberg-Moscow Coll.).

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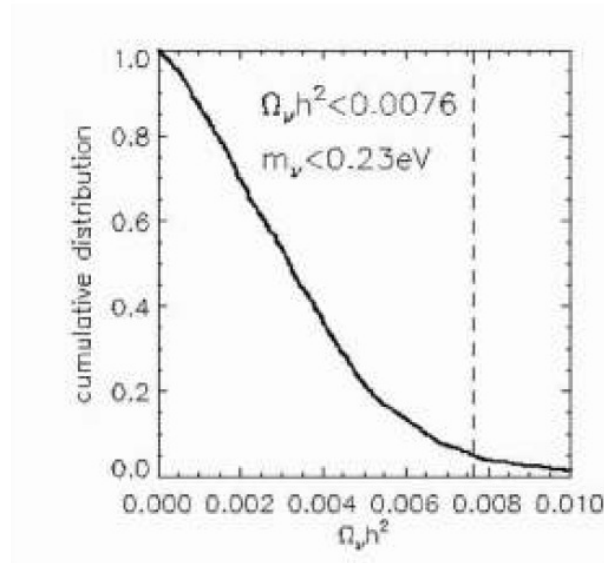
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UNCERTAINTY IN COSMOLOGICAL DATA DUE TO VARIETY OF SOURCES
& MEASUREMENTS

Model	Observables	Σm_ν (eV) 95% Bound
$\omega\text{CDM} + \Delta N_{\text{rel}} + m_\nu$	CMB+H0+SN+BAO	≤ 1.5
$\omega\text{CDM} + \Delta N_{\text{rel}} + m_\nu$	CMB+H0+SN+LSSPS	≤ 0.76
$\Lambda\text{CDM} + m_\nu$	CMB+H0+SN+BAO	≤ 0.61
$\Lambda\text{CDM} + m_\nu$	CMB+H0+SN+LSSPS	≤ 0.36
$\Lambda\text{CDM} + m_\nu$	CMB (+SN)	≤ 1.2
$\Lambda\text{CDM} + m_\nu$	CMB+BAO	≤ 0.75
$\Lambda\text{CDM} + m_\nu$	CMB+LSSPS	≤ 0.55
$\Lambda\text{CDM} + m_\nu$	CMB+H0	≤ 0.45

Model	Observables	Cosmo+Oscillations 95% Ranges		
		m_{ν_e} (eV)	m_{ee} (eV)	Σm_ν (eV)
ω CDM + $\Delta N_{\text{rel}} + m_\nu$	CMB+HO+SN+BAO	N [0.0047 – 0.51] I [0.047 – 0.51]	N [0.00 – 0.51] I [0.014 – 0.51]	N [0.056 – 1.5] I [0.098 – 1.5]
ω CDM + $\Delta N_{\text{rel}} + m_\nu$	CMB+HO+SN+LSSPS	N [0.0047 – 0.27] I [0.047 – 0.27]	N [0.00 – 0.25] I [0.014 – 0.25]	N [0.056 – 0.75] I [0.098 – 0.76]
Λ CDM + m_ν	CMB+H0+SN+BAO	N [0.0047 – 0.20] I [0.048 – 0.21]	N [0.00 – 0.20] I [0.014 – 0.21]	N [0.056 – 0.61] I [0.097 – 0.61]
Λ CDM + m_ν	CMB+H0+SN+LSSSP	N [0.0047 – 0.12] I [0.047 – 0.12]	N [0.00 – 0.12] I [0.014 – 0.12]	N [0.056 – 0.36] I [0.098 – 0.36]
Λ CDM + m_ν	CMB (+SN)	N [0.0047 – 0.40] I [0.047 – 0.40]	N [0.00 – 0.40] I [0.014 – 0.41]	N [0.056 – 1.2] I [0.098 – 1.2]
Λ CDM + m_ν	CMB+BAO	N [0.0052 – 0.25] I [0.047 – 0.25]	N [0.00 – 0.25] I [0.014 – 0.25]	N [0.056 – 0.75] I [0.099 – 0.75]
Λ CDM + m_ν	CMB+LSSPS	N [0.0047 – 0.18] I [0.048 – 0.19]	N [0.00 – 0.18] I [0.014 – 0.19]	N [0.056 – 0.55] I [0.099 – 0.55]
Λ CDM + m_ν	CMB+H0	N [0.0047 – 0.14] I [0.047 – 0.16]	N [0.00 – 0.14] I [0.014 – 0.16]	N [0.056 – 0.44] I [0.097 – 0.45]

WMAP excludes HOT Dark Matter



Contribution of neutrinos to energy density of Universe: $\Omega_\nu h^2 = \frac{\sum_i m_i}{94.0 \text{ eV}}$
(sum over light neutrino species (decouple while still relativistic)).

WMAP and other experiments (the Lyman α data etc) $\Omega_\nu h^2 < 0.0076 \Rightarrow \langle m_\nu \rangle_e < 0.23 \text{ eV}$:

Excludes HOT DM.

Model dependence...

NB: WMAP still consistent with Majorana neutrinos, and also marginally with $\beta\beta$ -decay (Heidelberg-Moscow Coll.).

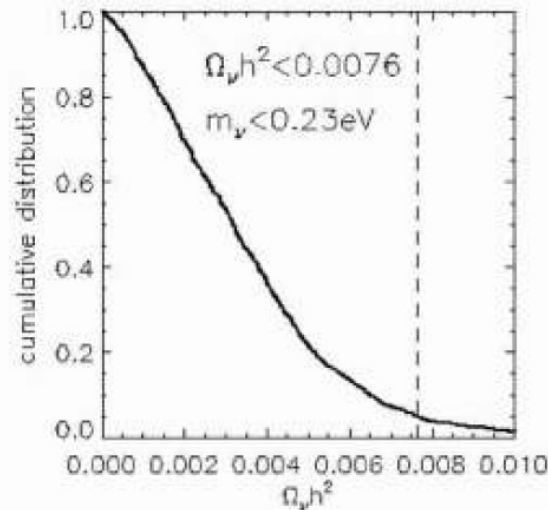
WMAP excludes HOT Dark Matter

Caution:

FRW- Comology & local Lorentz invariance **assumed.**

If Lorentz violated (TeV) ν of 2 eV mass could have $\Omega_\nu \sim 0.15$ to reproduce CMB spectrum

(Dodelson-Liguori 2006)



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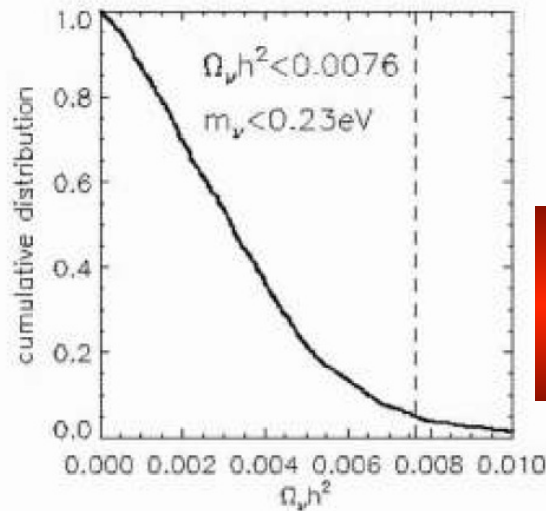
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BUT... TeV_S excluded by Gravitational Lensing Data (Sakellariadou, Yusaf, NEM)

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Sterile neutrinos and DARK MATTER

**Light Sterile Neutrinos (mass $> \text{keV}$) may provide
good dark Matter Candidates**

To be DM : life time $>$ age of Universe

Davidson, Widrow, Shi,
Fuller, Dolgov, Hansen,
Abazajian, Patel, Tucker ...

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ν MSM

Lightest of singlet fermions N_1 plays that role



Coupling with SM matter: superweak

$$\theta_1 G_F = \sum_{\alpha=e,\mu,\tau} \frac{v^2 |F_{\alpha I}|^2}{M_1^2}$$

Boyarski, Ruchayskiy,
Shaposhnikov...

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$$\Gamma_{N_1 \rightarrow \gamma \nu} = \frac{9 \alpha G_F^2}{1024 \pi^4} \sin^2(2\theta_1) M_1^5 \simeq 5.5 \times 10^{-22} \theta_1^2 \left[\frac{M_1}{\text{keV}} \right]^5 \text{ s}^{-1}$$

$$\theta_1^2 \lesssim 1.8 \times 10^{-5} \left(\frac{\text{keV}}{M_1} \right)^5$$

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Contributions to mass matrix of active neutrinos

$$\delta m_\nu \sim \theta_1^2 M_1$$

Sterile neutrinos and DARK MATTER

UNIVERSITÉ
DE GENÈVE



Also the
International Neutrino Summer School
Geneva 18–30 July 2011

dpnc.unige.ch/NeutrinoSummerSchool2011

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ν MSSM

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Coupling with SM matter: superweak

mass contribution smaller than solar mass diff experimental error

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ν MSSM

Boyarski, Ruchayskiy,
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$$\Delta m_{\text{sol}}^2 = 7.65_{-0.20}^{+0.23} \times 10^{-5} \text{ eV}^2$$
$$\Delta m_{\text{atm}}^2 = 2.40_{-0.11}^{+0.12} \times 10^{-3} \text{ eV}^2$$

$$M_1 \geq 2 \text{ keV} \Rightarrow$$

mass contribution smaller than
solar mass diff experimental error

$$\theta_1^2 \lesssim 1.8 \times 10^{-5} \left(\frac{\text{keV}}{M_1} \right)^5,$$

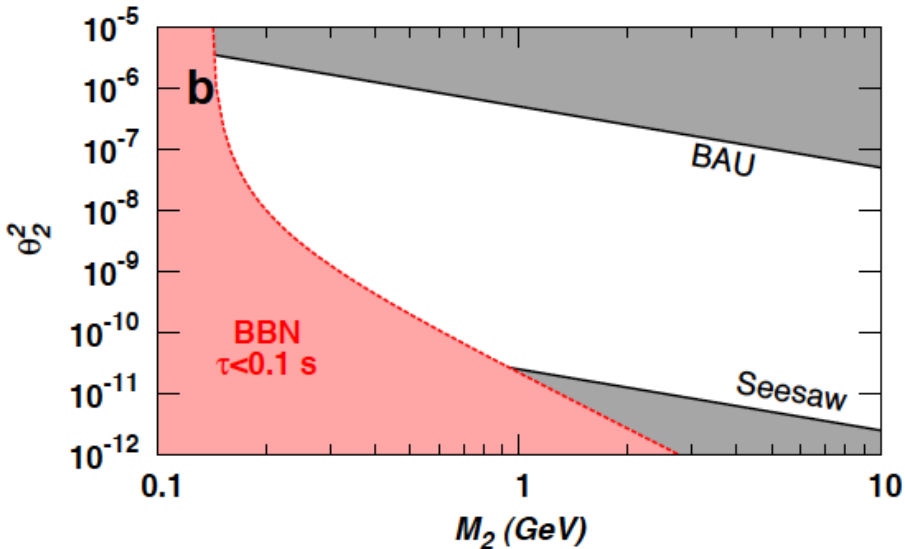
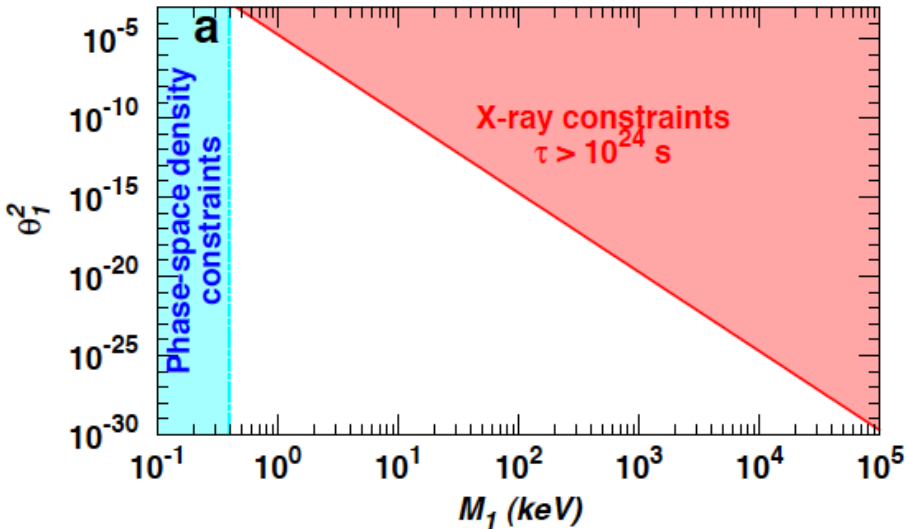
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Contributions to mass matrix of active neutrinos

More than one sterile neutrino needed to reproduce Observed oscillations

ν MSM

Boyarski, Ruchayskiy, Shaposhnikov...



Constraints on two heavy degenerate singlet neutrinos

N_1 DM production estimation in Early Universe must take into account its interactions with $N_{2,3}$ heavy neutrinos

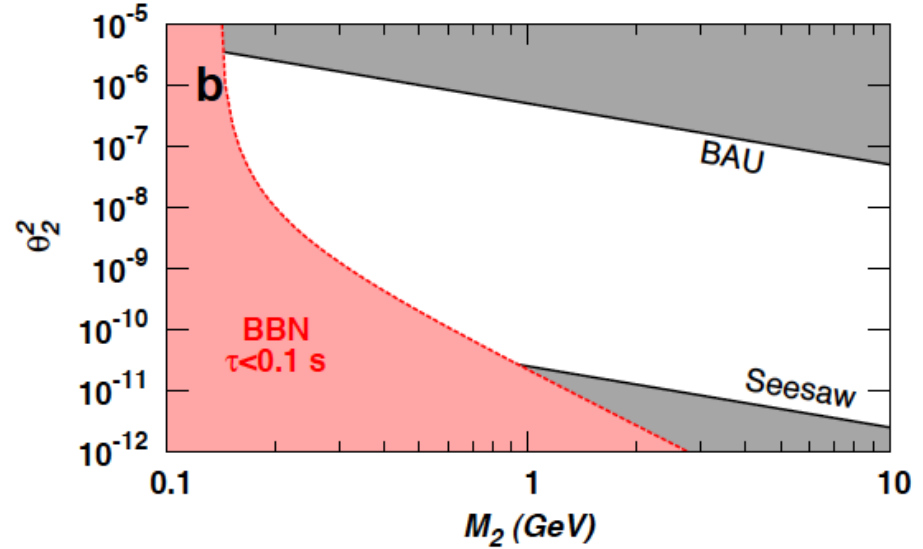
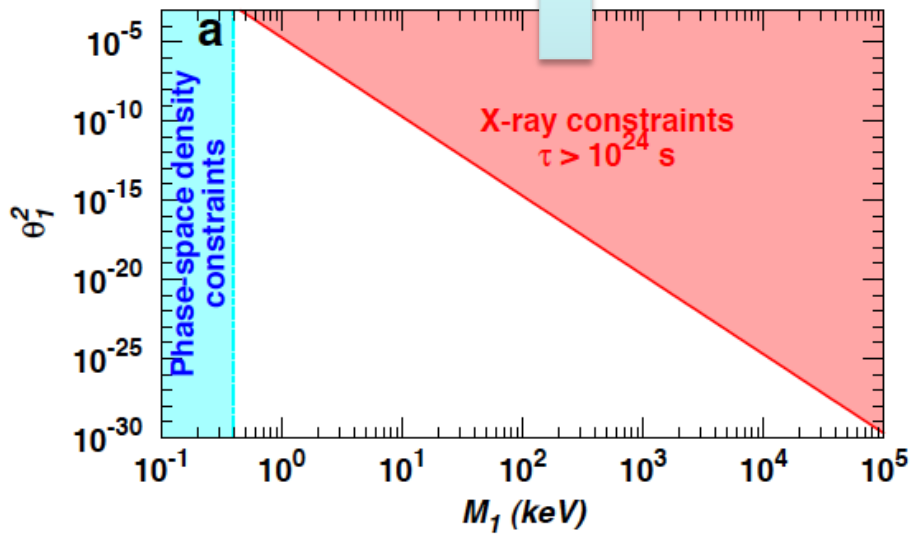


More than one sterile neutrino needed to reproduce Observed oscillations

ν MSM

Decaying N_1 produces narrow spectral line in spectra of DM dominated astrophysical objects

Boyarski, Ruchayskiy, Shaposhnikov...



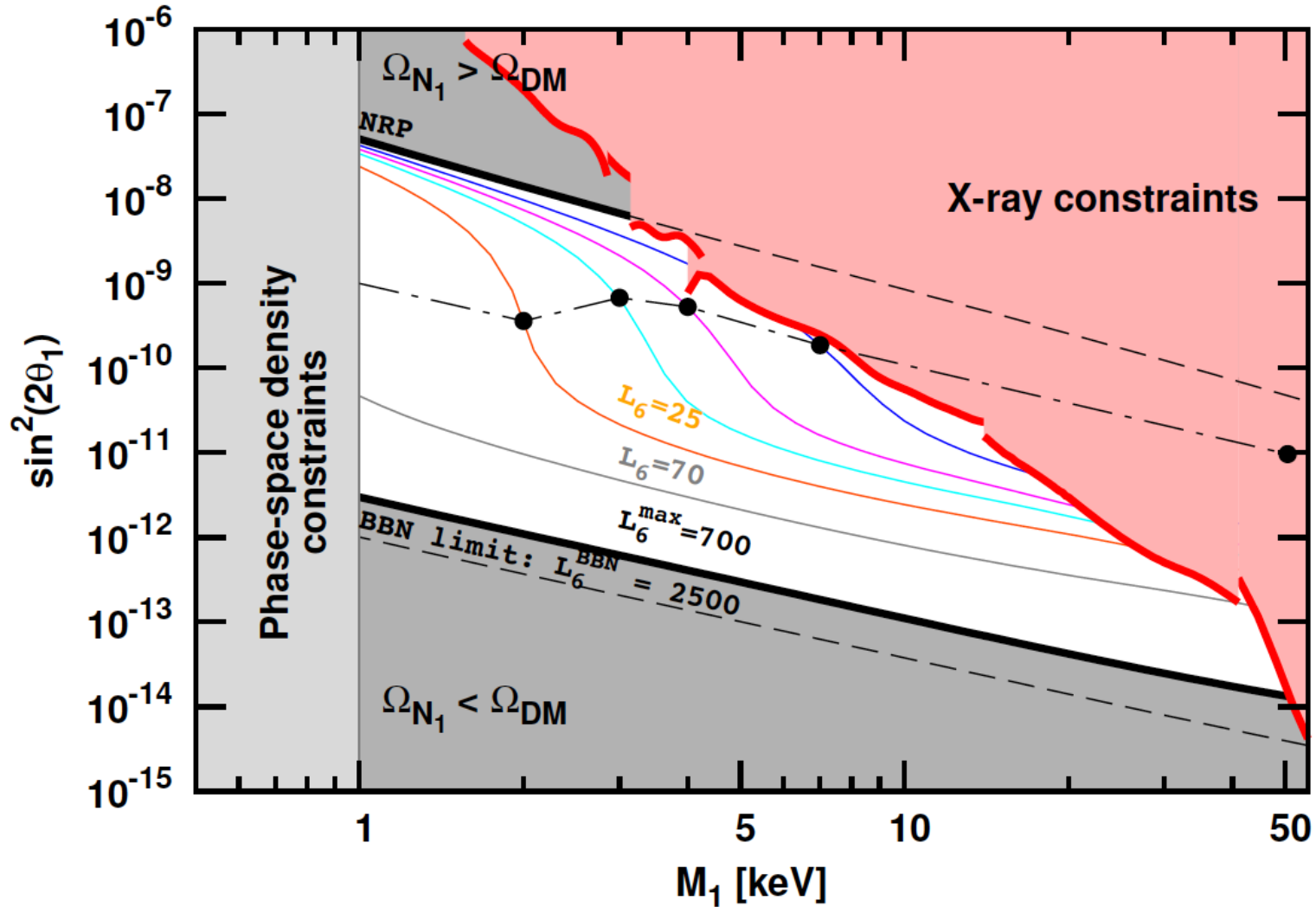
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vMSM

MODEL CONSISTENT WITH BBN, STRUCTURE FORMATION DATA IN THE UNIVERSE & ALL OTHER ASTROPHYSICAL CONSTRAINTS



MASS HIERARCHY ($N_1 \ll N_{2,3}$) AMONG STERILE NEUTRINOS

PHENOMENOLOGICAL...

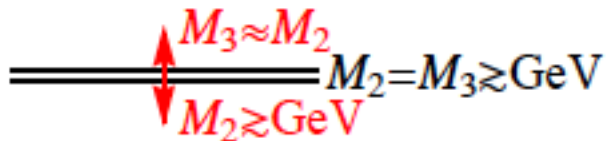
MICROSCOPIC EXPLANATIONS?

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PHENOMENOLOGICAL...

MICROSCOPIC EXPLANATIONS?

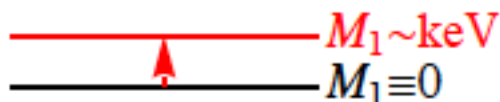
(I) **FLAVOUR SYMMETRIES** : $M_1 = 0$ if symmetry unbroken,
 Breaking of global
 Lepton symmetry generate
 singlet fermion mass hierarchy



$$L_e - L_\mu - L_\tau$$

~~$$L_e - L_\mu - L_\tau$$~~

Shaposhnikov
 Lindner, Merle, Niro



e.g. GUT models

Mohapatra, Senjanovic, Ross...

MASS HIERARCHY ($N_1 \ll N_{2,3}$) AMONG STERILE NEUTRINOS

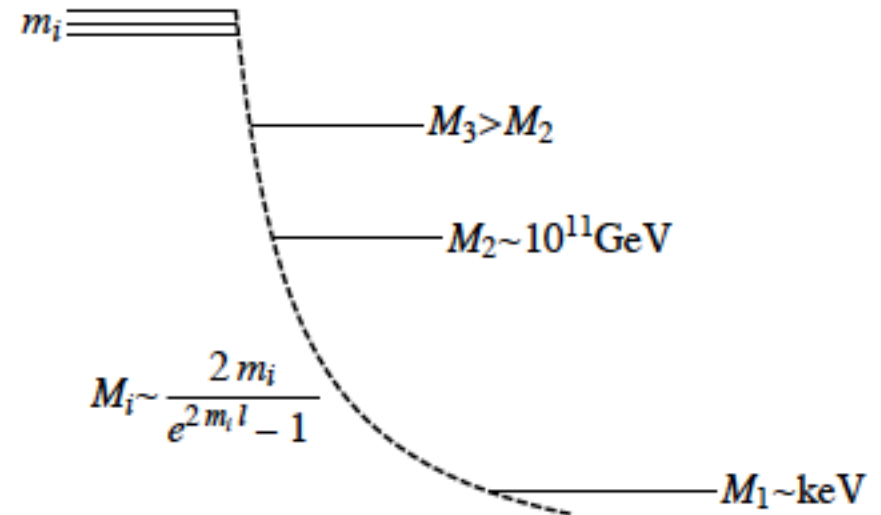
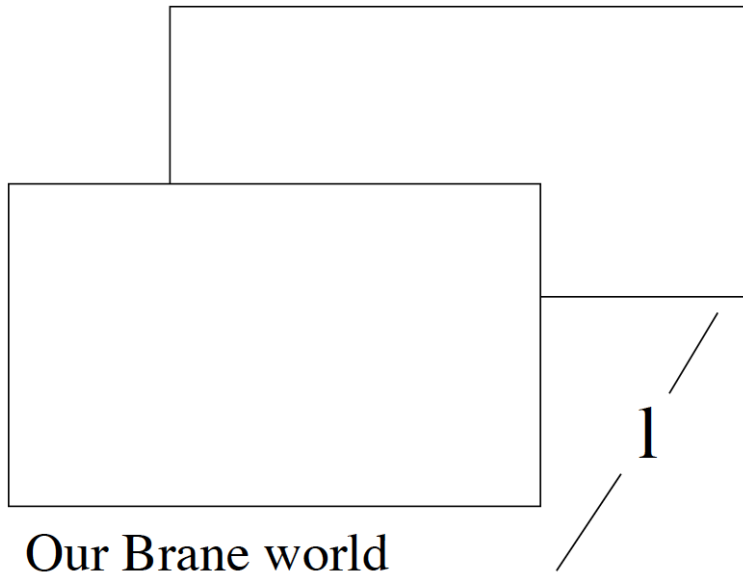
PHENOMENOLOGICAL...

MICROSCOPIC EXPLANATIONS?

(II) BRANE WORLD RANDALL-SUNDRUM MODELS: *Exponential Mass Suppression*

Kushenko,
Takahashi, Yanagida

Shadow world



MASS HIERARCHY ($N_1 \ll N_{2,3}$) AMONG STERILE NEUTRINOS

PHENOMENOLOGICAL...

MICROSCOPIC EXPLANATIONS?

(III) FROGATT – NIELSEN MECHANISM :

One fermion acquires mass via Higgs mechanism, others via higher order multiple see-saw

Merle Niro
Barry, Rodejohann, Zhang

$$\begin{aligned} \mathcal{L}_{\text{leptons}} = & -Y_e^{ij} \overline{e_{iR}} H L_{jL} \left(\frac{\Theta}{\Lambda}\right)^{k_i+f_j} + h.c. - Y_D^{ij} \overline{N_{iR}} \tilde{H} L_{jL} \left(\frac{\Theta}{\Lambda}\right)^{g_i+f_j} + h.c. \\ & - \frac{1}{2} \overline{N_{iR}} \tilde{M}_R^{ij} (N_{jR})^C \left(\frac{\Theta}{\Lambda}\right)^{g_i+g_j} + h.c. - \frac{1}{2} Y_L^{ij} \overline{(L_{iL})^C} (i\sigma_2 \Delta) L_{jL} \left(\frac{\Theta}{\Lambda}\right)^{f_i+f_j} + h.c. \end{aligned}$$

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$$- \frac{1}{2} \overline{N_{iR}} \tilde{M}_R^{ij} (N_{jR})^c \left(\frac{\Theta}{\Lambda}\right)^{g_i+g_j} + h.c. - \frac{1}{2} Y_L^{ij} \overline{(L_{iL})^c} (i\sigma_2 \Delta) L_{jL} \left(\frac{\Theta}{\Lambda}\right)^{f_i+f_j} + h.c.$$

“**Flavon**” field $\langle \Theta \rangle$

$\lambda = \frac{\langle \Theta \rangle}{\Lambda}$ being a small quantity of the order of the Cabibbo angle: $\lambda \simeq 0.22$

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Field	L_{iL}	$\overline{e_{iR}}$	$\overline{N_{iR}}$	H	Δ	Θ
SU(2) _L	<u>2</u>	<u>1</u>	<u>1</u>	<u>2</u>	<u>3</u>	<u>1</u>
U(1) _{FN}	f_i	k_i	g_i	0	0	-1

MASS MATRICES

Charged lepton

$$M_e = v \begin{pmatrix} Y_e^{11} \lambda^{|k_1+f_1|} & Y_e^{12} \lambda^{|k_1+f_2|} & Y_e^{13} \lambda^{|k_1+f_3|} \\ Y_e^{21} \lambda^{|k_2+f_1|} & Y_e^{22} \lambda^{|k_2+f_2|} & Y_e^{23} \lambda^{|k_2+f_3|} \\ Y_e^{31} \lambda^{|k_3+f_1|} & Y_e^{32} \lambda^{|k_3+f_2|} & Y_e^{33} \lambda^{|k_3+f_3|} \end{pmatrix}$$

Dirac neutrino

$$m_D = v \begin{pmatrix} Y_D^{11} \lambda^{|g_1+f_1|} & Y_D^{12} \lambda^{|g_1+f_2|} & Y_D^{13} \lambda^{|g_1+f_3|} \\ Y_D^{21} \lambda^{|g_2+f_1|} & Y_D^{22} \lambda^{|g_2+f_2|} & Y_D^{23} \lambda^{|g_2+f_3|} \\ Y_D^{31} \lambda^{|g_3+f_1|} & Y_D^{32} \lambda^{|g_3+f_2|} & Y_D^{33} \lambda^{|g_3+f_3|} \end{pmatrix}$$

MASS HIERARCHY ($N_1 \ll N_{2,3}$) AMONG STERILE NEUTRINOS

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Right-handed Neutrino sector

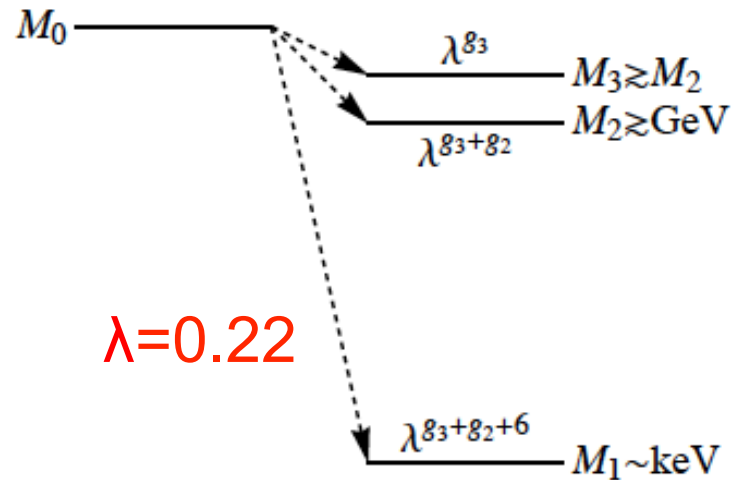
$$M_R = \begin{pmatrix} \tilde{M}_R^{11} \lambda^{|2g_1|} & \tilde{M}_R^{12} \lambda^{|g_1+g_2|} & \tilde{M}_R^{13} \lambda^{|g_1+g_3|} \\ \bullet & \tilde{M}_R^{22} \lambda^{|2g_2|} & \tilde{M}_R^{23} \lambda^{|g_2+g_3|} \\ \bullet & \bullet & \tilde{M}_R^{33} \lambda^{|2g_3|} \end{pmatrix}$$

then via see-saw

$$m_\nu^I = -m_D^T M_R^{-1} m_D = \begin{pmatrix} a_1 \lambda^{|2f_1|} & b_1 \lambda^{|f_1+f_2|} & c_1 \lambda^{|f_1+f_3|} \\ \bullet & d_1 \lambda^{|2f_2|} & e_1 \lambda^{|f_2+f_3|} \\ \bullet & \bullet & f_1 \lambda^{|2f_3|} \end{pmatrix}$$

MASS HIERARCHY ($N_1 \ll N_{2,3}$) AMONG

M_R	Eigenvalues
$\begin{pmatrix} A\lambda^6 & A\lambda^3 & A\lambda^3 \\ A\lambda^3 & A & B \\ A\lambda^3 & B & A \end{pmatrix}$	$M_1 = \mathcal{O}(\lambda^6) \simeq \mathcal{O}(\text{keV})$ $M_2 = A - B \simeq \mathcal{O}(\text{GeV})$ $M_3 = A + B \simeq \mathcal{O}(\text{GeV})$
$\begin{pmatrix} A\lambda^6 & A\lambda^3 & A\lambda^3 \\ A\lambda^3 & B & C \\ A\lambda^3 & C & B \end{pmatrix}$	$M_1 = \mathcal{O}(\lambda^6) \simeq \mathcal{O}(\text{keV})$ $M_2 = B - C \simeq \mathcal{O}(\text{GeV})$ $M_3 = B + C \simeq \mathcal{O}(\text{GeV})$
$\begin{pmatrix} A\lambda^6 & B\lambda^3 & B\lambda^3 \\ B\lambda^3 & C & D \\ B\lambda^3 & D & C \end{pmatrix}$	$M_1 = \mathcal{O}(\lambda^6) \simeq \mathcal{O}(\text{keV})$ $M_2 = C - D \simeq \mathcal{O}(\text{GeV})$ $M_3 = C + D \simeq \mathcal{O}(\text{GeV})$



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					0	-1

**BUT... presently
Lack of High Energy
(UV) COMPLETION**

$$\left. \begin{array}{l} |g_1 + g_3| \\ |g_2 + g_3| \\ |2g_3| \end{array} \right)$$

(IV) BEYOND SEE-SAW ? UV complete models?

ANOMALOUS GENERATION
OF RIGHT-HANDED MAJORANA
NEUTRINO MASSES THROUGH
TORSIONFUL QUANTUM GRAVITY

Mavromatos, Pilaftsis
arXiv: 1209.6387
(PRD 86, 124038 (2012))



- Field Theories with (Kalb-Ramond) torsion & axion fields : *String inspired models, loop quantum gravity effective field theories ...*

UV complete models

- Majorana Neutrino Masses from (three-loop) ***anomalous*** terms with axion-neutrino couplings

Microscopic UV complete underlying theory of quantum gravity :

STRINGS

Massless Gravitational multiplet of (closed) strings: spin 0 scalar (dilaton)
spin 2 traceless symmetric rank 2 tensor (graviton)
spin 1 antisymmetric rank 2 tensor

KALB-RAMOND FIELD $B_{\mu\nu} = -B_{\nu\mu}$

Effective field theories (low energy scale $E \ll M_s$) "gauge" invariant

$$B_{\mu\nu} \rightarrow B_{\mu\nu} + \partial_{[\mu}\theta(x)_{\nu]}$$

Depend only on field strength : $H_{\mu\nu\rho} = \partial_{[\mu}B_{\nu\rho]}$

Bianchi identity :

$$\partial_{[\sigma}H_{\mu\nu\rho]} = 0 \rightarrow d \star \mathbf{H} = 0$$

Anomaly (gravitational vs gauge) **cancellation** in strings require redefinition of H so that Bianchi identity now is extended to :

$$\mathbf{H} = \mathbf{d} \mathbf{B} + \frac{\alpha'}{8\kappa} \left(\Omega_L - \Omega_V \right)$$

$$\kappa^2 = 8\pi G_N = \frac{8\pi}{M_P^2}$$

Lorentz (L) & Gauge (V)
Chern-Simons three forms

EXTENDED BIANCHI IDENTITY

$$\mathbf{d} \mathbf{H} = \frac{\alpha'}{8\kappa} \text{Tr} \left(\mathbf{R} \wedge \mathbf{R} - \mathbf{F} \wedge \mathbf{F} \right)$$

ROLE OF H-FIELD AS TORSION

EFFECTIVE GRAVITATIONAL ACTION IN STRING LOW-ENERGY LIMIT CAN BE EXPRESSED IN TERMS OF **A GENERALIZED CURVATURE** RIEMANN TENSOR WHERE THE CHRISTOFFEL CONNECTION INCLUDES **TORSION** PROVIDED BY **H-FIELD**

4-DIM
PART

$$\begin{aligned} S^{(4)} &= \int d^4x \sqrt{-g} \left(\frac{1}{2\kappa^2} R - \frac{1}{6} H_{\mu\nu\rho} H^{\mu\nu\rho} \right) \\ &= \int d^4x \sqrt{-g} \left(\frac{1}{2\kappa^2} \bar{R} - \frac{1}{3} \kappa^2 H_{\mu\nu\rho} H^{\mu\nu\rho} \right) \end{aligned}$$

$$\bar{\Gamma}_{\nu\rho}^{\mu} = \Gamma_{\nu\rho}^{\mu} + \frac{\kappa}{\sqrt{3}} H_{\nu\rho}^{\mu} \neq \bar{\Gamma}_{\rho\nu}^{\mu}$$

IN 4-DIM DEFINE DUAL OF H AS $\mathbf{Y} = \star\mathbf{H}$

$$Y_\sigma = -3\sqrt{2}\partial_\sigma b = \sqrt{-g}\epsilon_{\mu\nu\rho\sigma}H^{\mu\nu\rho}$$

Pseudoscalar
(Kalb-Ramond (KR) axion)

IN 4-DIM DEFINE DUAL OF H AS $\mathbf{Y} = \star\mathbf{H}$

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REWRITE EXTENDED BIANCHI IDENTITY AS

$$\nabla_\sigma Y^\sigma = \frac{\alpha'}{32\kappa}\sqrt{-g}\epsilon_{\mu\nu\lambda\sigma}\left(R_{ad}{}^{\mu\nu}R^{\lambda\sigma ad} - F^{\mu\nu}F^{\lambda\sigma}\right)$$

HENCE EFFECTIVE ACTION

$$S^{(4)} = \int d^4x\sqrt{-g}\left[\frac{1}{2\kappa^2}R - \frac{1}{2}\partial_\mu b(x)\partial^\mu b(x) + \frac{\alpha'\sqrt{2}}{192\kappa}b(x)\epsilon_{\mu\nu\rho\lambda}\left(R_{ad}{}^{\mu\nu}R^{\rho\lambda ad} - F^{\mu\nu}F^{\rho\lambda}\right)\right]$$

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$bR\tilde{R}$ coupling

IN 4-DIM DEFINE DUAL OF H AS $Y = *H$

$$Y_\sigma = -3\sqrt{2}\partial_\sigma b = \sqrt{-g}\epsilon_{\mu\nu\rho\sigma}H^{\mu\nu\rho}$$

cf. axion-electromagnetic field coupling

$$a(x)F_{\mu\nu}\tilde{F}^{\mu\nu} = -F^{\mu\nu}F^{\lambda\sigma}$$

$$a(x)\vec{E} \cdot \vec{B}$$

$$+ \frac{\alpha'\sqrt{2}}{192\kappa} \left[b(x)\epsilon_{\mu\nu\rho\lambda} \left(R_{ad}^{\mu\nu} R^{\rho\lambda ad} - F^{\mu\nu}F^{\rho\lambda} \right) \right]$$

$bR\tilde{R}$ coupling

NB: Torsion Couples to fermions via gravitational covariant derivative \rightarrow integrating out torsion in path integral results in extra **fermion-fermion-axial current interactions**

$$\int Db \exp \left[-i \int \frac{1}{2} db \wedge * db - \frac{1}{f_b} b G(\mathbf{A}, \omega) + \frac{1}{2f_b^2} \mathbf{J}^5 \wedge \mathbf{J}^5 \right]$$

$$J_{\mu}^5 = \bar{\psi} \gamma_{\mu} \gamma_5 \psi \qquad f_b = (3\kappa^2/8)^{-1/2} = \frac{M_P}{\sqrt{3\pi}}$$

$$\begin{aligned} \nabla_{\mu} J^{5\mu} &= \frac{e^2}{8\pi^2} F^{\mu\nu} \tilde{F}_{\mu\nu} - \frac{1}{192\pi^2} R^{\mu\nu\rho\sigma} \tilde{R}_{\mu\nu\rho\sigma} \\ &\equiv G(\mathbf{A}, \omega) . \end{aligned}$$

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$$J_{\mu}^5 = \bar{\psi} \gamma_{\mu}$$

$$\nabla_{\mu} J^{5\mu} =$$

$$\equiv$$

above effective action
 generic... in loop quantum
 gravity *etc* \rightarrow b-field
 implements constraint
 of Bianchi identity (conservation of
 quantum torsion ``charge'')

$$= \frac{M_P}{\sqrt{3\pi}}$$

p) equation

ANOMALOUS MAJORANA NEUTRINO MASS TERMS from QUANTUM TORSION

SHIFT SYMMETRY $b(x) \rightarrow b(x) + c$

$c R^{\mu\nu\rho\sigma} \tilde{R}_{\mu\nu\rho\sigma}$ and $c F^{\mu\nu} \tilde{F}_{\mu\nu}$. total derivatives

OUR SCENARIO *Break* such *shift symmetry* by coupling first $b(x)$ to another pseudoscalar field such as QCD axion $a(x)$ (or e.g. other string axions)

$$\begin{aligned} \mathcal{S} = & \int d^4x \sqrt{-g} \left[\frac{1}{2} (\partial_\mu b)^2 + \frac{b(x)}{192\pi^2 f_b} R^{\mu\nu\rho\sigma} \tilde{R}_{\mu\nu\rho\sigma} \right. \\ & + \frac{1}{2f_b^2} J_\mu^5 J^{5\mu} + \gamma (\partial_\mu b) (\partial^\mu a) + \frac{1}{2} (\partial_\mu a)^2 \\ & \left. - y_a i a \left(\bar{\psi}_R^C \psi_R - \bar{\psi}_R \psi_R^C \right) \right], \end{aligned}$$

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Yukawa

neutrino fields

Field redefinition

$$b(x) \rightarrow b'(x) \equiv b(x) + \gamma a(x)$$

so, effective action becomes

$$\begin{aligned} \mathcal{S} = \int d^4x \sqrt{-g} & \left[\frac{1}{2} (\partial_\mu b')^2 + \frac{1}{2} (1 - \gamma^2) (\partial_\mu a)^2 \right. \\ & + \frac{1}{2f_b^2} J_\mu^5 J^{5\mu} + \frac{b'(x) - \gamma a(x)}{192\pi^2 f_b} R^{\mu\nu\rho\sigma} \tilde{R}_{\mu\nu\rho\sigma} \\ & \left. - y_a i a \left(\bar{\psi}_R^C \psi_R - \bar{\psi}_R \psi_R^C \right) \right]. \end{aligned}$$

must have

$$|\gamma| < 1$$

otherwise axion field $a(x)$ appears as a ghost \rightarrow canonically normalised kinetic terms

$$\begin{aligned} \mathcal{S}_a = \int d^4x \sqrt{-g} & \left[\frac{1}{2} (\partial_\mu a)^2 - \frac{\gamma a(x)}{192\pi^2 f_b \sqrt{1 - \gamma^2}} R^{\mu\nu\rho\sigma} \tilde{R}_{\mu\nu\rho\sigma} \right. \\ & \left. - \frac{i y_a}{\sqrt{1 - \gamma^2}} a \left(\bar{\psi}_R^C \psi_R - \bar{\psi}_R \psi_R^C \right) + \frac{1}{2f_b^2} J_\mu^5 J^{5\mu} \right]. \end{aligned}$$

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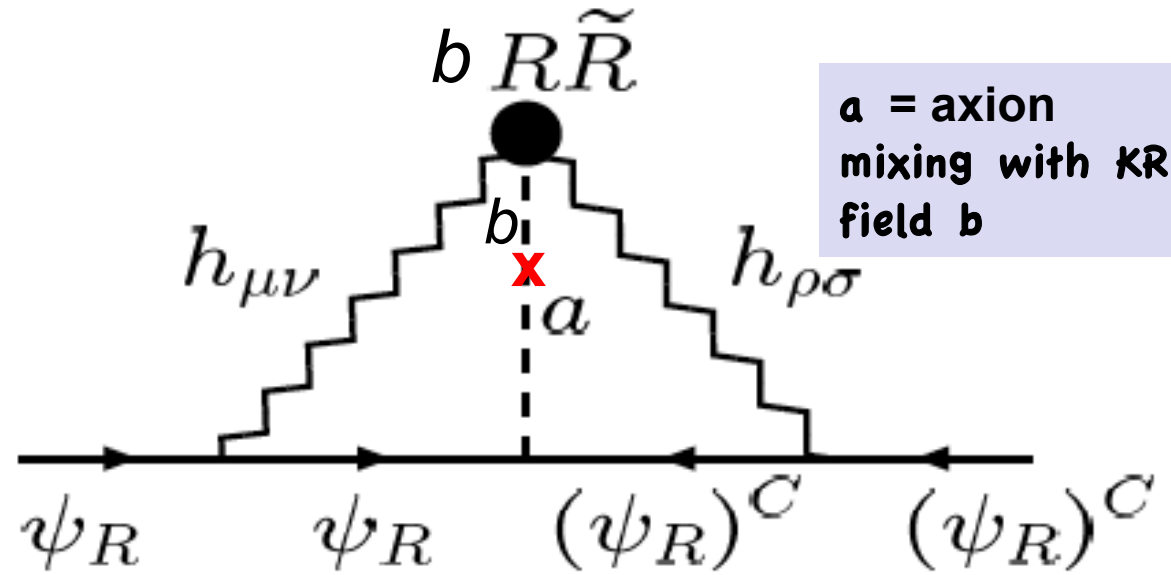
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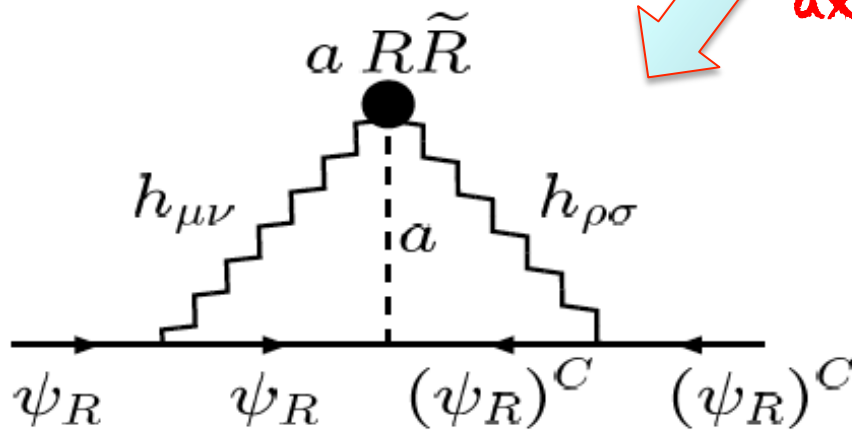
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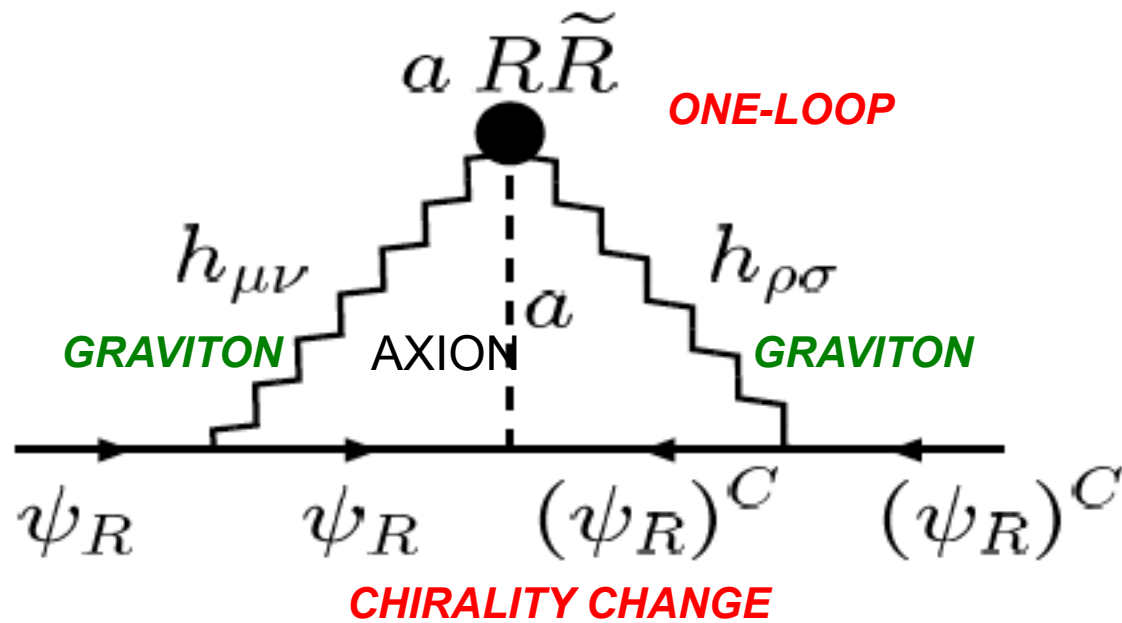
CHIRALITY CHANGE



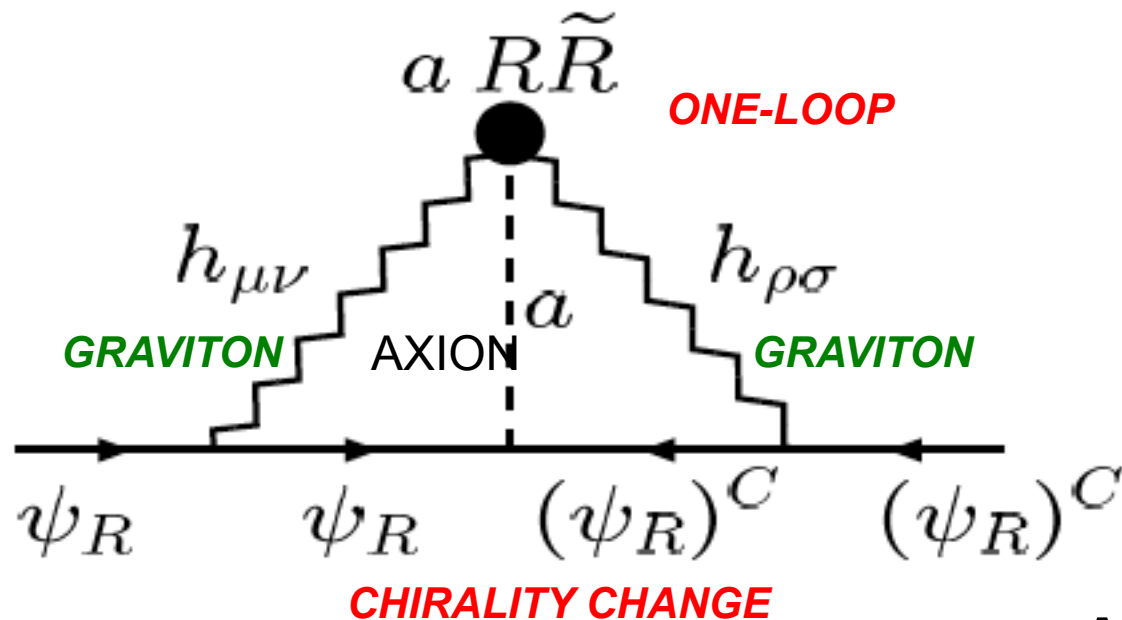
axion field redefinition



THREE-LOOP ANOMALOUS FERMION MASS TERMS



THREE-LOOP ANOMALOUS FERMION MASS TERMS



$\Lambda = \text{UV cutoff}$

$$M_R \sim \frac{1}{(16\pi^2)^2} \frac{y_a \gamma \kappa^4 \Lambda^6}{192\pi^2 f_b (1 - \gamma^2)} = \frac{\sqrt{3} y_a \gamma \kappa^5 \Lambda^6}{49152\sqrt{8} \pi^4 (1 - \gamma^2)}$$

SOME NUMBERS

$$\Lambda = 10^{17} \text{ GeV}$$

$$\gamma = 0.1$$

M_R is at the TeV

for $y_a = 10^{-3}$

$$\Lambda = 10^{16} \text{ GeV}$$

$M_R \sim 16 \text{ keV}$,

$y_a = \gamma = 10^{-3}$

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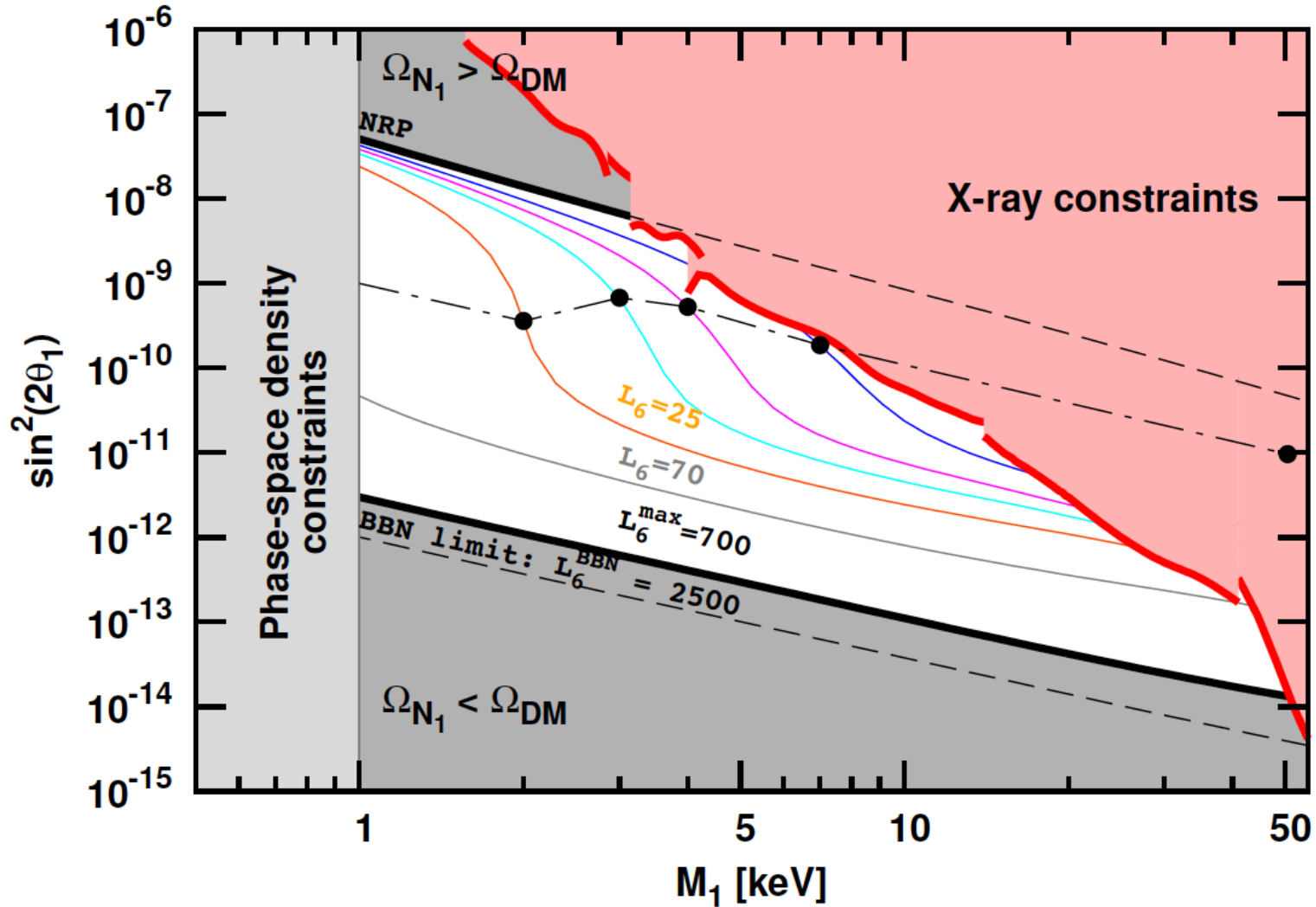
$y_a = \gamma = 10^{-3}$

**INTERESTING
WARM DARK MATTER
REGIME**

Appropriate Hierarchy for the other two massive Right-handed neutrinos for Leptogenesis-Baryogenesis & Dark matter constraints can be arranged by choosing Yukawa couplings

vMSM

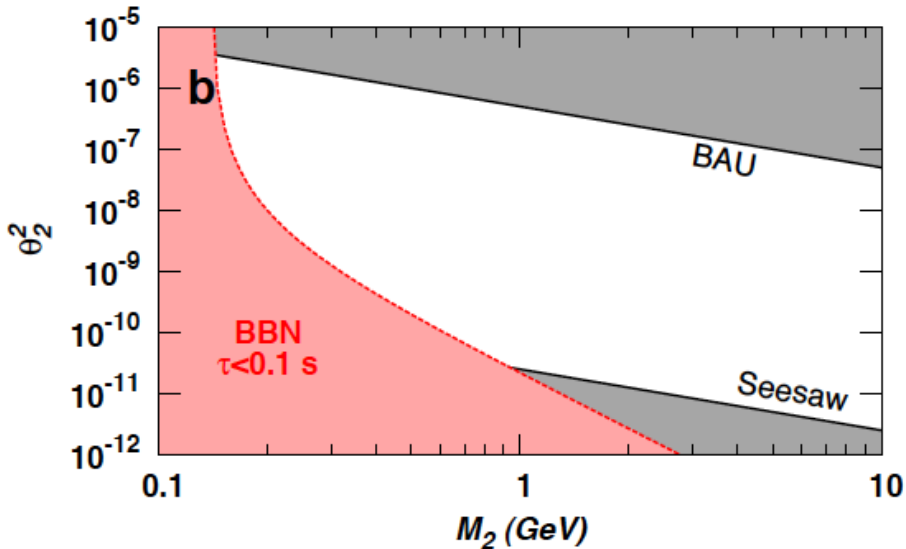
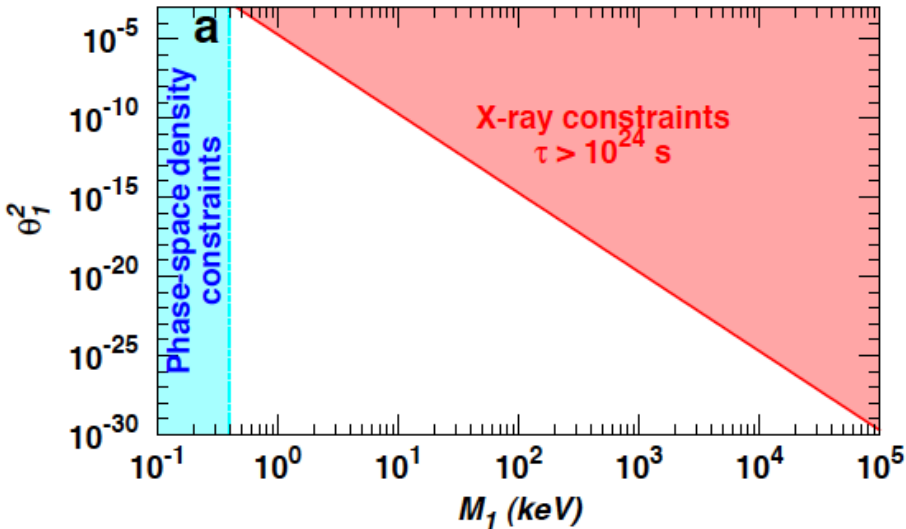
MODEL CONSISTENT WITH BBN, STRUCTURE FORMATION DATA IN THE UNIVERSE & ALL OTHER ASTROPHYSICAL CONSTRAINTS



More than one sterile neutrino needed to reproduce Observed oscillations

ν MSM

Boyarski, Ruchayskiy, Shaposhnikov...



Constraints on two heavy degenerate singlet neutrinos

N_1 DM production estimation in Early Universe must take into account its interactions with $N_{2,3}$ heavy neutrinos



FINITENESS OF THE MASS

MULTI-AXION SCENARIOS (e.g. string axiverse)

$$\mathcal{S}_a^{\text{kin}} = \int d^4x \sqrt{-g} \left[\frac{1}{2} \sum_{i=1}^n \left((\partial_\mu a_i)^2 - M_i^2 \right) + \gamma (\partial_\mu b) (\partial^\mu a_1) - \frac{1}{2} \sum_{i=1}^{n-1} \delta M_{i,i+1}^2 a_i a_{i+1} \right];$$

$$\delta M_{i,i+1}^2 < M_i M_{i+1} \quad \text{positive mass spectrum for all axions}$$

simplifying all mixing equals

$$M_R \sim \frac{\sqrt{3} y_a \gamma \kappa^5 \Lambda^{6-2n} (\delta M_a^2)^n}{49152 \sqrt{8} \pi^4 (1 - \gamma^2)} \quad n \leq 3$$

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M_R : UV finite for n=3 @ 2-loop independent of axion mass

In this **gravitationally-induced** right-handed neutrino mass scenario the ordinary (**active**) neutrinos are supposed to acquire their **Majorana masses** via standard Yukawa couplings & see-saw type mechanisms



$$y_e \bar{\nu}_{eR} \left(i\tau_2 \phi^* \right)^\dagger \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} + \text{h.c.} + \text{other flavours}$$

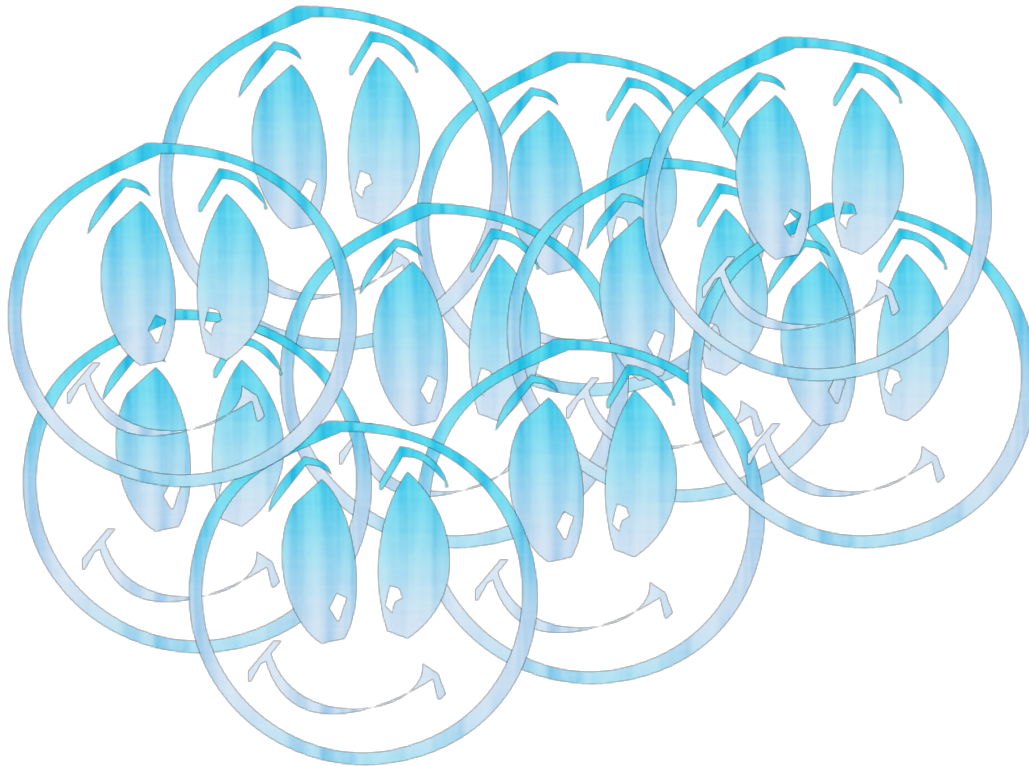
More thoughts and detailed analyses required...

COSMIC NEUTRINO CONDENSATES & DARK ENERGY

*Formation of fermion condensates dynamically in Early Universe
as in Nambu-Jona-Lasinio model*

Consider Models of (effective) four-fermion interactions of sterile Majorana neutrino in the early Universe

Kapusta, Antusch,
Kersten, Lindner, Ratz,
Barenboim, Rasero, Bhatt,
Desai, Ma, Rajasekaran,
U. Sarkar, NEM,



COSMIC NEUTRINO CONDENSATES & DARK ENERGY

Formation of fermion **condensates** dynamically in Early Universe as in Nambu-Jona-Lasinio model

Consider Models of (effective) four-fermion interactions of **light** sterile Majorana neutrino in the early Universe

Kapusta, Antusch, Kersten, Lindner, Ratz, Barenboim, **Bhatt, Desai, Ma, Rajasekaran, U. Sarkar** NEM,

$$H_I = -C (\bar{\nu}_M \nu_M) (\bar{\nu}_M \nu_M) \quad \text{e.g. through heavy scalar exchange} \quad C = \frac{f^2}{m_S^2}$$

$$\nu_M = \lambda \nu_R + \nu^c_L$$

One light ($O(10^{-3})$ eV) sterile Majorana neutrino forms the condensate

$$M_\nu = \begin{pmatrix} m_1 & 0 & 0 & 0 \\ 0 & m_2 & 0 & 0 \\ 0 & 0 & 0 & m_3 \\ 0 & 0 & m_3 & M \end{pmatrix}$$

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Dirac light masses of $O(0.1)$ eV

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pseudo Dirac light neutrino

Model consistent with solar neutrino data

COSMIC NEUTRINO CONDENSATES & DARK ENERGY

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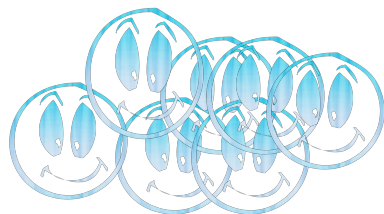
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$\nu_M = \lambda \nu_R + \nu^c_L$ **One light sterile Majorana neutrino forms condensate**

$$\nu_M = \begin{bmatrix} \chi \\ \lambda \bar{\chi} \end{bmatrix}$$

$$H_1^{MF} = -2 C \left[\lambda^{*2} \bar{\chi}_a^\dagger \chi_b D + \lambda^2 \chi_a^\dagger \bar{\chi}_b D^* \right] \epsilon_{ab}.$$



$$\langle \chi_a \bar{\chi}_b^\dagger \rangle = \epsilon_{ab} D$$



Coherence length,
Gap Equation in
FRW backgrounds
Dark Energy contribution

COSMIC NEUTRINO CONDENSATES & DARK ENERGY

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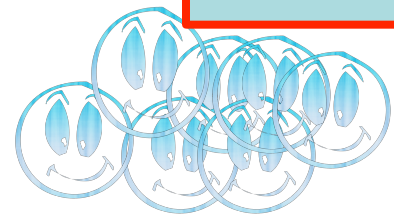
Better UV behaviour than flat space e.g. in de Sitter space time absorb UV infinities in Hubble H, and Planck scale M_P
Candelas, Reine (1975)

neutrino forms condensate

$[\dots + \lambda^2 \chi_a^\dagger \bar{\chi}_b D^*] \epsilon_{ab}$

coherence length, Gap Equation in FRW backgrounds **Dark Energy** contribution

$\langle \chi_a \bar{\chi}_b^\dagger \rangle = \epsilon_{ab} D$



PART III
CPT VIOLATION IN
THE
EARLY UNIVERSE
&
NEUTRINOS

CPT VIOLATION IN THE EARLY UNIVERSE

***GENERATE Baryon and/or Lepton ASYMMETRY
without Heavy Sterile Neutrinos?***

CPT Invariance Theorem :

- (i) Flat space-times
- (ii) Lorentz invariance
- (iii) Locality
- (iv) Unitarity

**Schwinger, Pauli,
Luders, Jost, Bell
revisited by:
Greenberg,
Chaichian, Dolgov,
Novikov...**

(ii)-(iv) Independent reasons for violation

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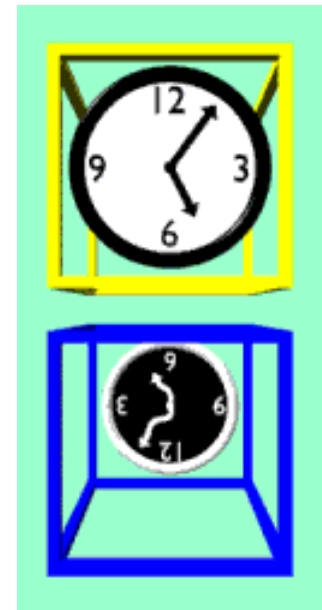
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CPT VIOLATION IN THE EARLY UNIVERSE

**GENERATE Baryon and/or Lepton ASYMMETRY
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Assume CPT Violation.
e.g. due to **Quantum Gravity** fluctuations,
strong in the Early Universe



physics.indiana.edu

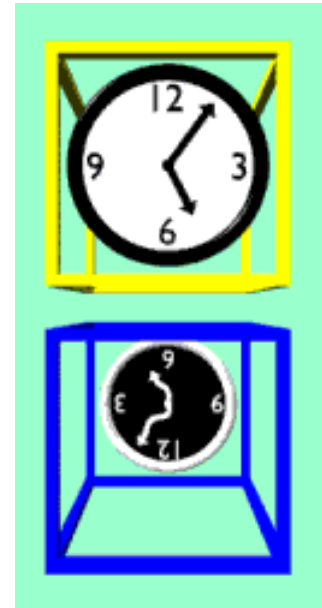
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ONE POSSIBILITY:
particle-antiparticle mass differences

$$m \neq \bar{m}$$



physics.indiana.edu

Equilibrium Distributions different between particle-antiparticles
Can these create the observed matter-antimatter asymmetry?

$$f(E, \mu) = \frac{1}{\exp[(E - \mu)/T] \pm 1} \quad m \neq \bar{m}$$
$$\delta m = m - \bar{m}$$

$$\delta n \equiv n - \bar{n} = g_{df} \int \frac{d^3p}{(2\pi)^3} [f(E, \mu) - f(\bar{E}, \bar{\mu})]$$

$$E = \sqrt{p^2 + m^2}, \bar{E} = \sqrt{p^2 + \bar{m}^2}$$

Dolgov, Zeldovich
Dolgov (2009)

Assume dominant contributions to Baryon asymmetry from quarks-antiquarks

$$m(T) \sim gT$$



High-T quark mass >> Lepton mass

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Assuming dominant contributions to Baryon asymmetry from quarks-antiquarks

$$\beta_T = \frac{n_B}{n_\gamma} = -8.4 \cdot 10^{-3} (18m_u \delta m_u + 15m_d \delta m_d) / T^2$$

Dolgov, Zeldovich
 Dolgov (2009)

$$n_\gamma = 0.24T^3 \quad \text{photon equilibrium density at temperature } T$$

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$$n_\gamma = 0.24T^3$$

Dolgov (2009)

Current bound
for proton-anti
proton mass diff.

$$\delta m_p < 7 \cdot 10^{-10} \text{ GeV}$$

ASACUSA Coll. (2011)

Reasonable to take: $\delta m_q \sim \delta m_p$  **Too small**
 $\beta^{T=0}$

NB: To reproduce the observed $\beta^{(T=0)} = 6 \cdot 10^{-10}$ need

$$\delta m_q(T = 100 \text{ GeV}) \sim 10^{-5} - 10^{-6} \text{ GeV} \gg \delta m_p$$

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**CPT Violating quark-antiquark Mass difference
alone CANNOT REPRODUCE observed BAU**



**GRAVITATIONALLY-
INDUCED
CPT VIOLATION**



GRAVITATIONAL BACKGROUNDS
GENERATING **CPT VIOLATING EFFECTS**
IN THE EARLY UNIVERSE:
PARTICLE-ANTIPARTICLE DIFFERENCES
IN DISPERSION RELATIONS →

Differences in populations

→ freeze out → Baryogenesis or
→ Leptogenesis → Baryogenesis

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Gravitational Baryogenesis

Davoudiasl, Kitano, Kribs,
Murayama, Steinhardt

Quantum Gravity (or something else (e.g. SUGRA)) may lead at low-energies (below Planck scale or a scale M_*) to a term in the effective Lagrangian (in curved back space-time backgrounds):

$$\frac{1}{M_*^2} \int d^4x \sqrt{-g} (\partial_\mu \mathcal{R}) J^\mu$$

Generation (flavour) #

$$\partial_\mu J_\mu^B = \partial_\mu J_\mu^L = \frac{n_f}{32\pi^2} \text{Tr} F_{\mu\nu} \tilde{F}_{\mu\nu} + U(1) \text{ part} \\ \text{SU}(2)$$

Current e.g.
baryon-number J_μ^B
current
(non-conserved
in Standard Model
due to anomalies)

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Term Violates CP but is CPT conserving *in vacuo*
It **Violates CPT** in the background space-time of an
expanding FRW Universe



$$\dot{\mathcal{R}} = -(1 - 3w) \frac{\dot{\rho}}{M_P^2} = \sqrt{3} (1 - 3w)(1 + w) \frac{\rho^{3/2}}{M_P^3}$$

Energy differences between particle vs antipartic $\pm \dot{\mathcal{R}}/M_*^2$: **Dynamical CPTV**

**LIKE A CHEMICAL
POTENTIAL FOR FERMIONS**

Gravitational Baryogenesis

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Energy differences between particle vs antipartic $\pm \dot{\mathcal{R}}/M_*^2$: **Dynamical CPTV**

Baryon Asymmetry $\frac{n_B}{s} \approx \left. \frac{\dot{\mathcal{R}}}{M_*^2 T} \right|_{T_D}$ Calculate for various w in some scenarios

@ $T < T_D$,
 $T_D = \text{Decoupling } T$

GRAVITATIONAL BACKGROUNDS
GENERATING **CPT VIOLATING EFFECTS**
IN THE EARLY UNIVERSE:
**PARTICLE-ANTIPARTICLE DIFFERENCES
IN DISPERSION RELATIONS** →

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REVIEW VARIOUS SCENARIOS

GRAVITATIONAL BACKGROUNDS
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B-L conserving GUT
or
Sphaleron

REVIEW VARIOUS SCENARIOS

NB: (

Standard Thermal Leptogenesis

Independent of
Initial Conditions
@ $T \gg T_{eq}$

Heavy Right-handed Majorana neutrinos enter *equilibrium at $T = T_{eq} > T_{decay}$*

Lepton number
Violation

$$N_I \rightarrow H\nu, \bar{H}\bar{\nu}$$

Out of Equilibrium Decays

$$T \simeq T_{decay} > T_{sph}$$



enhanced CP V

Produce Lepton asymmetry

Equilibrated electroweak
B+L violating sphaleron
interactions

*Independent of Initial
Conditions*

**Observed Baryon Asymmetry
In the Universe (BAU)**

$$L = \frac{2}{M} l_L l_L \phi \phi + \text{H.c.}$$

where

$$l_L = \begin{bmatrix} \nu_e \\ e \end{bmatrix}_L, \begin{bmatrix} \nu_\mu \\ \mu \end{bmatrix}_L, \begin{bmatrix} \nu_\tau \\ \tau \end{bmatrix}_L$$

**Estimate BAU by solving Boltzmann equations
for Heavy Neutrino Abundances**

CPT Violating Thermal Leptogenesis

Early Universe
 $T > 10^{15}$ GeV

CPT Violation



No need for enhanced CPV
from Heavy Right-handed
Majorana neutrinos?

already in thermal
equilibrium

Produce Lepton asymmetry

Equilibrated electroweak
B+L violating sphaleron
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Independent of Initial
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Estimate BAU by fixing CPTV background parameters
In some models this may imply fine tuning

NB:)

2. CPTV Effects of different Space-Time-Curvature/Spin couplings between neutrinos/antineutrinos

B. Mukhopadhyay, U. Debnath, N. Dadhich, M. Sinha Lambiase, Mohanty

Curvature Coupling to **fermion spin** may lead to different dispersion relations between neutrinos and antineutrinos (assumed **dominant** in the Early eras) in **non-spherically symmetric** geometries in the Early Universe.

Dirac Lagrangian

$$\mathcal{L} = \sqrt{-g} (i \bar{\psi} \gamma^a D_a \psi - m \bar{\psi} \psi)$$

$$D_a = \left(\partial_a - \frac{i}{4} \omega_{bca} \sigma^{bc} \right),$$

Gravitational covariant derivative including spin connection

$$\sigma^{ab} = \frac{i}{2} [\gamma^a, \gamma^b]$$

$$\omega_{bca} = e_{b\lambda} (\partial_a e_c^\lambda + \Gamma_{\gamma\mu}^\lambda e_c^\gamma e_a^\mu).$$

$$\mathcal{L} = \mathcal{L}_f + \mathcal{L}_I = \sqrt{-g} \bar{\psi} [(i \gamma^a \partial_a - m) + \gamma^a \gamma^5 B_a] \psi,$$

$$B^d = \epsilon^{abcd} e_{b\lambda} (\partial_a e_c^\lambda + \Gamma_{\alpha\mu}^\lambda e_c^\alpha e_a^\mu)$$

Dirac Lagrangian (for concreteness, it can be extended to Majorana neutrinos)

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for the Majorana neutrinos, above \mathcal{L}_I turns out explicitly as

$$\mathcal{L}_I = \psi_L^\dagger \gamma^a \psi_L B_a, \quad \mathcal{L}_I = -\psi_L^c \dagger \gamma^a \psi_L^c B_a$$

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For homogeneous and isotropic **Friedman-Robertson-Walker** geometries the resulting B^μ **vanish**

Dirac Lagrangian (for concreteness, it can be extended to Majorana neutrinos)

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$$B^d = \epsilon^{abcd} e_{b\lambda} (\partial_a e_c^\lambda + \Gamma_{\alpha\mu}^\lambda e_c^\alpha e_a^\mu)$$

Can be constant in a given local frame in Early Universe
axisymmetric (Bianchi) cosmologies
 or **near rotating Black holes**



DISPERSION RELATIONS OF NEUTRINOS ARE **DIFFERENT**
FROM THOSE OF ANTINEUTRINOS IN **SUCH** GEOMETRIES



$(p_a \pm B_a)^2 = m^2$, \pm refers to chiral fields (here neutrino/antineutrino)

CPTV Dispersion relations

$$E = \sqrt{(\vec{p} - \vec{B})^2 + m^2} + B_0, \quad \bar{E} = \sqrt{(\vec{p} + \vec{B})^2 + m^2} - B_0$$

but (bare) masses are equal between particle/anti-particle sectors

NOT the Effective fermion/antifermion masses ($p=0$) $m_{F\pm}^{\text{eff}} = m \pm B^0$

Abundances of neutrinos in Early Universe, then, **different** from those of antineutrinos if B_0 is **non-trivial**.

Abundances of neutrinos in Early Universe *different* from those of antineutrinos
if $B_0 \neq 0$

$$\Delta n = \frac{g}{(2\pi)^3} \int d^3\mathbf{p} \left[\frac{1}{1 + \exp(E_\nu/T)} - \frac{1}{1 + \exp(E_{\bar{\nu}}/T)} \right]$$

$$\Delta n = \frac{g}{(2\pi)^2} T^3 \int_0^\infty \int_0^\pi \left[\frac{1}{1 + e^u e^{B_0/T}} - \frac{1}{1 + e^u e^{-B_0/T}} \right] u^2 d\theta du$$

$$u = |\vec{p}|/T$$

$$\Delta n_\nu \equiv n_\nu - n_{\bar{\nu}} \sim g^* T^3 \left(\frac{B_0}{T} \right)$$

with g^* the number of degrees of freedom for the (relativistic) neutrino.

Case I: BARYOGENESIS VIA GUT LEPTOGENESIS

$$\Delta n_\nu \equiv n_\nu - n_{\bar{\nu}} \sim g^* T^3 \left(\frac{B_0}{T} \right)$$

@ $T = T_d$ (decoupling Temp. of Lepton number (L) Violating processes) there is a **constant ratio** of net neutrino/antineutrino asymmetry (ΔL) to entropy density ($\sim T^3$)

$$\Delta L(T < T_d) = \frac{\Delta n_\nu}{s} \sim \frac{B_0}{T_d}$$

for $T_d \sim 10^{15}$ GeV and $B_0 \sim 10^5$ GeV

$\Delta L \sim 10^{-10}$, in agreement with observations (**Leptogenesis**)

Communicated to Baryon sector, and thus generates BAU either via a B-L conserving symmetry as in GUT models or via B + L conserving sphaleron processes \rightarrow **BARYOGENESIS**

Case II : Black-Hole induced neutrino-antineutrino population difference

$$\Delta n = \frac{g}{(2\pi)^3} \int_{R_i}^{R_f} dV \int d^3|\vec{p}| \left[\frac{1}{1 + \exp(E_\nu/T)} - \frac{1}{1 + \exp(E_{\bar{\nu}}/T)} \right]$$

Consider Kerr black holes for which $\vec{B} \cdot \vec{p} \ll B_0 p^0$ and show that

$$\Delta n = \frac{g}{(2\pi)^2} T^3 \int_{R_i}^{R_f} \int_0^\infty \int_0^\pi \left[\frac{1}{1 + e^u e^{B_0/T}} - \frac{1}{1 + e^u e^{-B_0/T}} \right] u^2 d\theta du dV$$

$$u = |\vec{p}|/T$$

Then, if $B^0 \ll T$

$$\Delta n \sim g T^3 \left(\frac{\overline{B_0}}{T} \right)$$

averaged over
the spatial
volume V

$$\Delta n \sim g T^3 \left(\frac{\overline{B_0}}{T} \right)$$

REMARKS

Asymmetry depends on the sign of B^0



PRIMORDIAL BLACK HOLES WITH MASSES $M_{\text{BH}} < 10^{15}$ gm have evaporated today, only BH with masses $M_{\text{BH}} > 10^{15}$ gm may survive today

Hawking temperature $T = \frac{\hbar}{8\pi k_B M} \sim 10^{-7} K \left(\frac{M_\odot}{M} \right)$

$T \sim 10^{11}$ K $\sim 1.6 \times 10^{-5}$ erg, $\overline{B_0} \sim 1.6 \times 10^{-6}$ erg, then $\Delta n \sim 10^{-16}$.

To reproduce observed Baryon asymmetry $\Delta n = O(10^{-10})$ we need 10^6 BH with the same sign of $B^0 \rightarrow$ **fine tuning**

3. Fermions in Gravity with TORSION

Dirac Lagrangian (for concreteness, it can be extended to Majorana neutrinos)

$$\mathcal{L} = \sqrt{-g} (i \bar{\psi} \gamma^a D_a \psi - m \bar{\psi} \psi)$$

$$D_a = \left(\partial_a - \frac{i}{4} \omega_{bca} \sigma^{bc} \right),$$

Gravitational covariant derivative including spin connection

$$\sigma^{ab} = \frac{i}{2} [\gamma^a, \gamma^b]$$

$$\omega_{bca} = e_{b\lambda} (\partial_a e_c^\lambda + \Gamma_{\gamma\mu}^\lambda e_c^\gamma e_a^\mu).$$



If torsion then $\Gamma_{\mu\nu}^\lambda \neq \Gamma_{\nu\mu}^\lambda$
antisymmetric part is the
 contorsion tensor, contributes to

$$e^\mu_a e^\nu_b \eta^{ab} = g^{\mu\nu}$$

vielbeins (tetrads)
independent from
spin connection ω_μ^{ab}
now

$$B^d = \epsilon^{abcd} e_{b\lambda} (\partial_a e_c^\lambda + \Gamma_{\alpha\mu}^\lambda e_c^\alpha e_a^\mu)$$

ROLE OF Kalb-Ramond H-FIELD AS TORSION

EFFECTIVE GRAVITATIONAL ACTION IN STRING LOW-ENERGY LIMIT CAN BE EXPRESSED IN TERMS OF **A GENERALIZED CURVATURE** RIEMANN TENSOR WHERE THE CHRISTOFFEL CONNECTION INCLUDES **H-FIELD TORSION**

4-DIM
PART

$$\begin{aligned} S^{(4)} &= \int d^4x \sqrt{-g} \left(\frac{1}{2\kappa^2} R - \frac{1}{6} H_{\mu\nu\rho} H^{\mu\nu\rho} \right) \\ &= \int d^4x \sqrt{-g} \left(\frac{1}{2\kappa^2} \bar{R} - \frac{1}{3} \kappa^2 H_{\mu\nu\rho} H^{\mu\nu\rho} \right) \end{aligned}$$

$$\bar{\Gamma}_{\nu\rho}^{\mu} = \Gamma_{\nu\rho}^{\mu} + \frac{\kappa}{\sqrt{3}} H_{\nu\rho}^{\mu} \neq \bar{\Gamma}_{\rho\nu}^{\mu}$$

IN 4-DIM DEFINE DUAL OF H AS :

$$-3 \sqrt{2} \partial_{\sigma} b = \sqrt{-g} \epsilon_{\mu\nu\rho\sigma} H^{\mu\nu\rho}$$

**$b(x)$ = Pseudoscalar
(Kalb-Ramond (KR) axion)**

FERMIONS COUPLE TO H-TORSION VIA GRAVITATIONAL COVARIANT DERIVATIVE

$$S_\psi = \frac{i}{2} \int d^4x \sqrt{-g} \left(\bar{\psi} \gamma^\mu \bar{\mathcal{D}}_\mu \psi - (\bar{\mathcal{D}}_\mu \bar{\psi}) \gamma^\mu \psi \right)$$

TORSIONFUL CONNECTION, FIRST-ORDER FORMALISM

$$\bar{\mathcal{D}}_\mu = \bar{\nabla}_\mu - ieA_\mu$$

gauge field

$$\bar{\omega}_{ab\mu} = \omega_{ab\mu} + K_{ab\mu}$$

contorsion

$$K_{abc} = \frac{1}{2} \left(T_{cab} - T_{abc} - T_{bca} \right)$$

$$H_{cab}$$

Non-trivial contributions to B^μ

$$B^d = \epsilon^{abcd} e_{b\lambda} \left(\partial_a e_c^\lambda + \Gamma_{\alpha\mu}^\lambda e_c^\alpha e_a^\mu \right)$$

$$\bar{\Gamma}_{\nu\rho}^\mu = \Gamma_{\nu\rho}^\mu + \frac{\kappa}{\sqrt{3}} H_{\nu\rho}^\mu \neq \bar{\Gamma}_{\rho\nu}^\mu$$

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Constant H?

$$\bar{\Gamma}_{\nu\rho}^\mu = \Gamma_{\nu\rho}^\mu + \frac{\kappa}{\sqrt{3}} H_{\nu\rho}^\mu \neq \bar{\Gamma}_{\rho\nu}^\mu$$

Non-trivial contributions to B^μ

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Exact (conformal Field Theories on World-sheet) Solutions from String theory

Antoniadis, Bachas, Ellis, Nanopoulos

Cosmological Solutions, non-trivial time-dependent dilatons, axions

In Einstein frame E (Scalar curvature term in gravitational effective action has canonical normalisation):

$$ds^2 = g_{\mu\nu}^E(x) dx^\mu dx^\nu = dt^2 - a(t)^2 \delta_{ij} dx^i dx^j$$

$$a(t) = t$$

$$\Phi = -\ln a(t) + \phi_0$$

$$H_{\mu\nu\rho} = e^{2\Phi} \epsilon_{\mu\nu\rho\sigma} \partial^\sigma b(x) \quad b(x) = \sqrt{2} e^{-\phi_0} \sqrt{Q^2} \frac{M_s}{\sqrt{n}} t$$

Central charge of underlying world-sheet conformal field theory $n \in \mathbb{Z}^+$

$$c = 4 - 12Q^2 - \frac{6}{n+2} + c_I$$

“internal” dims
central charge

Kac-Moody
algebra level

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H-torsion & CPTV

Covariant Torsion tensor

$$\bar{\Gamma}^{\lambda}_{\mu\nu} = \Gamma^{\lambda}_{\mu\nu} + e^{-2\Phi} H^{\lambda}_{\mu\nu} \equiv \Gamma^{\lambda}_{\mu\nu} + T^{\lambda}_{\mu\nu}$$

$$T_{ijk} \sim \epsilon_{ijk} \dot{b} = \epsilon_{ijk} \sqrt{2Q^2} e^{-\phi_0} \frac{M_s}{\sqrt{n}}$$

Constant

$$B^0 \sim \sqrt{2Q^2} e^{-\phi_0} \frac{M_s}{\sqrt{n}} \text{ GeV} > 0.$$

constant B^0

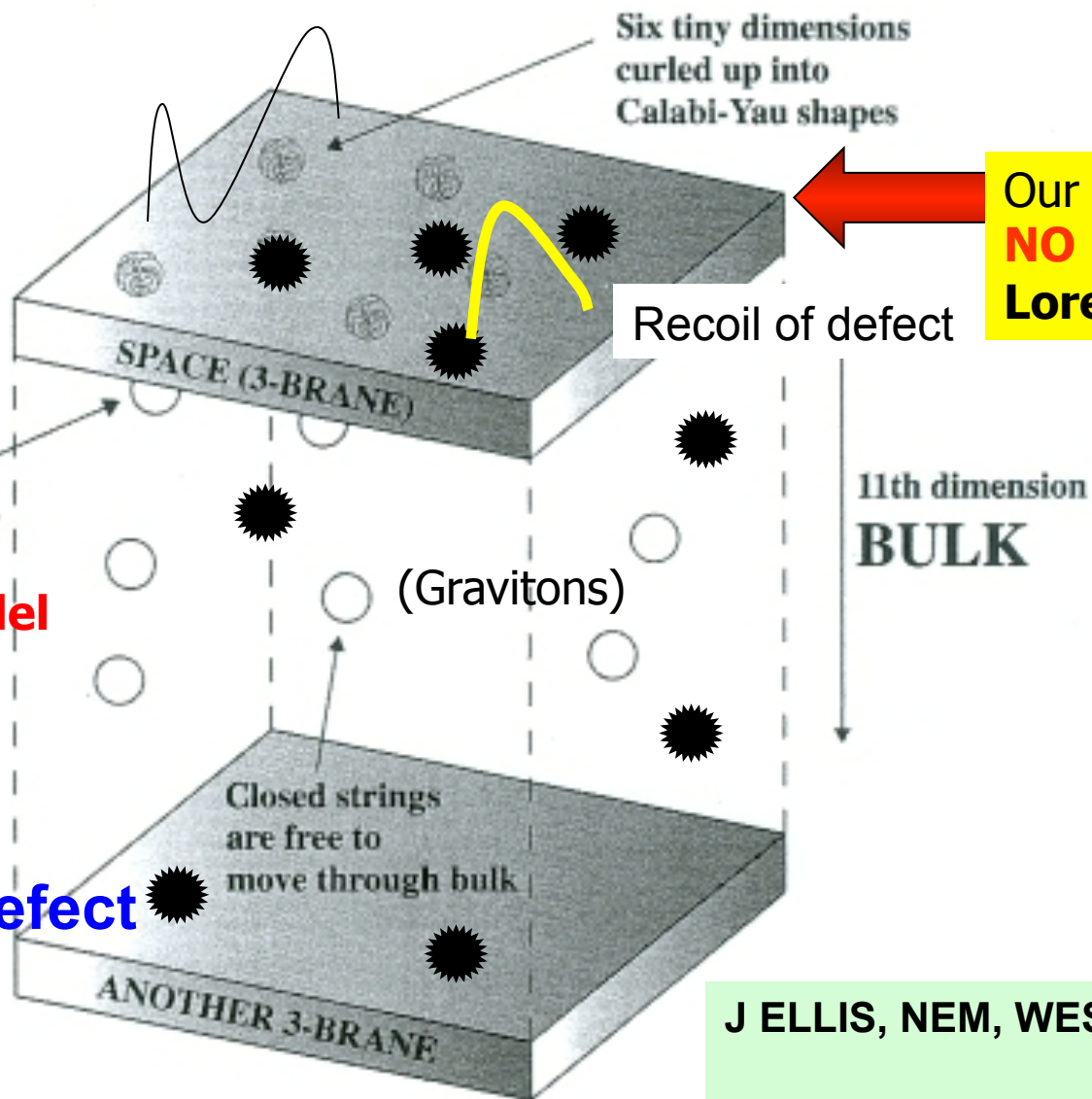
Lepton Asymmetry as in previous cases , e.g. for neutrinos

$$\Delta L(T < T_d) = \frac{\Delta n_\nu}{s} \sim \frac{B_0}{T_d} \quad \text{Requires } B^0 \sim 10^5 \text{ GeV. @ } T_d$$

4. STRINGY SPACE-TIME D(efect)-FOAM & CPTV

NEM & Sarben Sarkar, arXiv:1211.0968

BRANE-WORLDS with D-PARTICLE (POINT-LIKE BRANE) DEFECTS



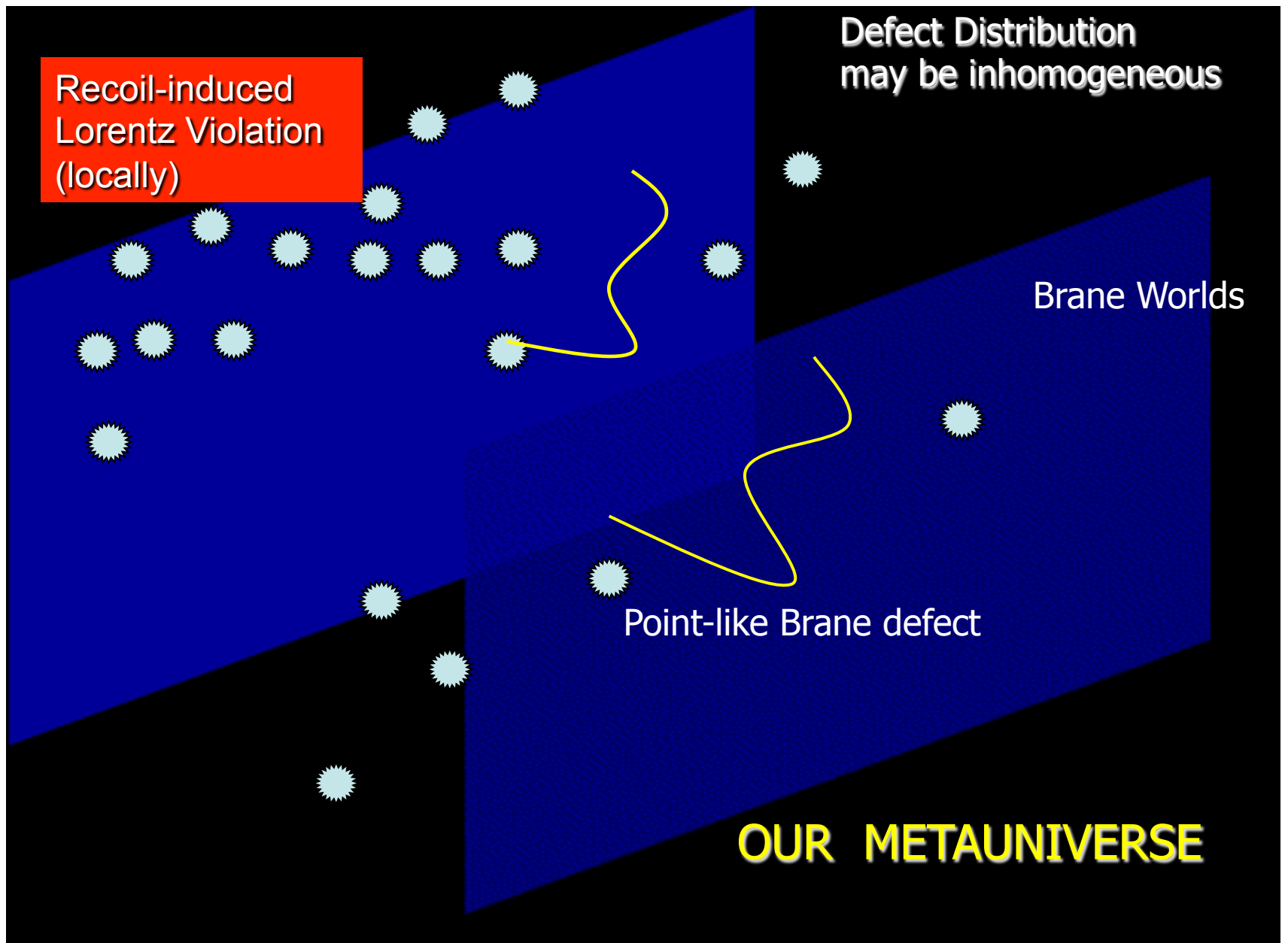
Our Universe
NO LONGER
Lorentz Invariant

(Standard Model particles)

D-particle defect

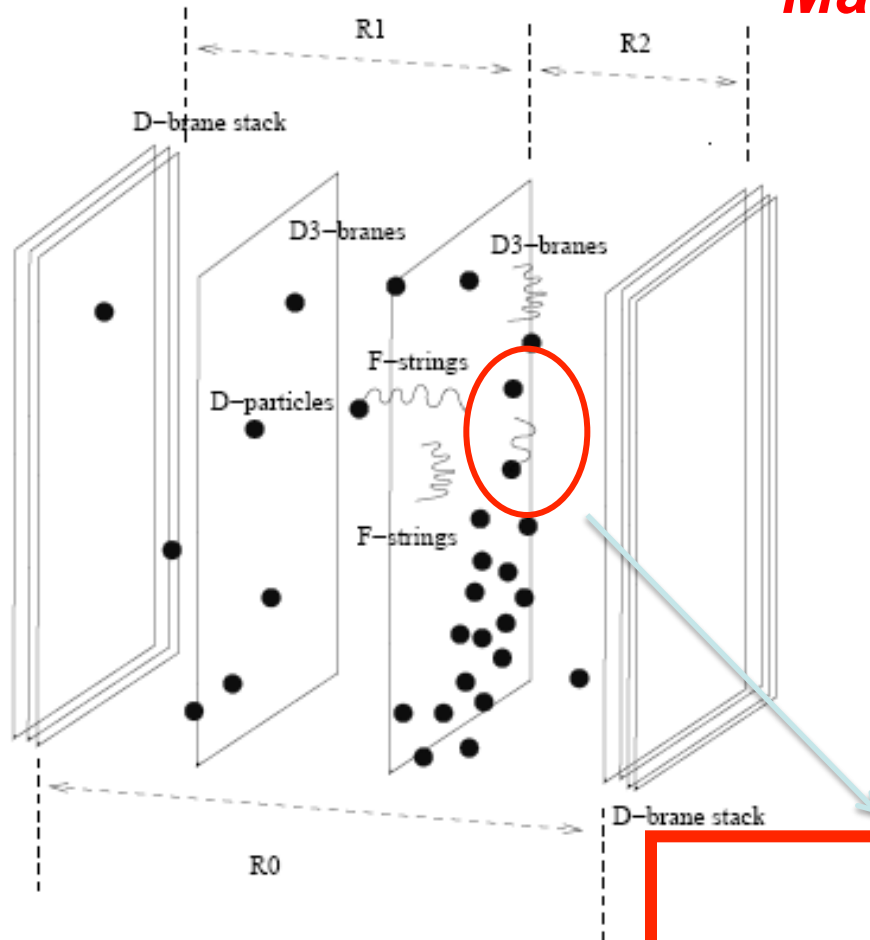
$$\text{Mass} \sim \frac{M_s}{g_s}$$

J ELLIS, NEM, WESTMUCKETT



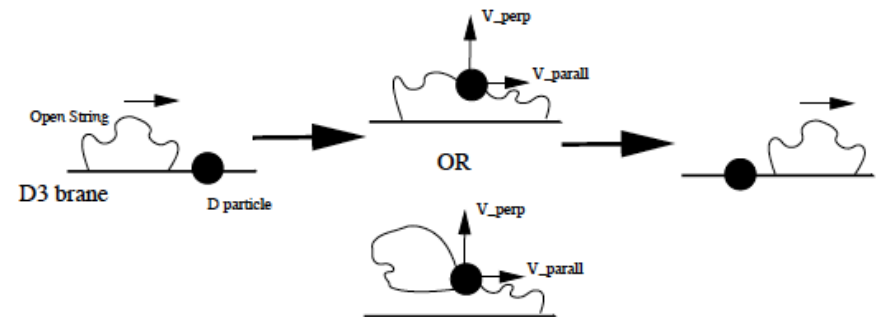
Colliding Brane world model of Space-Time with point-like space-time defects

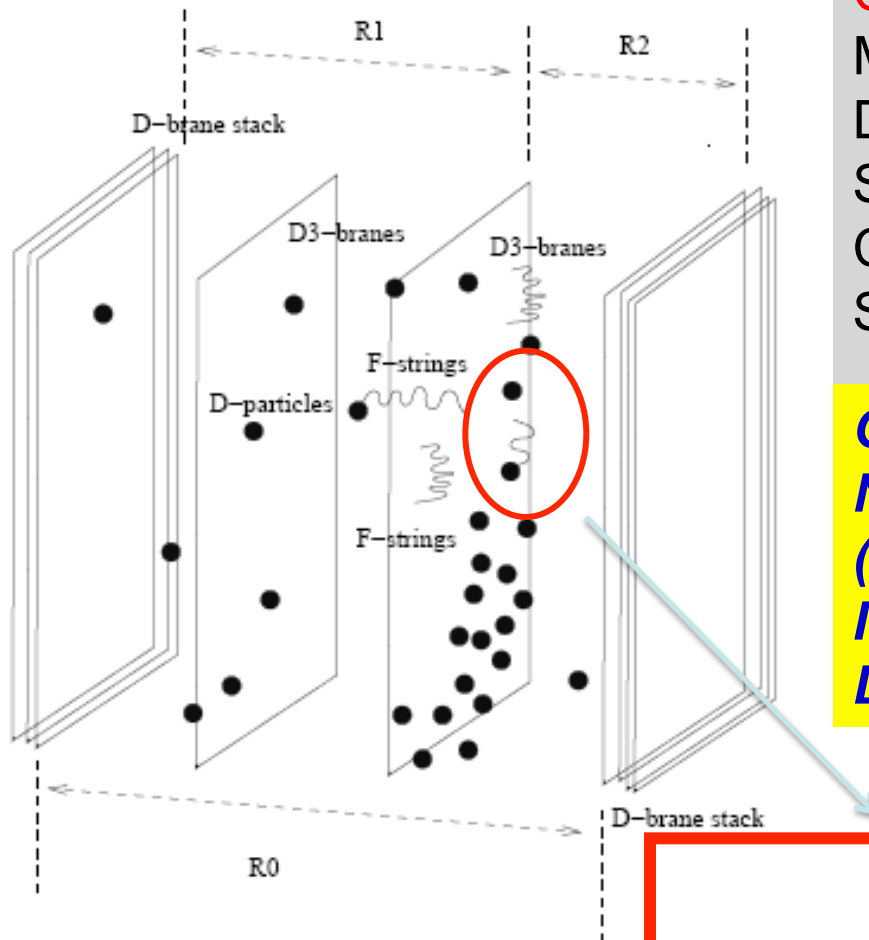
Matter/D-foam Interactions



String **Splitting** & (“momentary”) **Capture** by the defects

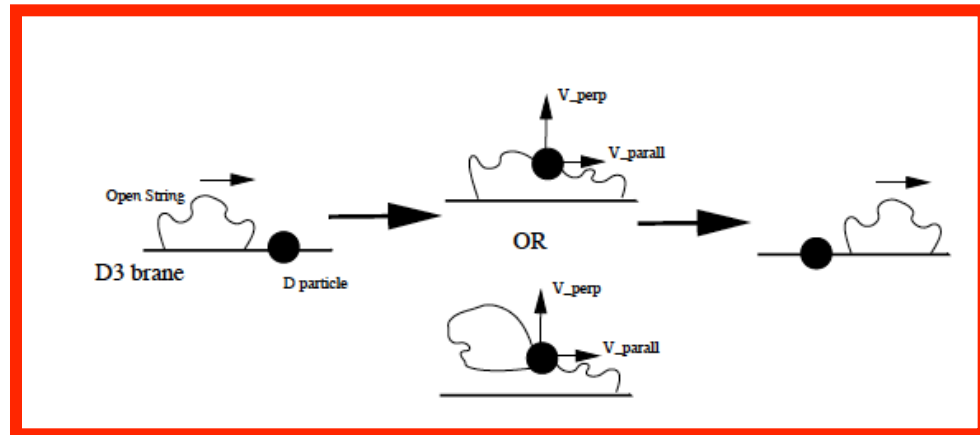
INTERMEDIATE STRING CREATION/EXCHANGE PURELY **NON-LOCAL** EFFECT

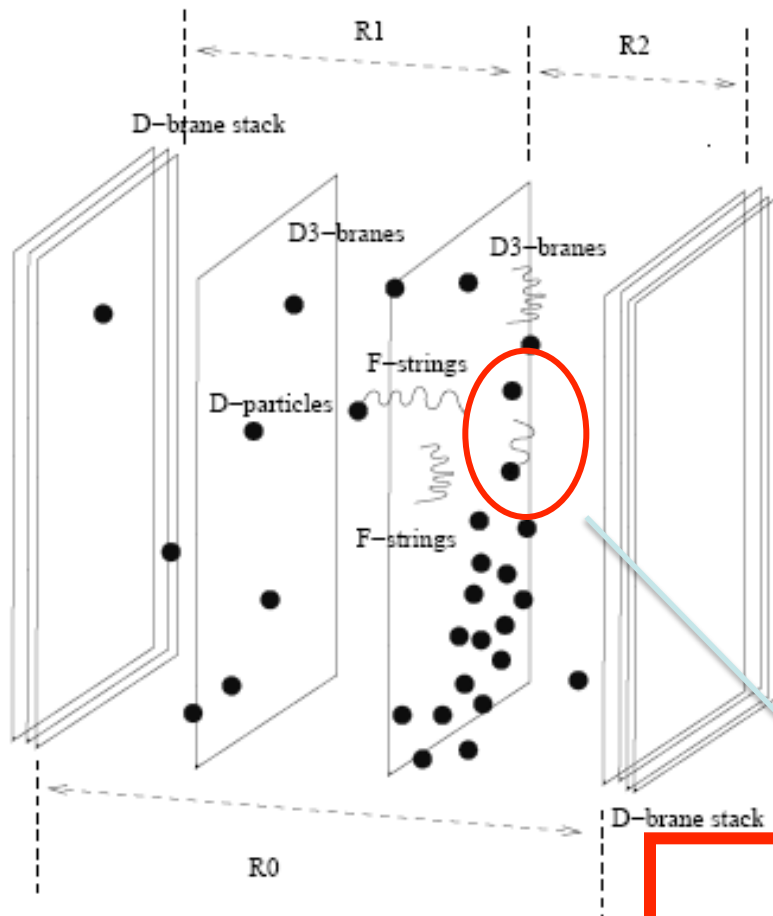




CHARGE CONSERVATION
MUST BE RESPECTED
DURING STRING
SPLITTING, INTERMEDIATE
CREATION AND
STRETCHING:

ONLY ELECTRICALLY
NEUTRAL EXCITATIONS
(e.g. Photons, Neutrinos)
INTERACT VIA CAPTURE
DOMINANTLY WITH FOAM

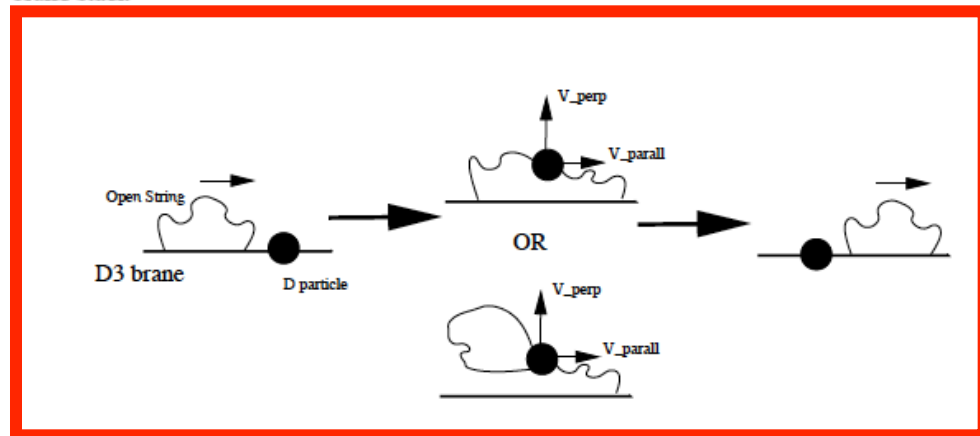




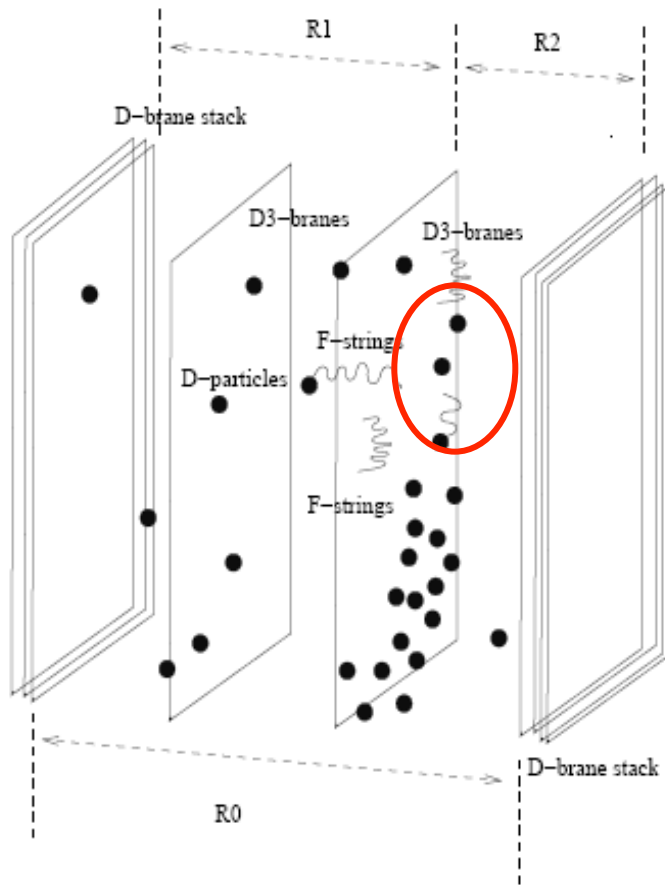
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INTERACT VIA CAPTURE
DOMINANTLY WITH FOAM
DEFECT RECOIL OCCURS

Time Delays due to
 Intermediate String Creation
 & Oscillations – **Subluminal**
Vacuum Refractive Index



J ELLIS, NEM, NANOPOULOS



Local Lorentz Violation due to direction of Defect recoil velocities

Induced metric depends on momenta as well as coordinates (Finsler type) : e.g. $u \parallel X_1$

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

Explicit local breaking of $SO(3,1)$ down to $SO(2,1)$ rotation and boosts in transverse directions

$$h_{01} = g_s \frac{\Delta k_i}{M_s} \equiv u_1$$

“Frame Dragging by recoiling D-particle”

**Space time Foam situations –
Average over both populations of defects & quantum fluctuations**

Isotropic & (in)homogeneous foam

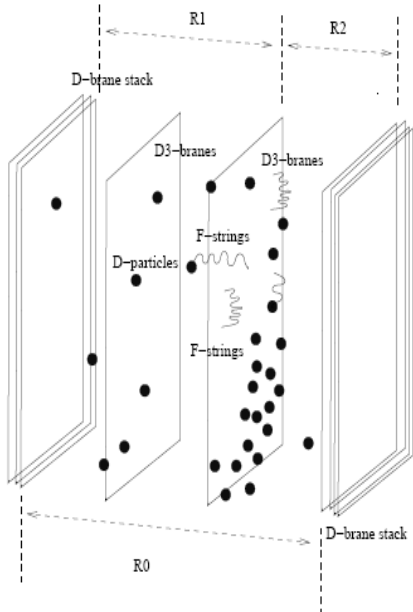
for a brane observer:

$$\langle u_i \rangle \equiv \frac{g_s}{M_s} \langle \Delta k_i \rangle = 0$$

**Lorentz Invariance
on Average**

$$\frac{g_s^2}{M_s^2} \langle \Delta k_i \Delta k_j \rangle = \sigma^2 \delta_{ij}$$

Violated in flcts



c.f. Stochastic Foam, through coherent graviton states
leading to light cone fluctuations **Ford (95)**

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

$$\langle h_{\mu\nu} \rangle = 0$$

$$\langle h_{\mu\nu} h_{\rho\sigma} \rangle \neq 0$$

D-foam Induced CPTV for Neutrinos

$$\langle\langle E_\nu \rangle\rangle = \sqrt{p^2 + m_\nu^2} \left(1 + \frac{1}{2} \sigma^2 \right) - \frac{1}{2} \frac{M_s}{g_s} \sigma^2$$
$$\langle\langle E_{\bar{\nu}} \rangle\rangle = \sqrt{p^2 + m_\nu^2} \left(1 + \frac{1}{2} \sigma^2 \right) + \frac{1}{2} \frac{M_s}{g_s} \sigma^2$$



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cf. "Frame Dragging"
by recoiling D-particle



D-foam Induced CPTV for Neutrinos

$$\begin{aligned}\langle\langle E_\nu \rangle\rangle &= \sqrt{p^2 + m_\nu^2} \left(1 + \frac{1}{2}\sigma^2 \right) \left(-\frac{1}{2} \frac{M_s}{g_s} \sigma^2 \right) \\ \langle\langle E_{\bar{\nu}} \rangle\rangle &= \sqrt{p^2 + m_\nu^2} \left(1 + \frac{1}{2}\sigma^2 \right) \left(+\frac{1}{2} \frac{M_s}{g_s} \sigma^2 \right)\end{aligned} \pm B_0$$

One can thus generate a Lepton asymmetry and, then through B-L conserving processes in the Early Universe a Baryon asymmetry.



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$\pm B_0$

constant > 0
if $\sigma^2 \approx \text{const}$
in an era

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Correct Sign for Matter dominance over Antimatter due to Energetics
no Fine Tuning



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$$\Delta n = \frac{g}{(2\pi)^3} \int d^3\mathbf{p} \left[\frac{1}{1 + \exp(E_\nu/T)} - \frac{1}{1 + \exp(E_{\bar{\nu}}/T)} \right]$$

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One can thus generate a Lepton asymmetry and through B+L conserving processes in the Early Universe a Baryon asymmetry.

The correct value (observed) for BAU is reproduced for, e.g. GUTs

$$\frac{1}{2} \frac{M_s}{g_s} \sigma^2 \sim 10^5 \text{ GeV}$$

for D-foam at $T_d \sim 10^{15} \text{ GeV}$

implying that in these scenarios, for $\sigma^2 < 1$, one must have $M_s/g_s > 200 \text{ TeV}$



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**PHENOMENOLOGY OF EARLY UNIVERSE NEEDS
TO BE CHECKED FOR COMPATIBILITY.... IN PROGRESS**



IS THIS CPTV ROUTE WORTH FOLLOWING?



CPT Violation

Construct Microscopic Quantum Gravity models with strong CPT Violation in Early Universe, but maybe weak today... Fit with all available data... Estimate in this way matter-antimatter asymmetry in Universe.



CONCLUSIONS-OUTLOOK


- ***Neutrinos (Sterile)***
may explain matter-antimatter origin in the Universe
- **May also provide interesting Dark matter Candidates**
- ***Neutrino condensates***
may contribute to dark energy



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 - Interesting CPTV Physics for the Early Universe to be investigated
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CONCLUSIONS-OUTLOOK

- *Neutrinos (Sterile) may explain matter-antimatter origin in the Universe*

- *Gravitationally-induced anomalous Right-handed Majorana neutrino*

- *May also be interesting matter Candidates*

THANK YOU !

- *Neutrino condensates may contribute to dark energy*

- *Interesting CPTV Physics for the Early Universe to be investigated*

SPARES

CONCLUSIONS-OUTLOOK

- **Neutrinos (Sterile)** may explain matter-antimatter origin in the Universe
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 - **Mass Varying ν**
 - **Gravitationally-induced anomalous Right-handed Majorana neutrino masses possible, beyond see-saw...**
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- 

MASS-VARYING NEUTRINOS & COSMOLOGY

OTHER INTERESTING TOPICS

Mass *Varying* neutrinos & the Dark Sector

Nelson, Fardon, Weiner
Chitov, August, Natarajan,
Kahniashvili

Couple Scalar cosmic fields with potential $U(\varphi, T)$ and massless fermions ψ through Yukawa couplings

$$\mathcal{S} = S_B^E + S_D^E|_{m=0} + g \int_0^\beta d\tau \int a^3 d^3x \varphi \bar{\psi} \psi$$
$$S_B^E = \int_0^\beta d\tau \int a(t)^3 d^3x \left[\frac{1}{2} (\partial_\tau \varphi)^2 + \frac{1}{2a^2} (\nabla \varphi)^2 + U(\varphi) \right]$$

Fermion mass: $m = g\phi_c$

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Thermodynamic potential density

Fermion mass: $m = g\phi_c$

$$\Omega(\phi_c) = U(\phi_c) + \Omega_D(\phi_c)$$

$$\mathcal{Z}_D \equiv \text{Tr} e^{-\beta(\hat{H} - \mu \hat{Q})} = \int \mathcal{D}\bar{\psi} \mathcal{D}\psi e^{-S_D^E}$$

$$\Omega_D \equiv -\frac{1}{\beta a^3 V} \log \mathcal{Z}_D$$

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$$S_B^E = \int_0^\beta d\tau \int a(t)^3 d^3x \left[\frac{1}{2} (\partial_\tau \varphi)^2 + \frac{1}{2a^2} (\nabla \varphi)^2 + U(\varphi) \right]$$

Thermodynamic potential density

Fermion mass: $m = g\phi_c$

$$\Omega(\phi_c) = U(\phi_c) + \Omega_D(\phi_c)$$

$$\mathcal{Z}_D \equiv \text{Tr} e^{-\beta(\hat{H} - \mu \hat{Q})} = \int \mathcal{D}\bar{\psi} \mathcal{D}\psi e^{-S_D^E}$$

$$\Omega_D \equiv -\frac{1}{\beta a^3 V} \log \mathcal{Z}_D$$

comoving volume

MASS-VARYING NEUTRINOS & COSMOLOGY

OTHER INTERESTING TOPICS

Mass *Varying* neutrinos & the Dark Sector

Nelson, Fardon, Weiner
Chitov, August, Natarajan,
Kahniashvili

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$$\left. \frac{\partial^2 \Omega(\varphi)}{\partial \varphi^2} \right|_{\varphi=\phi_c} > 0$$

T dependent !

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Fermion mass: $m = g\phi_c$

$$\phi_c = \langle \varphi \rangle$$

$$U'(\phi_c) + g\rho_s = 0$$

$$\rho_s \equiv \frac{\langle \hat{N} \rangle}{V} = \frac{\partial \Omega_D}{\partial m} = \rho_0 + \frac{m}{\pi^2} \int_0^\infty \frac{k^2 dk}{\varepsilon(k)} [n_F(\varepsilon_-) + n_F(\varepsilon_+)] \quad \hat{N} = \int d^3x \bar{\psi} \psi$$

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$$U'(\phi_c) + g\rho_s = 0$$

Fermion mass: $m = g\phi_c$

$$U(\varphi) = \frac{M^{\alpha+4}}{\varphi^\alpha}$$

$$\alpha > 0.$$

High T phase:

$$\frac{m}{M} \approx \left(\sqrt{6\alpha} \frac{M}{T} \right)^{\frac{2}{\alpha+2}} \propto T^{-\frac{2}{\alpha+2}}$$

Fermionic contribution to
thermodynamic potential **dominant**

Scalar mass $m_\phi \approx \sqrt{\frac{\alpha+1}{6}} T$

MASS-VARYING NEUTRINOS & COSMOLOGY

OTHER INTERESTING TOPICS

Mass *Varying* neutrinos & the Dark Sector

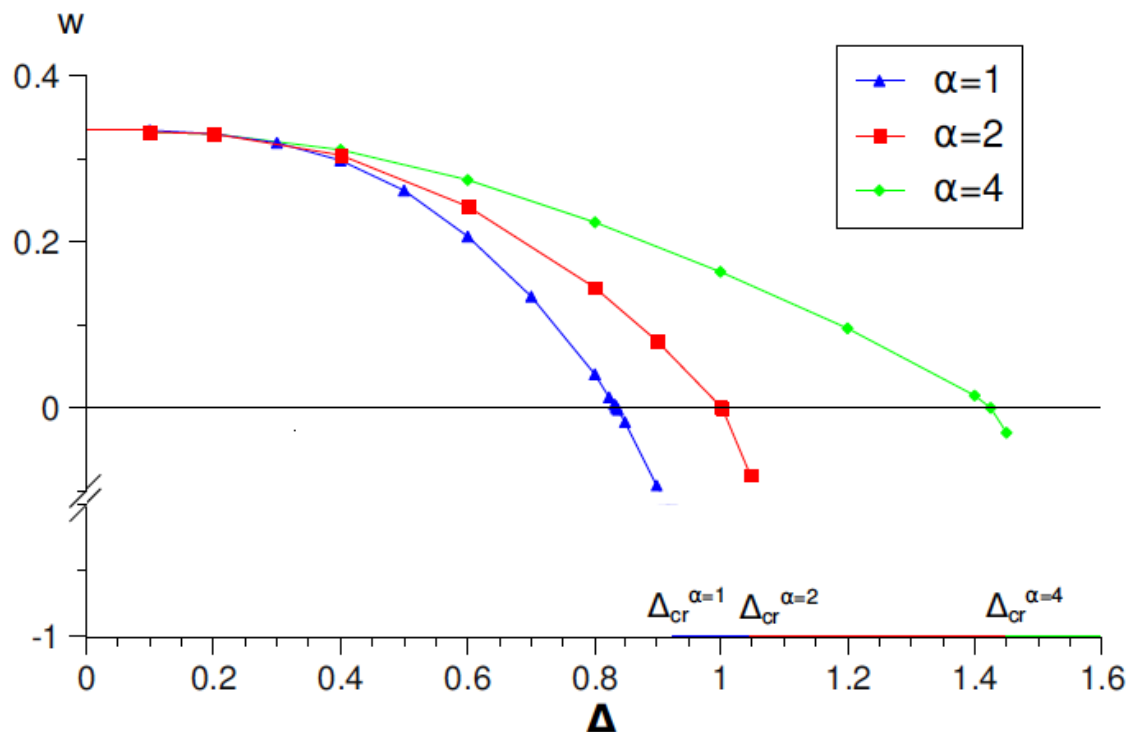
Nelson, Fardon, Weiner
Chitov, August, Natarajan,
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$$U'(\phi_c) + g\rho_s = 0$$

$$U(\varphi) = \frac{M^{\alpha+4}}{\varphi^\alpha} \quad \alpha > 0.$$

Equation of state:

$$\Delta = M/T$$



**Towards $w=-1$
for low T
(Cosmo. Const. like)**

MASS-VARYING NEUTRINOS & COSMOLOGY

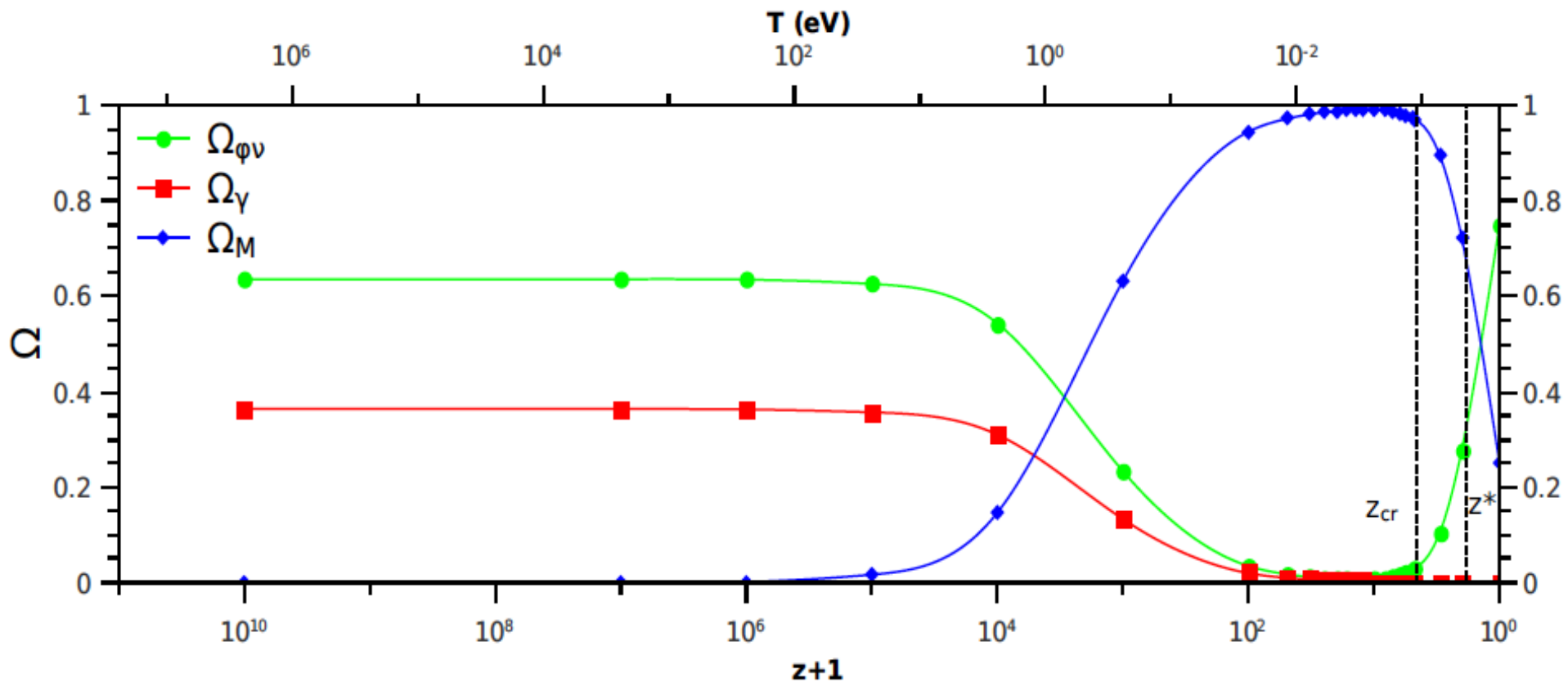
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Mass *Varying* neutrinos & the Dark Sector

Neutrino Dark Energy evolution vs Dark Matter Ω_M

$$M = 2.39 \cdot 10^{-3} \text{ eV } (\alpha = 0.01)$$



MASS-VARYING NEUTRINOS & COSMOLOGY

OTHER INTERESTING TOPICS

Chitov, August, Natarajan,
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Mass *Varying* neutrinos & the Dark Sector

$$M = 2.39 \cdot 10^{-3} \text{ eV} \quad (\alpha = 0.01)$$

$$U(\varphi) = \frac{M^{\alpha+4}}{\varphi^{\alpha}} \quad \alpha > 0.$$

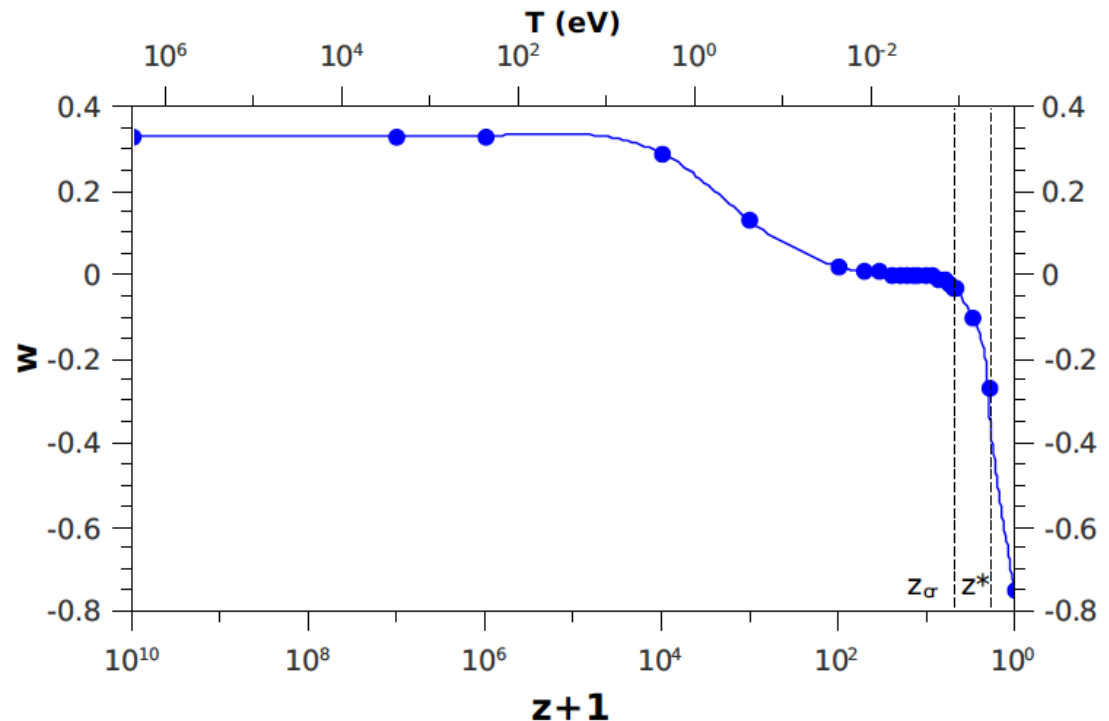
Equation of state of entire
Universe including
radiation contributions:

$$P_{\text{tot}} = w_{\text{tot}} \rho_{\text{tot}}$$

$$P_{\text{tot}} = P_{\gamma} + P_{\varphi\nu}$$

$$P_{\gamma} = \frac{1}{3} \rho_{\gamma}$$

**Towards $w=-1$ for low z
(Cosmo. Const. like)**



MASS-VARYING NEUTRINOS & COSMOLOGY

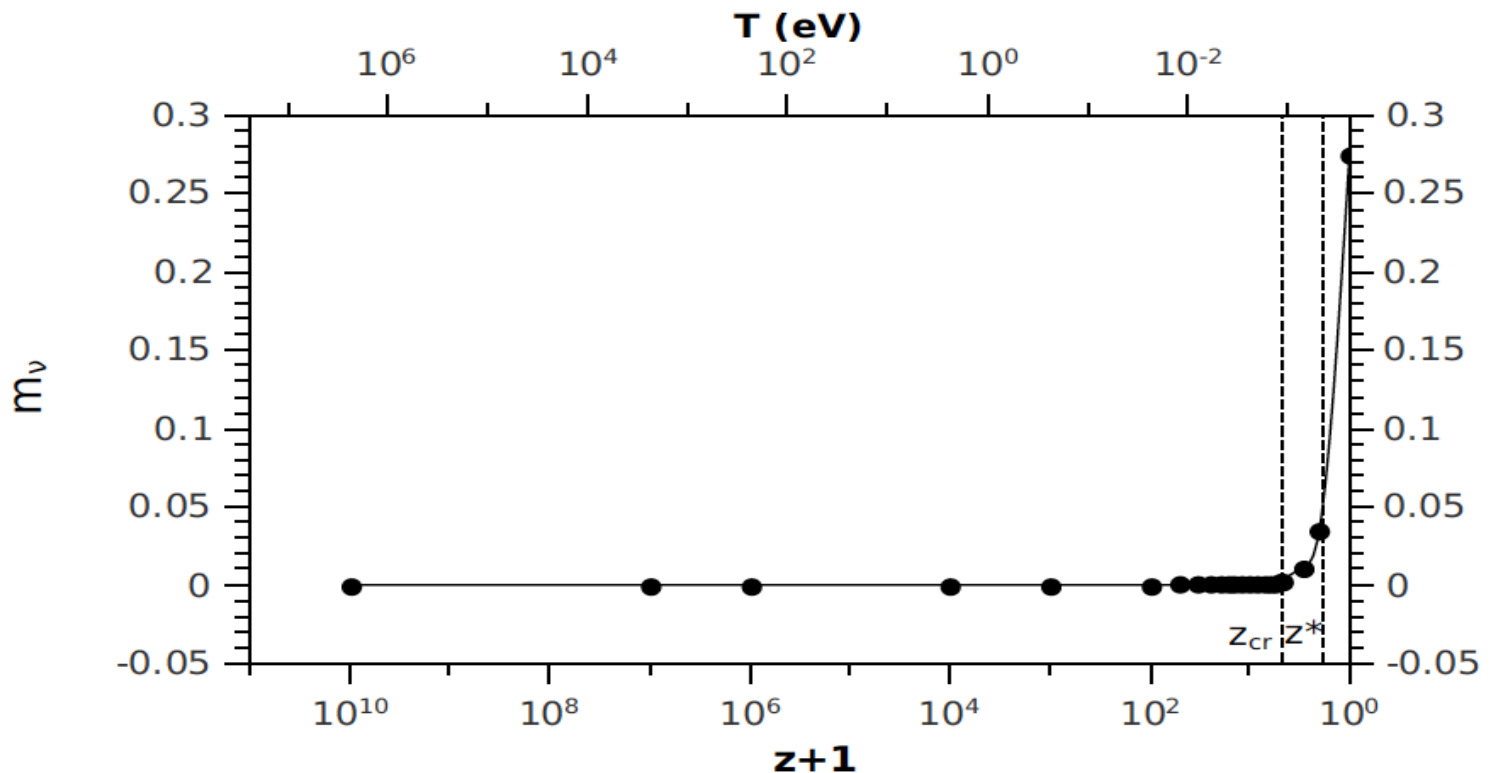
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Mass *Varying* neutrinos & the Dark Sector

$$U(\varphi) = \frac{M^{\alpha+4}}{\varphi^\alpha}$$

Neutrino mass evolution $M = 2.39 \cdot 10^{-3} \text{ eV} \ (\alpha = 0.01)$



MASS-VARYING NEUTRINOS & COSMOLOGY

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Acceptable Cosmology & Neutrino phenomenology

Effective (Anti)Neutrino CPTV D-foam Mass

No mixing $\nu - \bar{\nu}$

$$m_{\nu}^{\text{eff}} = m_{\nu} \left(1 + \frac{1}{2} \sigma^2(T)\right) - \frac{M_s}{g_s} \sigma^2(T) \simeq m_{\nu} - \frac{M_s}{g_s} \sigma^2(T)$$

$$m_{\bar{\nu}}^{\text{eff}} = m_{\nu} \left(1 + \frac{1}{2} \sigma^2(T)\right) + \frac{M_s}{g_s} \sigma^2(T) \simeq m_{\nu} + \frac{M_s}{g_s} \sigma^2(T)$$

$$\frac{1}{2} \frac{M_s}{g_s} \sigma^2 \sim 10^5 \text{ GeV} \quad @ T_d \sim 10^{15} \text{ GeV}$$

to generate BAU

Bounds from WMAP Cosmology ($z < 1000$)

$$\sum_{i=1}^3 m_{\nu}^{\text{eff}} < 0.69 \text{ eV}$$

$$\sigma^2(T) \sim \Delta^2(T) g_s^2 \frac{\bar{p}^2}{M_s^2} \sim \frac{g_s^2}{M_s^2} \beta_0 (1+z)^3$$

Dust type
D-particles

$$\sum m_{\nu} - 3 \frac{g_s}{M_s} \beta_0 < 10^{-3} \text{ eV}$$

is a safe not strong
bound on foam flets σ^2 today
(within current exp errors)

CPTV \rightarrow neutrino/ antineutrino mixing & oscillations

M Sinha & B. Mukhopadhyay
arXiv: 0704.2593

$$\mathcal{L} = \det(e) \bar{\Psi} \left(\frac{i}{2} \gamma^a \overleftrightarrow{\partial}_a - m + \gamma^a \gamma^5 B_a \right) \Psi$$

$$B^d = \epsilon^{abcd} \omega_{bca}$$

$$E_\nu = \sqrt{(\vec{p} - \vec{B})^2 + m^2} + B_0,$$

$$E_{\nu^c} = \sqrt{(\vec{p} + \vec{B})^2 + m^2} - B_0$$

Assume Majorana neutrino in Weyl rep (Lepton number violation unavoidable)

$$\Psi = \begin{pmatrix} \psi_L^c \\ \psi_L \end{pmatrix} \quad \mathcal{D}_\mu \equiv (\partial_0, \partial_i + \gamma^5 B_i).$$

**Majorana mass term
violates L number**

$$(-g)^{-1/2} \mathcal{L} = (\psi^{c\dagger} \ \psi^\dagger) \frac{i}{2} \gamma^0 \gamma^\mu \overleftrightarrow{\mathcal{D}}_\mu \begin{pmatrix} \psi^c \\ \psi \end{pmatrix} - (\psi^{c\dagger} \ \psi^\dagger) \begin{pmatrix} -B_0 & -m \\ -m & B_0 \end{pmatrix} \begin{pmatrix} \psi^c \\ \psi \end{pmatrix}$$

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Lead to
neutrino/antineutrino
mixing & oscillations

$$(-g)^{-1/2} \mathcal{L} = (\psi^{c\dagger} \ \psi^\dagger) \frac{i}{2} \gamma^0 \gamma^\mu \overleftrightarrow{\mathcal{D}}_\mu \begin{pmatrix} \psi^c \\ \psi \end{pmatrix} - (\psi^{c\dagger} \ \psi^\dagger) \begin{pmatrix} -B_0 & -m \\ -m & B_0 \end{pmatrix} \begin{pmatrix} \psi^c \\ \psi \end{pmatrix}$$

mass eigenstates ν_1 and ν_2 as

$$|\nu_1\rangle = \frac{1}{N} \left\{ \left(B_0 + \sqrt{B_0^2 + m^2} \right) |\psi^c\rangle + m |\psi\rangle \right\}$$

$$|\nu_2\rangle = \frac{1}{N} \left\{ -m |\psi^c\rangle + \left(B_0 + \sqrt{B_0^2 + m^2} \right) |\psi\rangle \right\}.$$

$$m_{1,2} = \mp \sqrt{B_0^2 + m^2}.$$

$$|\nu_1\rangle = \cos \theta |\psi^c\rangle + \sin \theta |\psi\rangle$$

$$|\nu_2\rangle = -\sin \theta |\psi^c\rangle + \cos \theta |\psi\rangle$$

NB: neutrino CPTV mass shifts

neutrino/antineutrino mixing

$$\tan \theta = \frac{m}{B_0 + \sqrt{B_0^2 + m^2}}.$$

$$|\psi^c\rangle = \cos \theta |\nu_1\rangle - \sin \theta |\nu_2\rangle$$

$$|\psi\rangle = \sin \theta |\nu_1\rangle + \cos \theta |\nu_2\rangle.$$

oscillations

$$\mathcal{P}(t) = \sin^2 2\theta \sin^2 \delta(t)$$

$$= \frac{m^2}{B_0^2 + m^2} \sin^2 \{ (B_0 - |\vec{B}|) t \}$$

$$\delta(t) = \frac{|E_\nu - E_{\nu^c}| t}{2},$$

mass eigenstates ν_1 and ν_2 as

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$$|\psi^c\rangle = \cos \theta |\nu_1\rangle - \sin \theta |\nu_2\rangle$$

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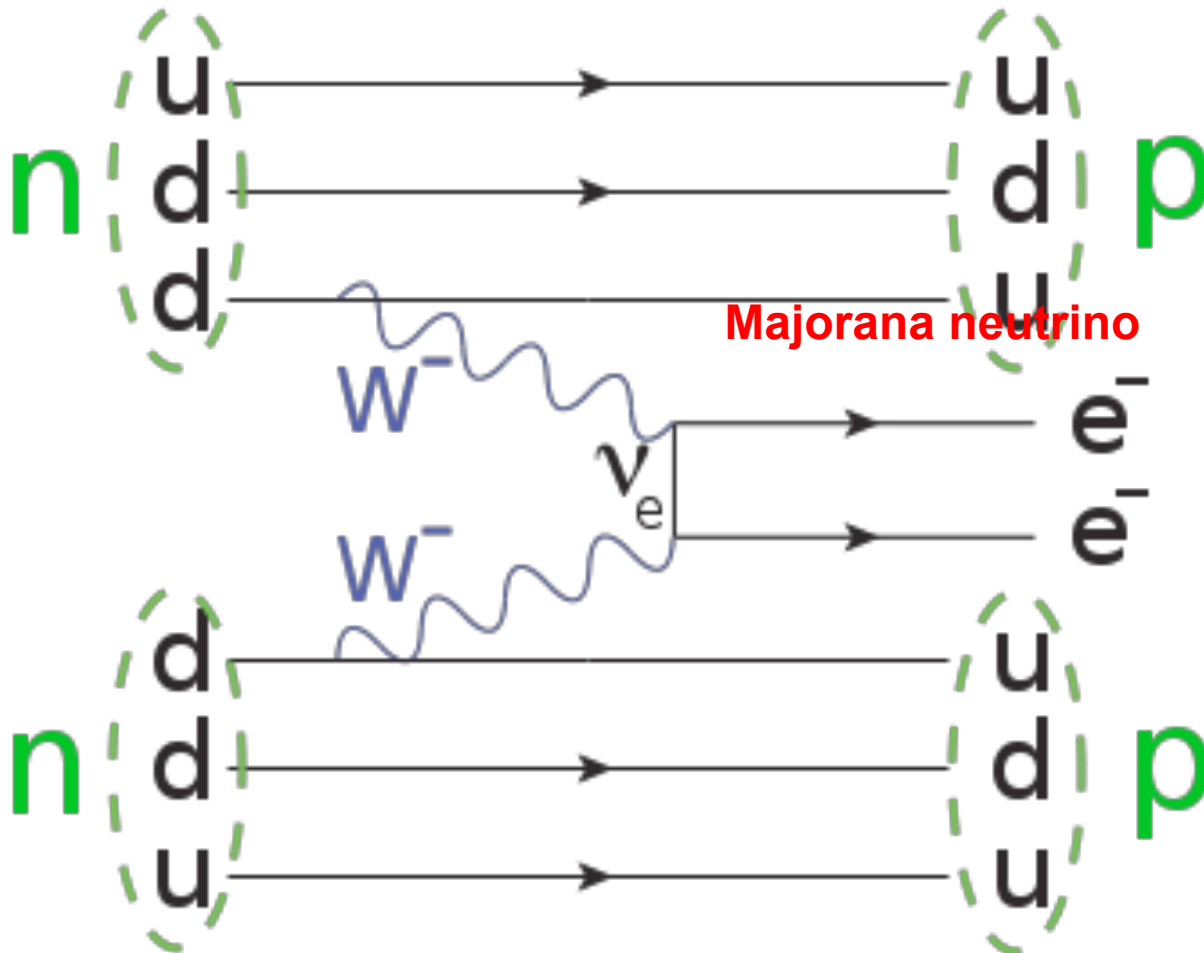
oscillation length $\lambda = \frac{\pi}{B_0 - |\vec{B}|}$.

oscillations $= \frac{\mathcal{P}(t)}{B_0^2 + m^2} = \frac{\sin^2 2\theta \sin^2 \delta(t)}{B_0^2 + m^2} \sin^2 \{ (B_0 - |\vec{B}|) t \}$

$$\delta(t) = \frac{|E_\nu - E_{\nu^c}| t}{2},$$

Modifications in Neutrinoless 2Beta decay rate in 2 flavour mixing
(due to CPTV modified effective mass)

M Sinha & B. Mukhopadhyay
arXiv: 0704.2593



ignore neutrino/
antineutrino
mixing here:

Amplitude $A \propto \sqrt{B_0^2 + m_e^2}$.

CPTV Effects of different Space-Time-Curvature/Spin couplings between ν , $\bar{\nu}$ in Bianchi Cosmologies

B. Mukhopadhyay, U. Debnath, N. Dadhich, M. Sinha Lambiase, Mohanty

Assumption: Neutrinos non clustering properties, dominant species in early Universe

$$ds^2 = -dt^2 + S(t)^2 dx^2 + R(t)^2 [dy^2 + f(y)^2 dz^2] - S(t)^2 h(y) [2dx - h(y) dz] dz$$

Bianchi II, VIII and IX models, respectively $f(y)$ and $h(y)$ are given as

$$f(y) = \{y, \sinh y, \sin y\}, \quad h(y) = \{-y^2/2, -\cosh y, \cos y\}.$$

$$B^0 = \frac{S[-f^2 R^2 (hf' R + Sh') + h^2 S^2 (hf' R + Sh') + 2fhRS(Rf' - hh'S)]}{f^4 R^4 + f^2 h^2 R^2 S^2}$$

$$B^2 = \frac{h[-f^2 R^2 + 2fRS + h^2 S^2][RS' - R'S]}{f^3 R^4 + fh^2 R^2 S^2}.$$

$$B^3 = B^1 = 0$$

Neutrino/antineutrino asymmetry around black holes

B. Mukhopadhyay, astro-ph/0505460

Consider the metric of a Kerr (**rotating**) black hole

$$ds^2 = \eta_{ij} dx^i dx^j - \left[\frac{2\alpha}{\rho} s_i v_j + \alpha^2 v_i v_j \right] dx^i dx^j$$

$$\alpha = \frac{\sqrt{2Mr}}{\rho}, \quad \rho^2 = r^2 + \frac{a^2 z^2}{r^2} \quad v_i = \left(1, \frac{ay}{a^2 + r^2}, \frac{-ax}{a^2 + r^2}, 0 \right)$$

$$s_i = \left(0, \frac{rx}{\sqrt{r^2 + a^2}}, \frac{ry}{\sqrt{r^2 + a^2}}, \frac{z\sqrt{r^2 + a^2}}{r} \right)$$

and we have $r^+ - r^- (x^- + y^- + z^- - a^-) - a^2 z^2 = 0$

Modified Neutrino dispersion relations due to locally induced metric

$$p^\mu p^\nu g_{\mu\nu} = -m^2 \Rightarrow E = \vec{p} \cdot \vec{u} \pm \sqrt{p^2 + m^2 + (\vec{p} \cdot \vec{u})^2}$$

Interpret (Dirac hole theory) negative energies as corresponding to anti-particles \leftrightarrow Fermions, exclusion principle

$$\langle\langle E \rangle\rangle = \langle\langle \vec{p} \cdot \vec{u} \rangle\rangle \pm \langle\langle \sqrt{p^2 + m^2 + (\vec{p} \cdot \vec{u})^2} \rangle\rangle$$

$$\langle\langle E \rangle\rangle \simeq \pm \sqrt{p^2 + m^2} \left(1 + \frac{1}{2} \sigma^2 \right), \quad p \gg m$$

Momentum-Energy conservation during ν scattering with D-particles

$$\langle\langle \vec{p}_1 + \vec{p}_2 \rangle\rangle = \frac{M_s}{g_s} \langle\langle \vec{u} \rangle\rangle = 0$$

$$\langle\langle E_1 \rangle\rangle = \langle\langle E_2 \rangle\rangle + \frac{1}{2} \frac{M_s}{g_s} \langle\langle u^2 \rangle\rangle \Rightarrow$$

$$\langle\langle E_2 \rangle\rangle = \pm \sqrt{p^2 + m^2} \left(1 + \frac{1}{2} \sigma^2 \right) - \frac{1}{2} \frac{M_s}{g_s} \sigma^2$$