String/M Theory – what is it?

Nick Evans

Strong Force Origins

Superstrings

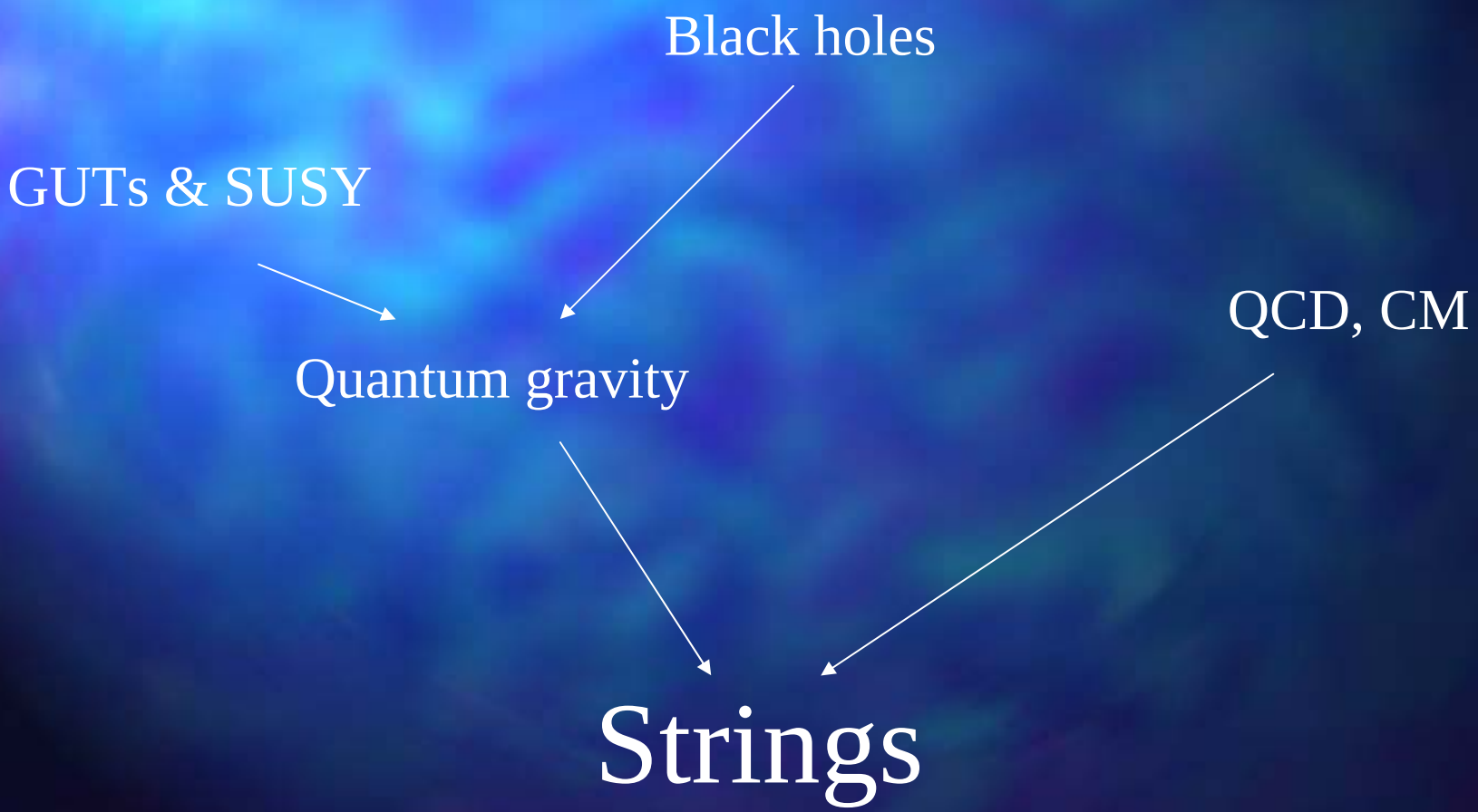
Ten & eleven dimensions

Branes and brane world

The Landscape

Holography

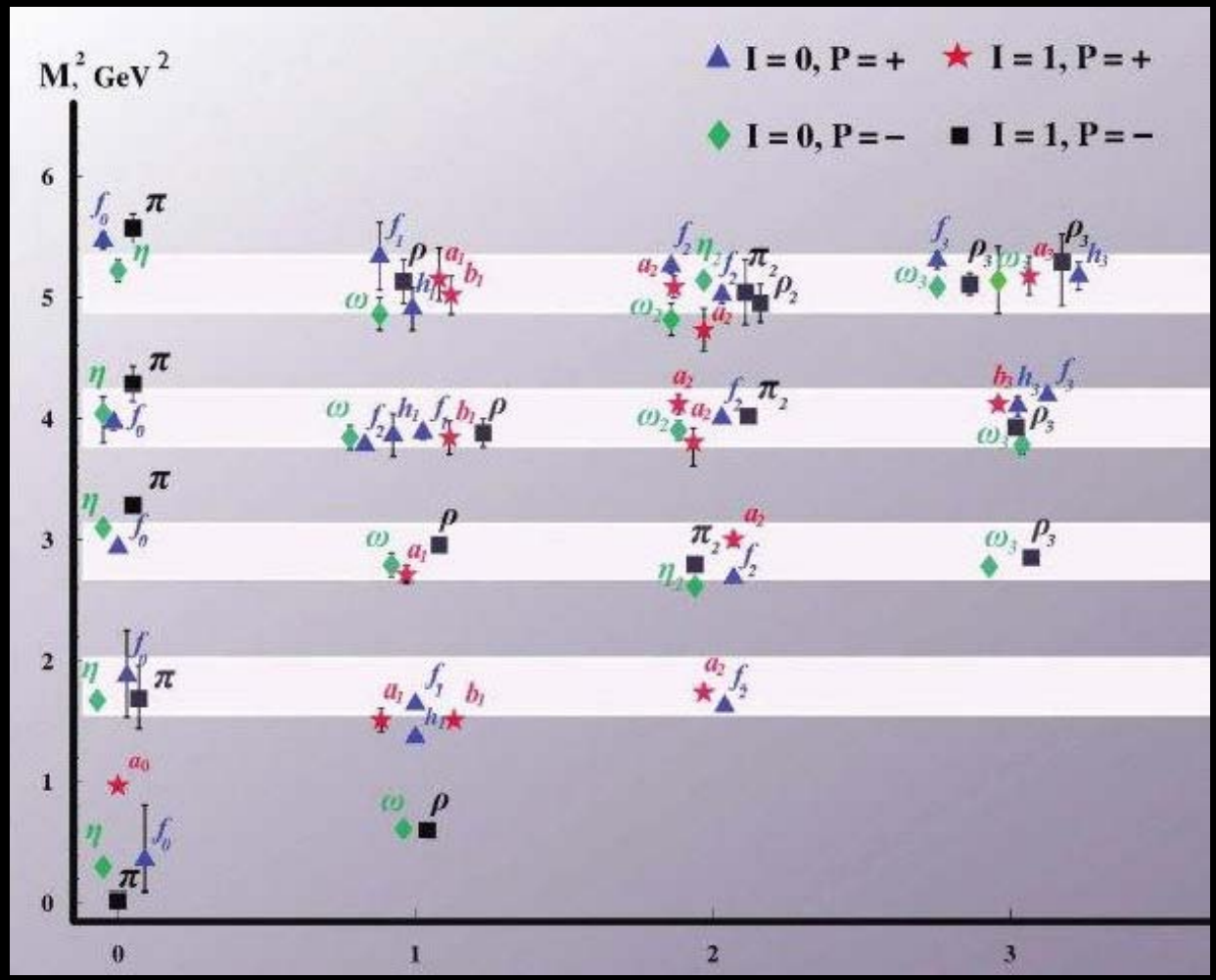
Paths to String Theory



Emergence from Confusion – 1960s

Regge trajectories for mesons

M^2 vs J



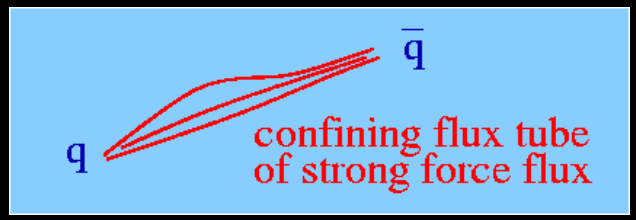
QCD

Quarks

Gluon mediated interactions

SU(3) gauge invariance

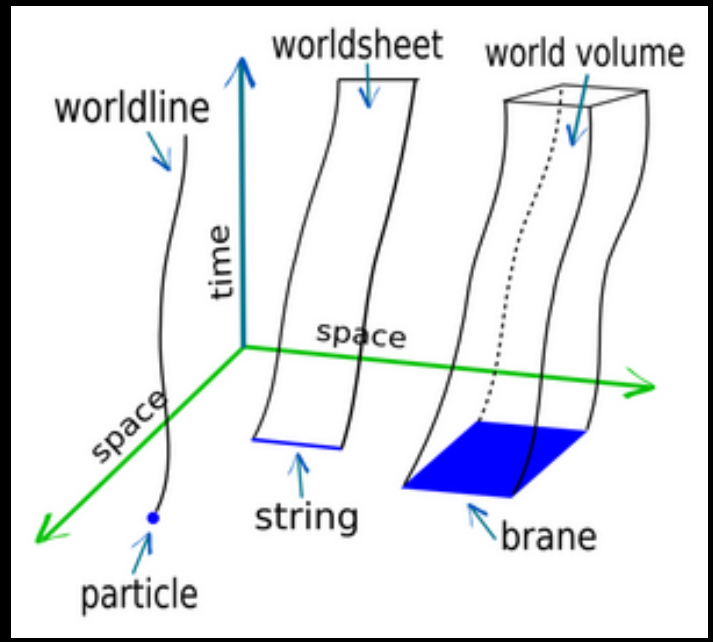
Confinement due to asymptotic freedom



String Theory

Fundamental strings

Basic property is Tension



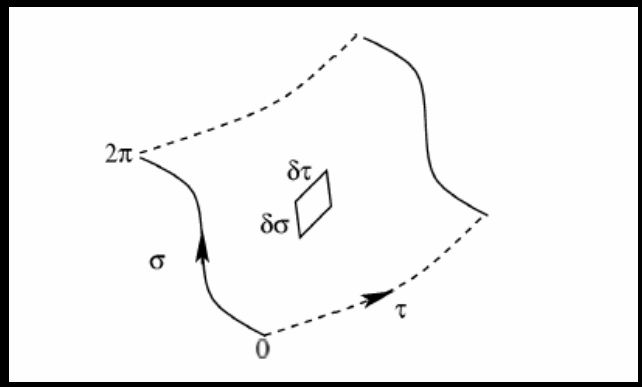
$$M^2 \sim J$$

String Theory

Fundamental strings

Basic property is Tension

$$\begin{aligned}\delta s &= \delta\sigma\delta\tau\sin\theta \\ &= \sqrt{\delta\sigma^2\delta\tau^2 - (\delta\sigma.\delta\tau)^2} \\ &= \delta\sigma\delta\tau\sqrt{\left(\frac{dX^\mu}{d\sigma}\right)^2\left(\frac{dX^\nu}{d\tau}\right)^2 - \left(\frac{dX^\mu}{d\sigma}\frac{dX_\mu}{d\tau}\right)^2}\end{aligned}$$



$$S = T\int d^2\sigma\sqrt{\det(\partial_\alpha X^\mu \partial_\beta X_\mu)}$$

Polyakov form:

$$S = -\frac{1}{4\pi\alpha'}\int d^2\sigma\sqrt{-h}h^{ab}\partial_a X^\mu \partial_b X_\mu$$

Here the world-sheet gravity is an auxilliary field…. note this is a 2d field theory….

Classical Solutions

$$\left(\frac{\partial^2}{\partial\sigma^2} - \frac{\partial^2}{\partial\tau^2}\right) X^\mu = 0$$

The solutions in flat space are not surprising:

- The CoM moves like a relativistic particle
- There are standing waves on the string
- The mass of the string depends on how many oscillations are excited

$$X_R^\mu = \frac{1}{2}x^\mu + \frac{1}{\sqrt{\pi T}}\alpha_0^\mu(\tau-\sigma) + \frac{i}{2\sqrt{\pi T}}\sum_{n\neq 0}\frac{1}{n}\alpha_n^\mu e^{-in(\tau-\sigma)}$$

$$X_L^\mu = \frac{1}{2}x^\mu + \frac{1}{\sqrt{\pi T}}\tilde{\alpha}_0^\mu(\tau+\sigma) + \frac{i}{2\sqrt{\pi T}}\sum_{n\neq 0}\frac{1}{n}\tilde{\alpha}_n^\mu e^{-in(\tau+\sigma)}$$

Compute the Hamiltonian

$$M^2 = \frac{1}{\alpha'}\sum_{n=1}^{\infty}\alpha_{-n}.\alpha_n$$

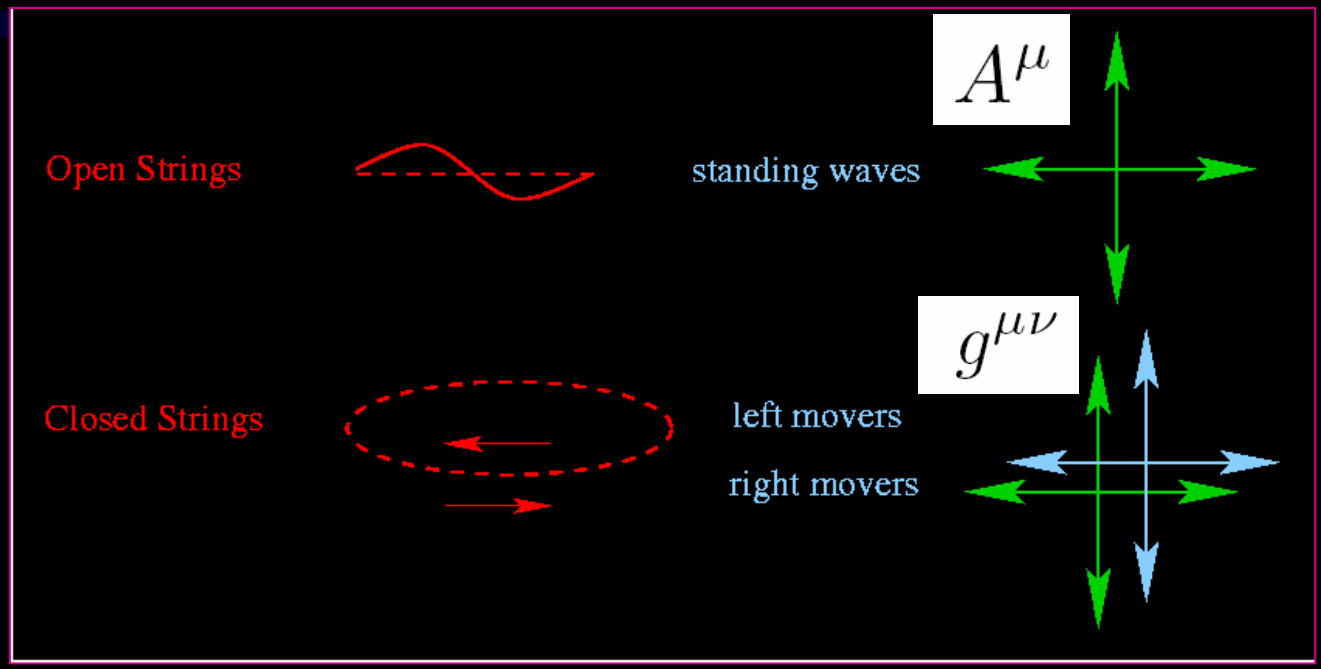
Quantum Zero Point Energy

$$\Delta M = \sum_w (D-2)\frac{1}{2}w = \infty + \frac{2-D}{12} + ..$$

After renormalization (trust us) this leads to a negative shift in the mass spectrum…

Hints of a TOE

In D=26 the massless states are a photon (one excitation) and a graviton (one plus one)



String theory unifies gravity and other forces

The Quantum Gravity Problems

Simplistic quantization of GR includes non-renormalizable interactions between gravitons – the theory becomes strongly coupled at the Planck Scale

$$M_{Pl} = \sqrt{\frac{\hbar c}{G}} \simeq 10^{19} GeV$$

The strings length at 10^{-34} m would cut off the theory

Quantum vacuum energies represent an enormous and unobserved dark energy component

$$\Lambda \sim \sum_{0}^{M_{Pl}} \frac{1}{2} \hbar w$$

Gravity is so weak that we've never measured it's effects between individual particles or on distance scales smaller than 0.1mm!

Superstrings

When shift the mass of the states in the quantum theory unexcited states get negative mass squared – tachyons are not allowed!

Supersymmetry is a symmetry that links fermions and bosons (unseen in nature as yet)

photon (spin 1)	photino (spin ½)
electron (spin ½)	selectron (spin 0)

Amongst the constraints is one that scalar masses must be positive.

GSO added fermions on the string world sheet and projected out a well behaved supersymmetric theory

– “first string revolution”

Superstrings

Fermion world sheet partners

$$S = -\frac{1}{2\pi}\int d^2\sigma(\partial_a X^\mu \partial^a X_\mu - i\bar{\psi}^\mu \gamma^a \partial_a \psi_\mu)$$

Anti-commutation

$$\{\psi^{\dagger\mu}, \psi^\nu\} = \eta^{\mu\nu}\delta(\sigma - \sigma')$$

Dirac equation

$$-i\not{\partial}\psi = 2(\partial_+\psi_- - \partial_-\psi_+) = 0$$

Wave solutions

$$\psi_+^\mu = \frac{1}{\sqrt{2}}\sum_{n=0}^{\infty} d_n^\mu e^{-in(\tau+\sigma)/2}$$

$$\psi_-^\mu = \frac{1}{\sqrt{2}}\sum_{n=0}^{\infty} d_n^\mu e^{-in(\tau-\sigma)/2}$$

$$\{d_0^\mu, d_0^\nu\} = \eta^{\mu\nu}$$

Constant functions don't change the string energy???

The d act as space time γ matrices.. fermions in space time…

Conformal Field Theory in 2d

$$S = -\frac{1}{4\pi\alpha}\int d^2\sigma g^{1/2} g^{ab} G_{\mu\nu}(x)\partial_a X^\mu \partial_b X^\nu$$

$$g^{ab} \to e^{\phi} g^{ab}$$

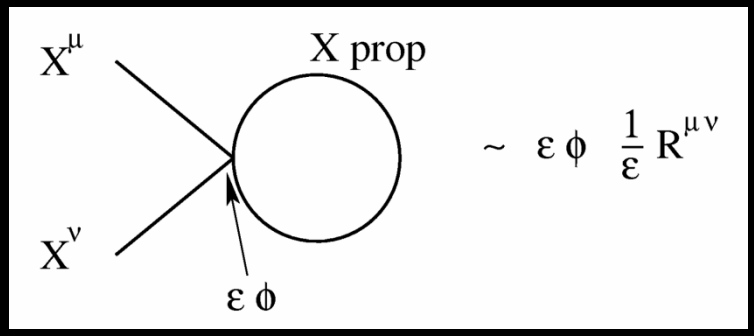
conformal invariance

$$g^{1/2} = \sqrt{det\begin{pmatrix} g_{tt} & g_{t\sigma} \\ g_{\sigma t} & g_{\sigma\sigma} \end{pmatrix}}$$

Away from flat space we get interactions…

$$G_{\mu\nu} = \eta_{\mu\nu} + \frac{1}{3} R_{\mu\lambda\nu\kappa} x^\lambda x^\kappa + \ldots$$

In the quantum theory we get loops… and use dimensional regulation (d=2-ε) to control them…



$$\sqrt{-g} g^{ab} \to e^{\epsilon\phi} \sqrt{-g} g^{ab}$$

Magically conformal invariance is only present with

Einstein's equations

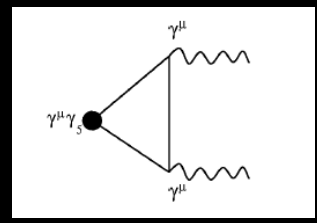
Maxwell's equations

D= 9 + 1

In the full computations there are ghost fields as in QCD computations… the D=10 emerges because we are dialling the number of fermions to cancel a quantum anomaly

Eg why is the right handed up quarks hyper charge = +4/3… because..

$$0 - 1 - 1 - 2 + 3x\ (1/3 + 1/3 - 2/3 + ?) = 0$$



Where is space-time emergence?

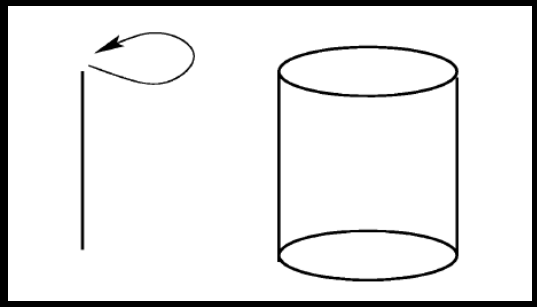
A full picture of spacetime would explain how it emerges from the underlying degrees of freedom…

String theory only watches one string in a classical background…

Five Consistent String Theories

There are five consistent anomaly free string theories

Type IIA – closed strings in 10d



Type IIB – closed strings in 10d but with an alternative choice of spin (LH vs RH) for the superpartner fermion

Type I – open and closed strings in 10d

Heterotic closed strings – left moving super-waves on the closed string live in 10d… right moving waves live in 26d… the extra dimensions can be compactified in two ways to give space time gauge groups

SO(32) or E8 x E8

These theories match the possible 10d supersymmetric (but non-renormalizable) supergravity theories – they are the low energy actions of these string theories

Extra Dimensions

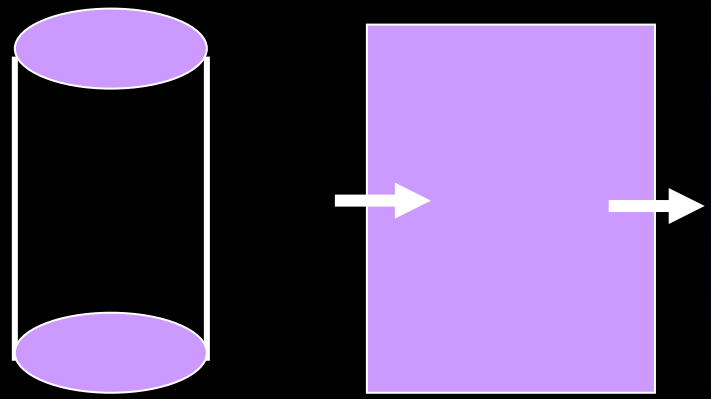
Surprisingly the mathematics of string theory only makes sense in 9 spatial dimensions and 1 time dimension!

A prediction…. But wrong!!

Compactification

We can imagine a space where directions are curled up

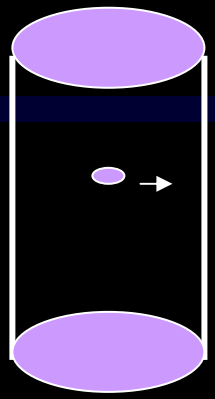
Why are 6 dimensions compact… by what mechanism? UNKNOWN!



There are a vast number of ways to compactify too – each generates a very different short distance theory…

$$\int d^6x d^4x \frac{1}{g^2} F^{\mu\nu} F_{\mu\nu} = \int d^4x \frac{L^6}{g^2} F^{\mu\nu} F_{\mu\nu}$$

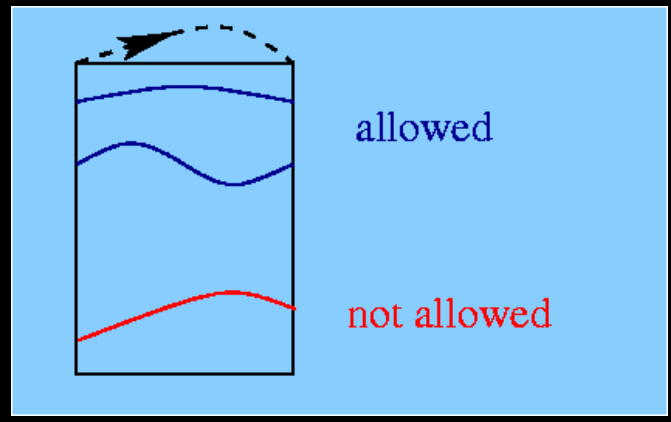
Shrinking Dimensions Away



Remember particles are really quanta of fields… only very special field configurations make sense on a compact dimension

$$e^{ip_r r} = e^{ip_r (r+2\pi R)}$$

$$p_r = \frac{n}{R}$$



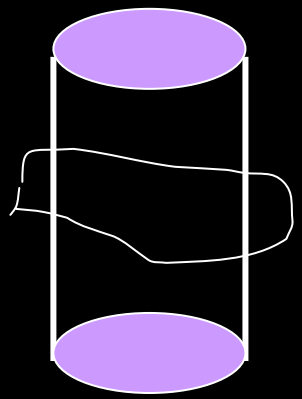
Energy = E, 2E, 3E, 4E, 5E,….

As we shrink the circumference away E grows – eventually you can't make the extra states… it's gone!

Standard string lore – E is 10^{19} proton mass

Wrapping Strings

Closed strings can do something new in a compact space



Wrapping modes have energies

E’, 2E’, 3E’, 4E’, 5E’…

As we shrink the circumference away these states are very low energy

T Duality

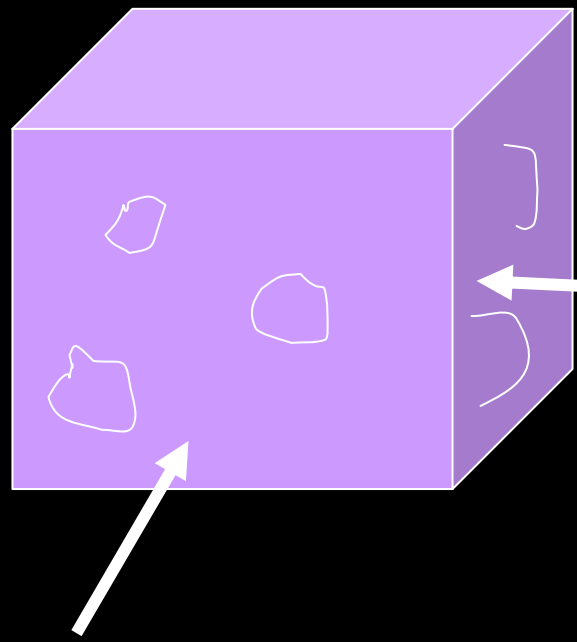
If we “confuse” winding modes and kinetic modes we can see a new dimension appear as another disappears!

$$M_{24}^2 = \frac{n^2}{R^2} + \frac{\omega^2 R^2}{\alpha'^2} - \frac{4}{\alpha'}$$

$$\omega \leftrightarrow n, \qquad R \leftrightarrow \frac{\alpha'}{R}$$

Membranes

Only closed strings see the new direction… not open strings…



Electromagnetic particles live on a sub-space or "brane"

Gravitons live in a higher dimension "bulk"

"Existence proof" for such a world

Unifying String Theories

IIA and IIB string theories were crying out for branes – they have forms that had no electric or magnetic charges

$$\Delta S = \int A^{\mu} \frac{\partial X_{\mu}}{\partial \tau} d\tau$$

$$\Delta S = \int B^{\mu\nu} \frac{\partial X_{\mu}}{\partial \tau} \frac{\partial X_{\nu}}{\partial \sigma} d\tau d\sigma$$

IIA:	$A^{\mu}, B^{5} \rightarrow$ D0, D4 branes
IIB:	$C^{2}, D^{4} \rightarrow$ D1, D3 branes

Solitonic supergravity solutions existed for branes…

T Duality consistently turns the branes of IIA into those of IIB and back again… IIA & IIB are T-dual descriptions of the same theory….

Type I theory is just IIB with a space-filling 9-brane..

M

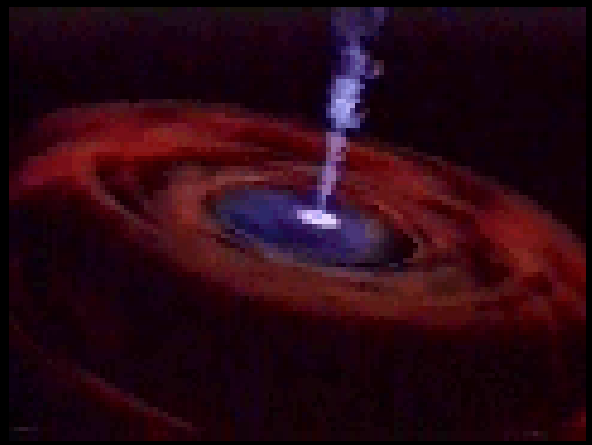
What are the fundamental objects in string theory – why strings and not the branes?

There was always an 11d supergravity that could only include M2 and M5 branes…

Witten proposed a 10+1 dimensional model of M2 and M5 branes that he showed compactifies to give all the string theories – but what is this theory? – Mystery….

Holography

General Relativity predicts there should be objects whose gravitational attraction is enough to stop even light escaping – BLACK HOLES



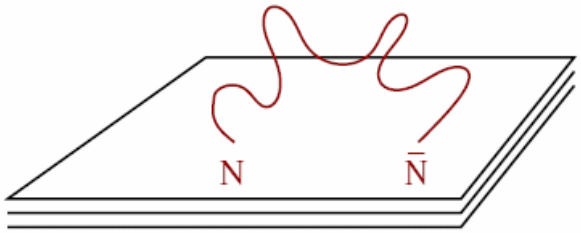
‘tHooft argued that any information dropped into a black hole must be

- lost to our Universe
- spread over the surface

If the surface can contain all the information of the contents the real theory of the Universe must be 2 + 1 dimensional!

Geometrical Engineering

Open strings described gauge fields in 10d... their ends can be restricted to D-branes though



EG D3 branes generate 3+1d N=4 gauge theory

A^{μ} 6ϕ 4ψ

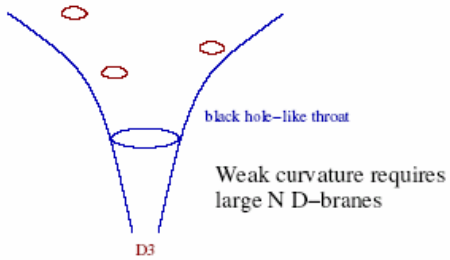
SO(1,9) –> SO(1,3) x SO(6)

In Tension $\rightarrow \infty$ limit all string modes decouple.

SU(N) results from N coincident branes

Gravity Solutions

3-branes have tension - a large N stack generates curvature like a black hole:



The Tension $\rightarrow \infty$ limit blows up the throat

$ds^2 = u^2 dx^2_{//} + \frac{du^2}{u^2} + d\Omega_5^2$

This is $AdS_5 \times S^5$

In this limit higher dimension operators linking the gauge theory on brane and gravity fields off are suppressed - the two descriptions decouple.

Duality - Maldacena

Geometry and gauge theory share global symmetries

SO(6) - S^5, 6 real salars
SO(2,4) - symmetry of AdS_5, conformal symmetry (susy non-renormalization)

DUAL?

eg dilatations

$\int d^4x(\partial\varphi)^2$ invariant to $x \to e^{\alpha}x, \quad \varphi \to e^{-\alpha}\varphi$

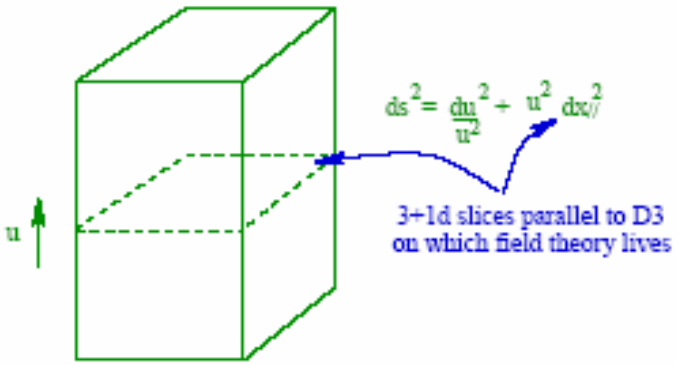
$ds^2 = u^2 dx^2_{//} + \frac{du^2}{u^2}$ invariant to
$x \to e^{\alpha}x, \quad u \to e^{-\alpha}u$

The radial direction is a field theory energy scale.

AdS/CFT Correspondence

Maldacena, Witten...

4d strongly coupled $\mathcal{N}$=4 SYM = IIB strings on $AdS_5 \times S^5$



u corresponds to energy (RG) scale in field theory

Masses, coupling constants etc are represented by five dimensional fields – the way they change with u tells you their RG flow

Witten checked every 5d field correctly corresponds to a gauge theory parameter…. Astonishing!

AdS/CFT Correspondence

Maldacena, Witten...

4d strongly coupled $\mathcal{N}$=4 SYM = IIB strings on $AdS_5 \times S^5$

Pretty well established by this point!

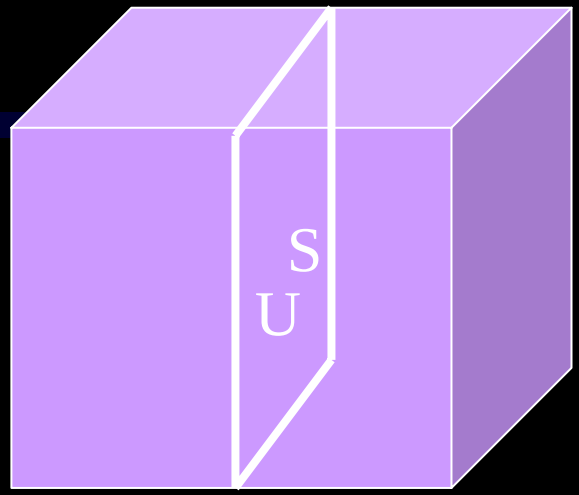
We have a renormalizable and complete non-perturbative definition of string theory on anti-de-Sitter space!

Any strongly coupled gauge theory defines a string theory?

We also are using this to better understand gauge theories (QCD?)

* confinement (hadronization)
* bound state spectra
* mass generation
* leading tool for heavy ion collision - quark gluon plasma
* first applications(?) to condensed matter systems….

Could Our Universe Be A Brane?



The strength of gravity is determined by the number of spatial dimensions (gravitons spread out around mass)

$$F = \frac{G M m}{r^2}$$

D=3+1

But…. we don't know anything about gravity on length scales below 0.1mm… R could = 0.1mm… and we wouldn't know it!

If so we've miscalculated the strength of gravity (G) – it could become strong in our particle accelerators at any new energy!!

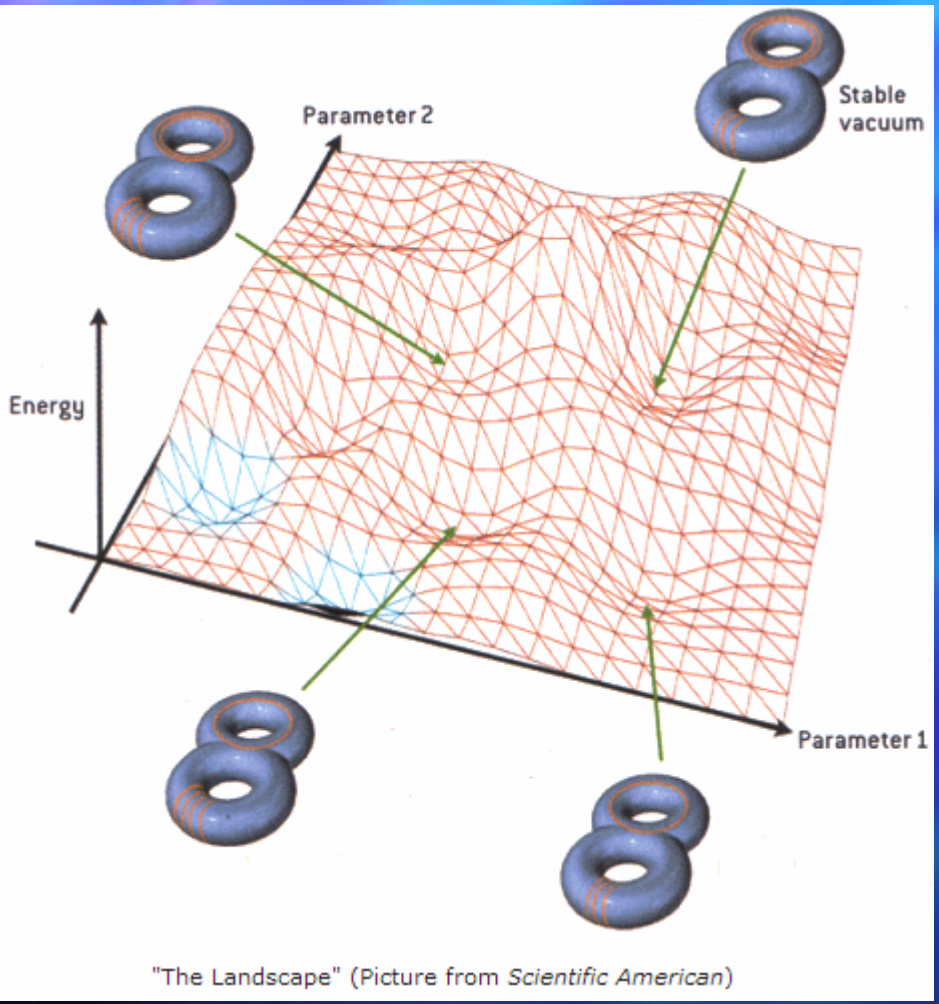
Which is all good fun.... BUT

String theory does not solve the cosmological constant problem… the vacuum is now full of quantum strings… and ought to roll up into a ball…

Measured Energy Density	=	$(10^{-2}eV)^4$
Standard Stringy prediction	=	$(10^{28}eV)^4$
Brane World	=	$(10^{12}eV)^4$

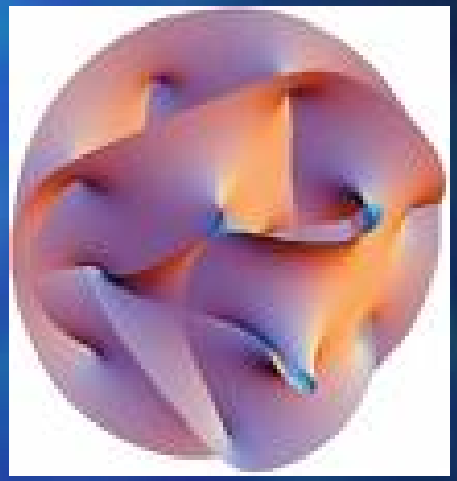
Roughly 120 orders of magnitude wrong!!!!

The Landscape



"The Landscape" (Picture from *Scientific American*)

There are billions and billions of ways to compactify six dimensions



Each choice gives a different Universe… and all are possible in string theory.

Inflation &/or many worlds QM means they can all exist at once!

The Anthropic Principle

When the vacuum energy is computed in each Universe there are many contributions that can cancel each other…

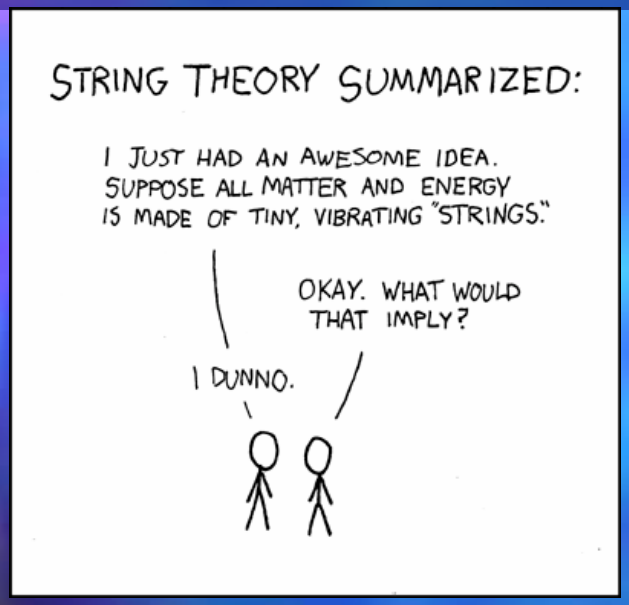
To get

$$10^{120} - 10^{120} \simeq 1$$

is very unlikely but in an infinite set of Universes there will be a few….

Only these large, open Universes can support life… so any life will naturally see a small Cosmological Constant!

What is String Theory?



- A failed theory of mesons
- A renormalizable theory of quantum gravity
- A theory of everything
- A landscape multi-verse
- A point particle theory or membrane theory
- An 11 dimensional theory of membranes
- A rewriting of QCD-like theories
- An ideas factory for particle theory
- A fanciful dead end in theory space??????????

Lot's of fun!!