KING'S College LONDON

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Gauge Theories from the 11th Dimension

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21 November 2018

Plan of Attack

- Symmetry and Quantum Field theory
- Supersymmetry
- String Theory and Quantum Field theory
- M-theory
- Back to Quantum Field theory

Symmetries in Physics

Symmetries underlie our deepest understanding of Physics

Special Relativity tells us that space and time are unified and the rotations of space extend to "rotations" of spacetime

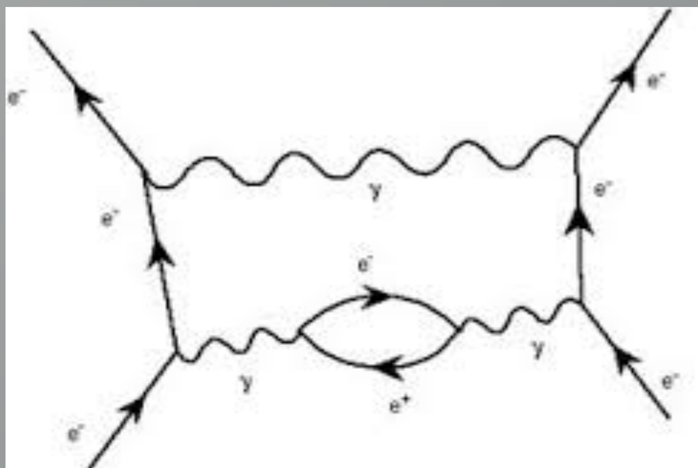
$x^2+y^2+z^2$ $\qquad$ $SO(3) \longrightarrow SO(1,3)$ $\qquad$ $-c^2t^2+x^2+y^2+z^2$

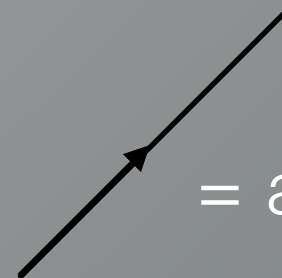
rotations $\qquad$ *Lorentz transformations*

e.g. electricity + SO(1,3) = electromagnetism

Marrying special relativity to quantum mechanics gives quantum field theory

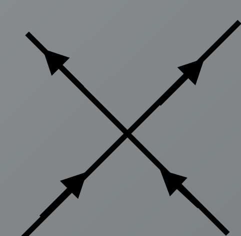
Quantum Field Theory

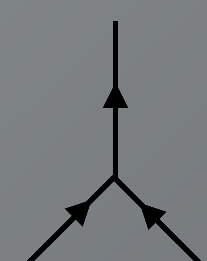




= a free particle

- an irreducible representation of the Lorentz group SO(1,3)
- and an internal symmetry group G_I (*e.g.* flavour symmetry)
- and some gauge group G_G where the symmetry is allowed to be local (*i.e.* spacetime dependent)





= interactions

- term/terms in the Lagrangian/Hamiltonian that are invariant under $SO(1,3) \times G_I \times G_G$

The standard model of particle physics lagrangian has

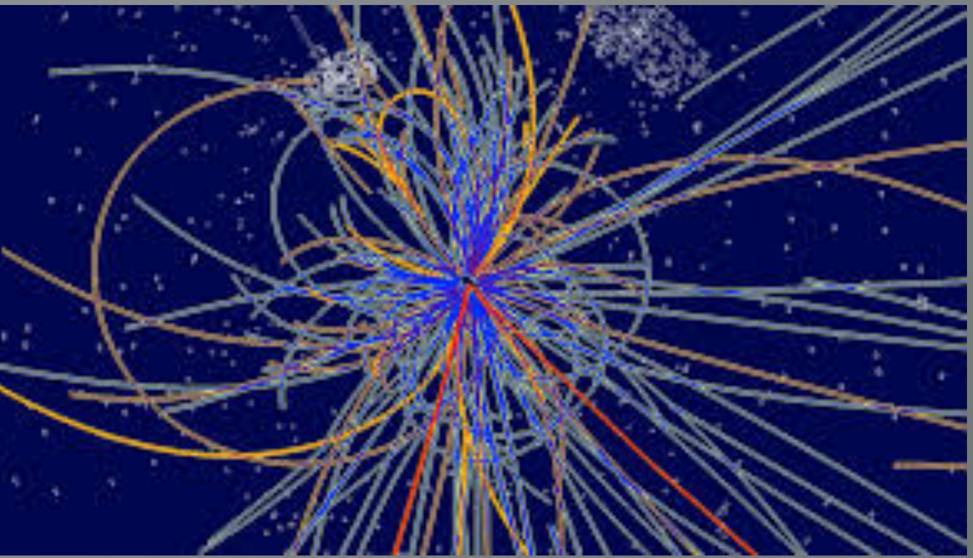
- $G_G = SU(3) \times SU(2) \times U(1)$
- $G_I = U(1) \times U(1)$

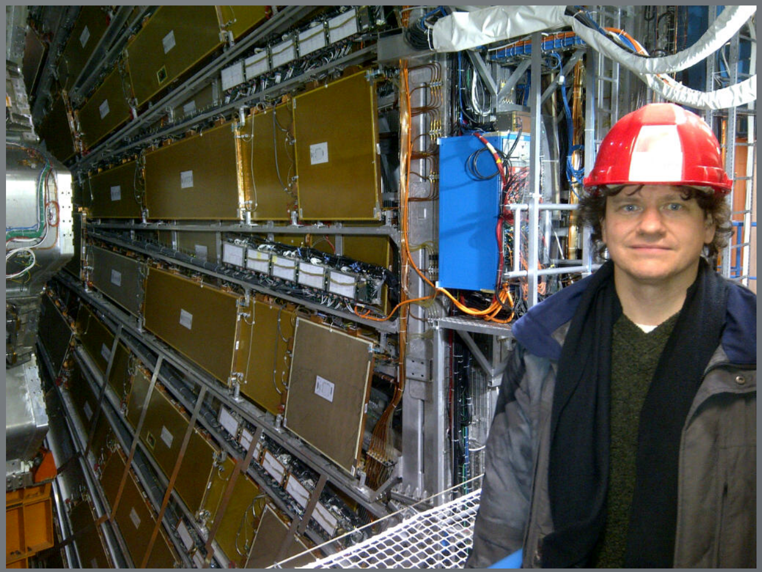
(but has bigger approximate flavour groups)

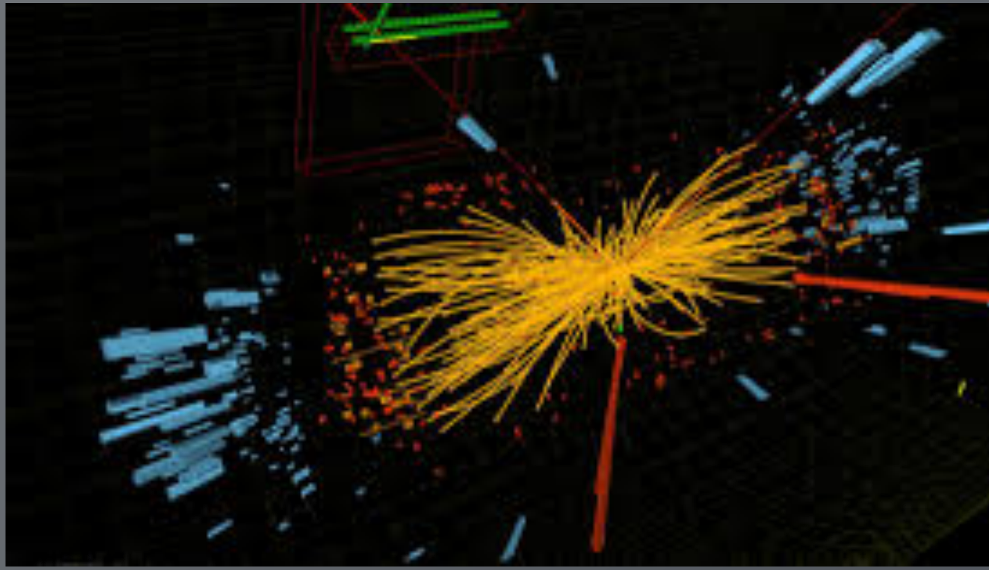
The group structure greatly restricts the possible interactions and relates many different interactions to each other

Gauge symmetry predicts particles, forces and interactions

All experimentally very well tested







Could there be something deeper?

Given the important role of symmetries could there be a bigger group that extends $SO(1,3) x G_I x G_G$?

Yes: Supersymmetry

Here one has anti-commuting generators (spinor representations of the Lorentz group)

Usual Lie-algebra $[T_i,T_j] = T_iT_j-T_jT_i = f_{ij}{}^kT_k$

Super-Lie-algebra $\{Q_a,Q_b\} = Q_aQ_b+Q_bQ_a = P_{ab}$

supersymmetry generators *translations (momentum)*

So What?

The associated conserved charges of these symmetries are not Lorentz invariant (*e.g.* they sensitive to rotations)

Acting on a particle in one representation of the Lorentz group produces a particle in another representation

This means that for every matter particle (fermion) there is an associated force particle (boson)

Not a symmetry of the standard model, not yet observed in any physical system

But of great interest as potential new physics at the LHC (*e.g.* dark matter, Higgs physics, GUT models)

Beautiful and mathematically deep with much greater control over computations (many exact results).

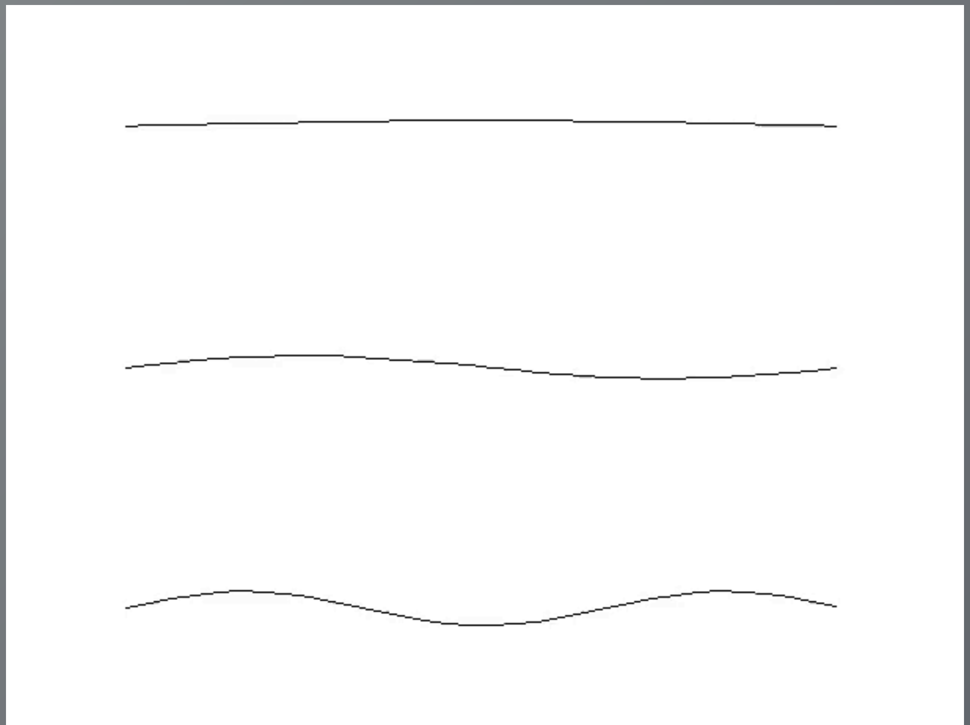
Could there be something still deeper?

But something is missing: what about gravity?

enter strings

Recipe: replace particles by strings and quantise

vibrations (standing waves) become particles in quantum theory



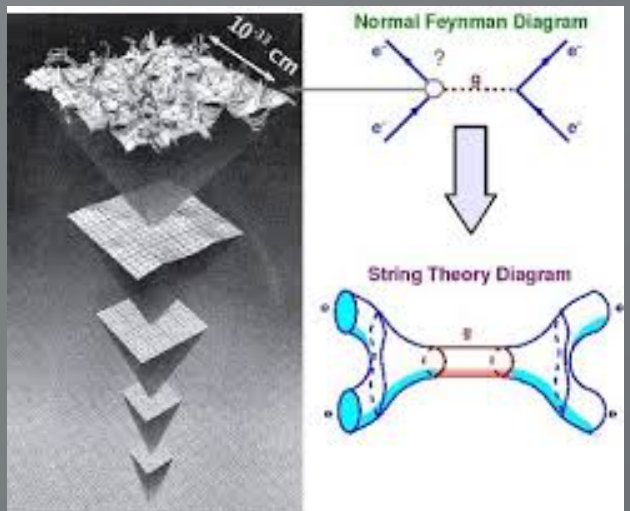
So What?

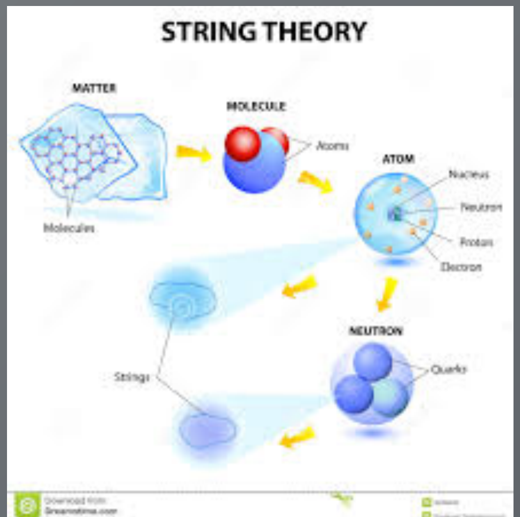
This is a surprisingly rich thing to do:

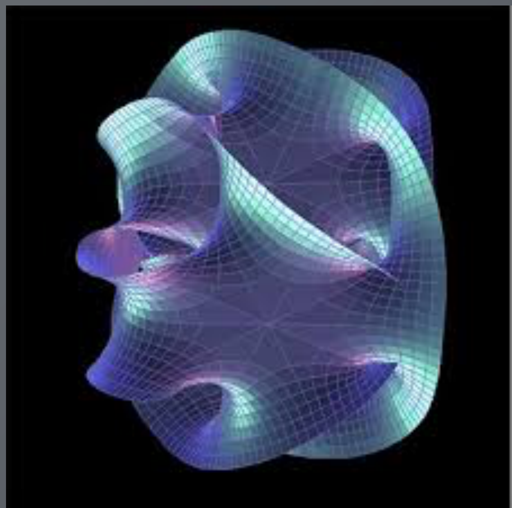
- produces an infinite tower of particles but only the lowest modes are relevant for low energy
- closed strings give gravity
- open strings give gauge forces

Unified, consistent quantum theory of all known physics (and more)

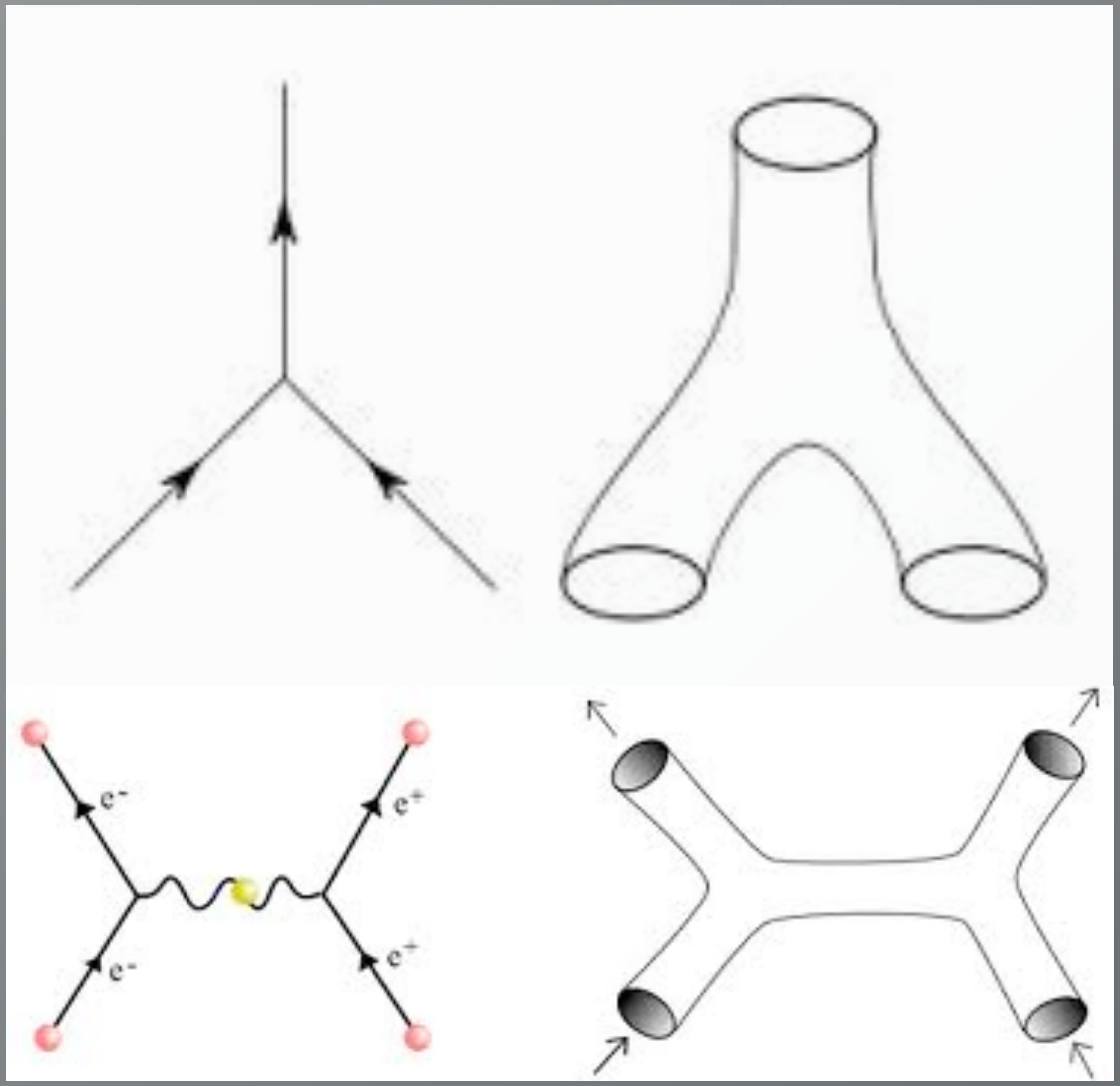
- with deep connections to mathematics







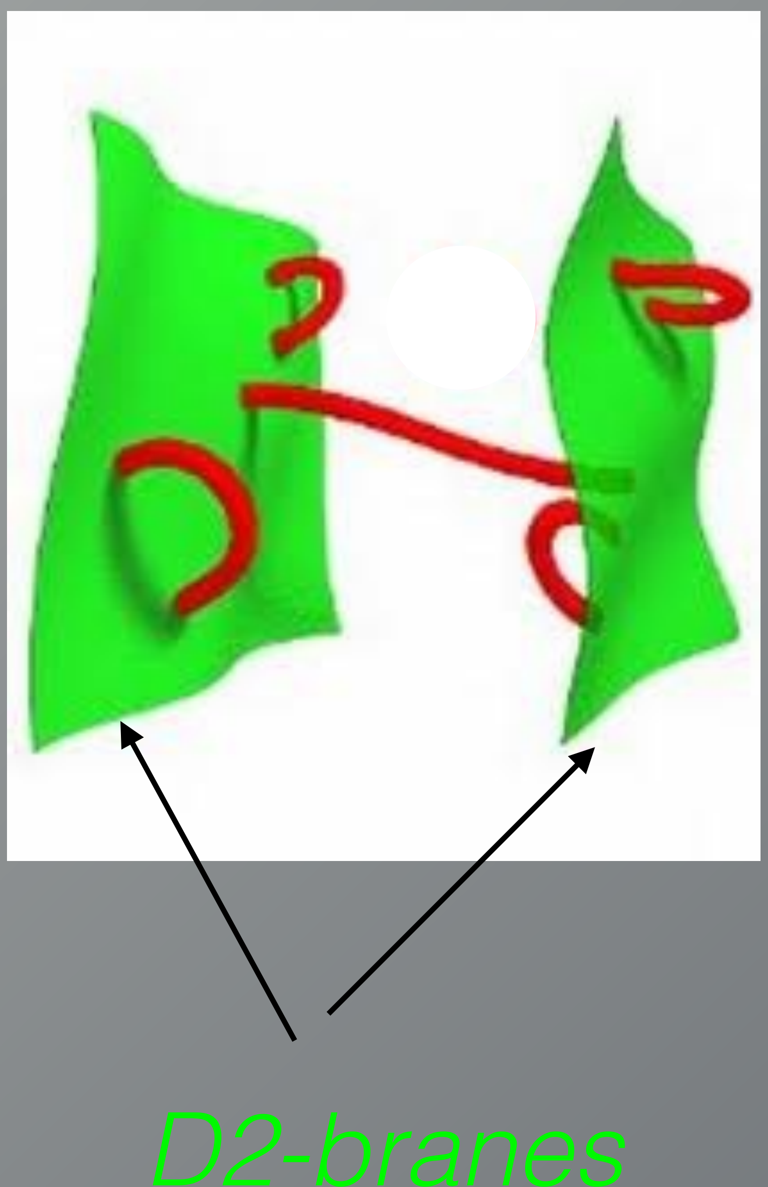
Strings are also (almost) unique: once you say how a single string behaves you also know how it interacts



This leads to just 5 of possible theories describing particle physics unified with gravity

- But all only in ten dimensions
- And with supersymmetry

So What Do Strings Say About QFT?



Open strings have to end somewhere

Such a surface is called a Dp-brane

- p=0,1,2,… is the spatial dimension

  so a particle is a D0-brane,
  a string a D1-brane *etc.*

- Their dynamics are governed by a p+1 dimensional quantum field theory, arising from dynamics of the open strings, that “lives” on the brane’s worldvolume.

What are these quantum field theories?

Consider the simplest cases consisting of flat parallel Dp-branes

Identify the symmetries:

1) Lorentz transformations ("rotations")

$$SO(1,9) \longrightarrow SO(1,p)xSO(9-p)$$

1+p+9-p=10

10D "rotations"

"rotations" along the brane

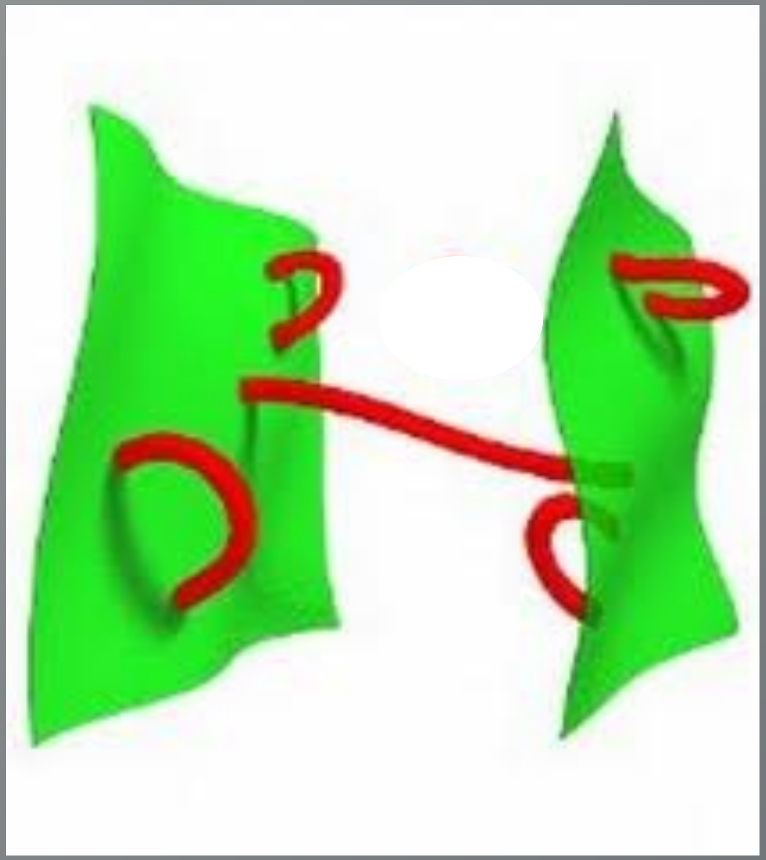
rotations off the brane

2) Supersymmetries: Q_a

these are in spinor representations of both SO(1,p) and SO(9-p), not just SO(1,p). Known as an R-symmetry

So even though we are in a lower dimension the field theory remembers that it comes from 10D

For example for the D2-brane we would need a supersymmetric quantum field theory in 2+1 dimensions with $G_I = SO(7)$ 1+2+7=10



The required theories have been known for 40 years:

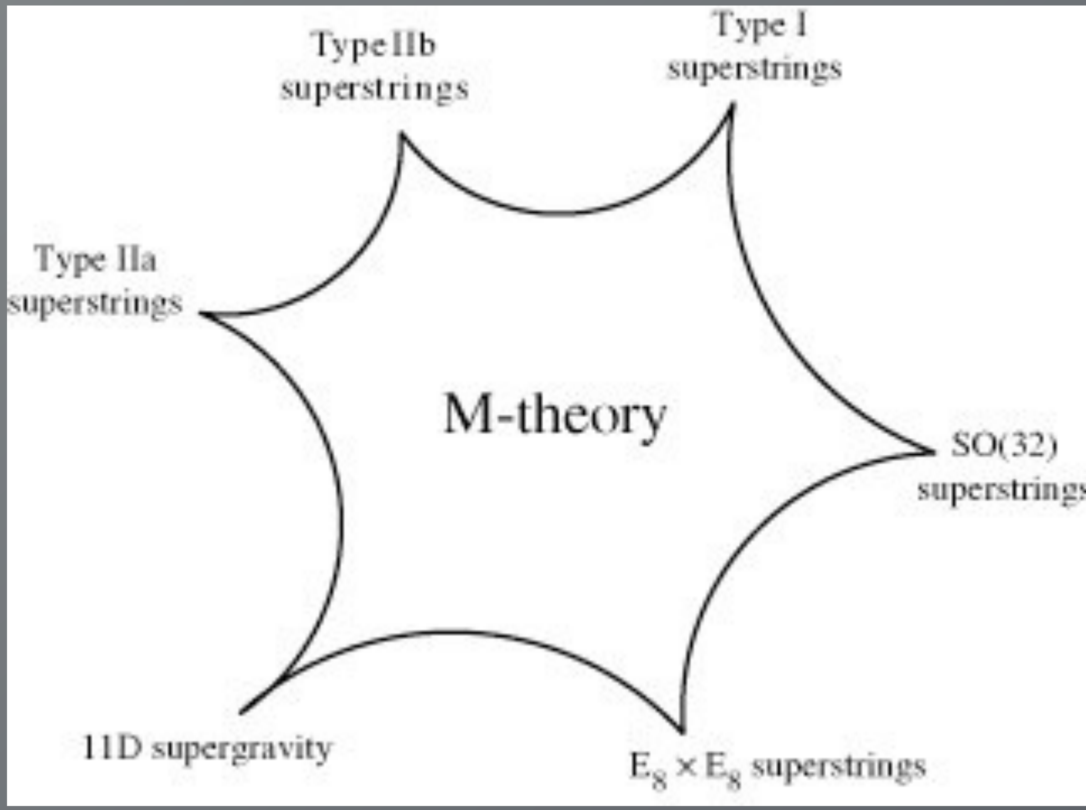
(maximally supersymmetric) Yang-Mills theories

- highly symmetric cousins of the gauge theories in the standard model of particle physics

Enter M-Theory

We now see the 5 String Theories as perturbative expansions of some deeper theory: M-theory

- 11-dimensional
- $R_{11} = g_s l_s \longrightarrow \infty$
- strongly coupled



No clear experimental predictions (like string theory) but M-theory has interesting predictions for Quantum Field Theory

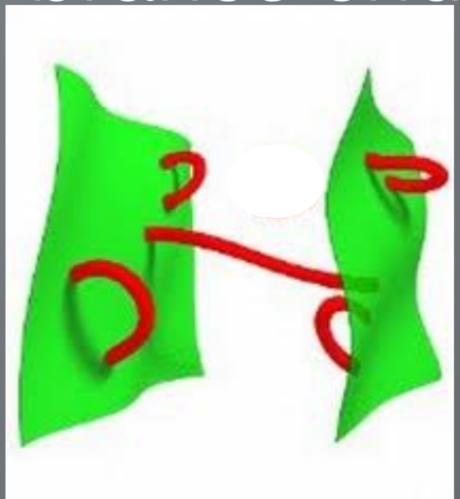
In the strong coupling limit D-branes migrate to M-branes

String theory branes

- F1 → M2 (wrapped)
- D2 → M2
- D4 → M5 (wrapped)
- NS5 → M5

M-theory branes

- no microscopic picture of M-theory or M-branes (no strings attached)
- formally open M2-branes ending on M2's,M5's



So What Does M-Theory say about Quantum Field Theory?

M-theory has M2 and M5-branes but now they live in 11D so it predicts quantum field theories with

M2-branes: 2+1 dimensional

- SO(1,2) x SO(8) symmetry (*c.f.* SO(1,2)xSO(7))

  1+2+8=11

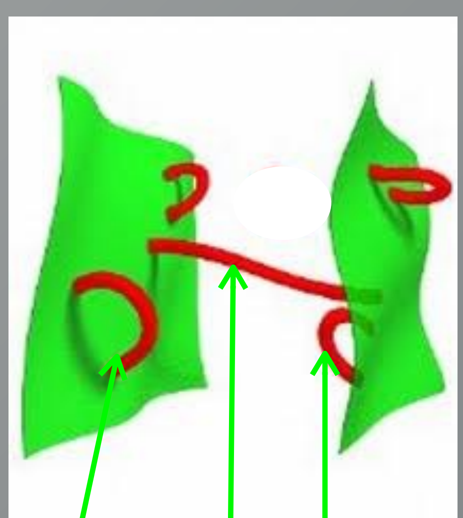
M5-branes: 5+1 dimensional

- SO(1,5) x SO(5) symmetry (*c.f.* SO(1,4)xSO(5))

  1+5+5=11

The first examples of these M2-brane theories is BLG to describe two M2's and the general case for N M2's is ABJM

To describe these theories let us first look in more detail at the theories string theory predicts:



- Fields associated to the open strings are naturally described by matrices X_{AB} where A,B=1,..,N labels which brane the ends of the string end on.
- Splitting and joining of strings is like matrix multiplication

$$\textit{e.g.}\ X,Y|_{12} \longrightarrow X_{11}Y_{12} + X_{12}Y_{22} = (XY)_{12}$$

- So one finds a theory of NxN matrices

In particular this gives maximally supersymmetric U(N) Yang-Mills gauge theory:

$$S=-\int d^{p+1}x\,\frac{1}{4}(F,F)+\frac{1}{2}\sum_I(DX^I,DX^I)-\frac{1}{4}\sum_{IJ}([X^I,X^J],[X^I,X^J])+fermions$$

Fields take values in the U(N) Lie algebra

- (,) is an invariant inner product on Lie(U(N))
- D = d + A is a connection
- F=dA+[A,A] is the curvature
- $[X^I,X^J] = X^IX^J - X^IX^J$
- Jacobi identity:

$[[X^I,X^J],X^K]+[[X^j,X^K],X^I]+[[X^K,X^I],X^J]=0$

To construct the M2-brane theory various symmetries imply that we need triple products

Fields take values in a 3-algebra V with triple product

- $[X^I, X^J, X^K] : V^3 \longrightarrow V$
- Fundamental identity:

$$[[X^I,X^J,X^K],X^L,X^M]+[X^K,[X^I,X^J,X^L],X^M]+[X^K,X^L,[X^I,X^J,X^M]]=0$$

$$S = -\int d^3x \, \frac{1}{2}\sum_I \langle DX^I, DX^I\rangle - \frac{1}{12}\sum_{IJK}\langle [X^I,X^J,X^K],[X^I,X^J,X^K]\rangle$$
$$+ (A\wedge, dA - \frac{2i}{3} A \wedge A) + fermions$$

- D=d+A is a connection on a lie-algebra Lie(G)
- (,) is an invariant inner product on Lie(G) (not positive definite)
- $\langle\ ,\ \rangle$ is an invariant inner production on the 3-algebra

3-algebras tell you the gauge algebra as the fundamental identity insures that

$$X \longrightarrow [A,B,X]$$

is the action of some Lie(G) on V (for any pair A,B in V)

Theorem (Faulkner): A 3-algebra V is equivalent to a vector space V and Lie algebra Lie(G) together with a representation of Lie(G) on V.

So these theories are Chern-Simons theories for some group G with matter fields in certain representations of G

The amount of supersymmetry is determined by the symmetry properties of the triple product [, ,] and hence by G and V.

We are after a maximally supersymmetric theory with SO(8) symmetry

This requires that [, ,] is totally anti-symmetric,

e.g. if T_i , i=1,2,3,4 are a basis for V then (k is an integer)

$$[T_i, T_j, T_k] = \frac{2\pi}{k}\varepsilon_{ijkl}T_l$$

The Lie algebra is that of SU(2)xSU(2) with matter fields in the ($\mathbf{2}$,$\overline{\mathbf{2}}$).

In fact this choice is the unique with SO(8) [Gauntlett, Gutowski, Papadopoulos]

Describes two M2-branes in eleven dimensions. [Bashkirov, Distler, Kapustin, NL, Mukhi, Papageorgagkis, Tong, van Raamsdonk]

Slightly less symmetry (SO(6)xSO(2), 3/4 supersymmetry) gives infinitely many choices

$$[X^I, X^J, X^K] = \frac{2\pi}{k}(X^I(X^K)^\dagger X^J - X^J(X^K)^\dagger X^I)$$

Here X^I are NxM matrices

The associated gauge Lie algebra is that of U(N)xU(M) with matter fields in the ($\mathbf{N}$,$\bar{\mathbf{M}}$) [Aharony, Bergman, Jafferis, Maldacena]

Describes an arbitrary number of M2-branes in eleven dimensions (with a spacetime Z_k orbifold)

- dual to $AdS_4 \times S^7/Z_k$

Curiously most of the SO(8) theories have no known role in M-theory: could there be something deeper?

Why did it take so long to find these theories?

They have at least two novel features:

1) The gauge fields are not in the same representation of the gauge group as the other fields

- okay since they are non-dynamical

2) The amount of supersymmetry depends on the choice of gauge group (the Lagrangians are essentially the same)

- SU(2) x SU(2) has maximal supersymmetry
- U(n) x U(m) have 3/4 of maximal supersymmetry
- other groups have less supersymmetry
  e.g. G_2xSU(2) has 5/8 supersymmetry

These are nicely encoded in the 3-algebra form but quite obscure in the usual Lie-algebra formulations.

The 6D Theory on M5-branes remains deeply mysterious

Until they were predicted it was thought that quantum field theories could not exist above four dimensions and we still have no systematic (text book) tools for them

The existence of this theory encapsulates a great number of highly non-trivial results about lower dimensional gauge theories (S-duality)

There are also several cousins in five and six dimensions

As well as relations to pure mathematics (Langlands Programme).

There is still no good understanding of this theory

It is not thought to have a 6D Lagrangian description

Reduction on S^1 gives 5D Super-Yang-Mills.

Reduction on $S^1 \times S^1$ gives 4D Super-Yang-Mills with manifest S-duality arising from modular transformations.

I have been exploring how different actions arise from choices of S^1 which arise from a single six-dimensional system with constraints.

No single action seems to capture all the physics but:

I [with Papageorgakis] have a (2,0) System:

$$
\begin{aligned}
0 &= D^2X^I - \frac{i}{2}[Y^\sigma, \bar{\Psi}, \Gamma_\sigma\Gamma^i\Psi] + [Y^\sigma, X^J, [Y_\sigma, X^J, X^I]] \\
0 &= D_{[\lambda}H_{\mu\nu\rho]} + \frac{1}{4}\varepsilon_{\mu\nu\lambda\rho\sigma\tau}[Y^\sigma, X^I, D^\tau X^I] + \frac{i}{8}\varepsilon_{\mu\nu\lambda\rho\sigma\tau}[Y^\sigma, \bar{\Psi}, \Gamma^\tau\Psi] \\
0 &= \Gamma^\rho D_\rho\Psi + \Gamma_\rho\Gamma^I[Y^\rho, X^I, \Psi] \ ,
\end{aligned}
$$

$$
\begin{aligned}
\delta X^I &= i\bar{\epsilon}\Gamma^I\Psi \\
\delta Y^\mu &= 0 \\
\delta\Psi &= \Gamma^\mu\Gamma^I D_\mu X^I\epsilon + \frac{1}{2\cdot 3!}H_{\mu\nu\lambda}\Gamma^{\mu\nu\lambda}\epsilon - \frac{1}{2}\Gamma_\mu\Gamma^{IJ}[Y^\mu, X^I, X^J]\epsilon \\
\delta H_{\mu\nu\lambda} &= 3i\bar{\epsilon}\Gamma_{[\mu\nu}D_{\lambda]}\Psi + i\bar{\epsilon}\Gamma^I\Gamma_{\mu\nu\lambda\rho}[Y^\rho, X^I, \Psi]
\end{aligned}
$$

$$
\delta A_\mu(\cdot) = i\bar{\epsilon}\Gamma_{\mu\nu}[Y^\nu, \Psi, \ \cdot\ ]
$$

There are also constraints:

$$
\begin{aligned}
D_\mu Y^\nu &= 0 \\
[Y^\mu, D_\mu, \ \cdot\ ] &= 0 \\
[Y^\mu, Y^\nu . \ \cdot\ ] &= 0 \\
F_{\mu\nu}(\ \cdot\ ) &= [Y^\lambda, H_{\mu\nu\lambda}, \ \cdot\ ]
\end{aligned}
$$

So in particular the vector Y is fixed to a constant and is non-dynamical.

There is also a generalization [with Sacco] to include M2-branes by introducing a constant abelian 3-form.

In this case some of the components of Y become dynamical

Fixing Y in different ways and solving the constraints leads to different actions, all associated to some proposal for the (2,0) theory:

Y spacelike: Constraints imply compactification on S^1

$$S_{SYM} = -\frac{4\pi^2}{R_5}\mathrm{tr}\int d^5x \left(\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}D_\mu X^I D^\mu X^I - \frac{1}{4}[X^I, X^J][X^I, X^J] + \frac{i}{2}\bar{\Psi}\Gamma^\mu D_\mu \Psi + \frac{1}{2}\bar{\Psi}\Gamma^I [X^I, \Psi]\right) .$$

This is 5D mSYM. But we [Douglas],[NL, Papageorgakis and Schmidt-Sommerfeld] have conjectured that this is in fact non-perturbatively well-defined and defines the (2,0) theory on S^1 with no additional UV degrees of freedom.

KK modes are given by soliton states whose spatial profiles are given by solutions by self-dual gauge fields:

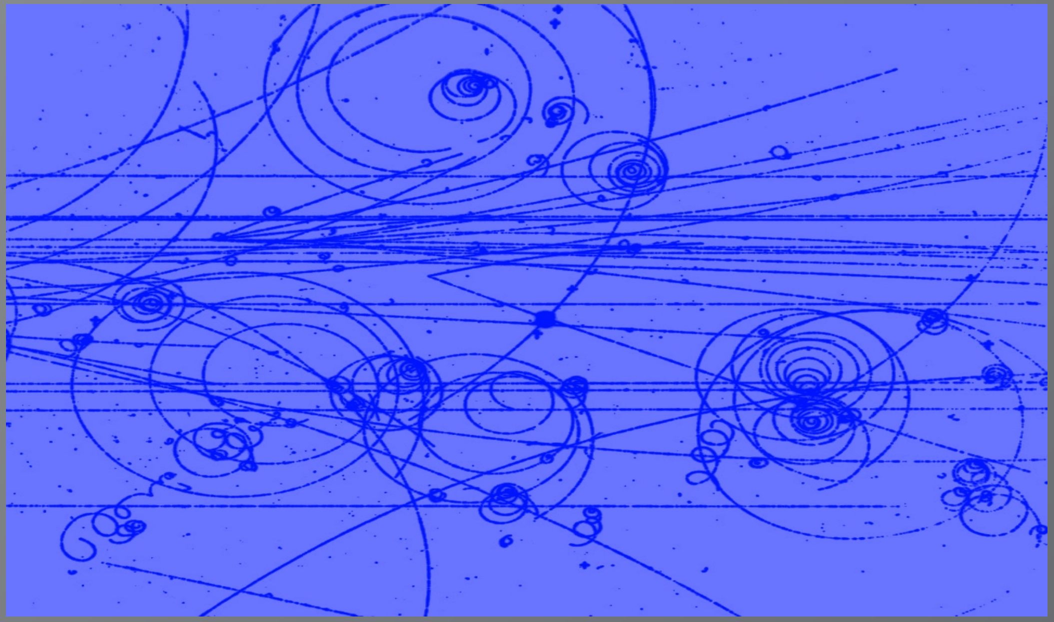
$$P_5 = \frac{1}{R_5} \underbrace{\frac{1}{8\pi^2}\mathrm{tr}\int F \wedge F}_{\text{instanton number} \in \mathbb{Z}}$$

Y Timelike implies compactification on a timelike S[1]

$$S_{ESYM} = -\frac{4\pi^2}{R_0}\mathrm{tr}\int d^5x\left(\frac{1}{4}F_{ij}F_{ij} - \frac{1}{2}D_iX^ID_iX^I - \frac{1}{4}[X^I,X^J][X^I,X^J] + \frac{i}{2}\bar{\Psi}\Gamma_iD_i\Psi - \frac{1}{2}\bar{\Psi}\Gamma^I[X^I,\Psi]\right).$$

This is a Wick-rotated version of the previous conjecture [Hull,NL].

Here one sees the entire world volume of the various states as solitons.



i.e. a particle appears as a 1D worldline and a string as a 2D world sheet

Y Null leads to compactification on a light like S^1

$$S_{NSYM} = \frac{4\pi^2}{R_-}\mathrm{tr}\int d^4x dx^+ \left(\frac{1}{2}F_{+i}F_{+i} - \frac{1}{2}D_iX^ID_iX^I + \frac{1}{2}F_{ij}G_{ij} + \frac{i}{2}\bar{\Psi}\Gamma_-D_+\Psi + \frac{i}{2}\bar{\Psi}\Gamma_iD_i\Psi - \frac{1}{2}\bar{\Psi}[X^I,\Gamma_-\Gamma^I\Psi]\right) ,$$

This is a curious non-Lorentzian theory with 16 supersymmetries.

G_{ij} is self-dual and its equation of motion implies that F_{ij} is anit-self-dual:

> Dynamics is restricted to the Manton approximation of motion on the ADHM moduli space

This reproduces and old DLCQ matrix model proposal of [Aharony, Berkooz, Kachru, Seiberg and Silverstein]

Conclusions

In this talk I have tried to show how M-theory leads to non-trivial predictions about ‘ordinary’ quantum field theory.

Predicts 3D CFT’s with enhanced symmetries:

> Now all constructed via Chern-Simons Lagrangian theories with novel gauge groups

Also predicts highly non-trivial 6D CFT’s

> No Lagrangian description with all symmetries
>
> But there are families of field theories and still much to be learnt


Thank you
BORING