Lattice QCD & the search for BSM physics in beauty

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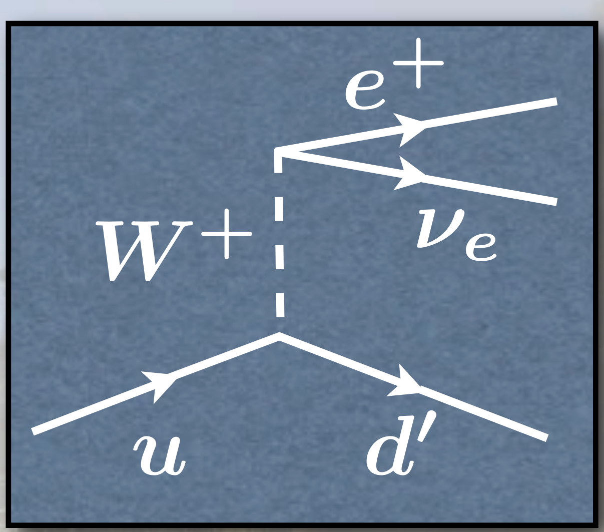
Outline

- Quark flavour
  - *Peering through the glue to study electroweak symmetry breaking*
- Lattice QCD
  - *Uniting the gauge theory, statistical physics, and effective field theory*

Quark flavour

- Discovery era & flavour
- High precision in flavour
- Rare decays

Quark flavour in the SM



- Only weak interactions change quark flavor

$$\begin{pmatrix} u \\ d' \end{pmatrix} \begin{pmatrix} c \\ s' \end{pmatrix} \begin{pmatrix} t \\ b' \end{pmatrix}$$

- Flavor mixing

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

- *V* is the CKM matrix. Unitarity + “rephasing” implies 4 free SM parameters (one of them a CP-violating phase)

CKM matrix from Higgs couplings

LH SU(2) doublets $Q^i_L = \begin{pmatrix} u'^i \\ d'^i \end{pmatrix}_L$ RH SU(2) singlets u^i_R d^i_R

Interact with gauge bosons in covariant derivative

$$\mathcal{L}_{\text{quark}} = \bar{Q}^i_L \, i\not{D} \, Q^i_L + \bar{u}^i_R \, i\not{D} \, u^i_R + \bar{d}^i_R \, i\not{D} \, d^i_R$$

Gives rise to weak current $J^{\mu,+}_{\text{weak}} = \bar{u}'^i_L \gamma^\mu d'^i_L$

The coupling to the Higgs field is not apparently diagonal in generation

$$\mathcal{L}_{\text{quark},\phi} = -\sqrt{2}\left[\lambda^{ij}_d \bar{Q}^i_L \, \phi \, d^j_R + \lambda^{ij}_u \bar{Q}^i_{La} \, \epsilon_{ab} \phi^\dagger_b \, u^j_R + \text{h.c.}\right]$$

Fields may be transformed to find mass eigenstates

$$\mathcal{L}_{\text{quark},\phi}|_{vev} = -\sum_i \left(m^i_d \bar{d}^i_L d^i_R + m^i_u \bar{u}^i_L u^i_R + \text{h.c.}\right)$$

Showing the weak current allows mixing between generations

$$J^{\mu,+}_{\text{weak}} = \bar{u}^i_L \gamma^\mu V^{ij}_{CKM} d^j_L$$

Physics Beyond the Standard Model

- Standard Model shortcomings: Higgs mass fine-tuning, dark matter, CP asymmetry & M/AM
- Direct production: BSM spectrum
- Indirect searches: BSM couplings
- Complementary approaches

Complementarity: top quark

Indirect

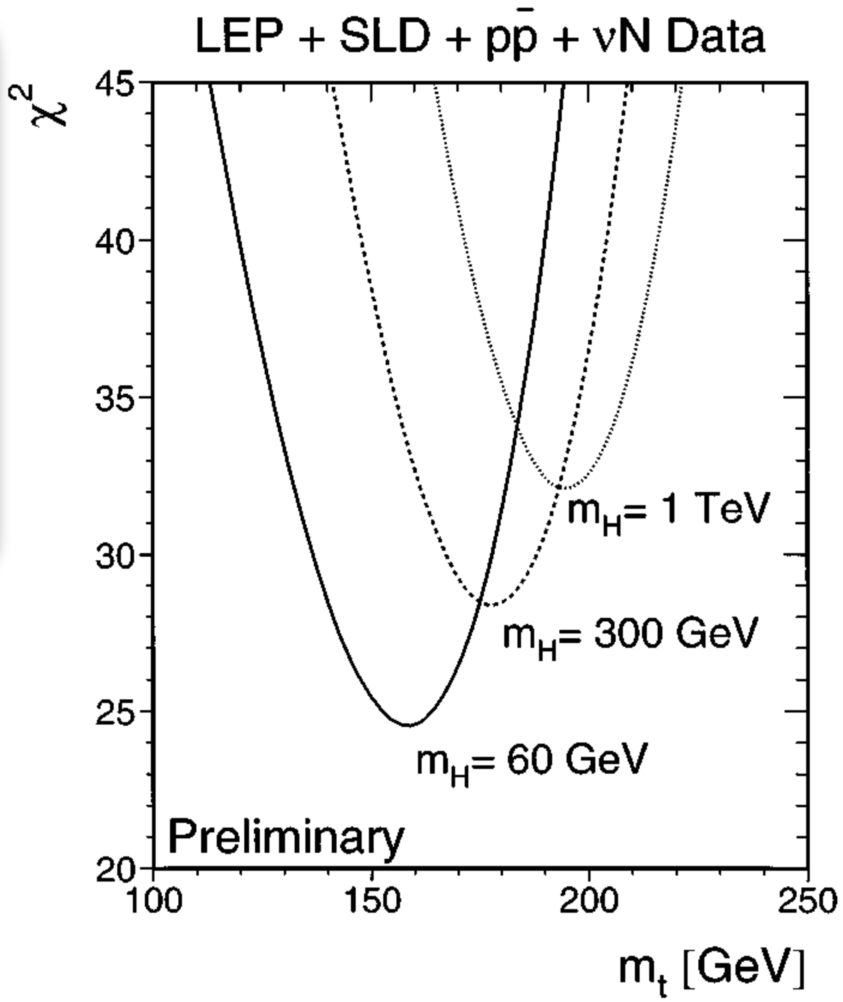


FIG. 13. The χ^2 curves for the standard model fit to the electroweak precision measurements from LEP, SLD, CDF, and D0 (W mass only) and neutrino-scattering experiments as a function of M_{top} for three different Higgs-mass values spanning the interval 60 GeV/$c^2 \leqslant M_{Higgs} \leqslant$1000 GeV/$c^2$. The number of degrees of freedom is 14 (LEP Collaborations, 1995).

Direct

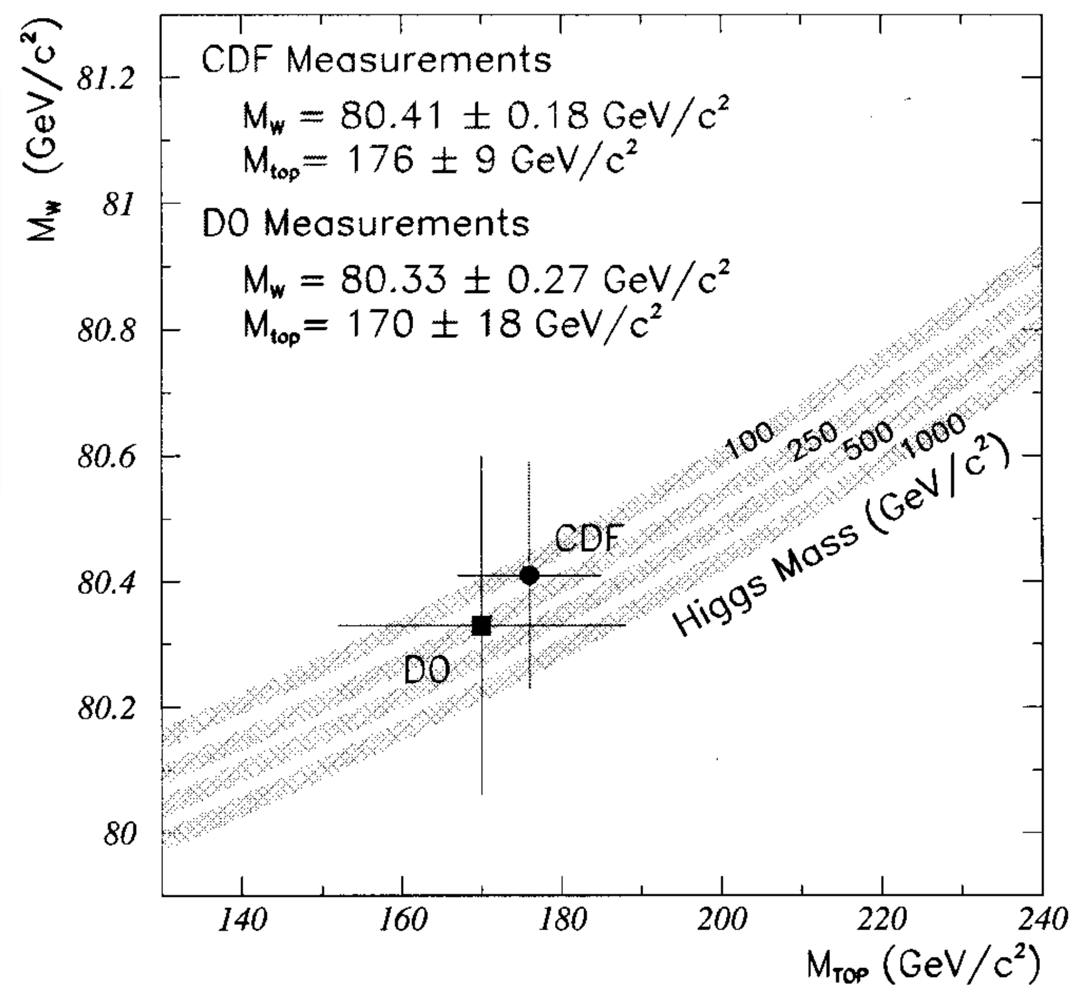
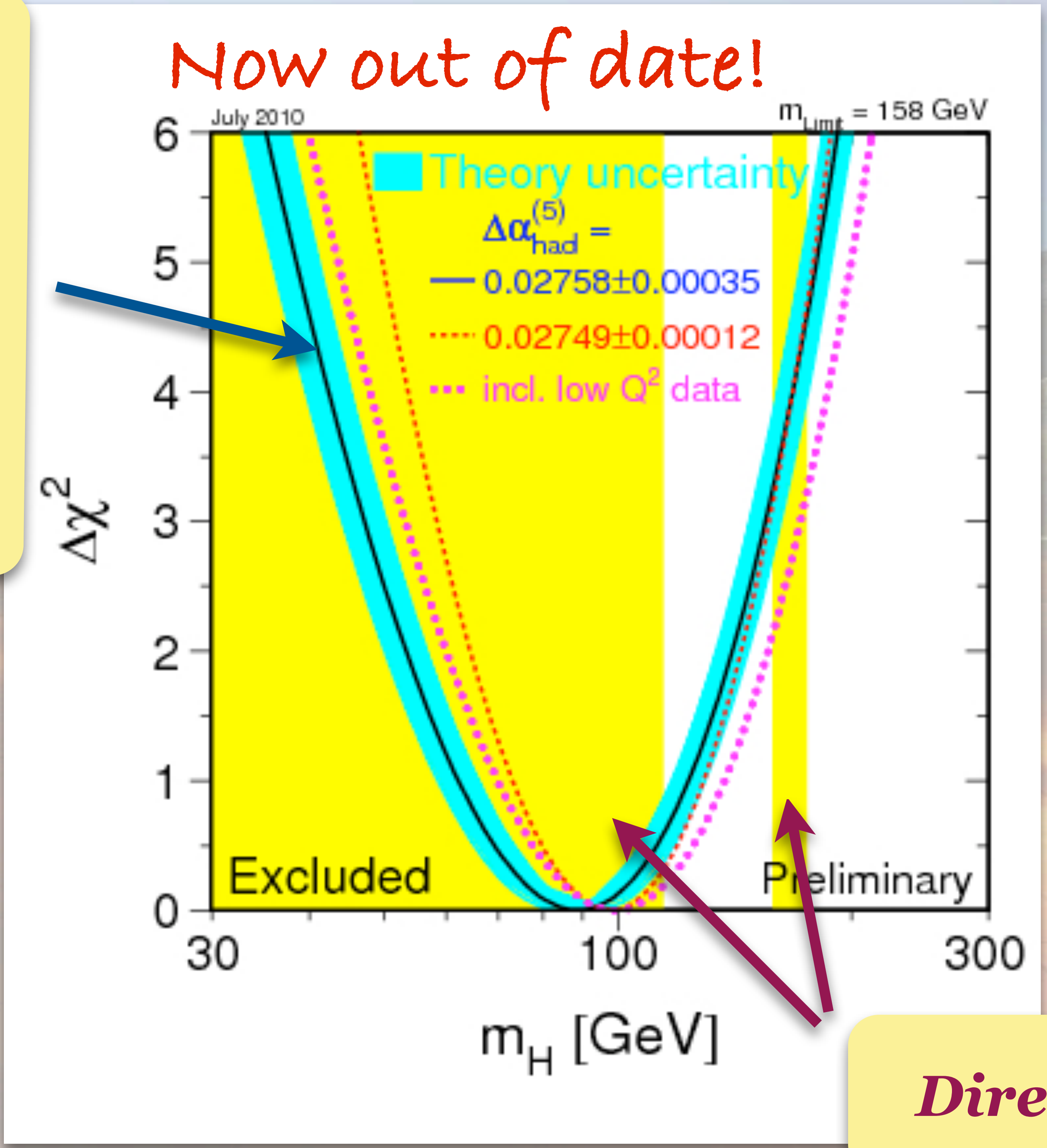


FIG. 12. W mass and top-quark mass measurements from the Fermilab collider experiments (CDF and D0). The top-mass values are from the full Tevatron data sets, with an integrated luminosity of ≈ 100 pb^{-1}. The W mass values are derived from analyses of the first 15–20 pb^{-1} only. The lines are standard model predictions for four different Higgs masses (Flattum, 1996).

from Campagnari and Franklin, Rev. Mod. Phys. **69**, 137 (1997)

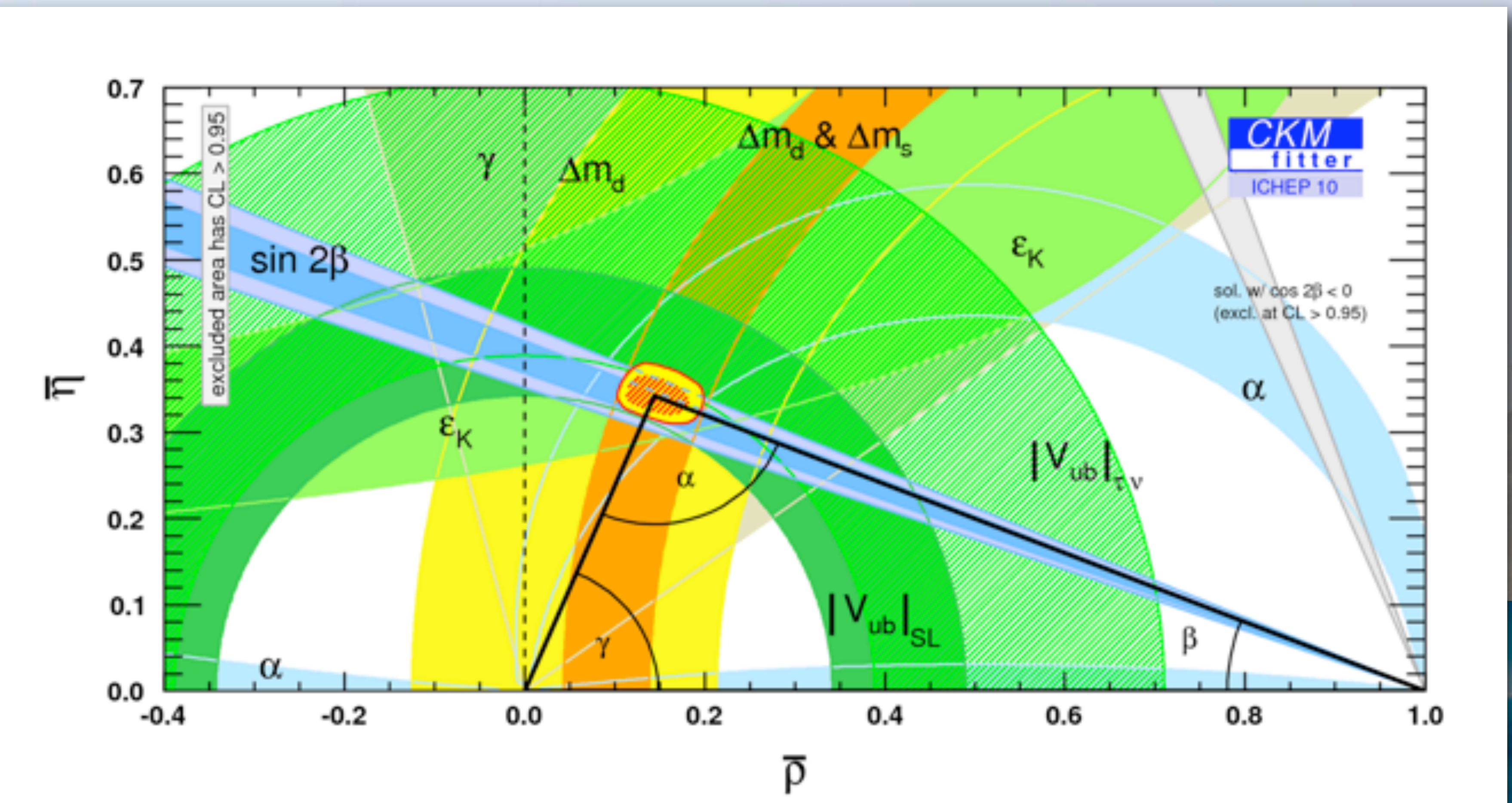
Complementarity: Higgs boson

Indirect inference

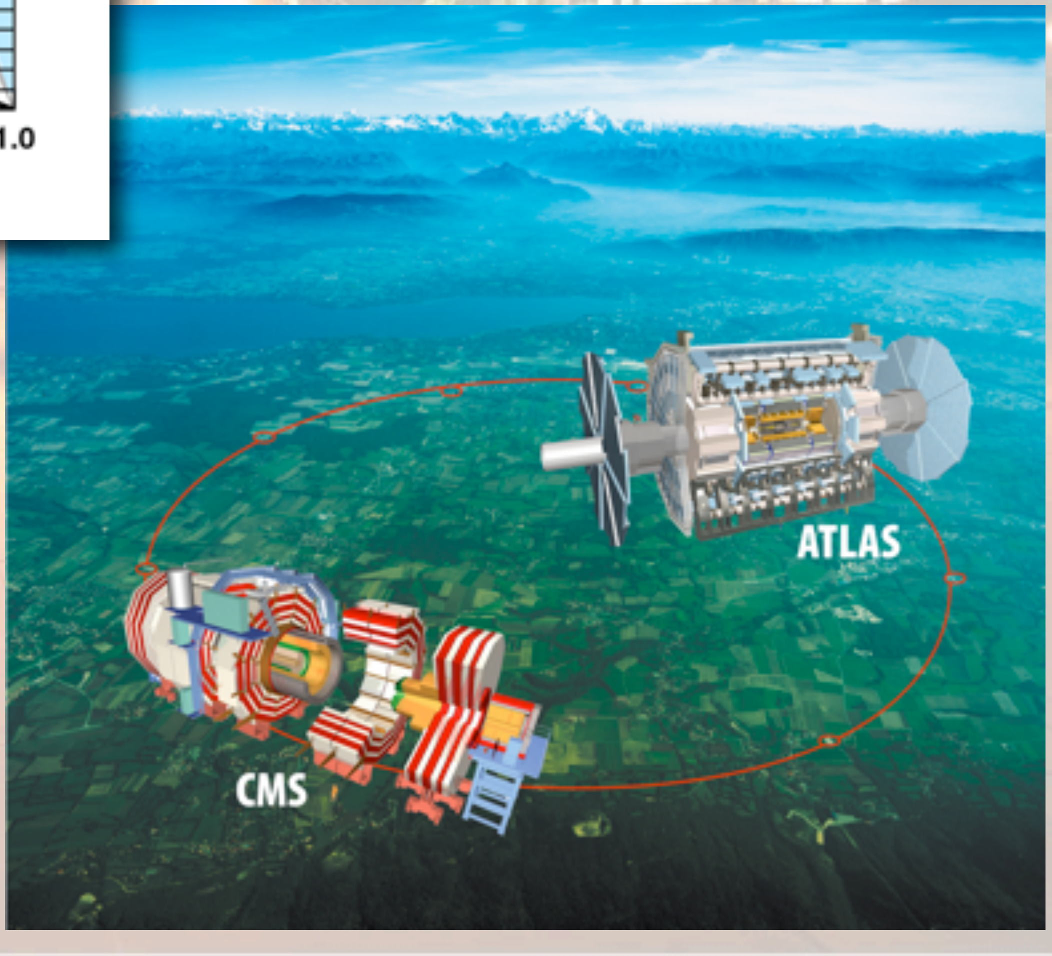


Direct exclusion

Complementarity in BSM searches



Indirect constraints on CKM params



Direct measurements (please?)

Peering through the glue

Model builder:

Lattice theorist

Experimentalist:

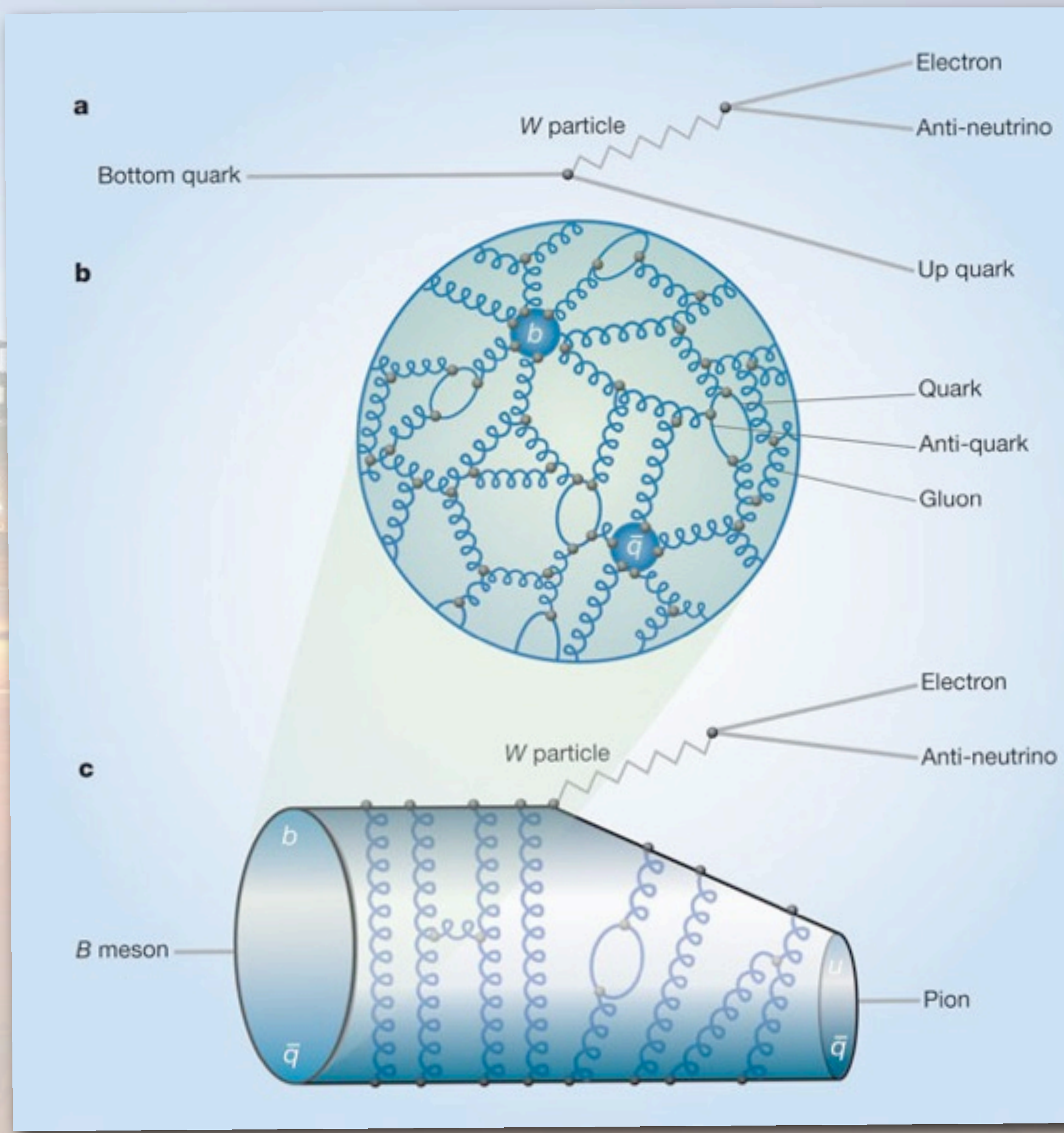


Illustration from I. Shipsey, Nature 427, 591 (2004)

Snapshot of recent work (Q2, 2011)

f_B, f_{B_s}

ETM, PoS(LAT2009);
HPQCD, PRL 92 (2004);
FNAL/MILC, PoS(LAT2008);
HPQCD, PRD 80 (2009)

B_{B_d}, B_{B_s}

HPQCD, PRD 76 (2007);
RBC-UKQCD, PoS(LAT2007);
HPQCD, PRD 80 (2009);
RBC-UKQCD, PRD 82 (2010)

$f_+^{B\to\pi}(q^2)$

HPQCD, PRD 73 (2006);
FNAL/MILC, PRD 79 (2009) 054507;
FNAL/MILC, PRD 80 (2010)

$\mathcal{F}^{B\to D}(1)$

FNAL/MILC, NPB Proc Suppl (2005)

$\mathcal{F}^{B\to D^*}(1)$

FNAL/MILC, PRD 79 (2009) 014506

$\hat{B}_K$

JLQCD, PRD 77 (2008);
HPQCD, PRD 73 (2006);
RBC-UKQCD, PRL 100 (2008);
Aubin et al., PRD 81 (2010)

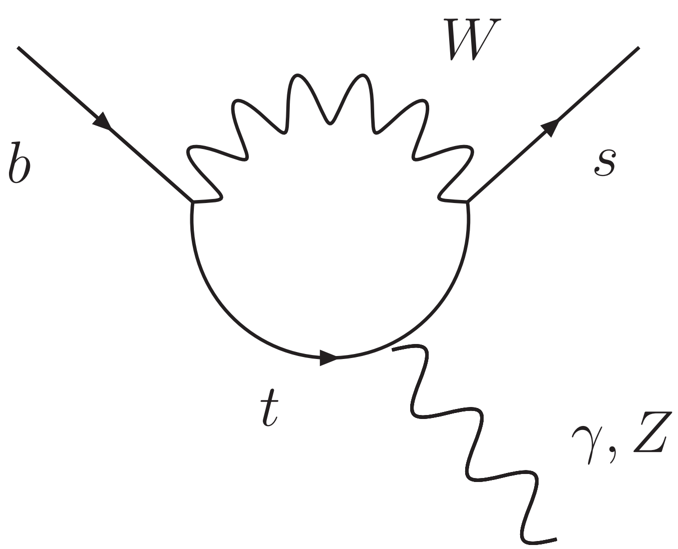
$f_+^{K\to\pi}(0)$

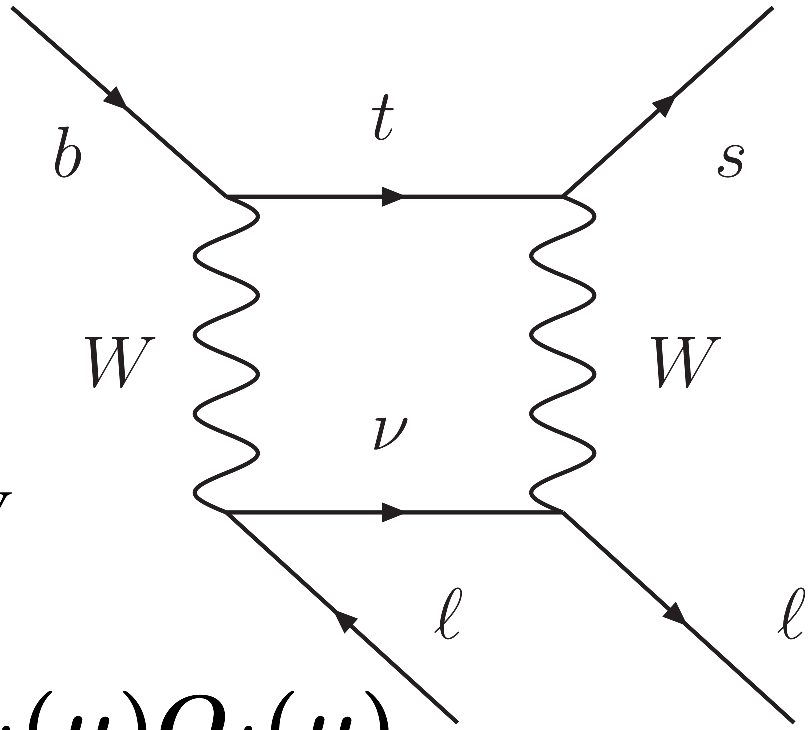
RBC-UKQCD, PRL 100 (2008);
ETM, PRD 80 (2009);
RBC-UKQCD, EPJ C69 (2010)

f_π, f_K

NPLQCD, PRD 75 (2007);
HPQCD, PRL 100 (2008);
QCDSF, PoS(LAT2007);
PACS-CS, PoS(LAT2008);
PACS-CS, PRD 79 (2009);
RBC-UKQCD, PRD 78 (2008);
Aubin et al., PoS(LAT2008);
MILC, PoS(CD09);
MILC, RMP 82 (2010);
JLQCD/TWQCD, PoS(LAT2009);
ETM, JHEP 07 (2009);
BMW, PRD 82 (2010)

$b \to s$ is rare in the SM



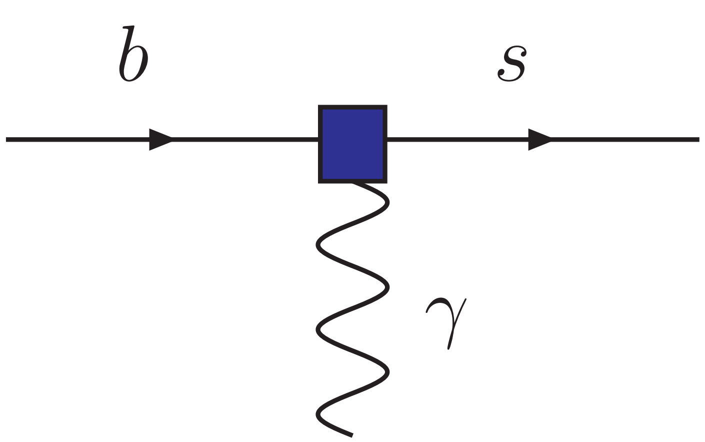


For energies $\ll m_W$

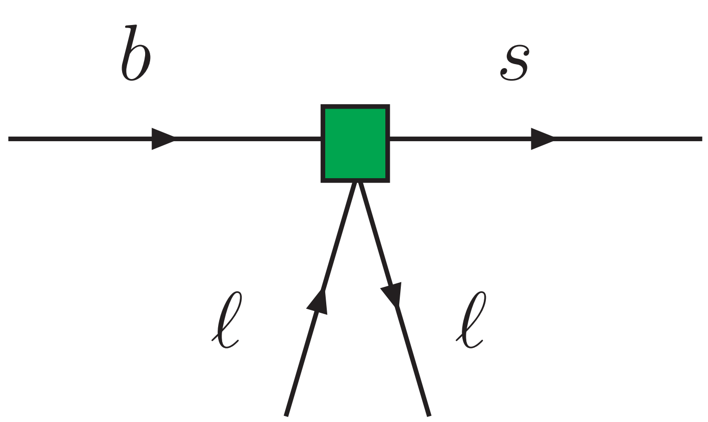
$$\mathcal{H}_{\text{eff}} = -\frac{G_F}{\sqrt{2}} V_{tb} V_{ts}^* \sum_{i=1}^{10} C_i(\mu) Q_i(\mu)$$

Wilson coefficients

Local operators



$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8m_W^2}$$



Dominant operators

Decays

$$B \to K^*\gamma$$
$$B_s \to \phi\,\gamma$$
$$B \to (\rho/\omega)\gamma$$

$$B \to K^{(*)}\ell^+\ell^-$$
$$B_s \to \phi\,\ell^+\ell^-$$

$$\Lambda_b \to \Lambda\,\gamma$$
$$\Lambda_b \to \Lambda\,\ell^+\ell^-$$

SM operators

$$Q_{7\gamma} = \frac{e}{8\pi^2}\, m_b\, \bar{s}_i \sigma^{\mu\nu}(1+\gamma_5) b_i F_{\mu\nu}$$

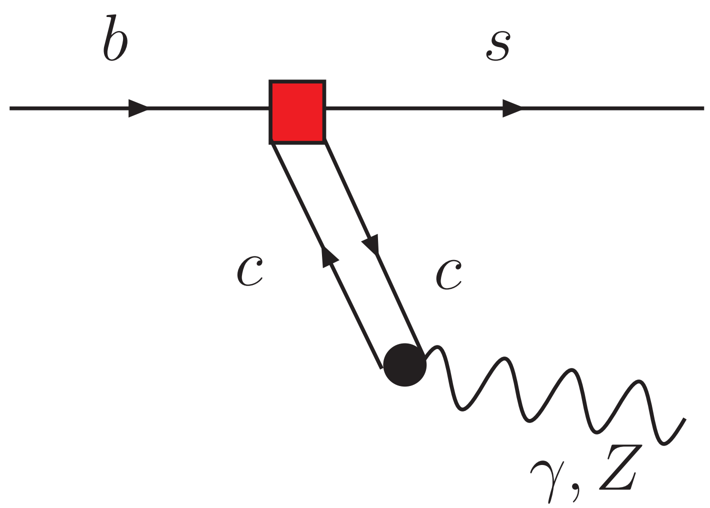
$$Q_{9V} = \frac{e}{8\pi^2}\,(\bar{s}\,b)_{V-A}\,(\bar{\ell}\,\ell)_V$$

$$Q_2 = (\bar{s}\,c)_{V-A}\,(\bar{c}\,b)_{V-A}$$

Long distance effects

Phenomenological calculations necessary

Charmonium resonances



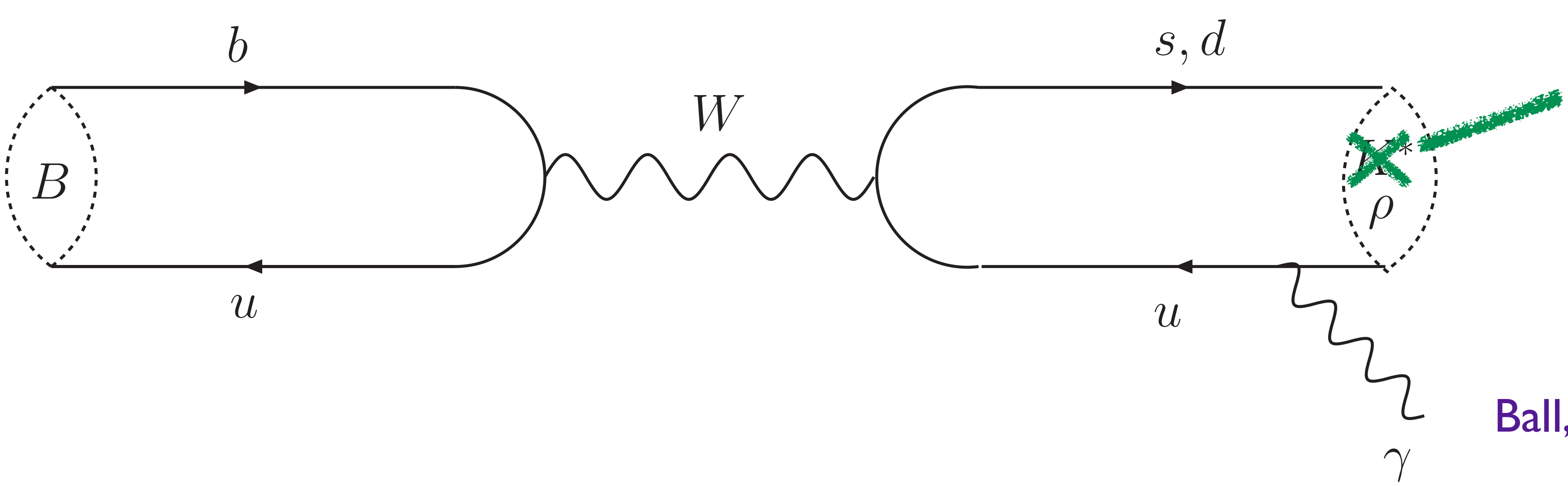
Low q^2
Large recoil

Khodjamirian, et al, PLB **402** (1997)
Khodjamirian, et al, arXiv:1006.4945

High q^2
Low recoil

Buchalla & Isidori, NPB **525** (1998)
Grinstein & Pirjol, PRD **62** (2000), PRD **70** (2004)
Beylich, Buchalla, Feldmann, arXiv:1101.5118

Weak annihilation

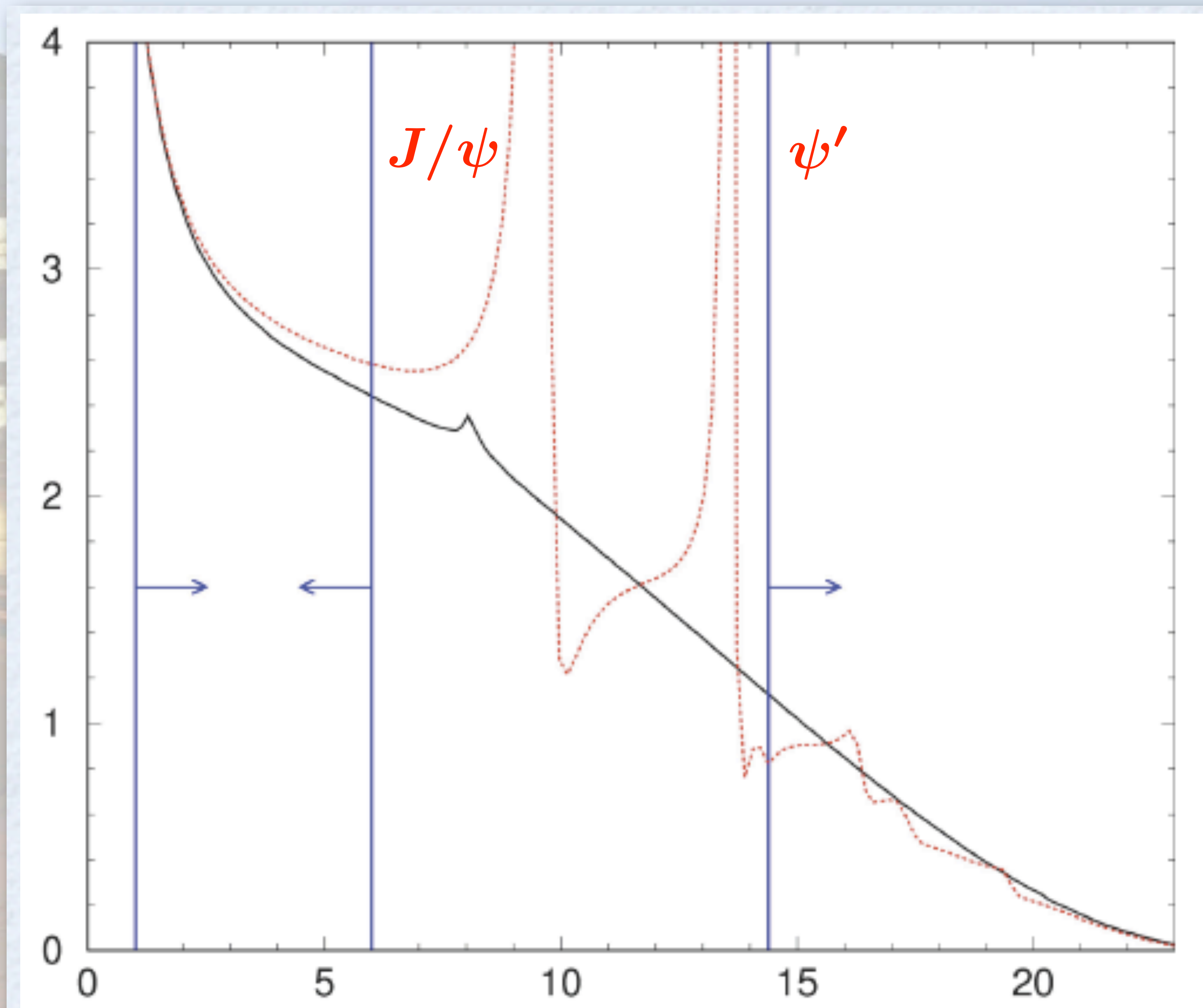


doubly Cabibbo-suppressed

Ball, Jones, Zwicky, PRD **75** (2007)

Regions of applicability

$B \to X_s \ell^+ \ell^-$



$q^2(\mathrm{GeV}^2)$

Plot from E Lunghi's CKM2008 talk

large recoil

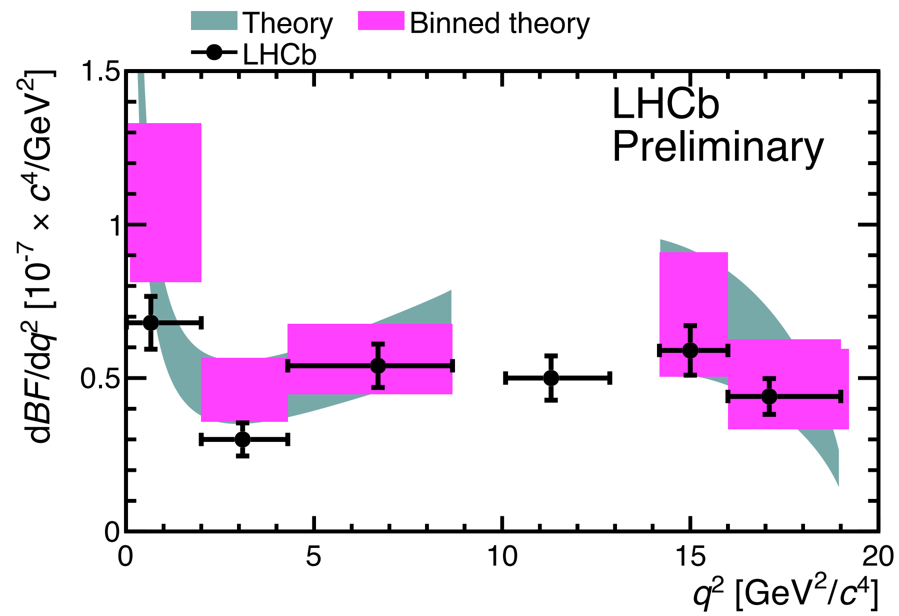
- ✤ Short distance effects dominate at low q^2
- ✤ Short distance effects dominate at high q^2 (Grinstein-Pirjol, Beylich-Buchalla-Feldmann)

low recoil

Latest from LHC*b*

$B^0 \to K^{*0}\mu^+\mu^-$ and $B_s^0 \to \phi\mu^+\mu^-$ differential branching fractions

- LHCb(1.0 fb^{-1}) : $B^0 \to K^{*0}\mu^+\mu^-$: 900 ± 34 signal events



- Measurement of the $B_s^0 \to \phi\mu^+\mu^-$ branching fraction reported at Moriond EW
- LHCb(1.0 fb^{-1}) : $B_s^0 \to \phi\mu^+\mu^-$: 77 ± 10 signal events
- $\mathcal{B}(B_s^0 \to \phi\mu^+\mu^-) = (0.778 \pm 0.097(stat) \pm 0.061(syst) \pm 0.278(B)) \times 10^{-6}$ [preliminary]
- The most precise measurements to-date and are consistent with SM expectations [4]

Lattice QCD

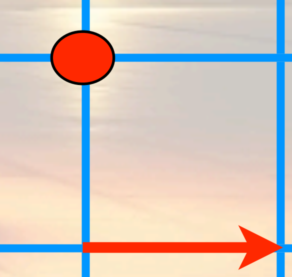
- Field theory as statistical mechanics
- Mending errors
- [Decisions, decisions]
- Work in progress (rare *B* decay form factors)

Lattice QCD in a nutshell

- QCD Lagrangian

$$\mathcal{L} = -\frac{1}{4}F^{a}_{\mu\nu}F^{a,\mu\nu} - \sum_q \overline{\psi}_q \left[\gamma^{\mu}(\partial_{\mu} - igA^{a}_{\mu}t^{a}) + m_q\right]\psi_q$$

$$= \mathcal{L}_g - \overline{\psi}Q\psi$$



- Break spacetime up into a grid
- Maintains gauge invariance
- Regulates the QFT nonperturbatively
- Breaking of Lorentz and translational symmetries scales like the lattice spacing a^p (p=2, usually)

Lattice QCD in a nutshell

QFT : Imaginary-time path integral

$$\langle J(z')J(z)\rangle = \frac{1}{Z}\int [d\psi][d\bar{\psi}][dU]\, J(z')J(z)\, e^{-S_E}$$

SFT : Sum over all microstates

$$\langle J(z')J(z)\rangle = \frac{1}{Z}\,\mathrm{Tr}\,\left[J(z')J(z)\, e^{-\beta H}\right]$$

Use the same numerical methods!

Monte Carlo Calculation : Find and use field "configurations" which dominate the integral/sum

Lattice QCD in a nutshell

Gluonic expectation values

$$\begin{aligned}\langle \Theta \rangle &= \frac{1}{Z}\int [d\psi][d\bar{\psi}][dU]\,\Theta[U]\,e^{-S_g[U]-\bar{\psi}Q[U]\psi} \\ &= \frac{1}{Z}\int [dU]\,\Theta[U]\,\det Q[U]\,e^{-S_g[U]}\end{aligned}$$

Probability weight

Fermionic expectation values

$$\langle \bar{\psi}\Gamma\psi \rangle = \int [dU]\,\frac{\delta}{\delta\bar{\zeta}}\Gamma\frac{\delta}{\delta\zeta}\,e^{-\bar{\zeta}Q^{-1}[U]\zeta}\,\det Q[U]e^{-S_g[U]}\Big|_{\zeta,\bar{\zeta}\to 0}$$

Determinant in probability weight difficult

1) Requires nonlocal updating; 2) Matrix

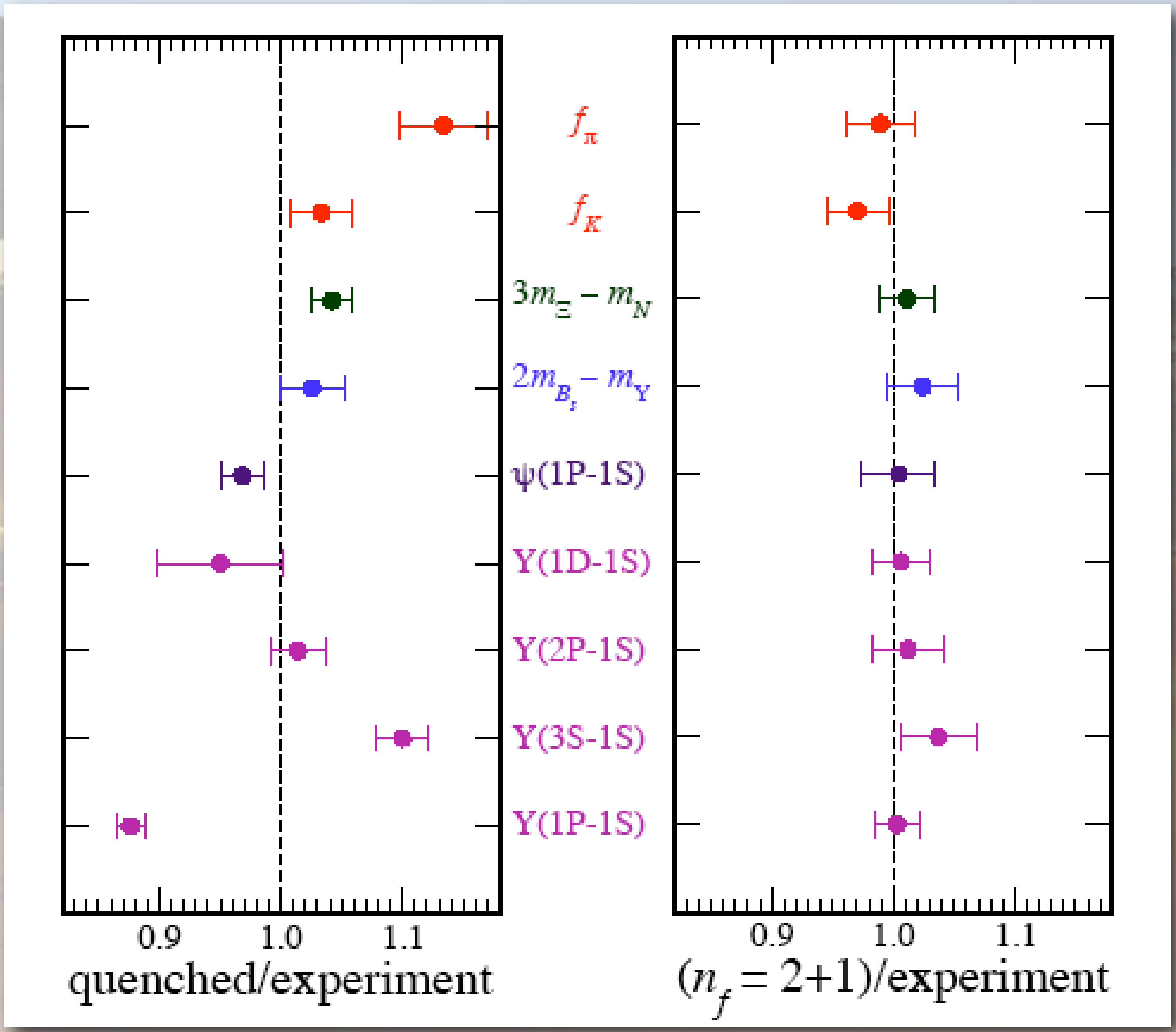
Quenched approximation

Set $\det Q = 1$

Partial quenching =

different mass for valence Q^{-1} than for sea $\det Q$

Lattice QCD progress



C. Davies, *et al.*, PRL 92 (2004)

- ✤ Effects of light sea *u*+*d*+*s* quarks important
- ✤ Much progress using staggered quarks (+ 4th root hypothesis)
- ✤ Single set of lattice inputs (quark masses)
- ✤ [MILC Collab'n lattices]

Systematic errors

Source of error	Controllable limit	Theory
Lattice volume	$L \gg 1/m_\pi$	Chiral pert. th. Brute force
Lattice spacing	$a \ll 1/\Lambda_{\mathrm{QCD}}$	Symanzik EFT
Heavy quark mass	$m_Q \gg 1/a$ $m_Q < 1/a$ $m_Q \approx 1/a$	NRQCD, HQET Extra-fine, extra-improvement Fermilab
Light quark mass	$m_\pi \ll m_\rho, 4\pi f_\pi$	Chiral pert. th.

Choice of discretizations

- Gluon field: improved actions, w/ various criteria (perturbative/nonperturbative Symanzik, RG)
- Light quarks: staggered, Wilson (clover), domain-wall, overlap, twisted-mass, ...
- Heavy quarks: static, nonrelativistic, relativistic (Fermilab (perturbative/nonperturbative), extrapolated light quarks)

HPQCD approach

with **Stefan Meinel, Zhaofeng Liu, Eike Müller,**
A. Hart, R. Horgan

- NRQCD formulation to calculate QCD dynamics of physically heavy *b* quark
- Improved staggered light quarks
- Matching to MSbar scheme in pert. th. (Müller, Hart, Horgan, PRD **83**, 2011)
- Can work in lattice frame boosted relative to *B* (Horgan *et al.*, PRD **80**, 2009)
- Stat. and EFT errors mandate working at low recoil
- $N_f = 2 + 1$ (MILC) configurations. No unquenched calculations of $B \rightarrow V$ form factors published yet.

Lattice data

High statistics

MILC lattices (2+1 asqtad staggered)

	a(fm)	am_{sea}	Volume	$N_{conf} \times N_{src}$	am_{val}	m_π (MeV)
coarse	$\sim$0.12	0.007/0.05	$20^3 \times 64$	2109×8	0.007/0.04	~300
		0.02/0.05	$20^3 \times 64$	2052×8	0.02/0.04	~460
fine	$\sim$0.09	0.0062/0.031	$28^3 \times 96$	1910×8	0.0062/0.031	~320

Light meson momenta (units of $2\pi/L$)

Momenta	$p^2/(2\pi/L)^2$
$(p_x, p_y, p_z) = (0, 0, 0)$.	0
$(\tilde{q}$,0,0), (0,$\tilde{q}$,0), (0,0,$\tilde{q}$), where $\tilde{q}$=1 or 2.	1 or 4
(1,1,0), (1,-1,0), (1,0,1), (1,0,-1), (0,1,1), (0,1,-1).	2
(1,1,1), (1,1,-1), (1,-1,1), (1,-1,-1).	3

Many Source/Sink separations (16 coarse, 22 fine)

So far, only v=0 NRQCD used (B at rest).

Leading order (HQET) current presently used.
$1/m_b$ current matrix elements computed, analysis in progress

Full set of form factors

Matrix element	Form factor	Relevant decay(s)
$\langle P\|\bar{q}\gamma^\mu b\|B\rangle$	f_+, f_0	$B \to \pi\ell\nu$ $B \to K\ell^+\ell^-$
$\langle P\|\bar{q}\sigma^{\mu\nu}q_\nu b\|B\rangle$	f_T	$B \to K\ell^+\ell^-$
$\langle V\|\bar{q}\gamma^\mu b\|B\rangle$	V	$\left\{ \begin{array}{l} B \to (\rho/\omega)\ell\nu \\ B \to K^*\ell^+\ell^- \end{array} \right.$
$\langle V\|\bar{q}\gamma^\mu\gamma^5 b\|B\rangle$	A_0, A_1, A_2	
$\langle V\|\bar{q}\sigma^{\mu\nu}q_\nu b\|B\rangle$	T_1	$\left\{ \begin{array}{l} B \to K^*\gamma \\ B \to K^*\ell^+\ell^- \end{array} \right.$
$\langle V\|\bar{q}\sigma^{\mu\nu}\gamma^5 q_\nu b\|B\rangle$	T_2, T_3	

... also make the spectator an s quark for B_s decays

Form factor definitions

T

$$q^\nu \langle K^*(p',\lambda)|\bar{s}\sigma_{\mu\nu} b|B(p)\rangle = 4\,T_1(q^2)\epsilon_{\mu\nu\rho\sigma} e_\lambda^{*\nu} p^\rho p'^\sigma,$$

PT

$$q^\nu \langle K^*(p',\lambda)|\bar{s}\sigma_{\mu\nu}\gamma_5 b|B(p)\rangle = 2i\,T_2(q^2)\left[e^*_{\lambda\mu}(M_B^2 - M_{K^*}^2) - (e_\lambda^* \cdot q)(p+p')_\mu\right] + 2i\,T_3(q^2)(e_\lambda^* \cdot q)\left[q_\mu - \frac{q^2}{M_B^2 - M_{K^*}^2}(p+p')_\mu\right].$$

V

$$\langle K^*(p',\lambda)|\bar{s}\gamma^\mu b|B(p)\rangle = \frac{2i\,V(q^2)}{M_B + M_{K^*}}\epsilon^{\mu\nu\rho\sigma} e^*_{\lambda\nu} p'_\rho p_\sigma,$$

AV

$$\langle K^*(p',\lambda)|\bar{s}\gamma^\mu\gamma_5 b|B(p)\rangle = 2M_{K^*} A_0(q^2)\frac{e_\lambda^*\cdot q}{q^2} q^\mu + (M_B + M_{K^*})A_1(q^2)\left[e_\lambda^{*\mu} - \frac{e_\lambda^*\cdot q}{q^2} q^\mu\right] - A_2(q^2)\frac{e_\lambda^*\cdot q}{M_B+M_{K^*}}\left[p^\mu + p'^\mu - \frac{M_B^2 - M_{K^*}^2}{q^2} q^\mu\right].$$

Correlation functions

3-point function

$$C_{FJB}(\mathbf{p}',\, \mathbf{p},\, x_0,\, y_0,\, z_0) = \sum_{\mathbf{y}} \sum_{\mathbf{z}} \left\langle \Phi_F(x)\, J(y)\, \Phi_B^\dagger(z) \right\rangle e^{-i\mathbf{p}'\cdot(\mathbf{x}-\mathbf{y})} e^{-i\mathbf{p}\cdot(\mathbf{y}-\mathbf{z})}$$

2-point functions

$$C_{BB}(\mathbf{p},\, x_0,\, y_0) = \sum_{\mathbf{x}} \left\langle \Phi_B(x)\, \Phi_B^\dagger(y) \right\rangle e^{-i\mathbf{p}\cdot(\mathbf{x}-\mathbf{y})},$$

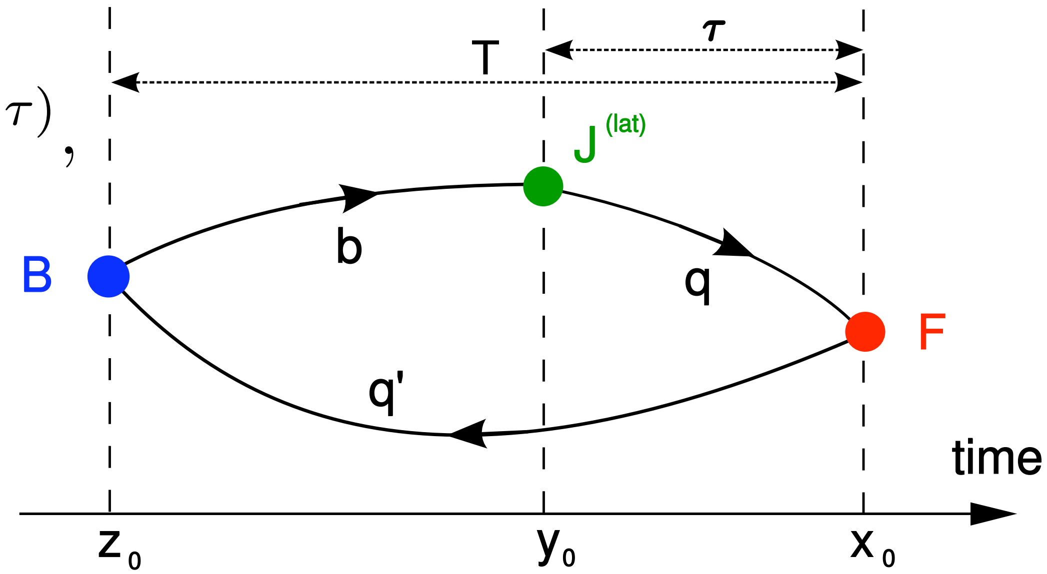
$$C_{FF}(\mathbf{p}',\, x_0,\, y_0) = \sum_{\mathbf{x}} \left\langle \Phi_F(x)\, \Phi_F^\dagger(y) \right\rangle e^{-i\mathbf{p}'\cdot(\mathbf{x}-\mathbf{y})}.$$

Large Euclidean-time behavior

$$C_{FJB}(\mathbf{p}',\, \mathbf{p},\, \tau,\, T) \rightarrow A^{(FJB)} e^{-E_F\tau}\, e^{-E_B(T-\tau)},$$

$$C_{FF}(\mathbf{p},\, \tau) \rightarrow A^{(FF)}\, e^{-E_F\tau},$$

$$C_{BB}(\mathbf{p},\, \tau) \rightarrow A^{(BB)}\, e^{-E_B\tau},$$



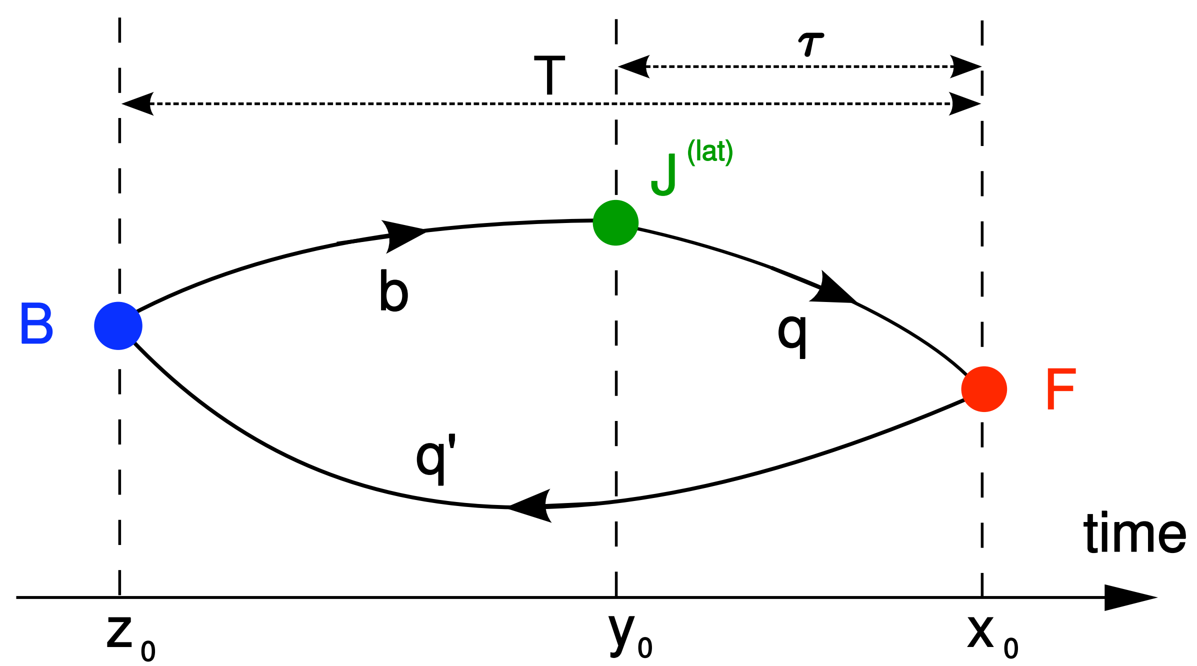
Correlation functions

Matrix element from amplitudes

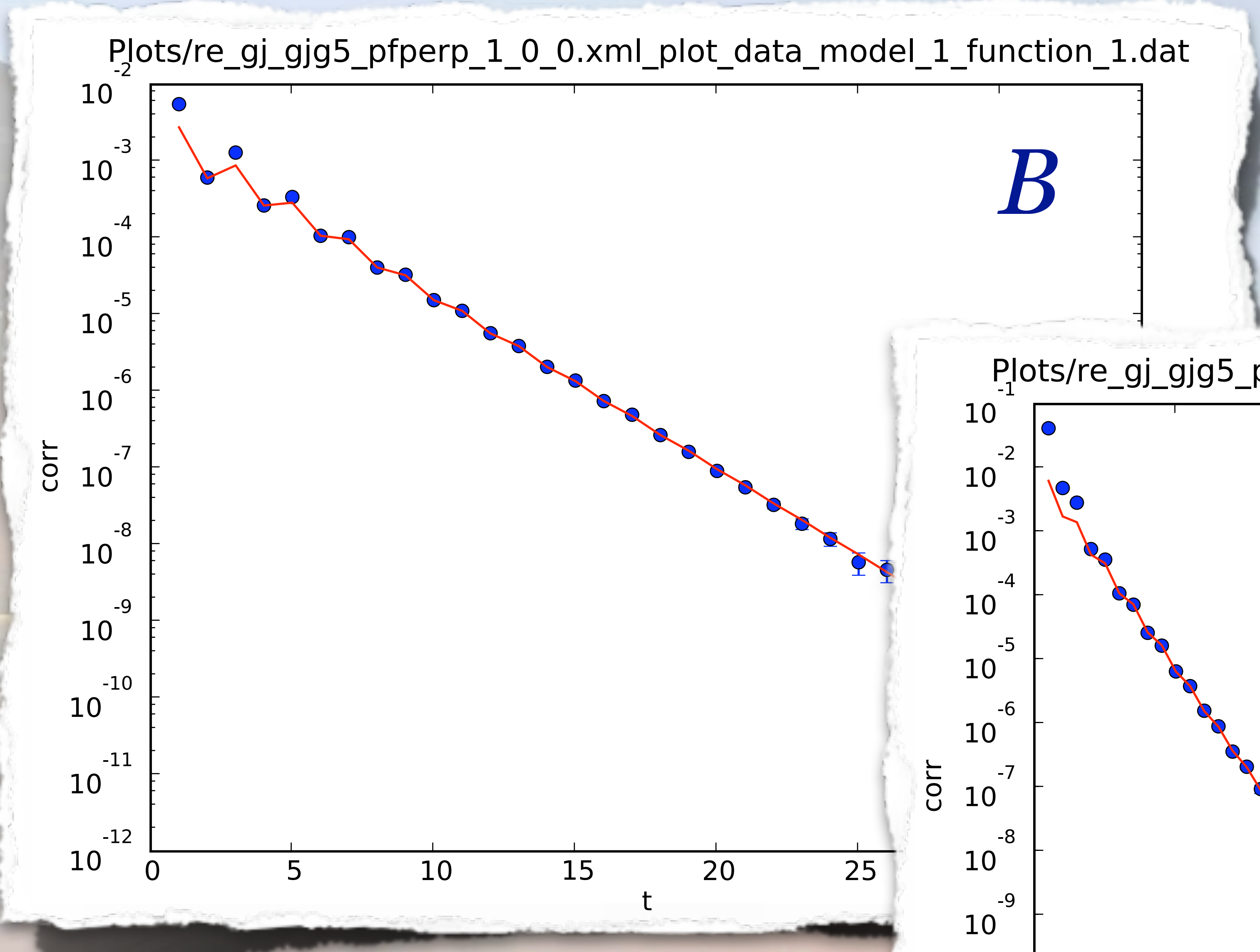
$$A^{(FJB)} = \frac{\sqrt{Z_V}}{2E_V}\frac{\sqrt{Z_B}}{2E_B}\sum_s \varepsilon_j(p',s)\,\langle V\left(p',\varepsilon(p',s)\right)|\,J\,|B(p)\rangle,$$

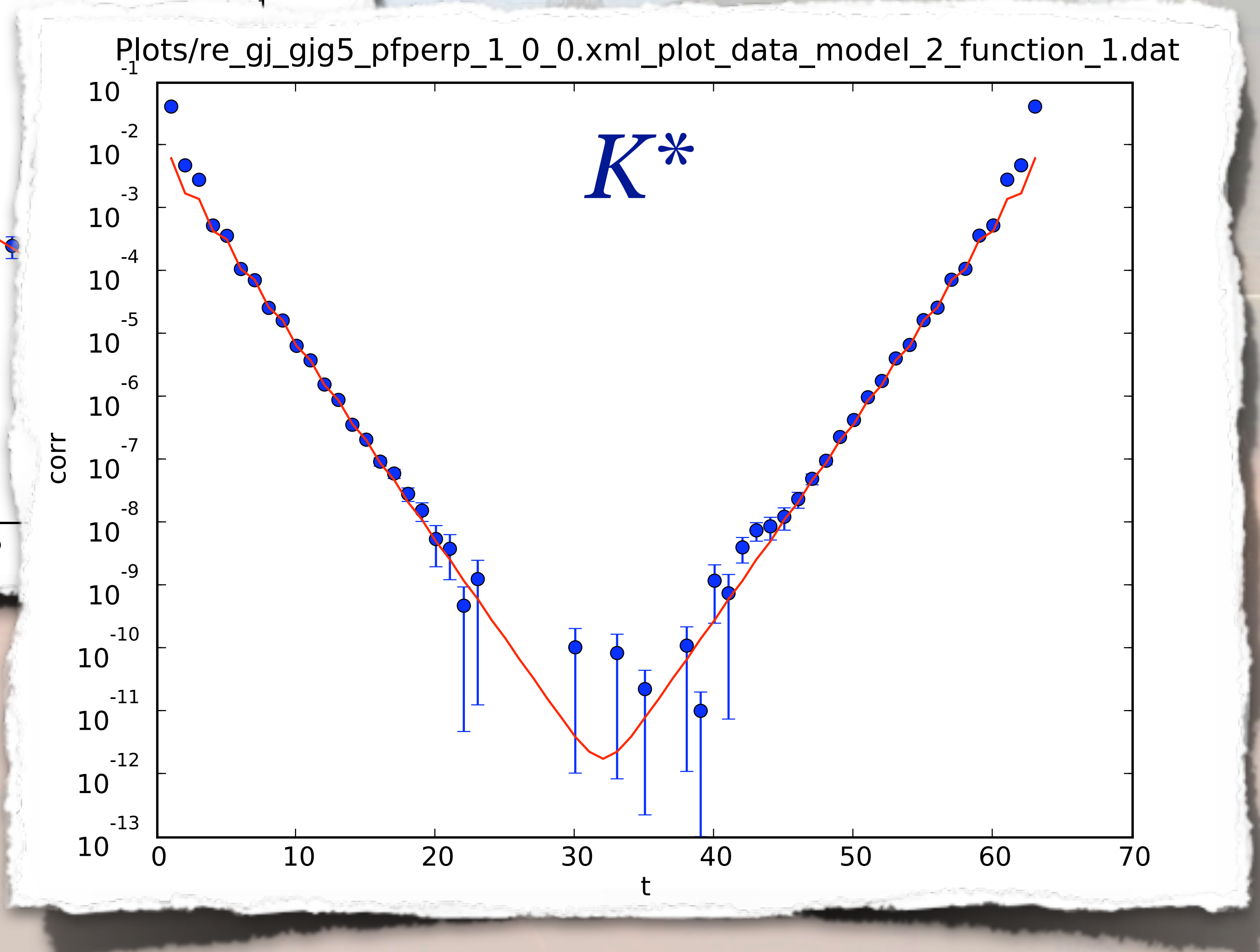
$$A^{(BB)} = \frac{Z_B}{2E_B},$$

$$A^{(FF)} = \sum_s \frac{Z_V}{2E_V}\,\varepsilon_j^*(p',s)\varepsilon_j(p',s)$$

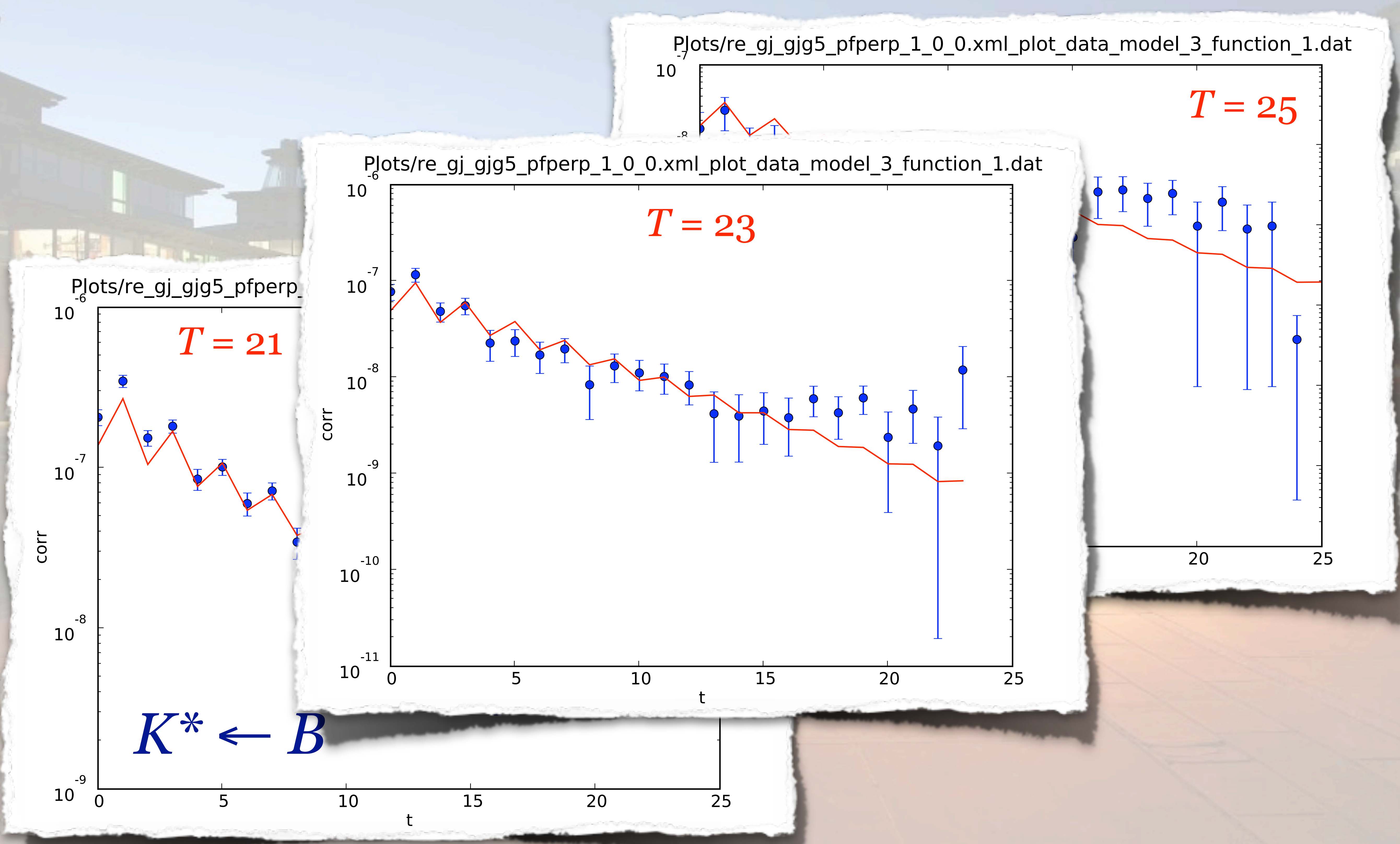


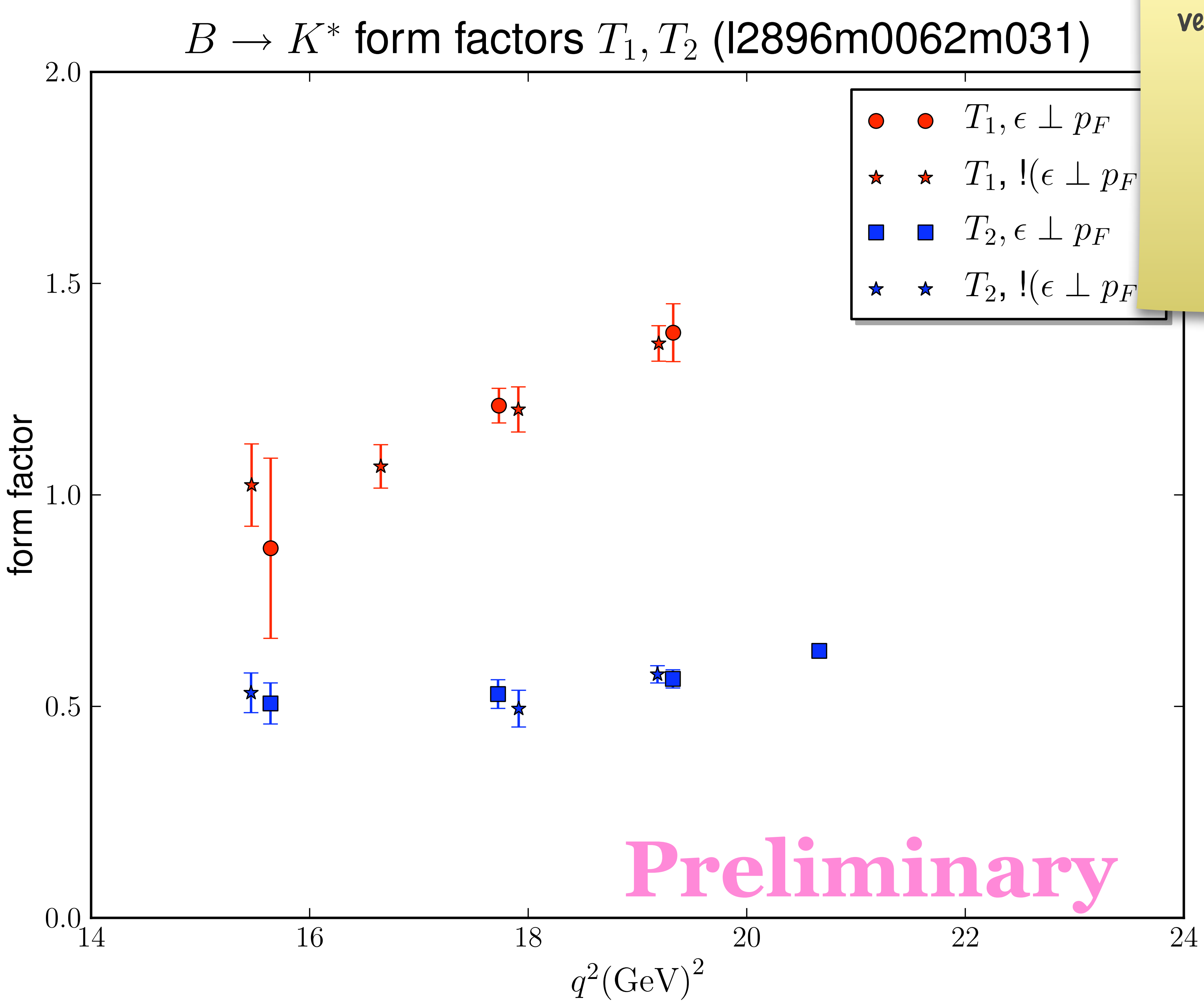
Example plots





Example plots





version of 16 Nov 2011

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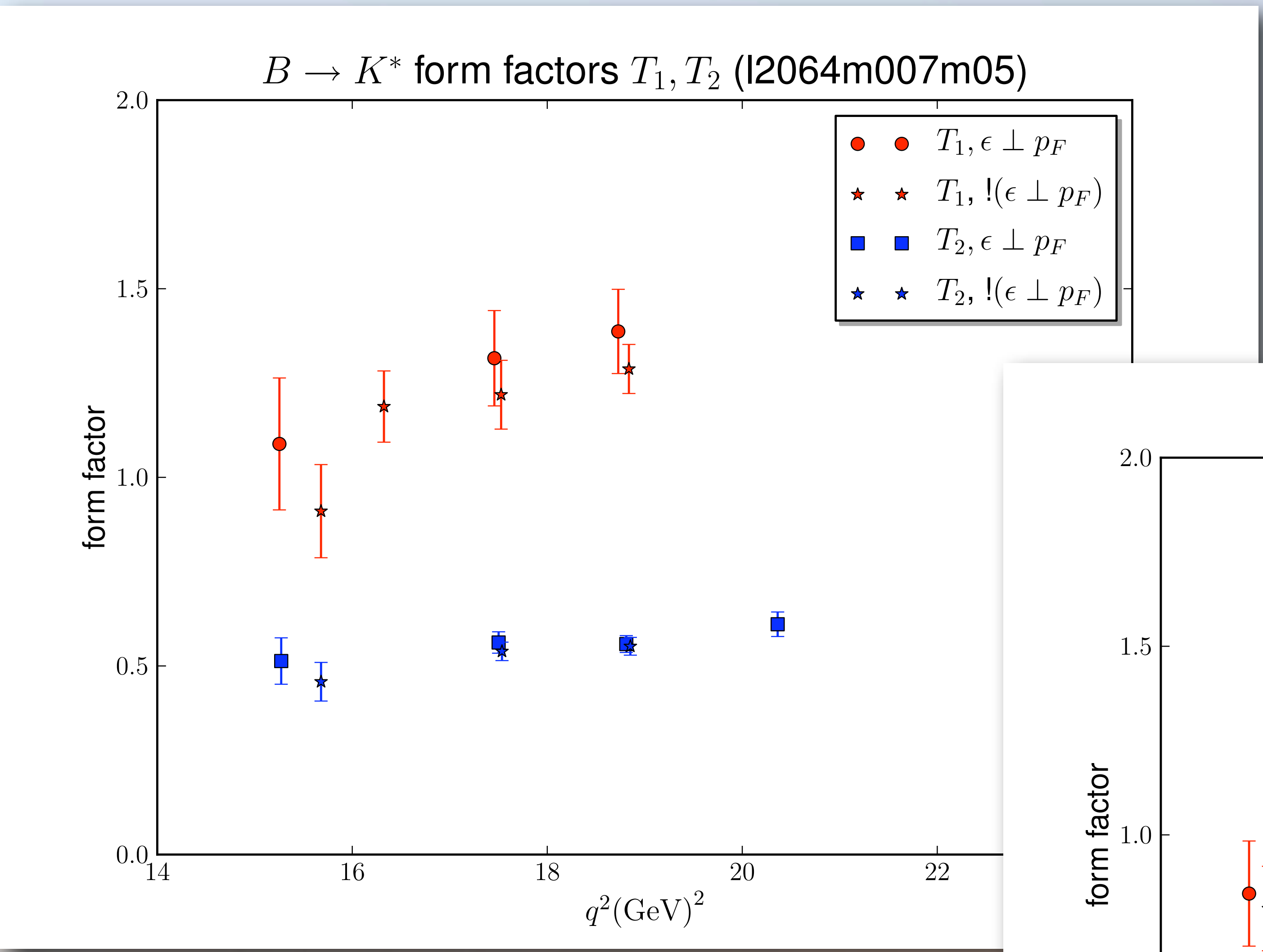
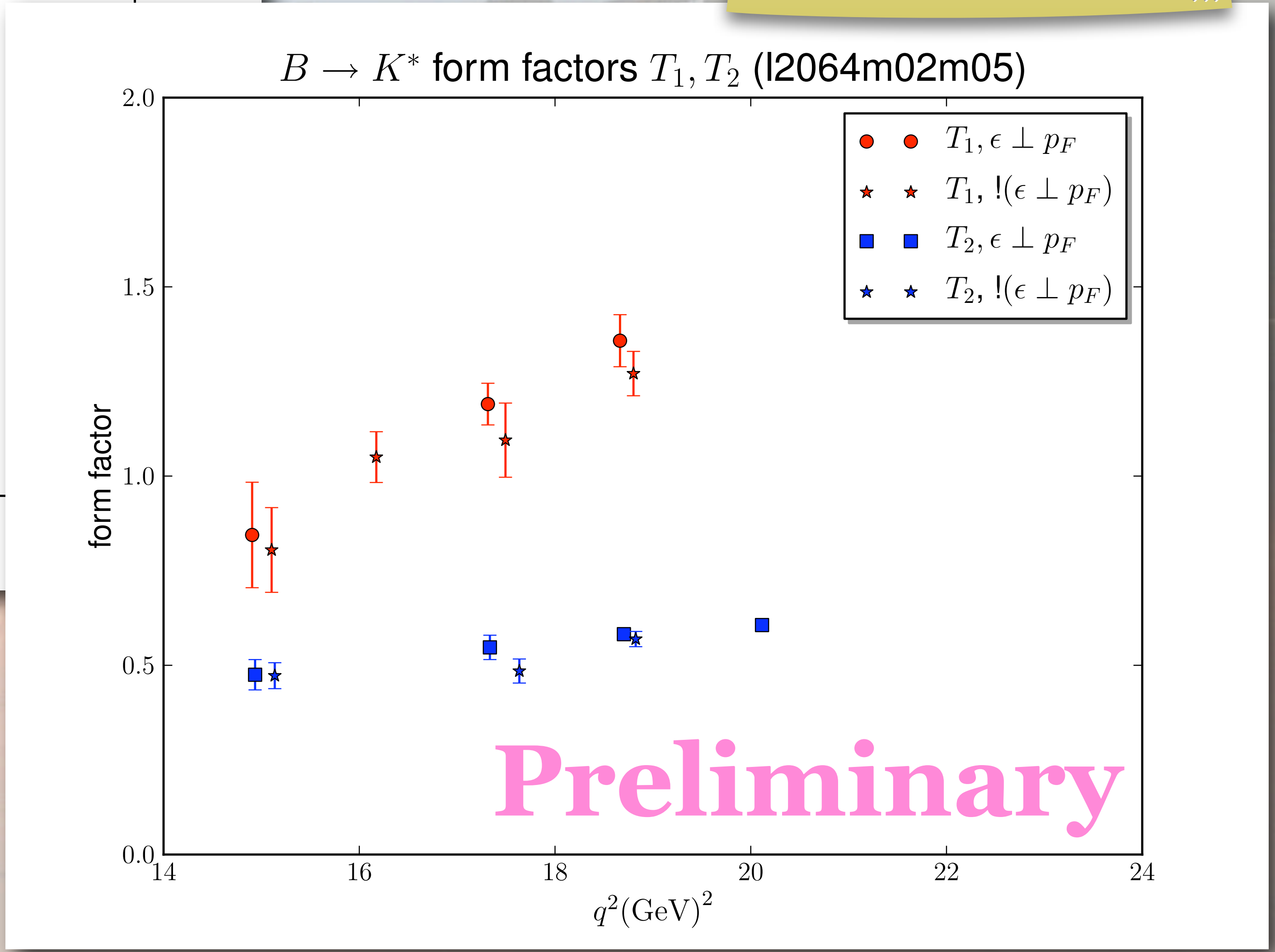
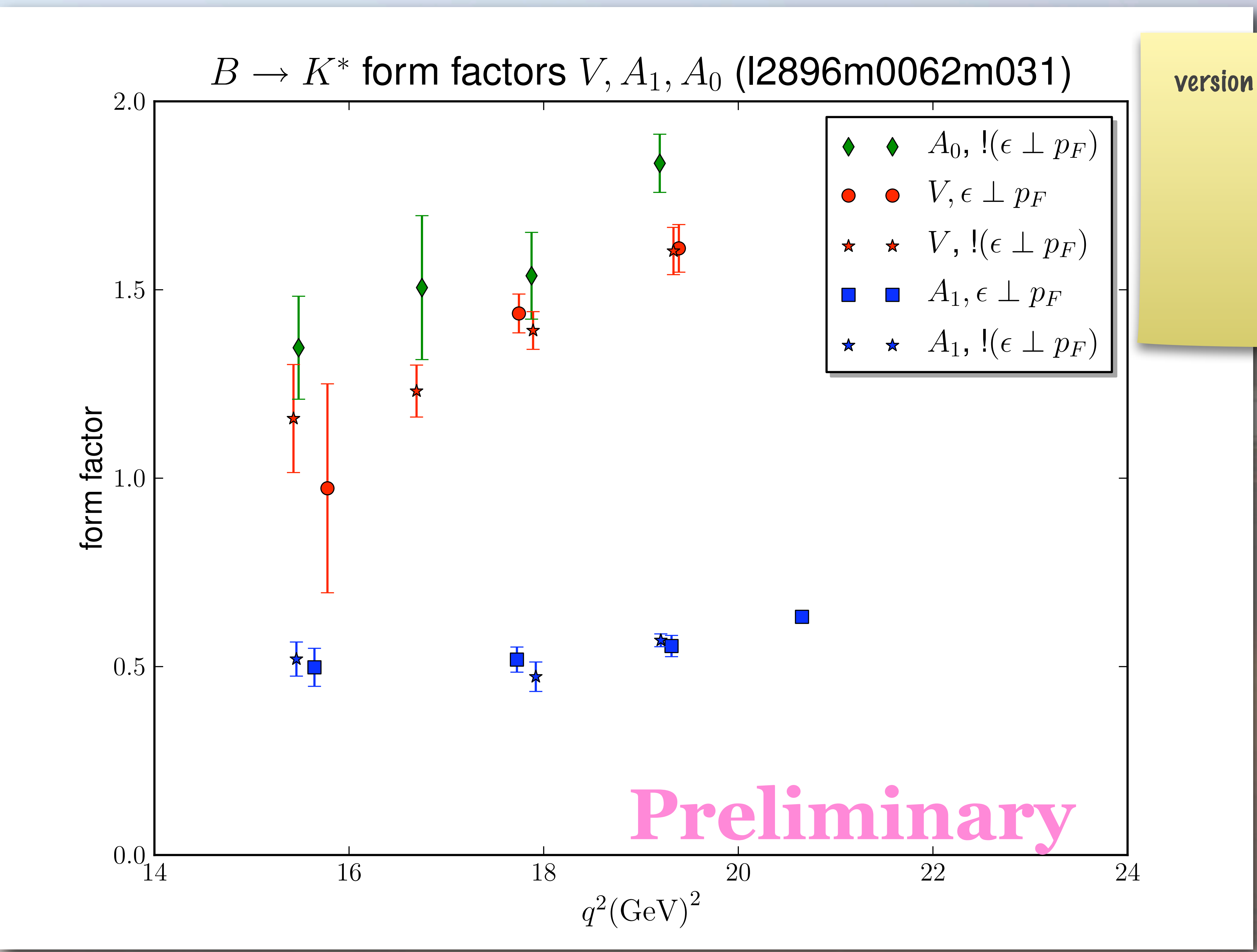


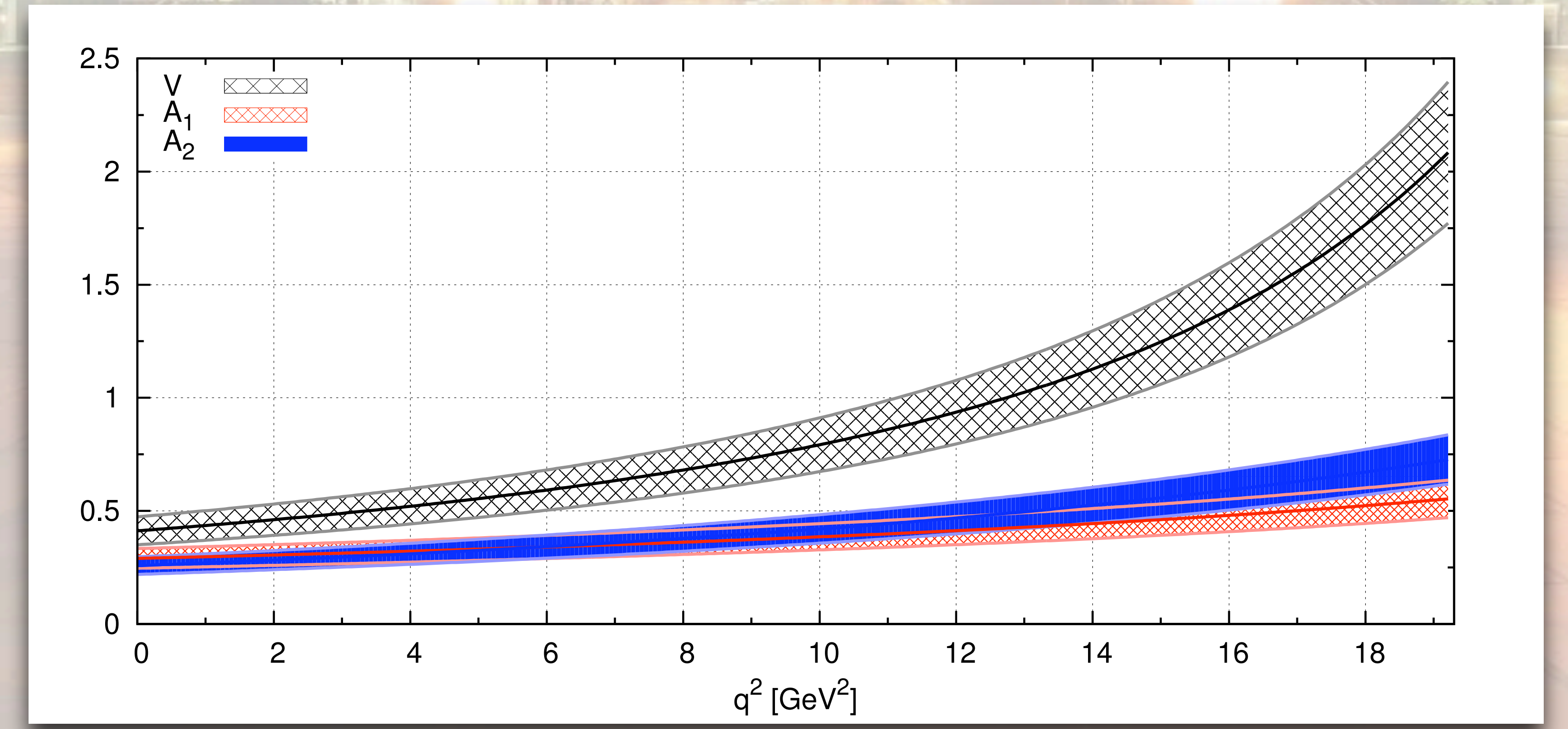




version of 16 Nov 2011

To do

- Fit & extrapolate in light quark mass, lattice spacing, kinematic variable
- Compare/include sum rule calculations from low q^2



Bobeth, Hiller, van Dyk, extrapolating from Ball & Zwicky's sum rule f.f.

New Physics: SM + Corrections

What is the fate of the Standard Model?

Add higher dimension operators

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{SM}} + \mathcal{L}_{\text{NP}}$$

$$\mathcal{L}_{\text{NP}} = \sum_{d>4} \sum_{n} \frac{c_n^{(d)}}{\Lambda_{\text{NP}}^{d-4}} \mathcal{O}_n^{(d)}$$

$\mathcal{O}_n^{(d)}$ local operator built from SM fields

Goal: perform enough precise expts + calculations to discover or constrain coefficients of $\mathcal{O}_n^{(d)}$

Standard Model agreement pushes FCNC Λ_{NP} to $10^{2\text{-}5}$ TeV unless an ad hoc flavour symmetry is imposed

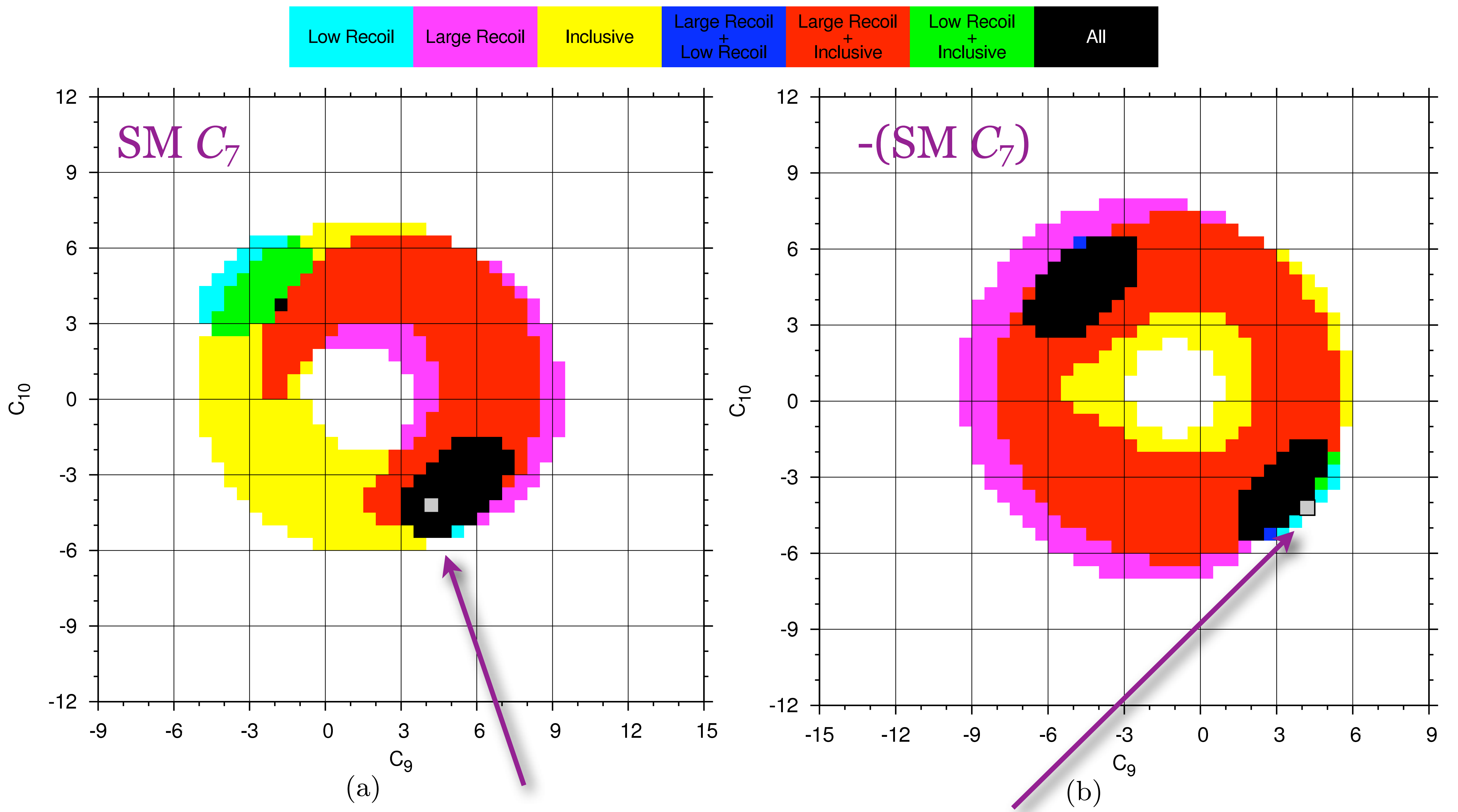
SM vs. BSM Wilson Coefficients

$$\mathcal{H}_{\text{eff}} = -\frac{G_F}{\sqrt{2}} V_{tb} V_{ts}^* \sum_{i=1}^{10} C_i(\mu) Q_i(\mu)$$

- Short distance Wilson coefficients C_i calculable perturbatively, given a model
- LQCD, sum rules, etc. compute matrix elements of local operators
- Combine experiment with theory to find "allowed" C_i's given a model framework

Constraints on Wilson Coefficients

Note how tight the constraints are from low recoil



Standard Model C_9, C_{10}

Bobeth, Hiller, van Dyk (2010)

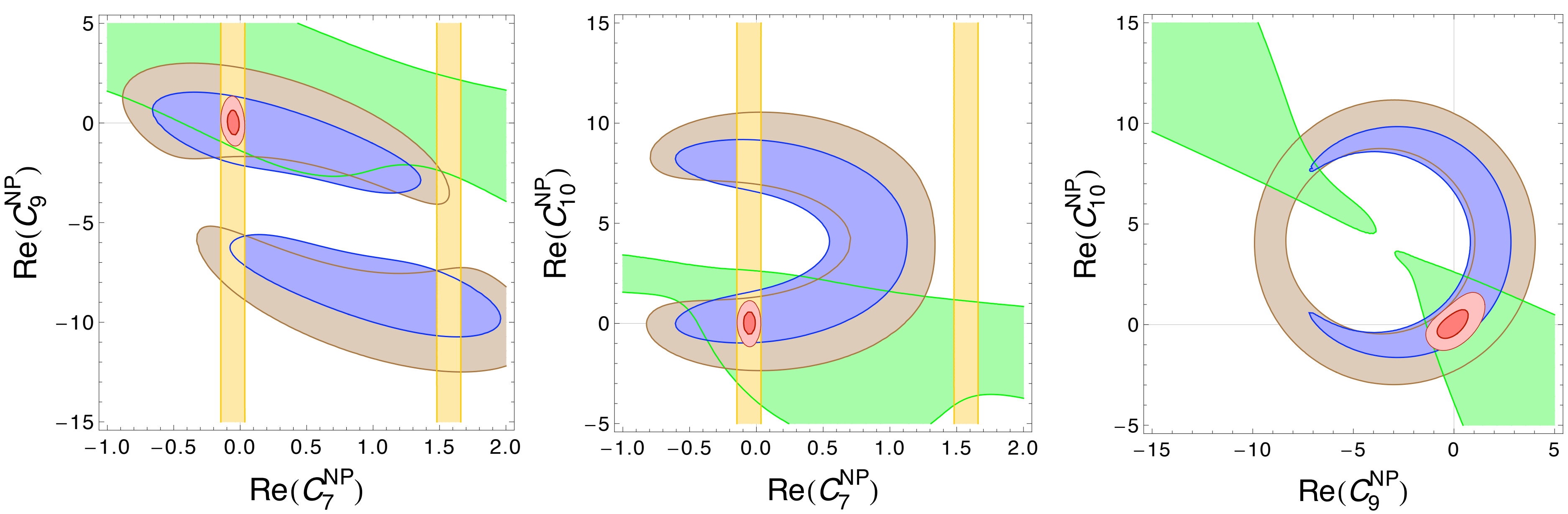
Constraints on Wilson Coefficients

Constraints on C_7 vs. C_9 vs. C_{10}

Std Model in red

LQCD form factors aim to reduce blue region

Now: correlations between 2 real Wilson coefficients



$\text{BR}(B \to X_s \ell^+ \ell^-)$ $\text{BR}(B \to X_s \gamma)$ $\text{BR}(B \to K^* \mu^+ \mu^-)$ $A_{\text{FB}}(B \to K^* \mu^+ \mu^-)$

- $B \to K^* \mu^+ \mu^-$ data exclude various "mirror solutions"

D Straub, Moriond EW, Mrach 2012

Summary

- First unquenched LQCD calculations of $B \to K^*$ & $B_s \to \varphi$ (as well as $B \to \rho$ & $B_s \to K^*$) form factors nearing completion
- Rare decays search for corrections to SM
- LQCD playing an important role in precision flavour physics (e.g. CKM unitarity)
- Flavour physics continues to play an important role in the discovery era