

The Quest for Precision in Simulations for the LHC

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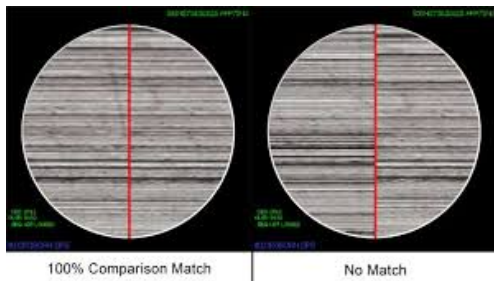
- what the talk is about
- fixed order & matching & merging with parton showers
- revisiting parton showers
- where we are and where we (should/could/would) go

motivation & introduction

motivation: the need for (more) accurate tools

- to date no survivors in searches for new physics & phenomena
(a pity, but that's what Nature hands to us)
- push into precision tests of the Standard Model
(find it or constrain it!)
- statistical uncertainties approach zero
(because of the fantastic work of accelerator, DAQ, etc.)
- systematic experimental uncertainties decrease
(because of ingenious experimental work)
- theoretical uncertainties are or become dominant
(it would be good to change this to fully exploit LHC's potential)

need more precise & accurate tools for more precise physics



matching @ (N)NLO
and
merging @ (N)LO

the aftermath of the NLO (QCD) revolution

- establishing a wide variety of automated tools for NLO calculations

BLACKHAT, GoSAM, MADGRAPH, NJET, OPENLOOPS, RECOLA + automated IR subtraction methods (MADGRAPH, SHERPA)

- first full NLO (EW) results with automated tools

- technical improvements still mandatory

(higher multis, higher speed, higher efficiency, easier handling, . . .)

- start discussing scale setting prescriptions

(simple central scales for complicated multi-scale processes? test smarter prescriptions?)

- steep learning curve still ahead: “NLO phenomenology”

(example: methods for uncertainty estimates beyond variation around central scale)

matching at NLO and NNLO

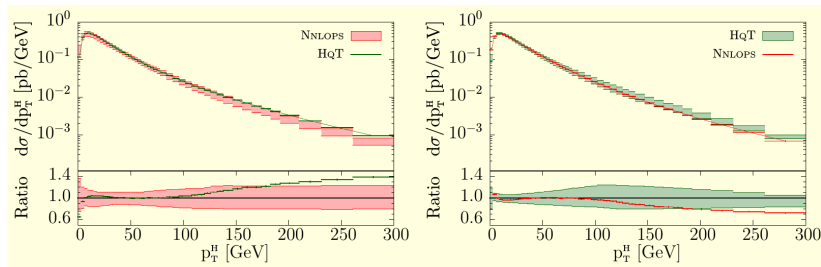
- avoid double-counting of emissions
- two schemes at NLO: MC@NLO and POWHEG
 - mismatches of K factors in transition to hard jet region
 - MC@NLO: \rightarrow visible structures, especially in $gg \rightarrow H$
 - POWHEG: \rightarrow high tails, cured by h dampening factor
 - well-established and well-known methods

(no need to discuss them any further)

- two schemes at NNLO: MINLO & UN²LOPS (singlets S only)
 - different basic ideas
 - MINLO: $S + j$ at NLO with $p_T^{(S)} \rightarrow 0$ and capture divergences by reweighting internal line with analytic Sudakov, NNLO accuracy ensured by reweighting with full NNLO calculation for S production
 - UN²LOPS identifies and subtracts and adds parton shower terms at FO from $S + j$ contributions, maintaining unitarity
 - available for two simple processes only: DY and $gg \rightarrow H$

NNLOs for H production: MINLO

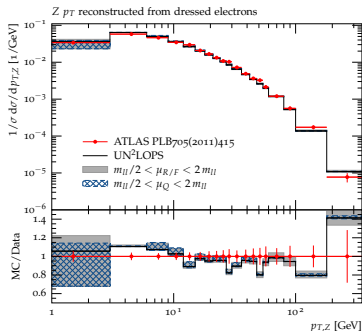
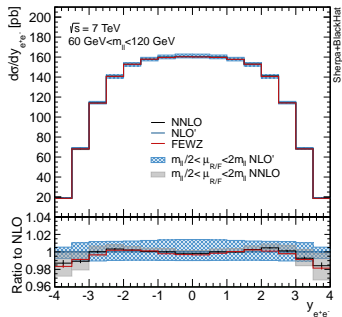
K. Hamilton, P. Nason, E. Re & G. Zanderighi, JHEP 1310



- also available for $Z/W/VH$ production

NNLOs for Z production: UN²LOs

S. Hoche, Y. Li, & S. Prestel, Phys.Rev.D90 & D91



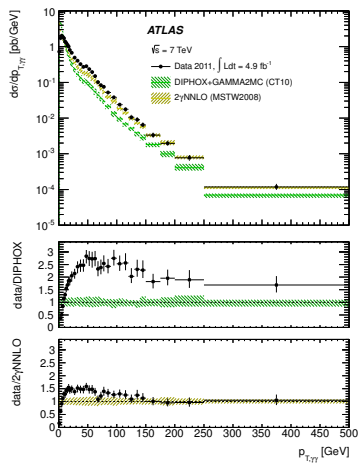
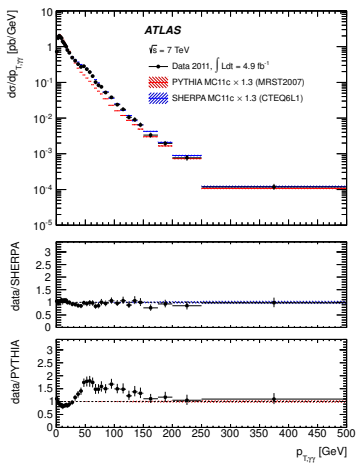
- also available for H production

NNLOs: shortcomings/limitations

- MINLO relies on knowledge of B_2 terms from analytic resummation
→ to date only known for colour singlet production
- MINLO relies on reweighting with full NNLO result
→ one parameter for H (y_H), more complicated for Z , ...
- UN²LOs relies on integrating single- and double emission to low scales and combination of unresolved with virtual emissions
→ potential efficiency issues, need NNLO subtraction
- UN²LOs puts unresolved & virtuals in “zero-emission” bin
→ no parton showering for virtuals (?)

merging example: $p_{\perp, \gamma\gamma}$ in MEPS@LO vs. NNLO

(arXiv:1211.1913 [hep-ex])



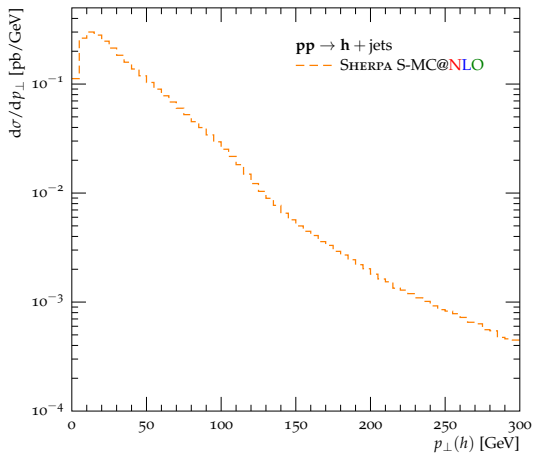
multijet-merging at NLO

- sometimes “more legs” wins over more loops
- basic idea like at LO: towers of MEs with increasing jet multiplicity (but this time at NLO)
- combine them into one sample, remove overlap/double-counting
- maintain NLO and LL accuracy of ME and PS
- this effectively translates into a merging of MC@NLO simulations and can be further supplemented with LO simulations for even higher final state multiplicities
- different implementations, parametric accuracy not always clear
- starts being used, still lacks careful cross-validation

(MEPs@NLO, FxFx, UNLOPs)

illustration: p_{\perp}^H in MEPS@NLO

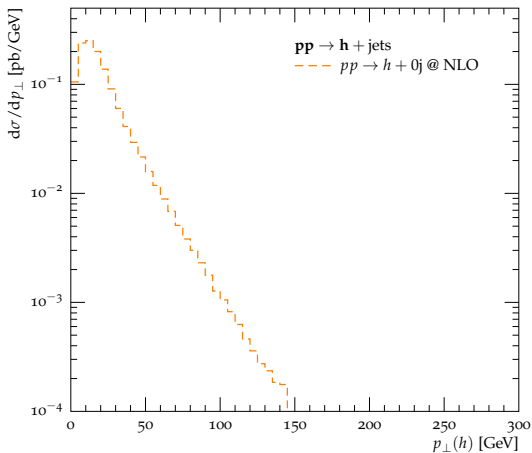
Transverse momentum of the Higgs boson



- first emission by MC@NLO

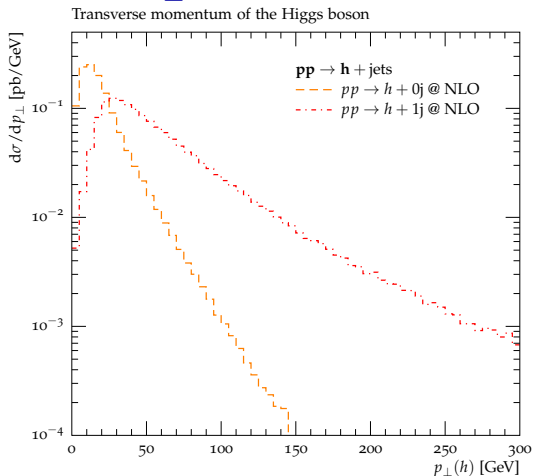
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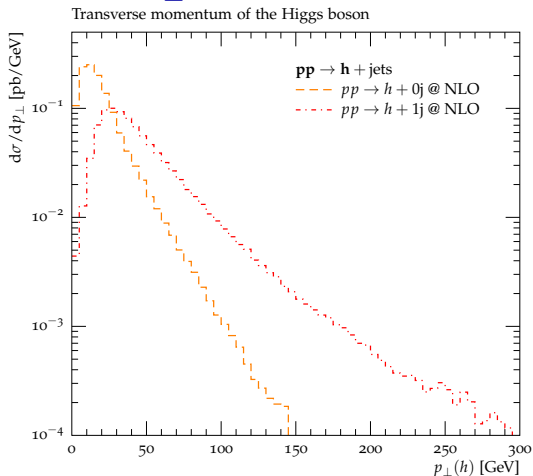
- first emission by MC@NLO, restrict to $Q_{n+1} < Q_{\text{cut}}$

illustration: p_{\perp}^H in MEPS@NLO



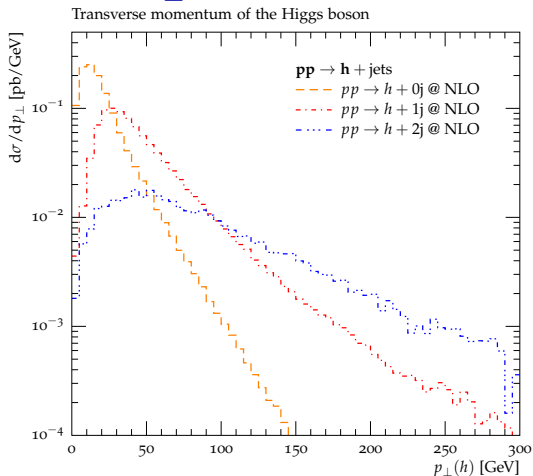
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illustration: p_{\perp}^H in MEPS@NLO



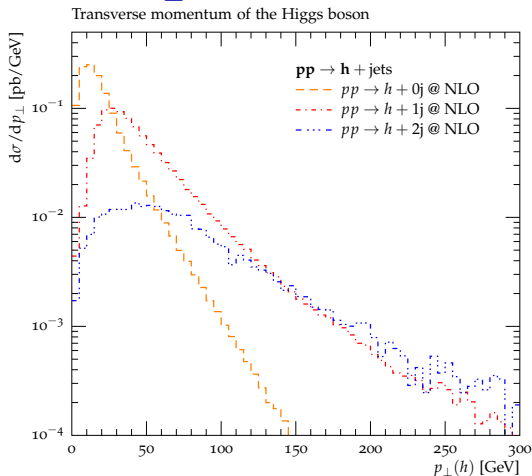
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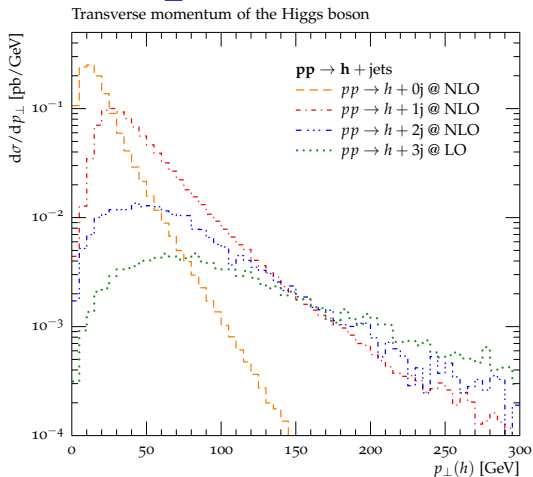
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illustration: p_{\perp}^H in MEPS@NLO



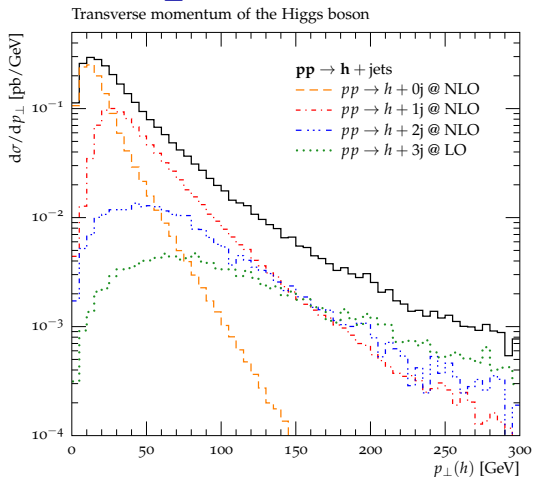
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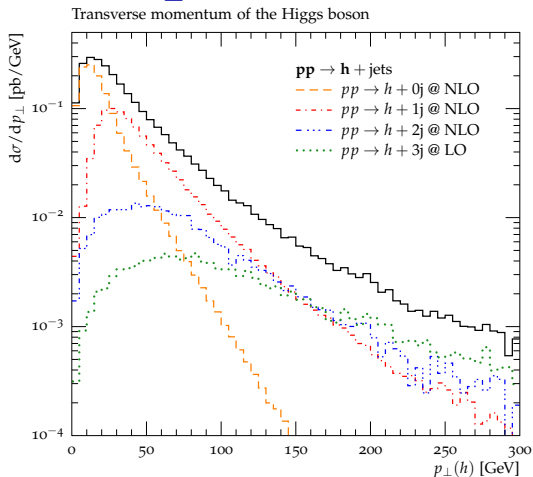
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- iterate
- sum all contributions

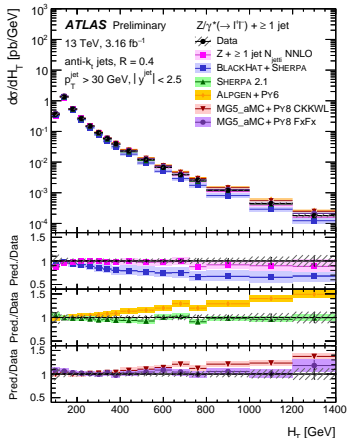
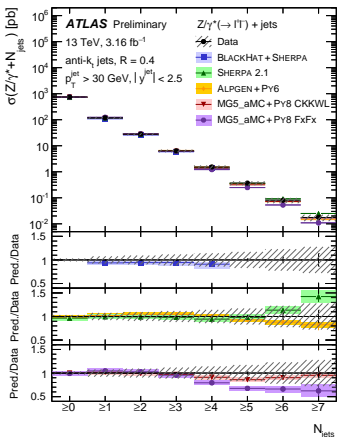
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- MC@NLO $pp \rightarrow h + 2\text{jets}$ for $Q_{n+2} > Q_{\text{cut}}$
- iterate
- sum all contributions
- eg. $p_{\perp}(h) > 200$ GeV has contributions fr. multiple topologies

Z+jets at 13 TeV: comparison with ATLAS data

- various merging codes at LO and NLO



including EW corrections

EW corrections

- EW corrections sizeable $\mathcal{O}(10\%)$ at large scales: **must include them!**
- but: more painful to calculate
- need EW showering & possibly corresponding PDFs

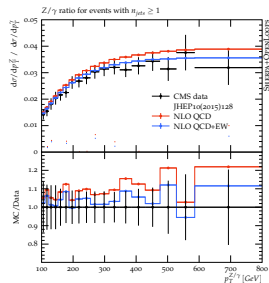
(somewhat in its infancy: chiral couplings)

- example: $Z\gamma$ vs. p_T (right plot)

(handle on p_{\perp}^Z in $Z \rightarrow \nu\bar{\nu}$)

(Kallweit, Lindert, Pozzorini, Schoenherr for LH'15)

- difference due to EW charge of Z
- no real correction (real V emission)
- improved description of $Z \rightarrow \ell\ell$



inclusion of electroweak corrections in simulation

- incorporate approximate electroweak corrections in MEPS@NLO
 - ① using electroweak Sudakov factors

$$\tilde{B}_n(\Phi_n) \approx \tilde{B}_n(\Phi_n) \Delta_{EW}(\Phi_n)$$

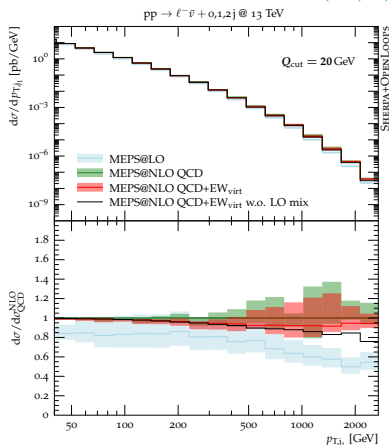
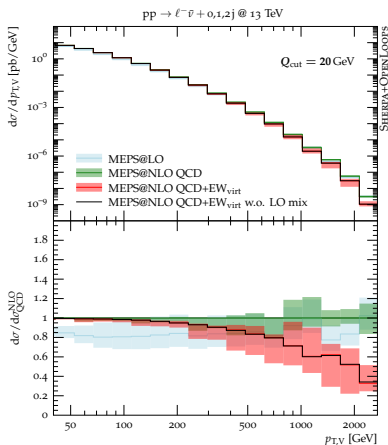
- ② using virtual corrections and approx. integrated real corrections

$$\tilde{B}_n(\Phi_n) \approx \tilde{B}_n(\Phi_n) + V_{n,EW}(\Phi_n) + I_{n,EW}(\Phi_n) + B_{n,mix}(\Phi_n)$$

- real QED radiation can be recovered through standard tools (parton shower, YFS resummation)
- simple stand-in for proper $QCD \oplus EW$ matching and merging
→ validated at fixed order, found to be reliable,
difference $\lesssim 5\%$ for observables not driven by real radiation

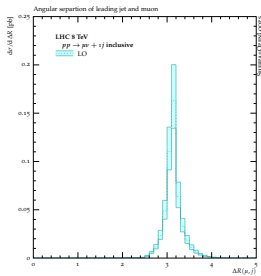
results: $pp \rightarrow \ell^- \bar{\nu} + \text{jets}$

(Kallweit, Lindert, Maierhöfer, Pozzorini, Schoenherr JHEP04(2016)021)



⇒ particle level events including dominant EW corrections

NLO EW predictions for $\Delta R(\mu, j_1)$

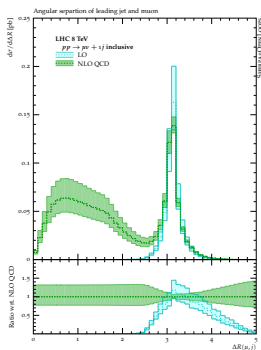


measure collinear W emission?

LHC@8TeV, $p_{\perp}^{j_1} > 500$ GeV, central μ and jet

- LO $pp \rightarrow Wj$ with $\Delta\phi(\mu, j) \approx \pi$

NLO EW predictions for $\Delta R(\mu, j_1)$

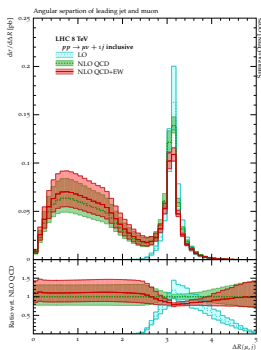


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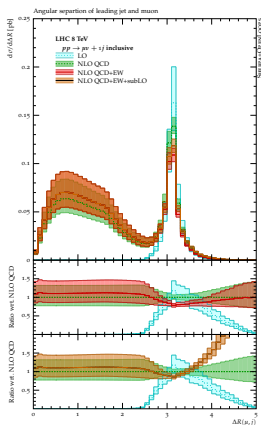


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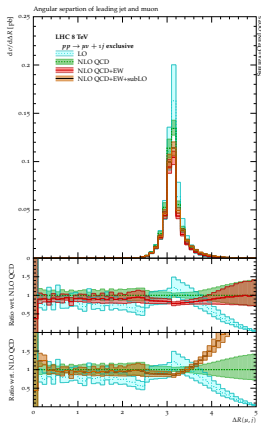
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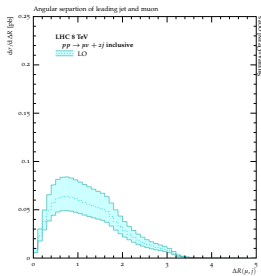
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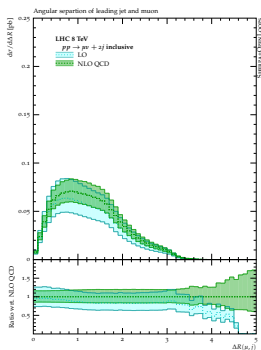


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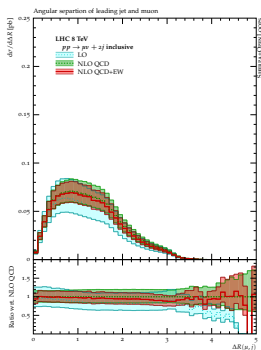


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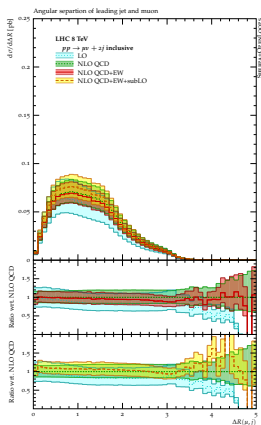


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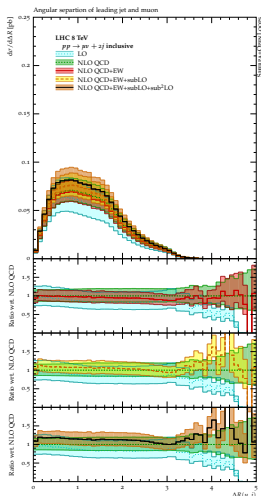
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→ possible double counting with BG

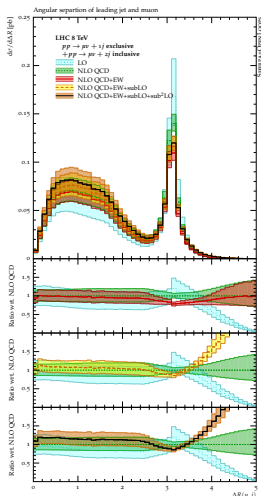


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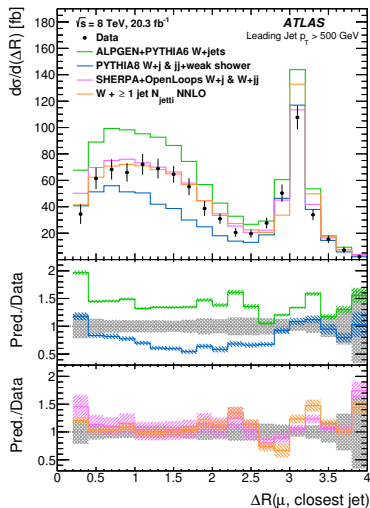
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- merge using exclusive sums



NLO EW predictions for $\Delta R(\mu, j_1)$

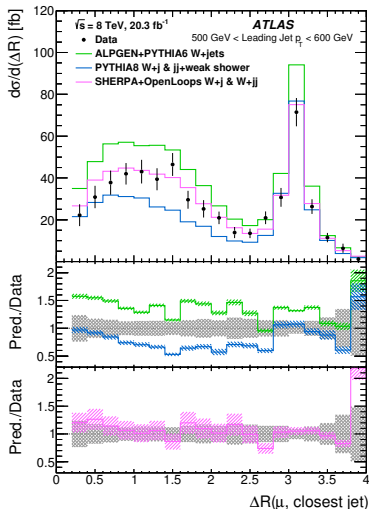


Data comparison

(M. Wu ICHEP'16, ATLAS arXiv:1609.07045)

- ALPGEN+PYTHIA
 $pp \rightarrow W + \text{jets}$ MLM merged
 (Mangano et al., JHEP07(2003)001)
- PYTHIA 8
 $pp \rightarrow Wj + \text{QCD shower}$
 $pp \rightarrow jj + \text{QCD+EW shower}$
 (Christiansen, Prestel, EPJC76(2016)39)
- SHERPA+OPENLOOPS
 NLO QCD+EW+subLO
 $pp \rightarrow Wj/Wjj$ excl. sum
 (Kallweit, Lindert, Maierhöfer,
 (Pozzorini, Schoenherr, JHEP04(2016)021)
- NNLO QCD $pp \rightarrow Wj$
 (Boughezal, Liu, Petriello, arXiv:1602.06965)

NLO EW predictions for $\Delta R(\mu, j_1)$

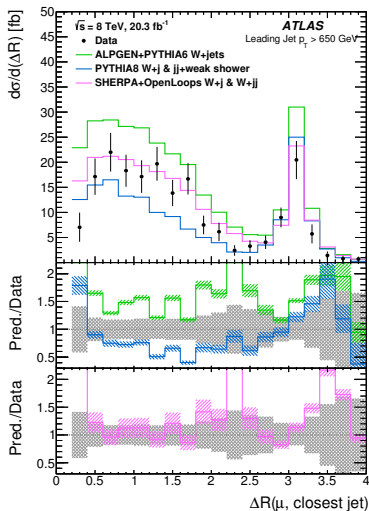


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improving parton showers

motivation: why care?

- QCD radiation omnipresent at the LHC
- enters as signal (and background) in high- p_{\perp} analyses
 - multi-jet signatures
 - multijet merging & higher-order matching (not the topic today)
 - inner-jet structures e.g. from “fat jets”
 - **parton shower algorithms**
- begs the question:
 - can we improve on parton showers and increase their precision?
 - (keep in mind: accuracy vs. precision)

another systematic uncertainty

- parton showers are approximations, based on
 - leading colour, leading logarithmic accuracy, spin-average
- parametric accuracy by comparing Sudakov form factors:

$$\Delta = \exp \left\{ - \int \frac{dk_{\perp}^2}{k_{\perp}^2} \left[A \log \frac{k_{\perp}^2}{Q^2} + B \right] \right\},$$

where A and B can be expanded in $\alpha_S(k_{\perp}^2)$

- Q_T resummation includes $A_{1,2,3}$ and $B_{1,2}$

(transverse momentum of Higgs boson etc.)
- showers usually include terms $A_{1,2}$ and B_1

A = cusp terms (“soft emissions”), $B \sim$ anomalous dimensions γ

connection to fragmentation functions

- DGLAP for FFs:

$$\frac{d x D_a(x, t)}{d \log t} = \sum_{b=q,g} \int_0^1 d\tau \int_0^1 dz \delta(x - \tau z) \frac{\alpha_S}{2\pi} [z P_{ab}(z)]_+ \tau D_b(\tau, t).$$

- rewrite for definition of “+”-function, $[z P_{ab}(z)]_+ = \lim_{\epsilon \rightarrow 0} z P_{ab}(z, \epsilon)$:

$$P_{ab}(z, \epsilon) = P_{ab}(z) \Theta(1 - z - \epsilon) - \delta_{ab} \sum_{c=q,g} \frac{\Theta(1 - z - \epsilon)}{\epsilon} \int_0^1 d\xi \xi P_{ac}(\xi)$$

$$\frac{d \log D_a(x, t)}{d \log t} = \underbrace{- \sum_{c=q,g} \int_0^{1-\epsilon} d\xi \frac{\alpha_S}{2\pi} \xi P_{ac}(\xi)}_{\text{derivative of Sudakov}} + \sum_{b=q,g} \int_x^{1-\epsilon} \frac{dz}{z} \frac{\alpha_S}{2\pi} P_{ac}(z) \frac{D_b(\frac{x}{z}, t)}{D_a(x, t)}$$

- re-introduce Sudakov form factor

$$\Delta_a(t, t_0) = \exp \left\{ - \int_{t_0}^t \frac{dt'}{t'} \sum_{c=q,g} \int_0^{1-\epsilon} d\xi \frac{\alpha_S}{2\pi} \xi P_{ac}(\xi) \right\}$$

to express equation above through generating functional

$\mathcal{D}_a(x, t, \mu^2) = D_a(x, t) \Delta_a(\mu^2, t)$:

$$\frac{d \log \mathcal{D}_a(x, t, \mu^2)}{d \log t} = \sum_{b=q,g} \int_x^{1-\epsilon} \frac{dz}{z} \frac{\alpha_S}{2\pi} P_{ac}(z) \frac{D_b(\frac{x}{z}, t)}{D_a(x, t)}$$

- add initial states (PDFs) & arrive at argument(s) for Sudakov form factors when jets not measured

$$\sum_{i \in IS} \sum_{b=q,g} \int_{x_i}^{1-\epsilon} \frac{dz}{z} \frac{\alpha_S}{2\pi} P_{bai}(z) \frac{f_b(\frac{x_i}{z}, t)}{f_{ai}(x, t)} + \sum_{j \in FS} \sum_{b=q,g} \int_{x_j}^{1-\epsilon} dz z \frac{\alpha_S}{2\pi} P_{ajb}(z).$$

subtle symmetry factors

- observations for LO PS in final state:
 - only $P_{qq}^{(0)}$ used but not $P_{qg}^{(0)}$
 - $P_{gg}^{(0)}$ comes with “symmetry factor” 1/2
- challenge this way of implementing symmetry through:

(Jadach & Skrzypek, hep-ph/0312355)

$$\sum_{i=q,g} \int_0^{1-\epsilon} dz z P_{qi}^{(0)}(z) = \int_{\epsilon}^{1-\epsilon} dz P_{qq}^{(0)}(z) + \mathcal{O}(\epsilon)$$

$$\sum_{i=q,g} \int_0^{1-\epsilon} dz z P_{gi}^{(0)}(z) = \int_{\epsilon}^{1-\epsilon} dz \left[\frac{1}{2} P_{gg}^{(0)}(z) + n_f P_{gq}^{(0)}(z) \right] + \mathcal{O}(\epsilon)$$

- net effect: replace symmetry factors by parton marker z

implementation in DIRE

- evolution and splitting parameter ($((ij) + k \rightarrow i + j + k)$):

$$\kappa_{j,ik}^2 = \frac{4(p_i p_j)(p_j p_k)}{Q^4} \quad \text{and} \quad z_j = \frac{2(p_j p_k)}{Q^2}.$$

- splitting functions including IR regularisation

(a la Curci, Furmanski & Petronzio, Nucl.Phys. B175 (1980) 27-92)

$$P_{qq}^{(0)}(z, \kappa^2) = 2C_F \left[\frac{1-z}{(1-z)^2 + \kappa^2} - \frac{1+z}{2} \right],$$

$$P_{qg}^{(0)}(z, \kappa^2) = 2C_F \left[\frac{z}{z^2 + \kappa^2} - \frac{2-z}{2} \right],$$

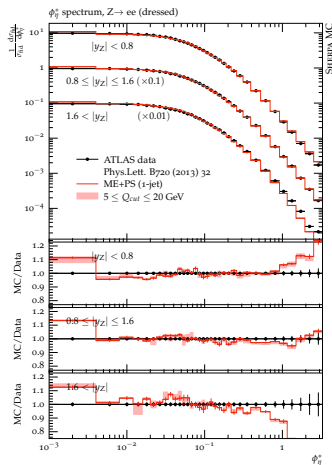
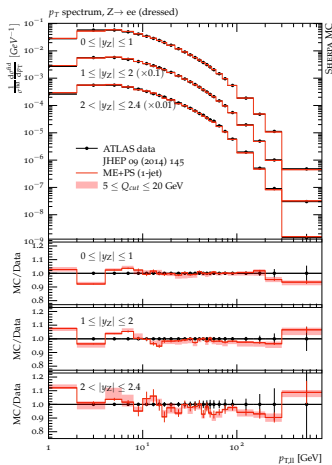
$$P_{gg}^{s(0)}(z, \kappa^2) = 2C_A \left[\frac{1-z}{(1-z)^2 + \kappa^2} - 1 + \frac{z(1-z)}{2} \right],$$

$$P_{gq}^{(0)}(z, \kappa^2) = T_R \left[z^2 + (1-z)^2 \right]$$

- renormalisation/factorisation scale given by $\mu = \kappa^2 Q^2$
- combine gluon splitting from two splitting functions with different spectators $k \rightarrow$ accounts for different colour flows

LO results for Drell-Yan

(example of accuracy in description of standard precision observable)



including NLO splitting kernels

including NLO splitting kernels

(Hoeche, FK & Prestel, 1705.00982, and Hoeche & Prestel, 1705.00742)

- expand splitting kernels as

$$P(z, \kappa^2) = P^{(0)}(z, \kappa^2) + \frac{\alpha_S}{2\pi} P^{(1)}(z, \kappa^2)$$

- aim: reproduce DGLAP evolution at NLO
include all NLO splitting kernels
- three categories of terms in $P^{(1)}$:
 - cusp (universal soft-enhanced correction) (already included in original showers)
 - corrections to $1 \rightarrow 2$
 - new flavour structures (e.g. $q \rightarrow q'$), identified as $1 \rightarrow 3$
- new paradigm: **two independent implementations**

implementation details: $1 \rightarrow 2$ splittings

- problem: new pole structure $1/z$ appears
- in final-state shower: symmetrisation yields extra factor z
(such a factor is present in IS shower)
- this factor accounts for $1/2$ typically applied to $g \rightarrow gg$
- include also $q \rightarrow gq$ splitting
- physical interpretation:
 - “unconstrained” (without) vs. “constrained” evolution
(DGLAP evolution for fragmentation functions)
 - factor z explicitly guarantees (momentum) sum rules
 - it also identifies final state particle

- symmetry factors not so clear at NLO \rightarrow more care needed

$$\begin{aligned}
 & \sum_{b=q,g,b \neq a} \int_0^{1-\epsilon} dz_1 \int_0^{1-\epsilon} dz_2 \frac{z_1 z_2}{1-z_1} \Theta(1-z_1-z_2) \\
 & \quad \times \left[P_{a \rightarrow ba\bar{b}}(z_1, z_2, \dots) + P_{a \rightarrow b\bar{b}a}(z_1, z_2, \dots) \right] \\
 & = \sum_{b=q,g,b \neq a} \int_0^{1-\epsilon} dz_1 \int_0^{1-z_1} dz_2 \frac{1}{\prod_{i=q,g} n_i!} P_{a \rightarrow ba\bar{b}}(z_1, z_2, \dots) + \mathcal{O}(\epsilon)
 \end{aligned}$$

1 \rightarrow 3 flavour changing kernels

- start with triple-collinear splitting functions

(Campbell & Glover, hep-ph/9710255 & Catani & Grazzini, hep-ph/9908523)

- re-interpret splitting as sequential:
 $(aij) + k \rightarrow (ai) + j + k \otimes (ai) + k \rightarrow a + i + k$
- kinematic mappings from CDST

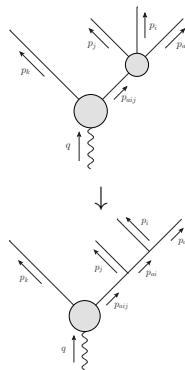
(Catani, Dittmaier, Seymour & Trocsanyi, hep-ph/0201036)

- evolution and splitting parameters:

$$t = \frac{4(p_j p_{ai})(p_{ai} p_k)}{q^2 - m_{aij}^2 - m_k^2}, \quad z_a = \frac{2p_a p_k}{q^2 - m_{aij}^2 - m_k^2}$$

$$s_{ai} = 2p_a p_i + m_a^2 + m_i^2, \quad x_a = \frac{p_a p_k}{p_{ai} p_k}$$

(Hoeche & Prestel, 1705.00742)



- phase space factorised by successive s -channels:

(Dittmaier, hep-ph/9904440)

$$d\Phi_{+2} = \left[\frac{1}{4(2\pi)^3} \frac{dt}{t} dz_a d\phi_j J_{FF}^{(1)} \right] \left[\frac{1}{4(2\pi)^3} ds_{ai} \frac{dx_a}{x_a} d\phi_i J_{FF}^{(2)}(2p_{ai}p_j) \right]$$

- combine with ME in coll. limit \rightarrow diff. branching probability:

$$\frac{d \log \Delta_{(aij)a}^{1 \rightarrow 3}}{d \log t} = \int dz_a \int ds_{ai} \int \frac{dx_a}{x_a} \int \frac{d\phi}{2\pi} \left(\frac{\alpha_S}{2\pi} \right)^2 \frac{z_a z_i}{1 - z_a} \frac{P_{(aij)a}(p_a, p_i, p_j)}{s_{ai}^2 / (2p_a p_j)}$$

with $P_{(aij)a} =$ triple-collinear splitting function

subtractions

- must subtract spin-correlated iterated $1 \rightarrow 2$ splittings

$$\frac{d \log \Delta_{(aij)a}^{(1 \rightarrow 2)^2}}{d \log t} = \int dz_a \int \frac{ds_{ai}}{s_{ai}} \int \frac{d\xi}{\xi} \left(\frac{\alpha_S}{2\pi} \right)^2 \frac{z_a z_i}{1 - z_a} \frac{P_{(aij)(ai)}^{(0)}(\xi) P_{(ai)a}^{(0)}\left(\frac{z_a}{\xi}\right)}{s_{aij}/(2p_a p_j)}$$

- must subtract convolution of one-loop matching coefficient with fixed-order renormalisation of fragmentation function, \mathcal{I}

$$\mathcal{I}_{qq'}(z) = 2C_F \int_z \frac{dx}{x} \left(\frac{1 + (1-x)^2}{x} \log[x(1-x)] + x \right) P_{gq'}^{(0)}\left(\frac{z}{x}\right)$$

(this is the finite part of convoluting $1 \rightarrow 2$ in D dimensions with another $1 \rightarrow 2$ in 4 dimensions)

final result

- arrive at final expression, ready for MC implementation

$$P_{qq'}(z) = \left(I + \frac{1}{\epsilon} \mathcal{P} - \mathcal{I} \right)_{qq'}(z) + \int d\Phi_{+1} \left(R - S \right)_{qq'}(z, \Phi_{+1}),$$

where

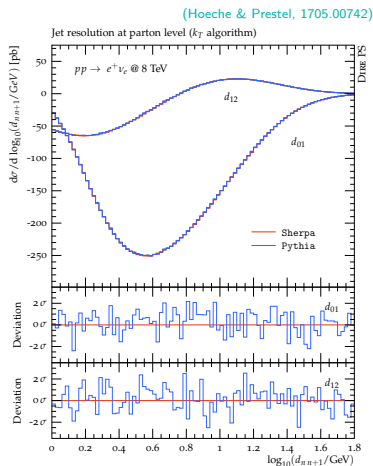
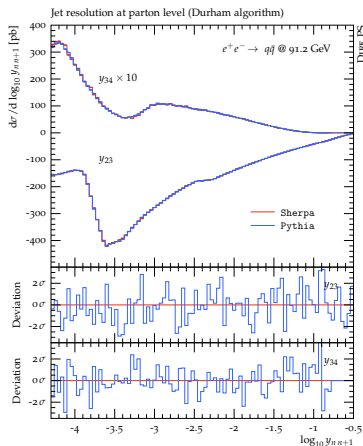
$$\left(I + \frac{1}{\epsilon} \mathcal{P} \right)_{qq'}(z) = \int d\Phi_{+1} S_{qq'}(z, \Phi_{+1})_{\text{finite}}$$

$$R_{qq'}(z, \Phi_{+1}) = P_{qq'}^{1 \rightarrow 3}(z, \Phi_{+1})$$

$$S_{qq'}(z, \Phi_{+1}) = \frac{s_{aij}}{s_{ai}} \left(P_{qg}^{(0)} \otimes P_{gq'}^{(0)} \right) (z, \Phi_{+1})$$

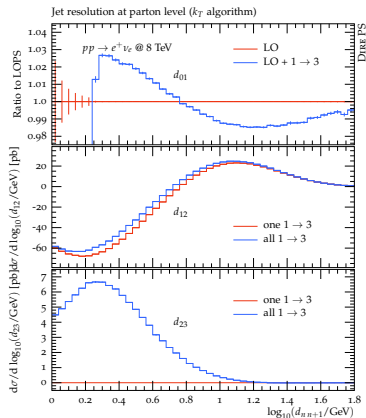
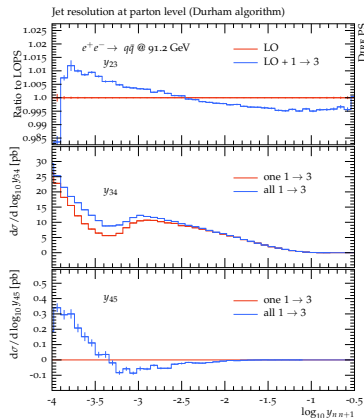
- this looks like MC@NLO inside the Sudakov exponent

validation of $1 \rightarrow 3$ splittings



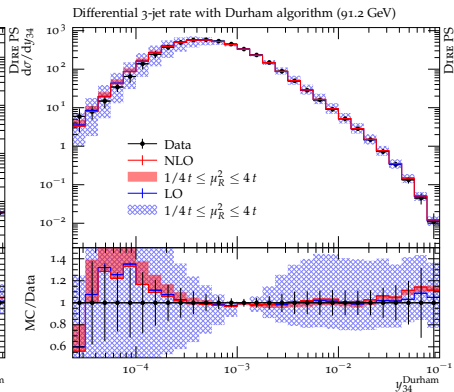
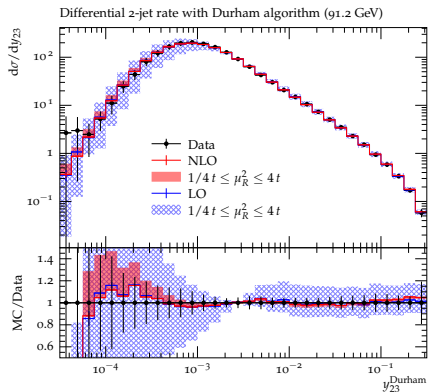
impact of $1 \rightarrow 3$ splittings

(Hoeche & Prestel, 1705.00742)



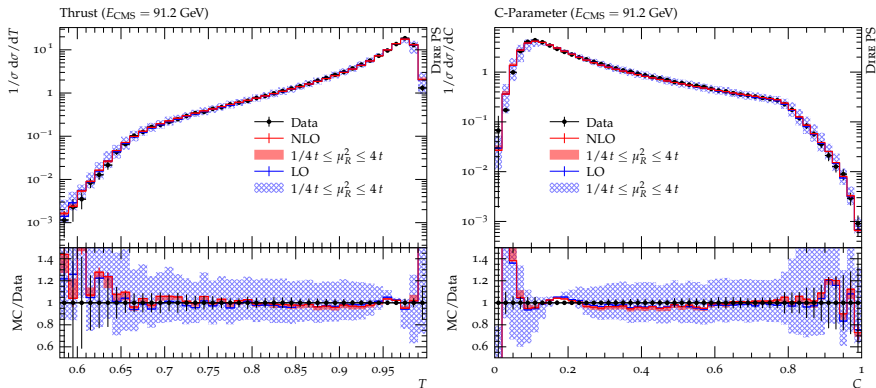
physical results: $e^-e^+ \rightarrow \text{hadrons}$

(Hoeche, FK & Prestel, 1705.00982)



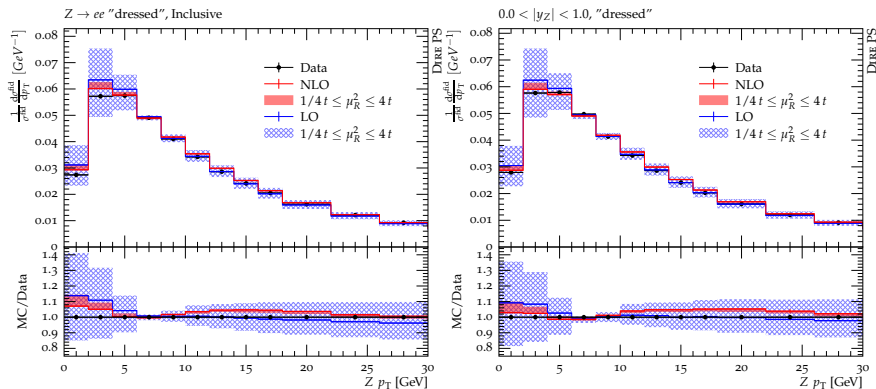
physical results: $e^-e^+ \rightarrow \text{hadrons}$

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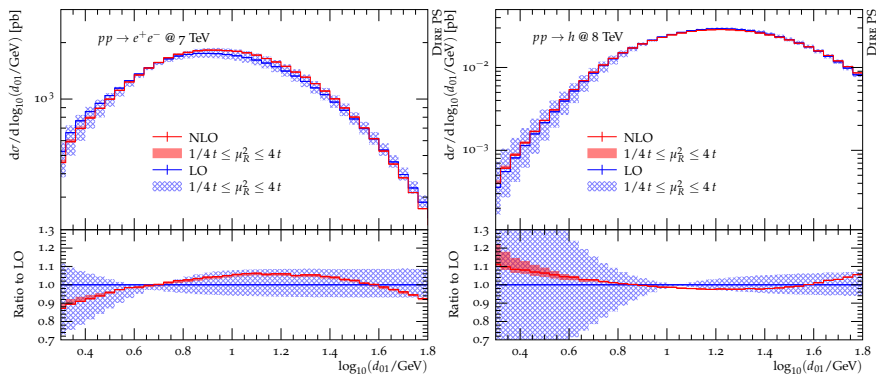
physical results: DY at LHC

(Hoeche, FK & Prestel, 1705.00982)



physical results: diff. jet rates at LHC

(Hoeche, FK & Prestel, 1705.00982)

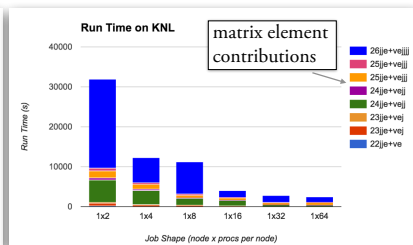
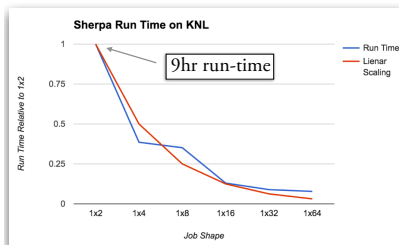


limitations
and
future challenges

limitation: computing short-distance cross sections – LO

(Childers, Uram, LeCompte, Benjamin, Hoeche, CHEP 2016)

- challenge of efficiency on tomorrow's (& today's) computers
- 2000's paradigm: memory free, flops expensive
(example: 16-core Xeon, 20MB L2 Cache, 64GB RAM)
- 2020's paradigm: flops free, memory expensive & must be managed
(example: 68-core Xeon KNL, 34MB L2 Cache, 16GB HBM, 96GB RAM)
- may **trigger rewrites of code to account for changing paradigm**



(figures stolen from Taylor-Childers' talk at CHEP)



theory limitations/questions

- we have constructed lots of tools for precision physics at LHC
 - **but** we did not cross-validate them careful enough (yet)
 - **but** we did not compare their theoretical foundations (yet)
- we also need unglamorous improvements on existing tools:
 - account for new computer architectures and HPC paradigms
 - systematically check advanced scale-setting schemes (MINLO)
 - automatic (re-)weighting for PDFs & scales
 - scale compensation in PS is simple (implement and check)
- 4 vs. 5 flavour scheme → **really?**
- how about α_S : range from 0.113 to 0.118

(yes, I know, but still - it bugs me)

→ is there any way to settle this once and for all (measurements?)

achievable goals (I believe we know how to do this)

- NLO for loop-induced processes:
 - MC@NLO tedious but straightforward → around the corner
 - EW NLO corrections with tricky/time-consuming calculation setup
 - but important at large scales: effect often \sim QCD, but opposite sign
 - need maybe faster approximation for high-scales (EW Sudakovs)
 - work out full matching/merging instead of approximations
 - improve parton shower:
 - beyond (next-to) leading log, leading colour, spin-averaged
 - HO effects in shower and scale uncertainties
- NLO DGLAP nearly done, now soft?
- start including next-to leading colour
 - include spin-correlations → important for EW emissions

more theory uncertainties/issues?

- with NNLOs approaching 5% accuracy or better:
 - non-perturbative uncertainties start to matter:
 - PDFs, MPIs, hadronization, etc.
 - question (example): with hadronization tuned to quark jets (LEP)
 - how important is the “chemistry” of jets for JES?
 - can we fix this with measurements?
 - example PDFs: to date based on FO vs. data
 - will we have to move to resummed/parton showered?

(reminder: LO* was not a big hit, though)
- $g \rightarrow q\bar{q}$ at accuracy limit of current parton showers:
 - how bad are $\sim 25\%$ uncertainty on $g \rightarrow b\bar{b}$?
 - can we fix this with measurements?

the looming revolution: going beyond NLO

- H in ggF at N³LO (Anastasiou, Duhr and others)
- explosive growth in NNLO (QCD) $2 \rightarrow 2$ results

(apologies for any unintended omissions)

- $t\bar{t}$ (1303.6254; 1508.03585;1511.00549)
 - single- t (1404.7116)
 - VV (1507.06257; 1605.02716;1604.08576; 1605.02716)
 - HH (1606.09519)
 - VH (1407.4747; 1601.00658;1605.08011)
 - $V\gamma$ (1504.01330)
 - $\gamma\gamma$ (1110.2375; 1603.02663)
 - Vj (1507.02850; 1512.01291; 1602.06965; 1605.04295; 1610.07922)
 - Hj (1408.5325; 1504.07922; 1505.03893; 1508.02684; 1607.08817)
 - jj (1310.3993; 1611.01460)
- NLO corrections to $gg \rightarrow VV$ (1605.04610)
 - WBF at NNLO (1506.02660) and N³LO (1606.00840)
 - different IR subtraction schemes:
N-jettiness slicing, antenna subtraction, sector decomposition,

living with the revolution

- we will include them into full simulations

(I am willing to place a bet: 5 years at most!)

- practical limitations/questions to be overcome:

- dealing with IR divergences at NNLO: slicing vs. subtracting

(I'm not sure we have THE solution yet)

- how far can we push NNLO? are NLO automated results stable enough for NNLO at higher multiplicity?
- matching for generic processes at NNLO?

(MINLO or UN²LOs or something new?)

- more scales (internal or external) complicated – need integrals

- philosophical questions:

- going to higher power of N often driven by need to include larger FS multiplicity – maybe not the most efficient method
- limitations of perturbative expansion:
 - breakdown of factorisation at HO (Seymour et al.)
 - higher-twist: compare $(\alpha_S/\pi)^n$ with Λ_{QCD}/M_Z

