



Hyper-k

David Hadley, University of Warwick

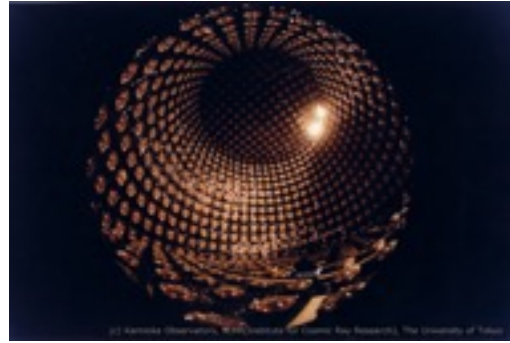
Outline

Hyper-K Detector

Long baseline neutrino oscillation status and prospects

Systematic uncertainty challenges and solutions

Kamiokande Detectors



Kamiokande
680 tonne
fiducial mass
(1983)

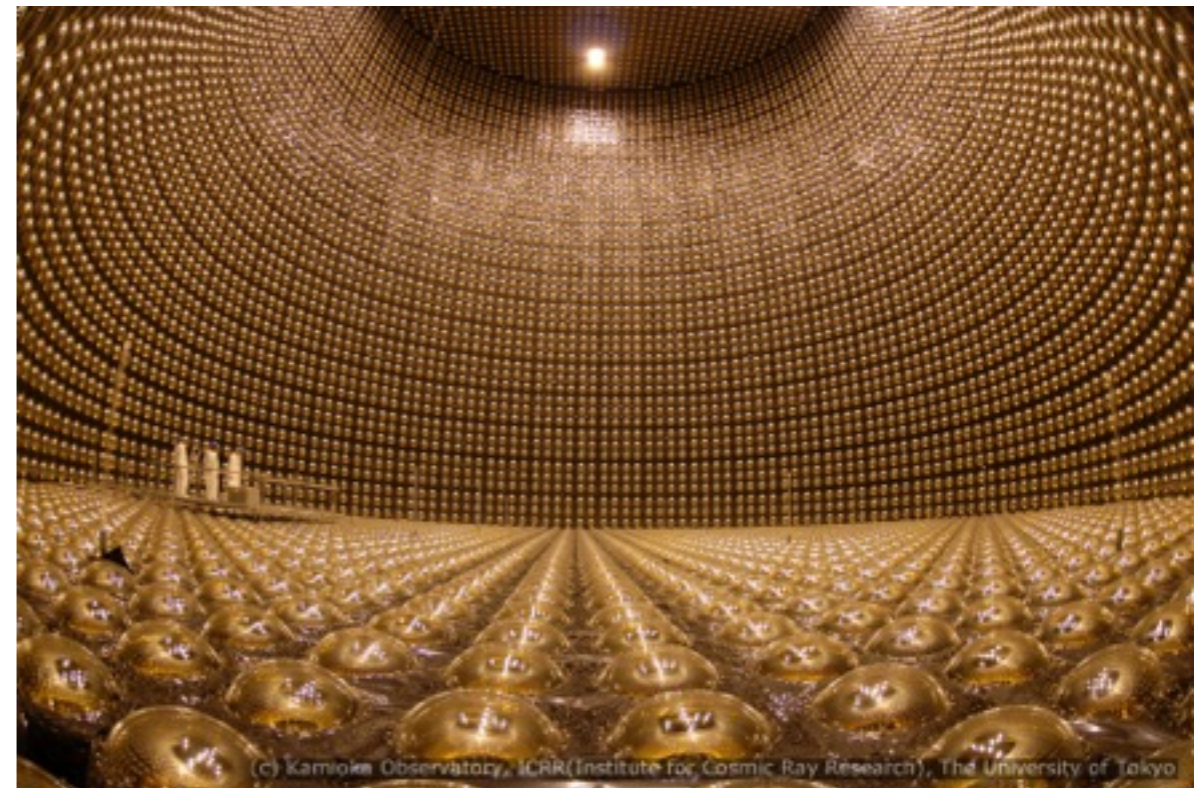
Kamiokande Detectors



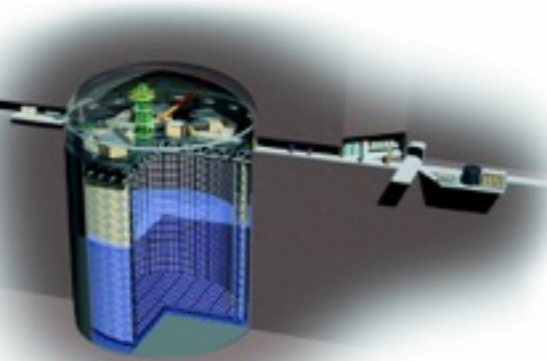
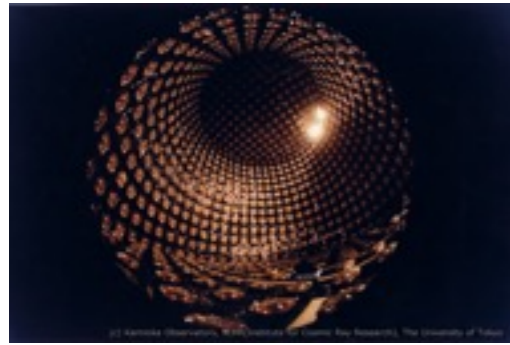
Super-Kamiokande
22.5kt fiducial mass
(33x Kamiokande)

(1996)

Kamiokande
680 tonne
fiducial mass
(1983)



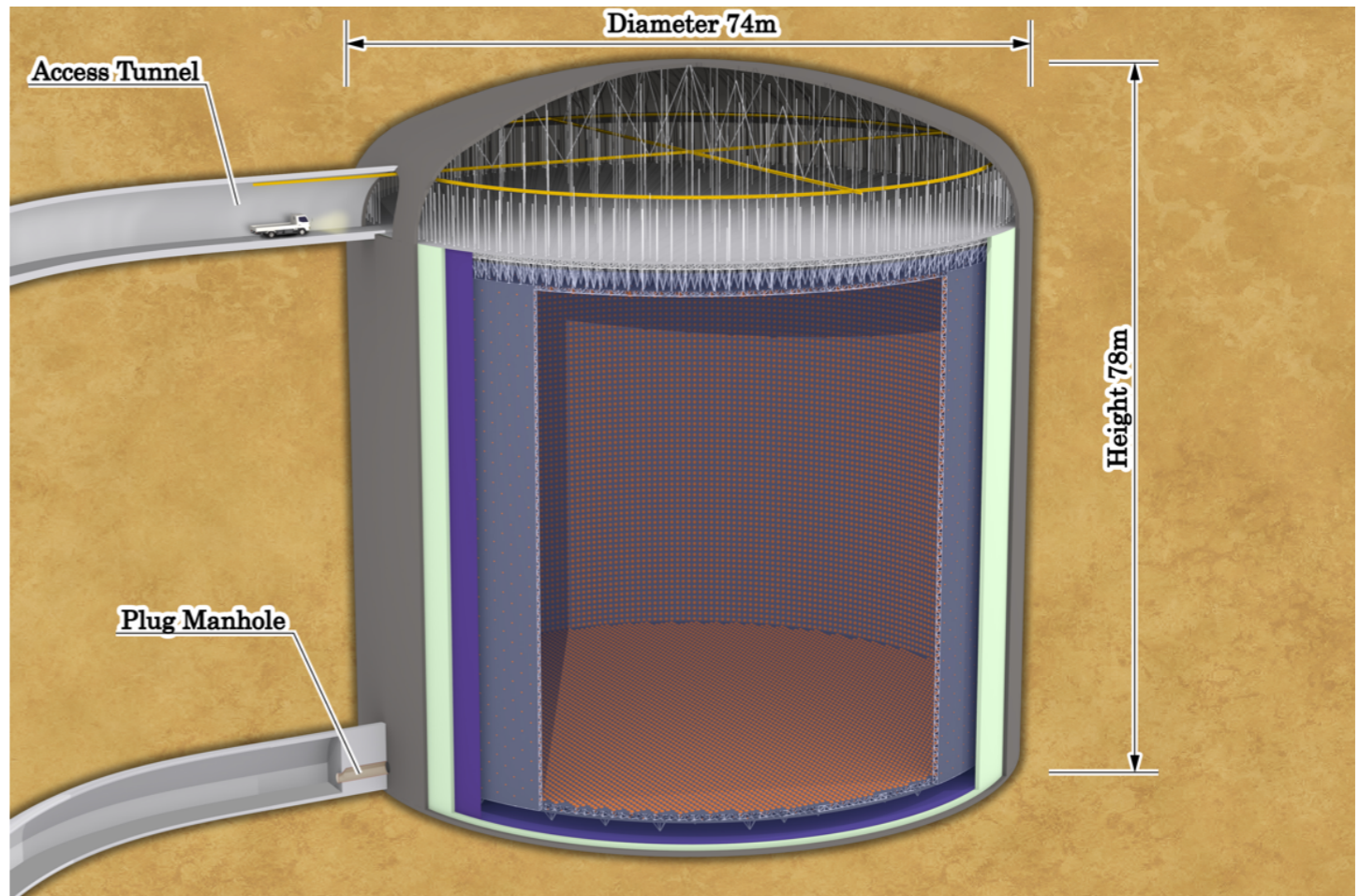
Kamiokande Detectors



Super-Kamiokande
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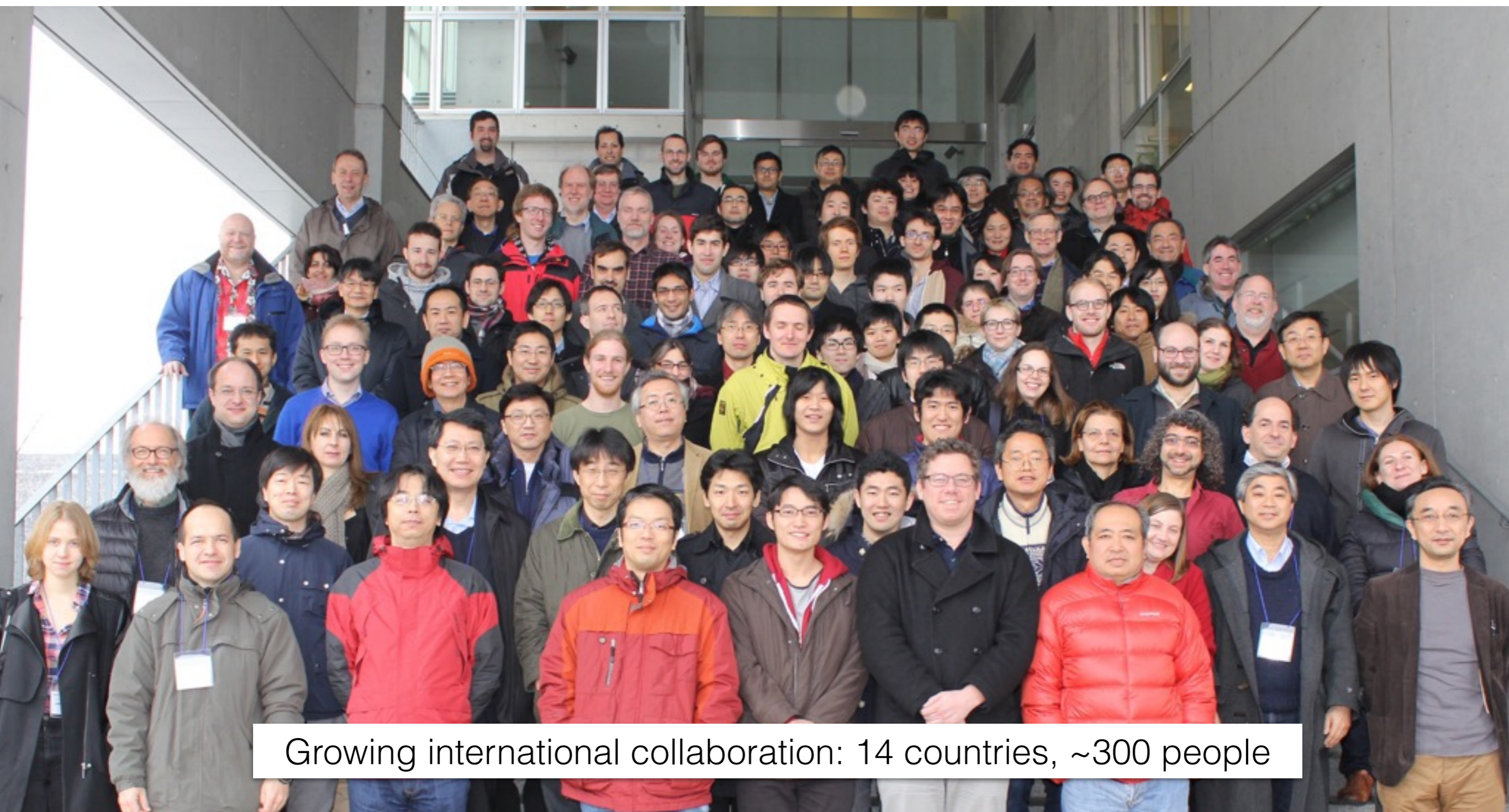
(1996)

Kamiokande
680 tonne
fiducial mass
(1983)



Hyper-Kamiokande
187 kt fiducial mass per tank
(2026?)

Hyper-K Collaboration

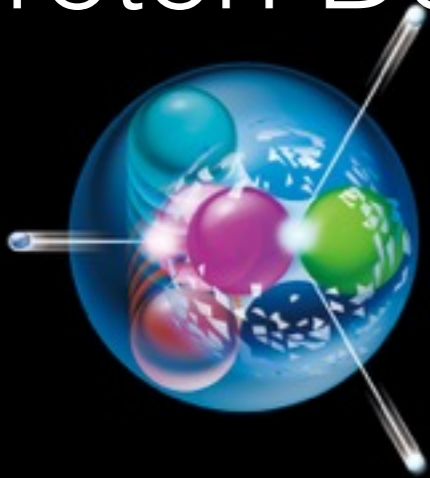


Growing international collaboration: 14 countries, ~300 people



Physics at Hyper-K

Proton Decay

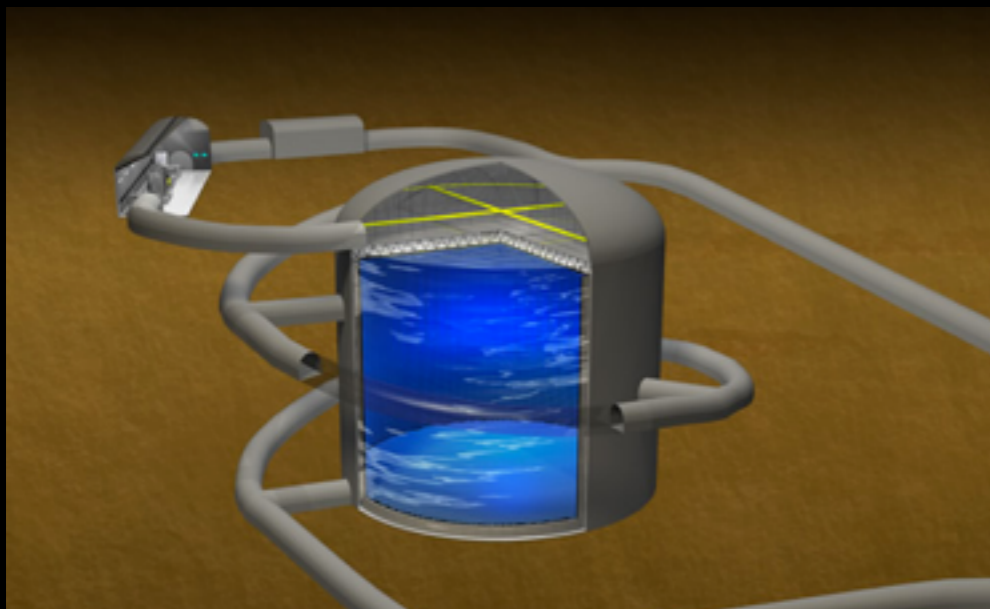
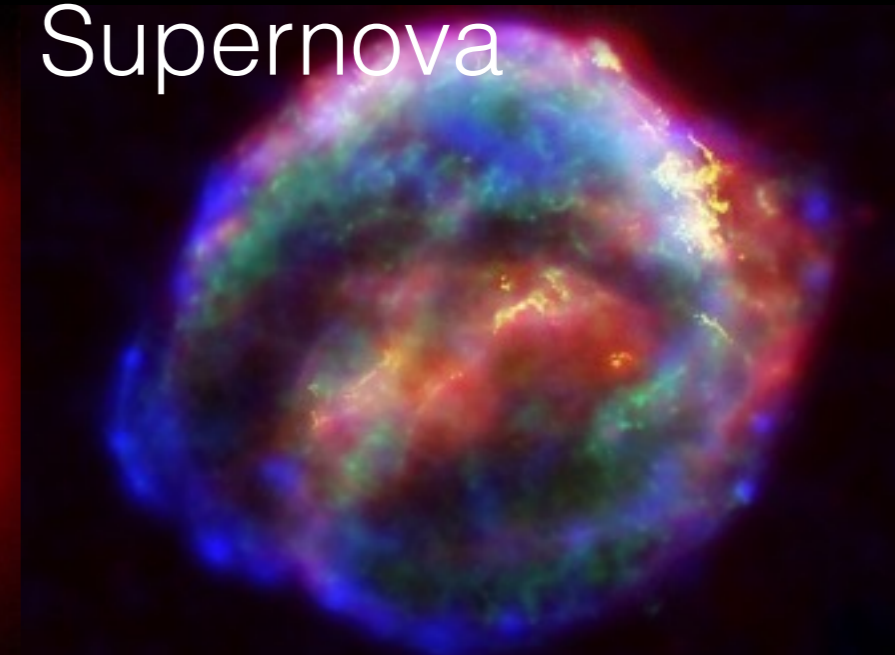


Neutrinos

Solar



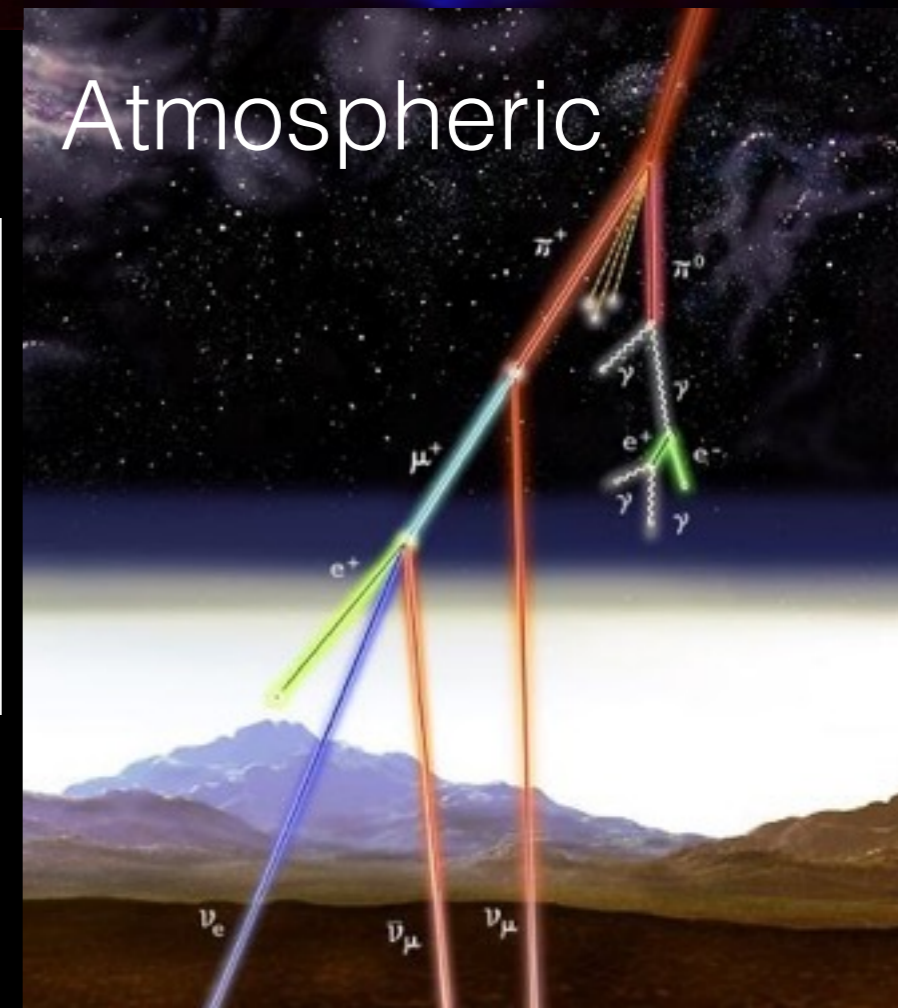
Supernova



Accelerator



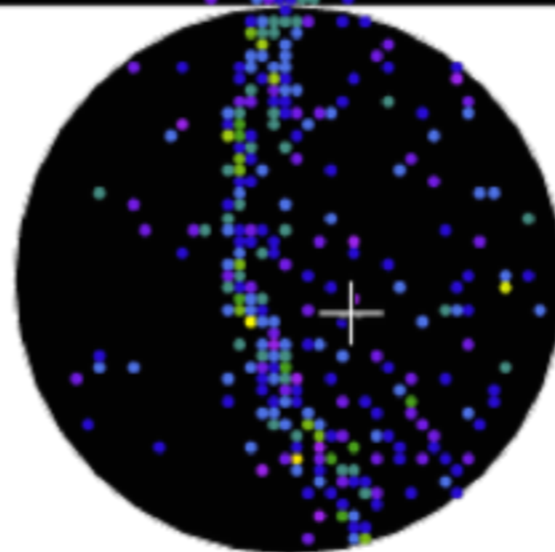
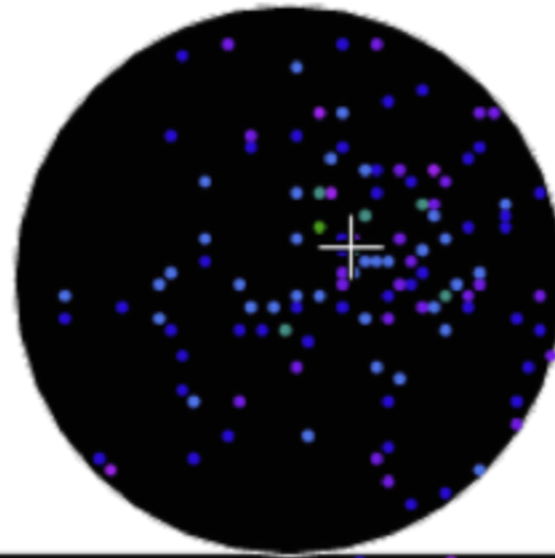
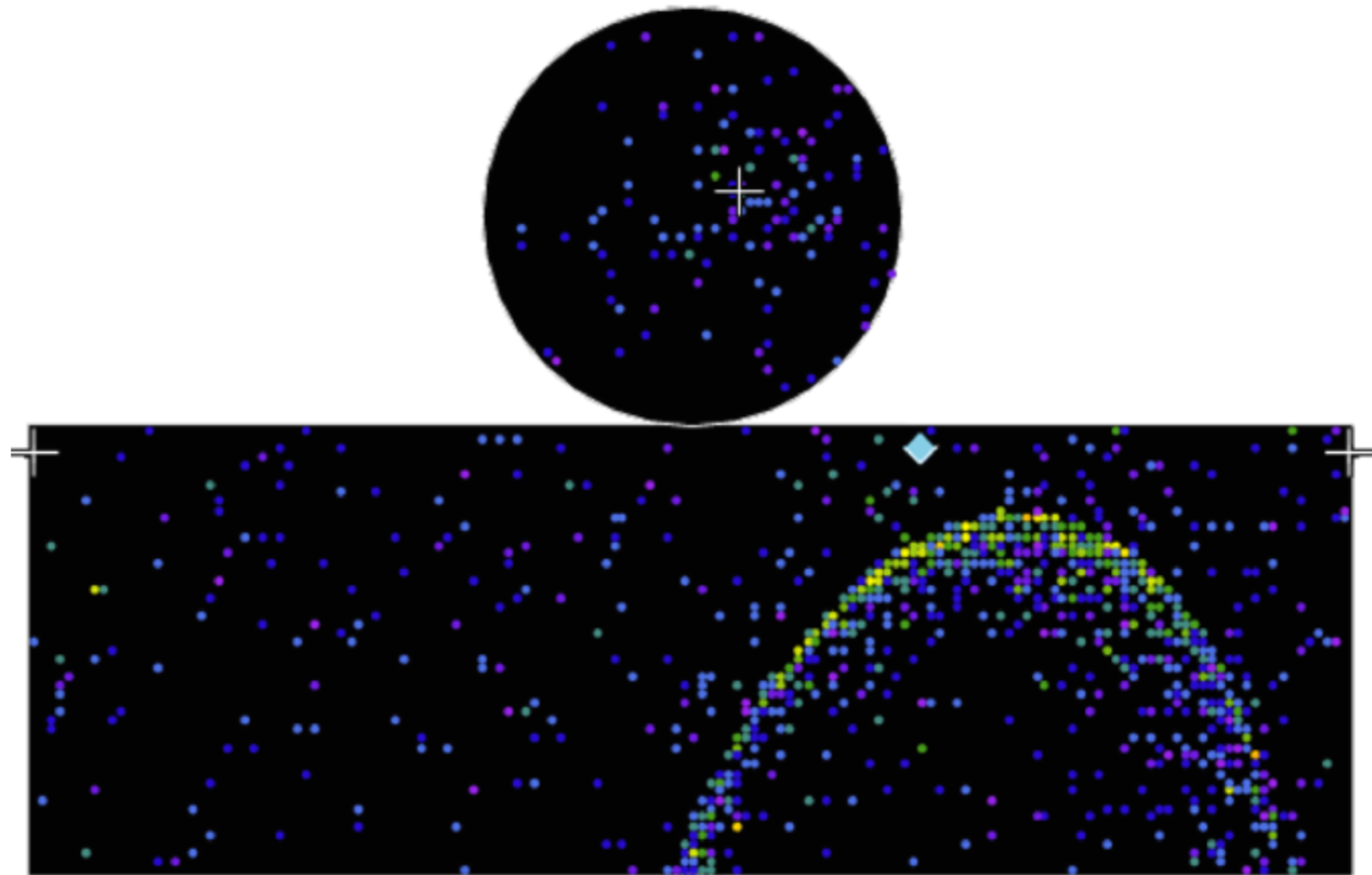
Atmospheric



Broad physics programme.

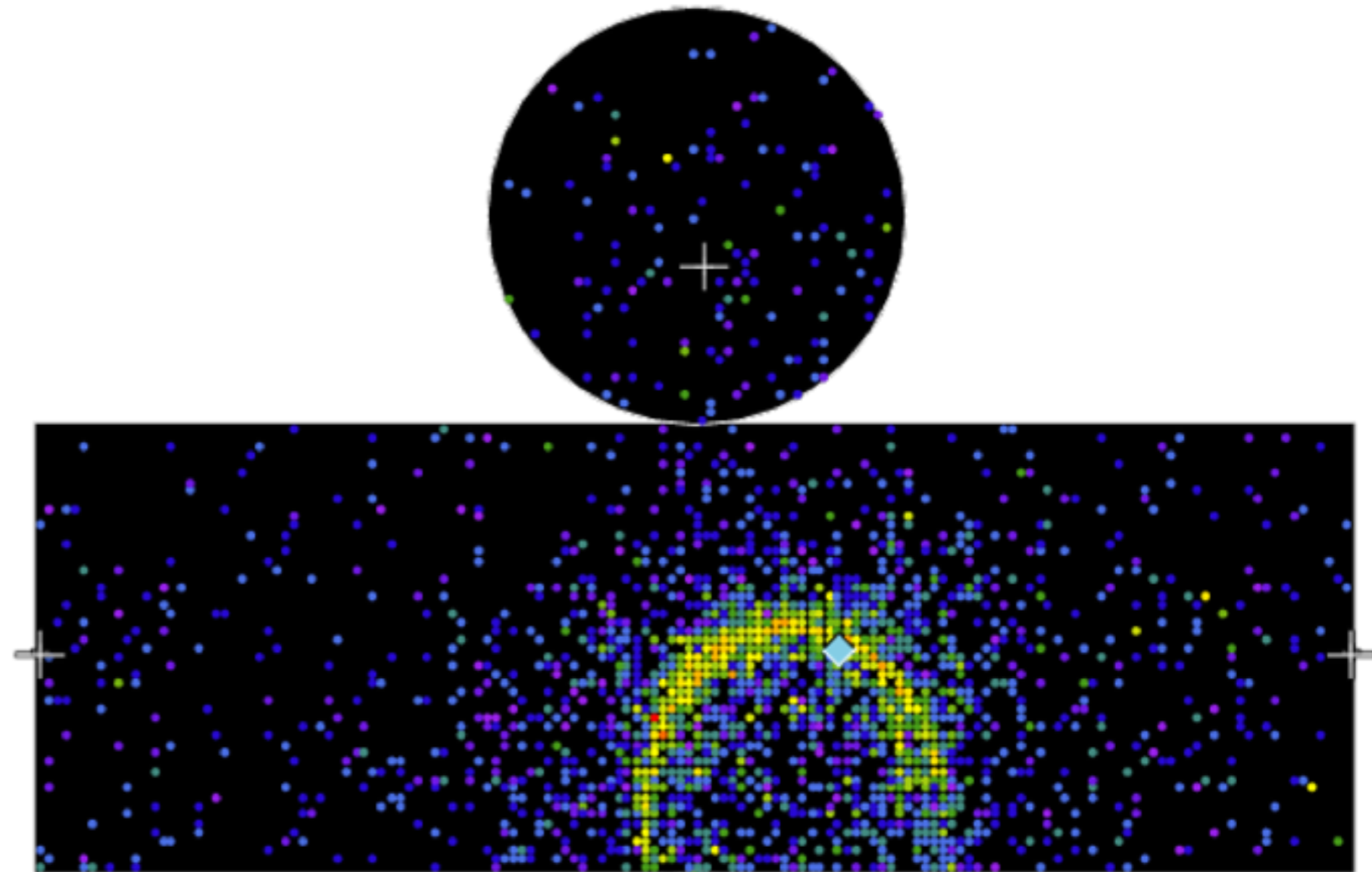
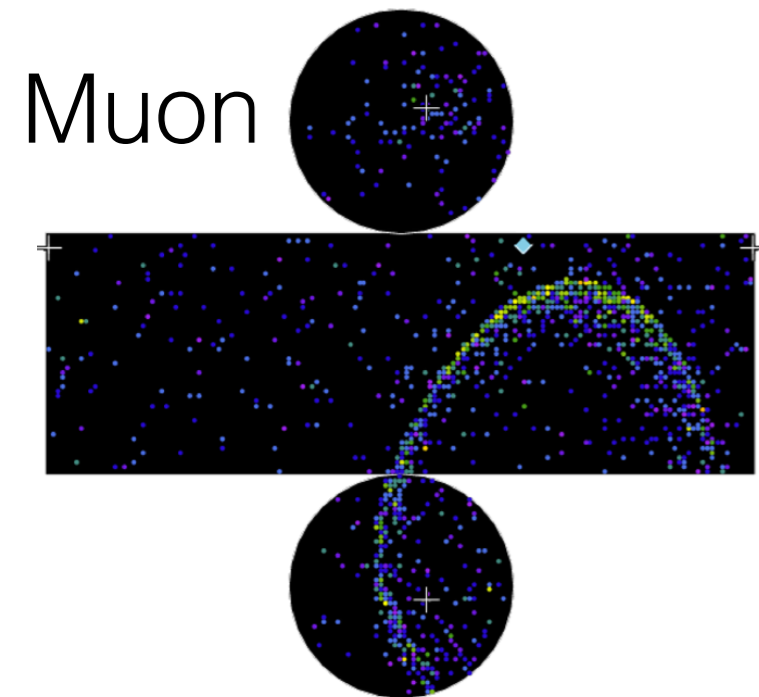


Water Cherenkov Technique



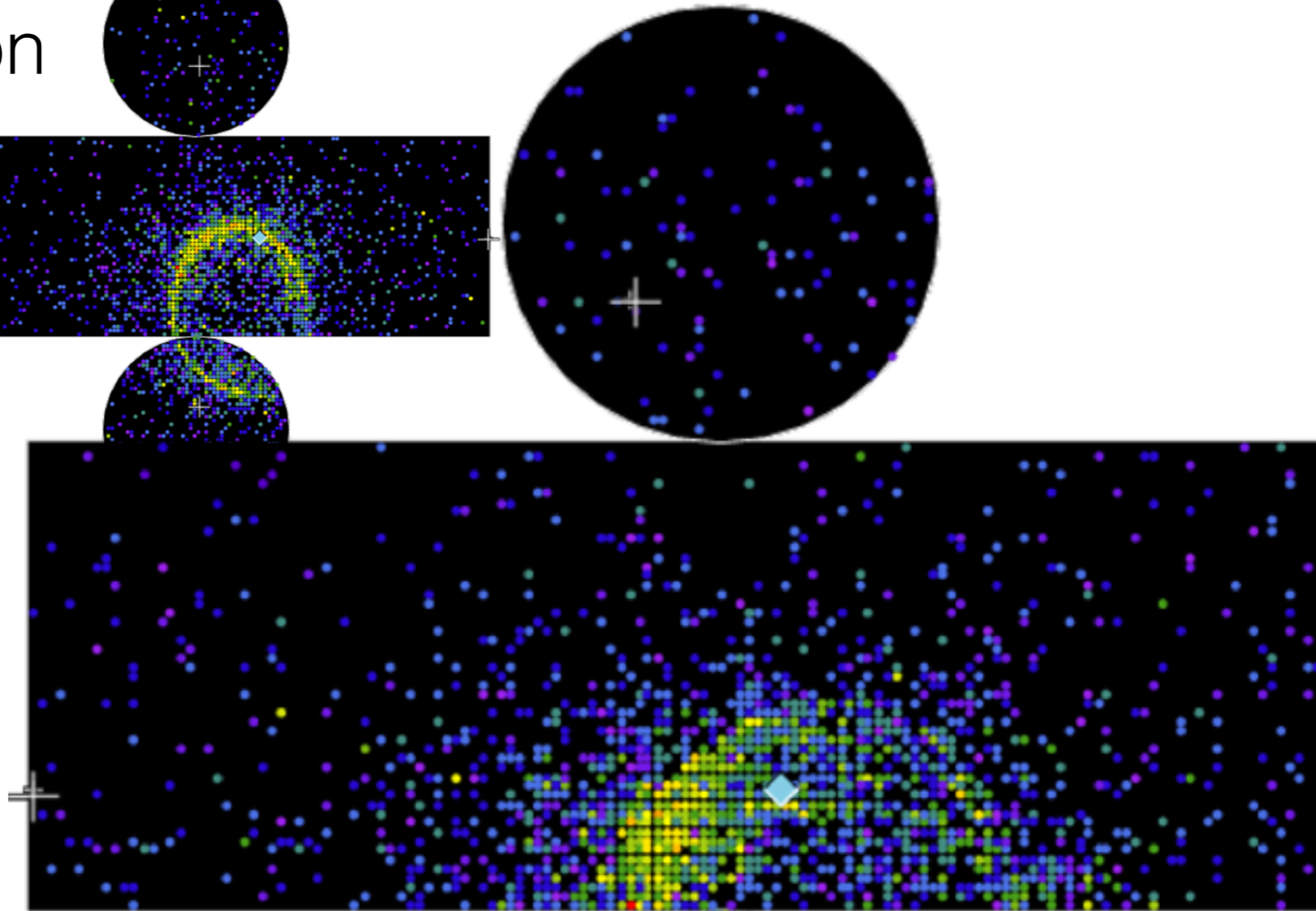
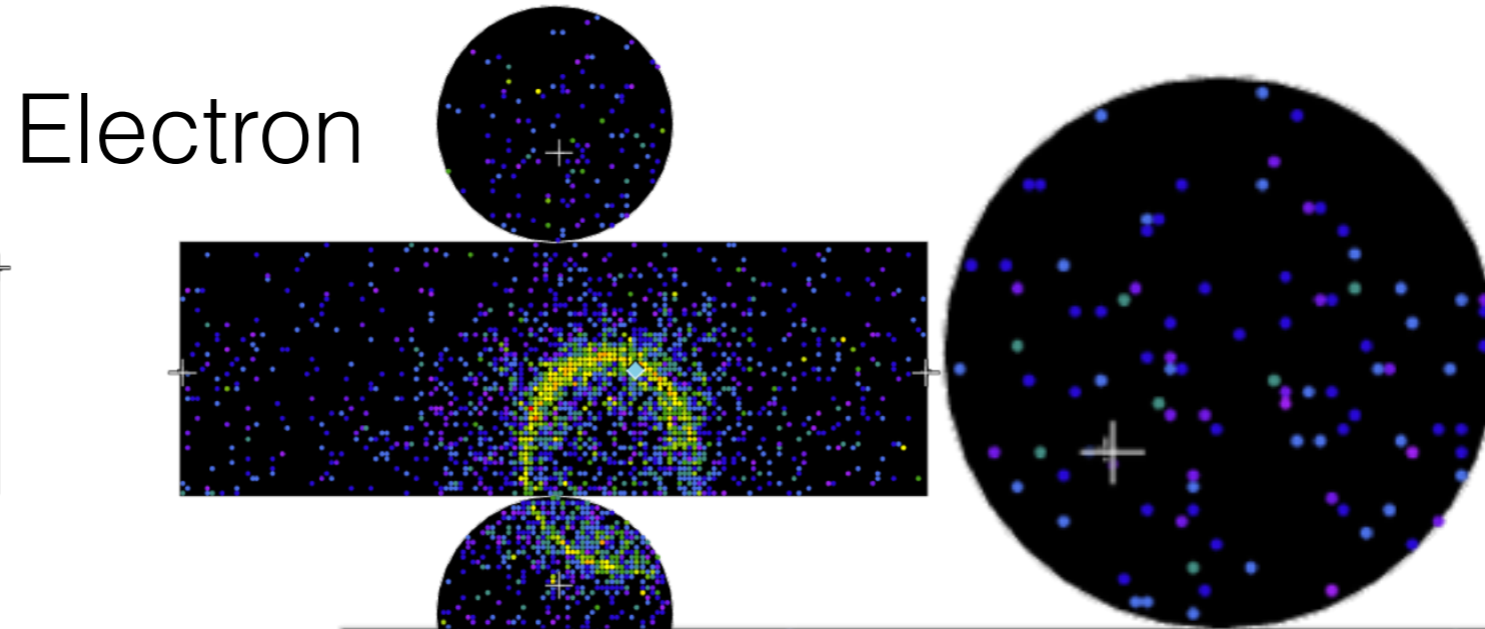
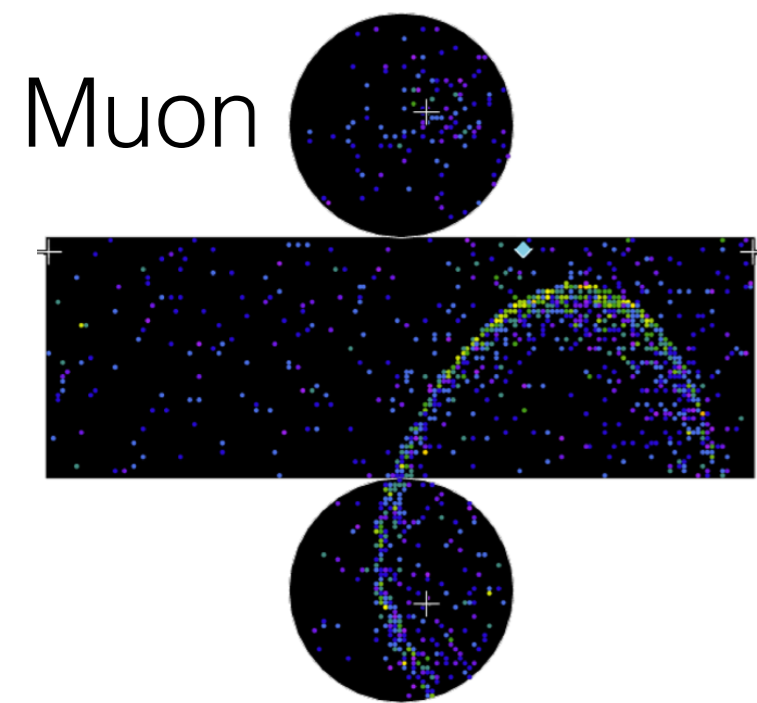
Muon

Water Cherenkov Technique

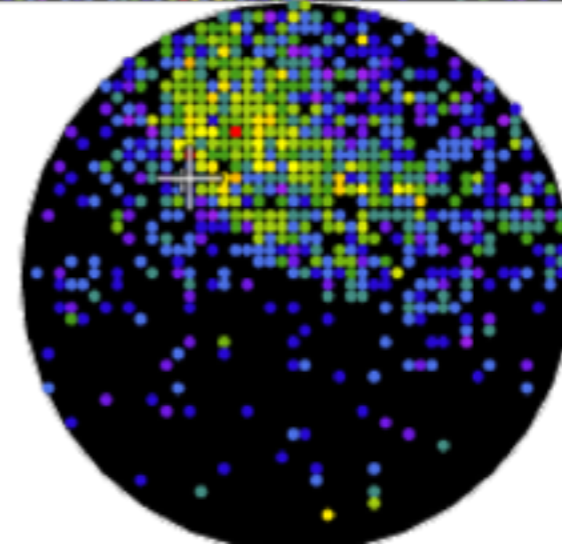


Electron

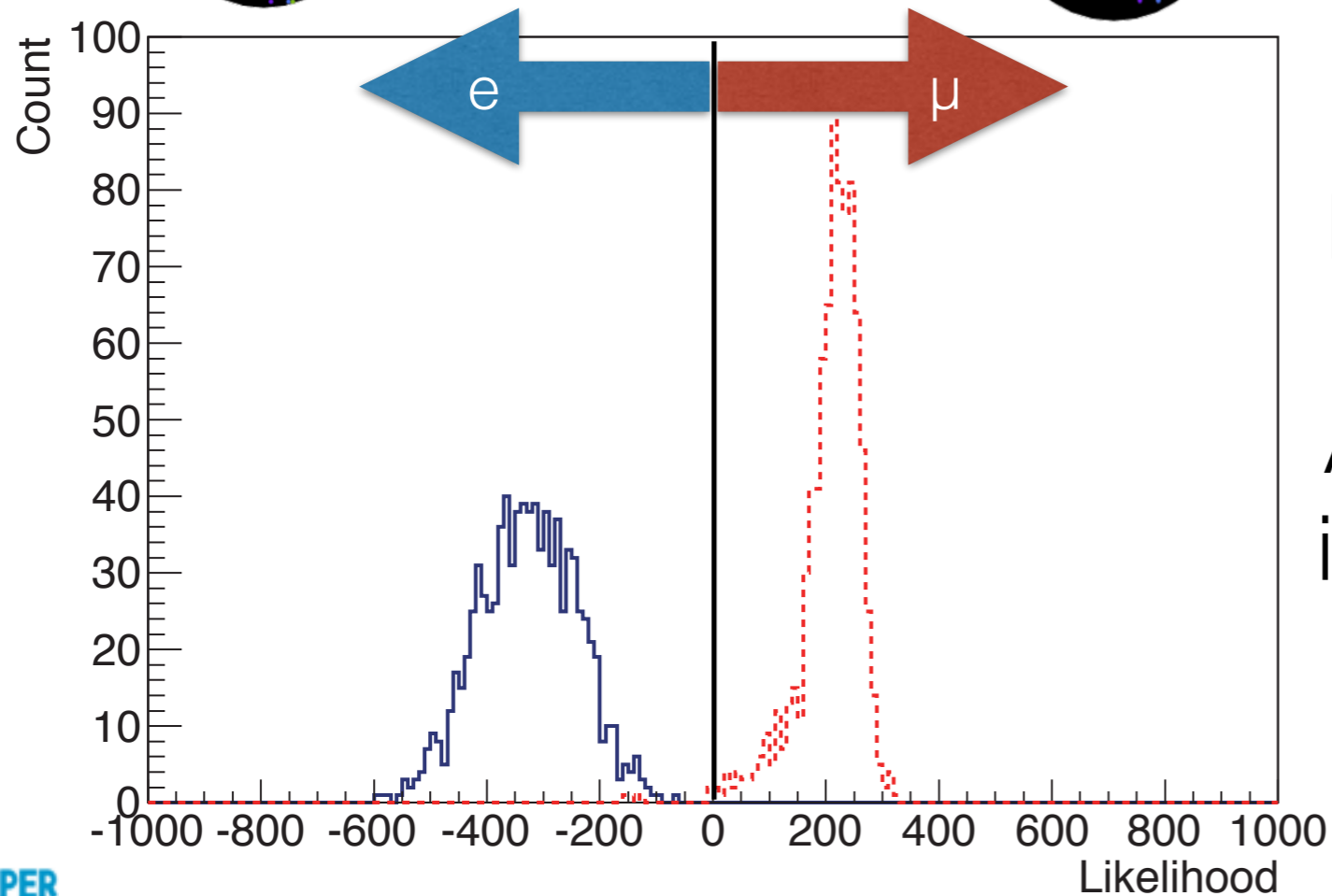
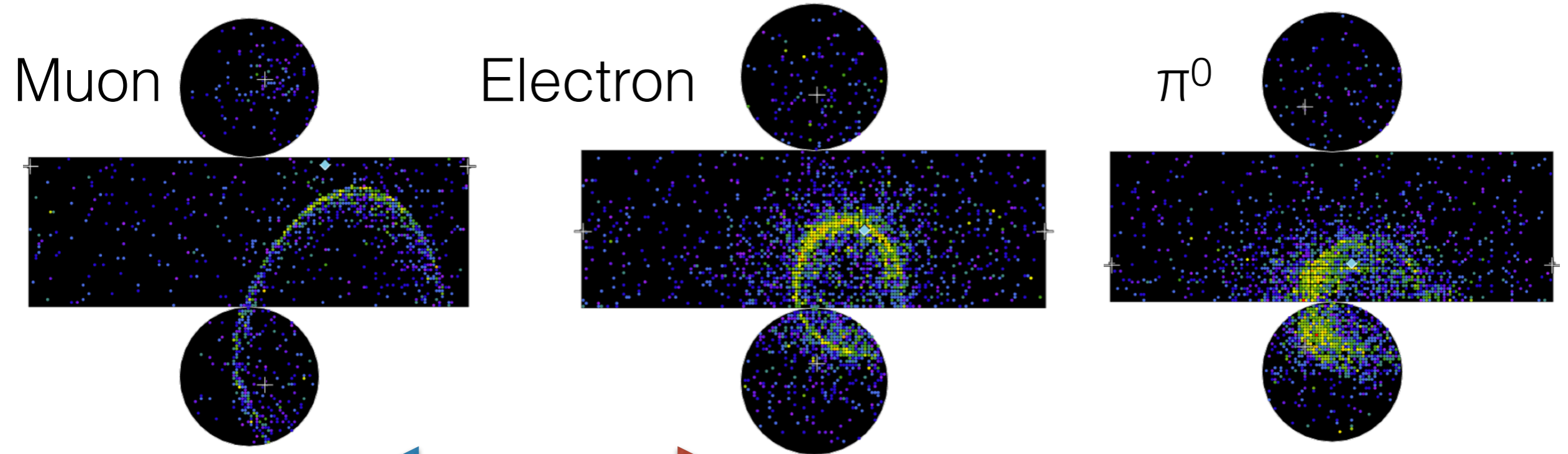
Water Cherenkov Technique



Neutral Pion



Water Cherenkov Technique



Excellent PID performance

Accelerator ν_e background is dominated by irreducible intrinsic ν_e .

Why Water Cherenkov?

Scalability

Water is cheap, non-toxic, liquid at room temperature
we already know how to build big water WC detectors

Proven technology

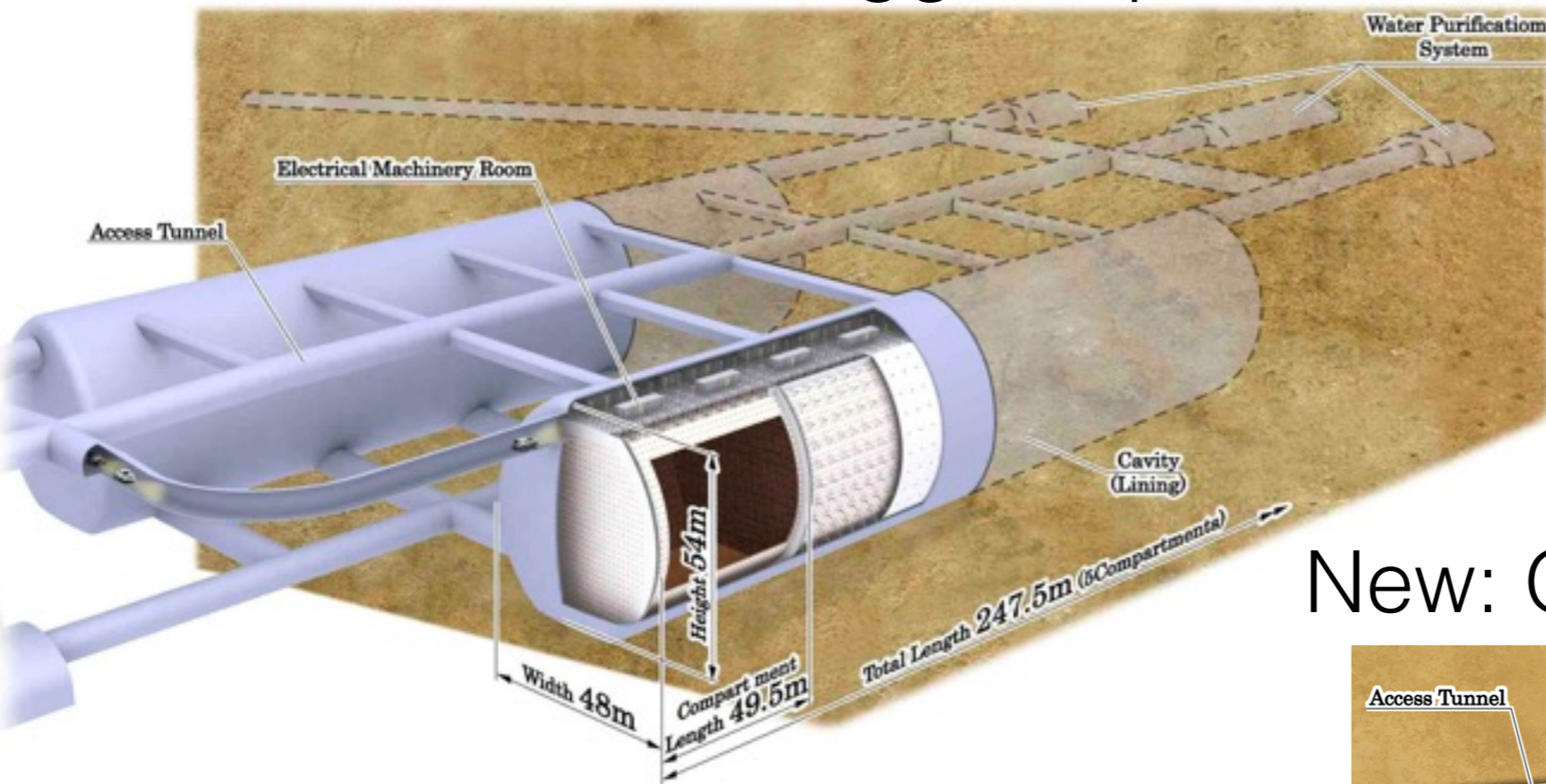
many years of experience from Super-K
low risk

Excellent performance

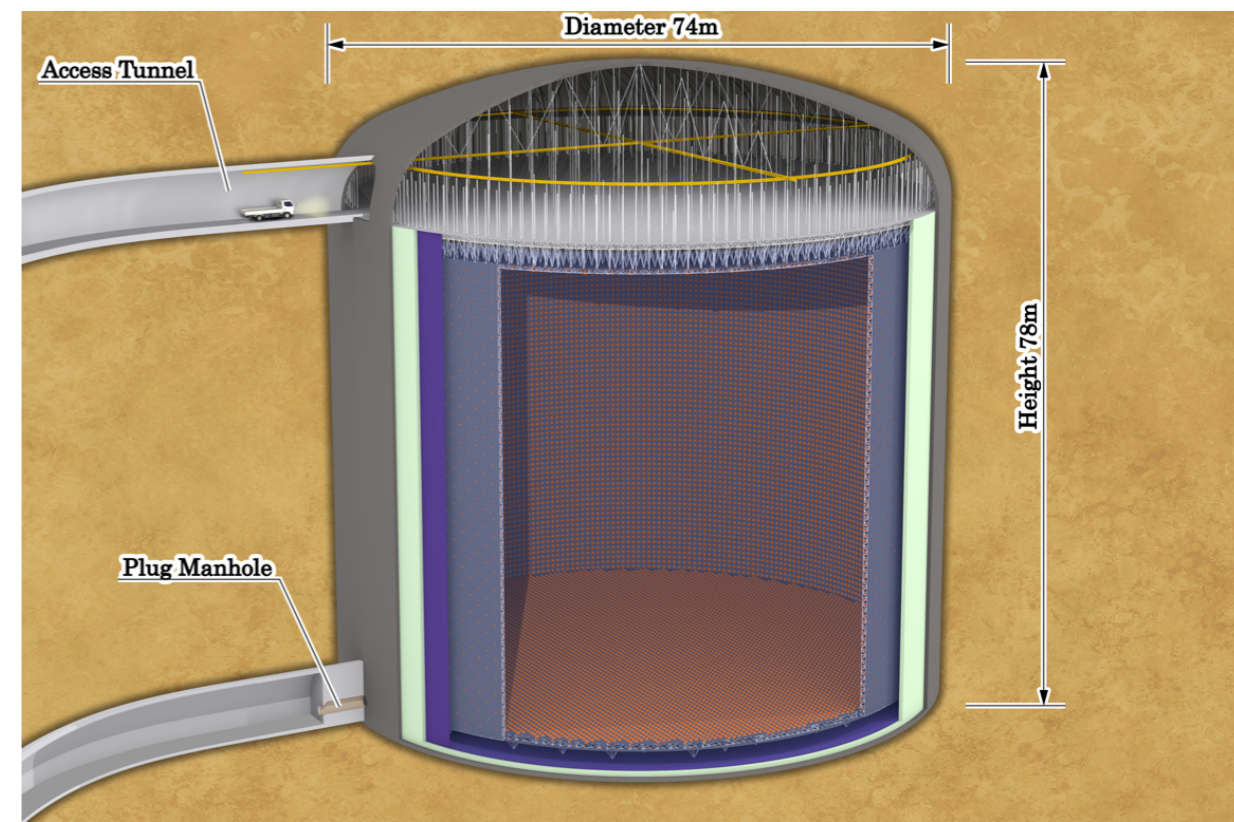
based on real Super-K and T2K performance

Tank Design

Old: Horizontal Egg-shaped Tank



New: Optimised Vertical Tank



Tank Design

ID: 40% photo-coverage
40,000 photo sensors per tank
OD:

Structure of bottom part

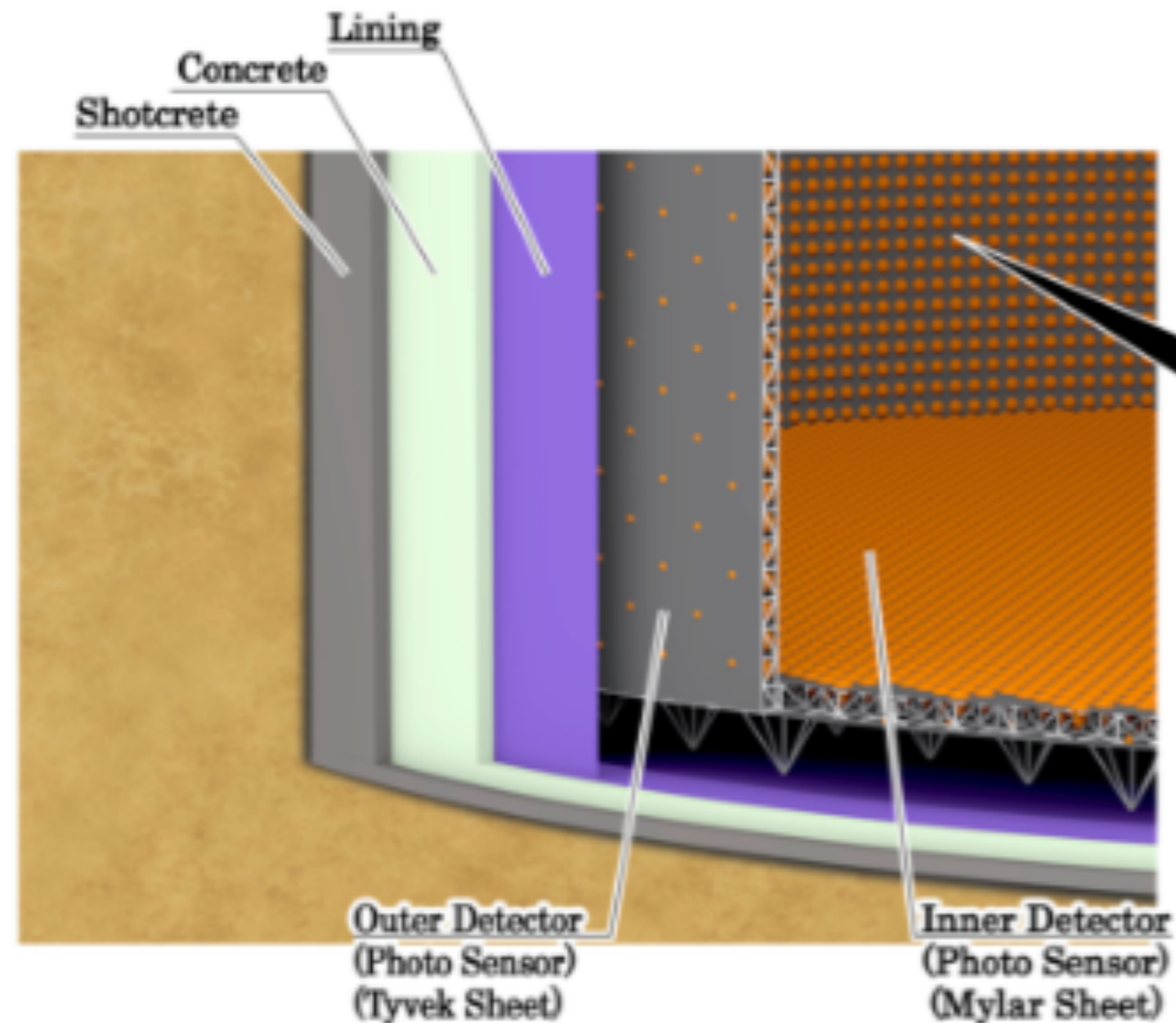
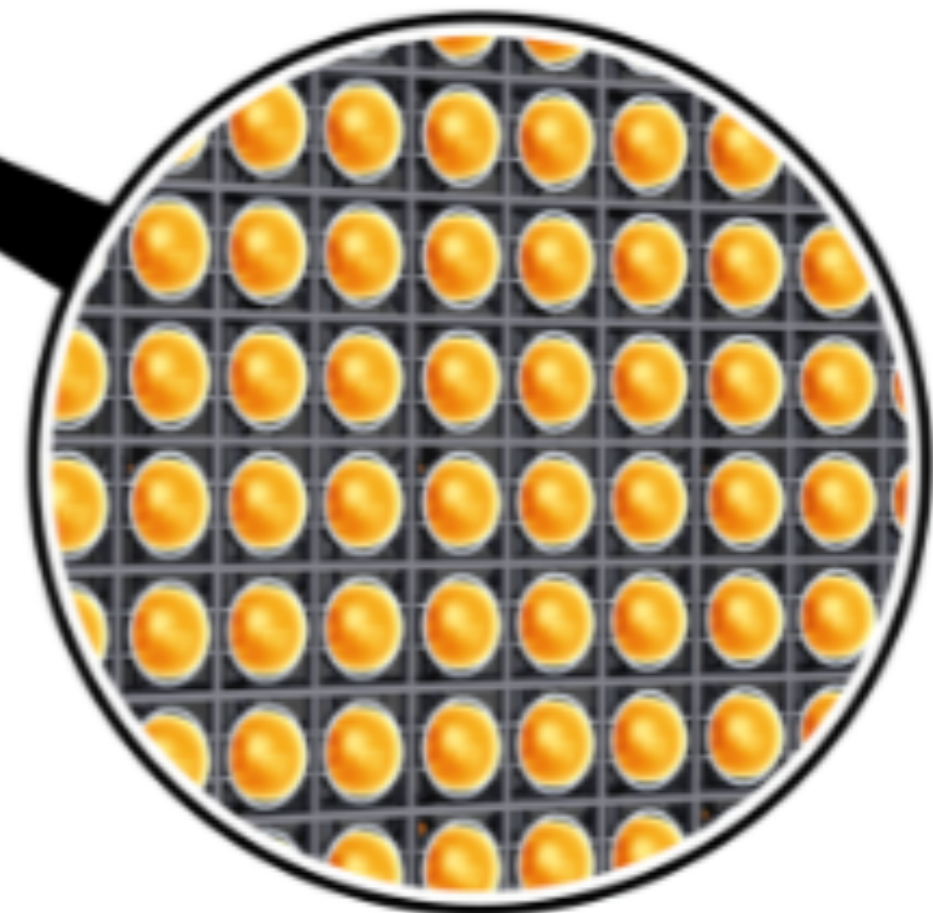


Photo-Sensors

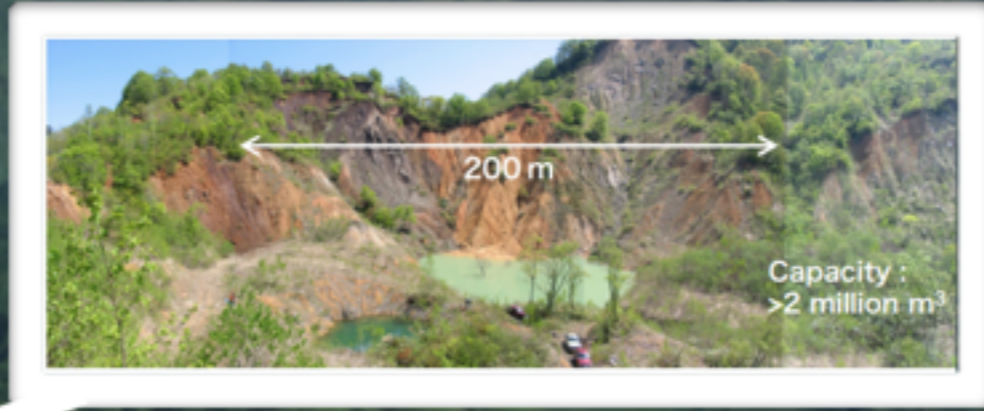


Mt. Ikeno-yama

SK

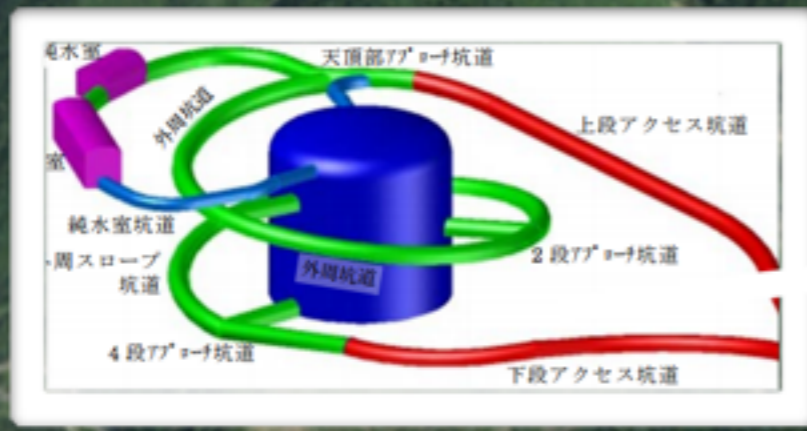
1000 m

Maruyama



Excavated rock disposal site

Mt. Nijyugo-yama



650 m

HK



Tunnel Entrance

Route 41

Kamioka Town

Wasabo

Funatsu Bridge

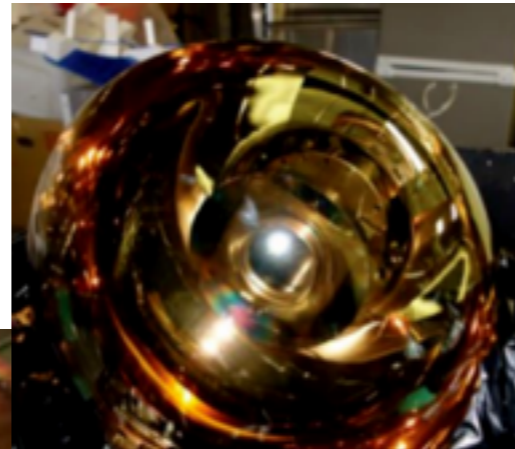
Google

Photo Sensors



Super-K PMT

QE 22%
CE 80%



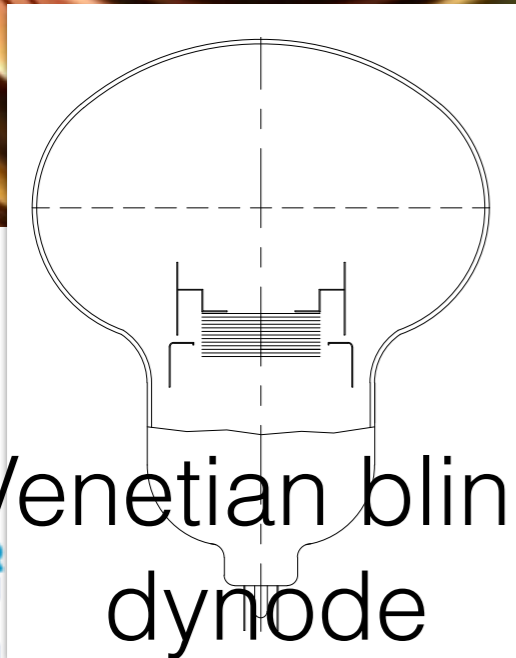
High QE/CE PMT

QE 30%
CE 93%

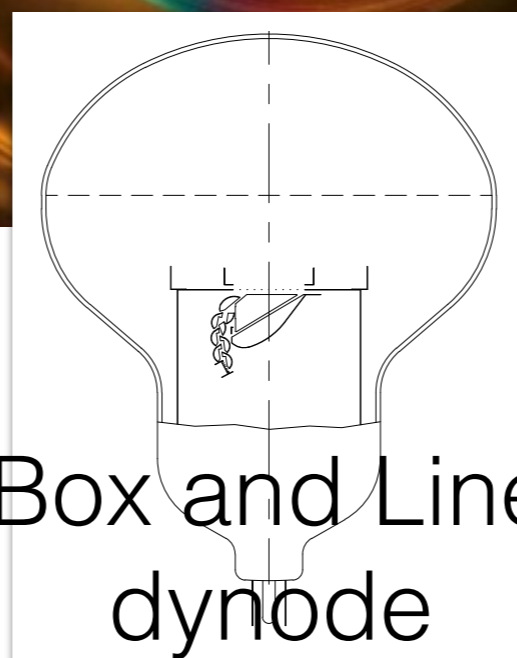


High QE/CE Hybrid PD

QE 30%
CE 95%



Venetian blind
dynode



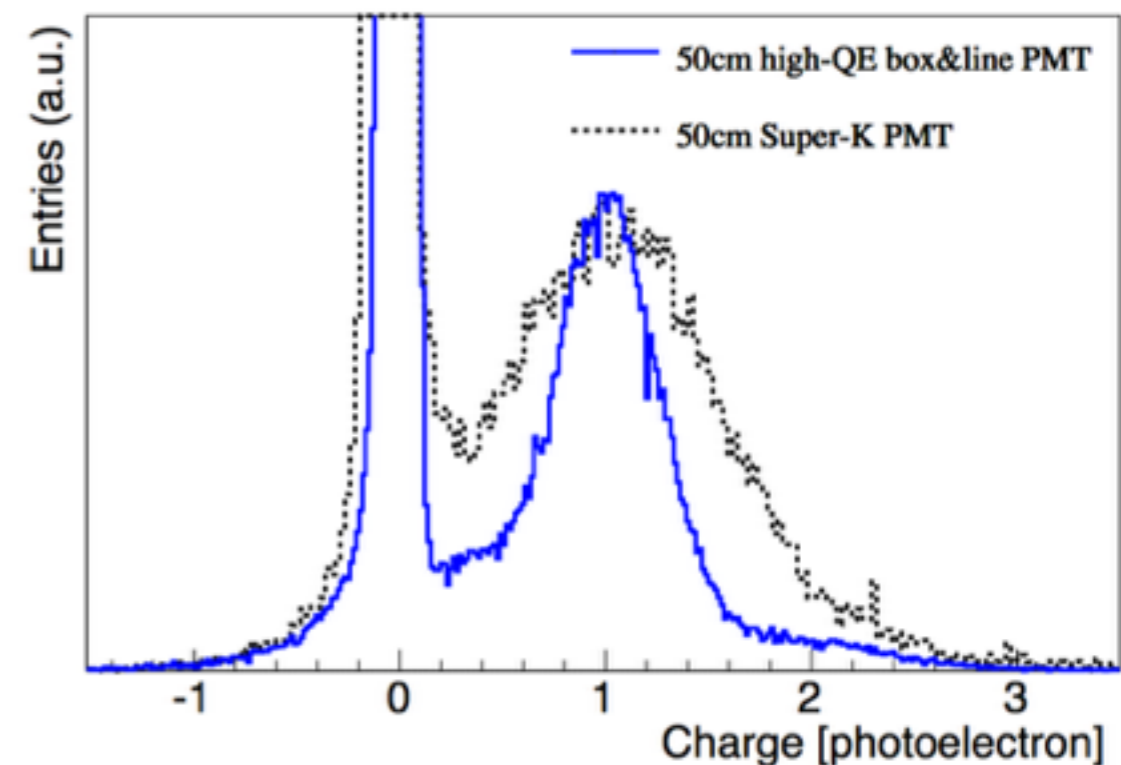
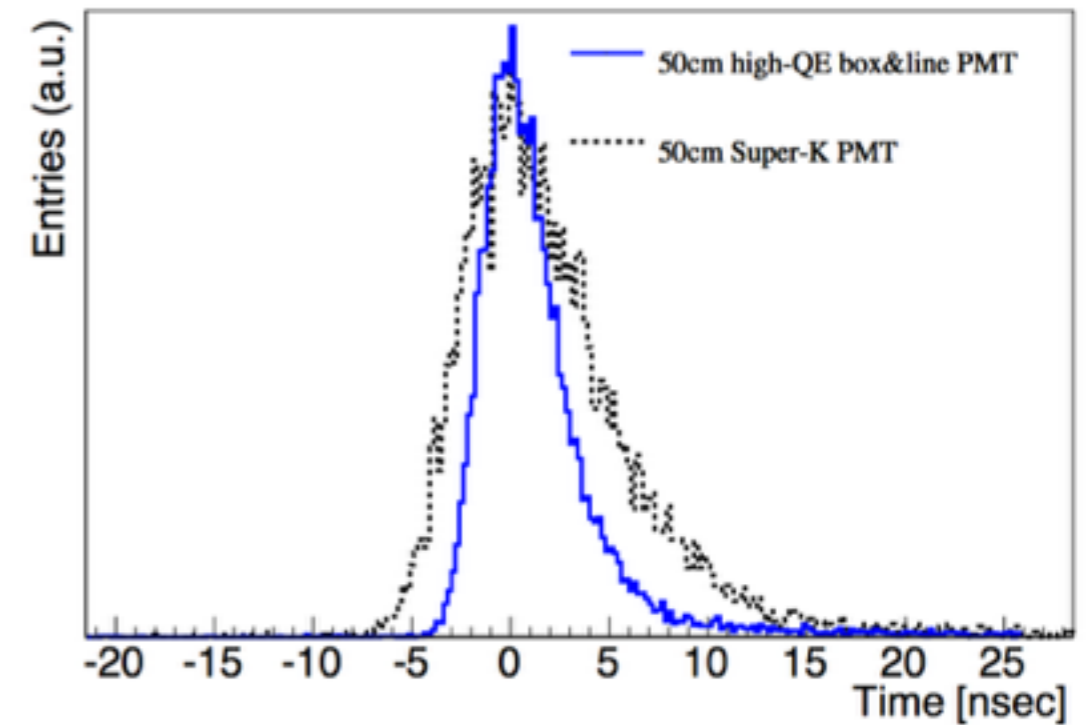
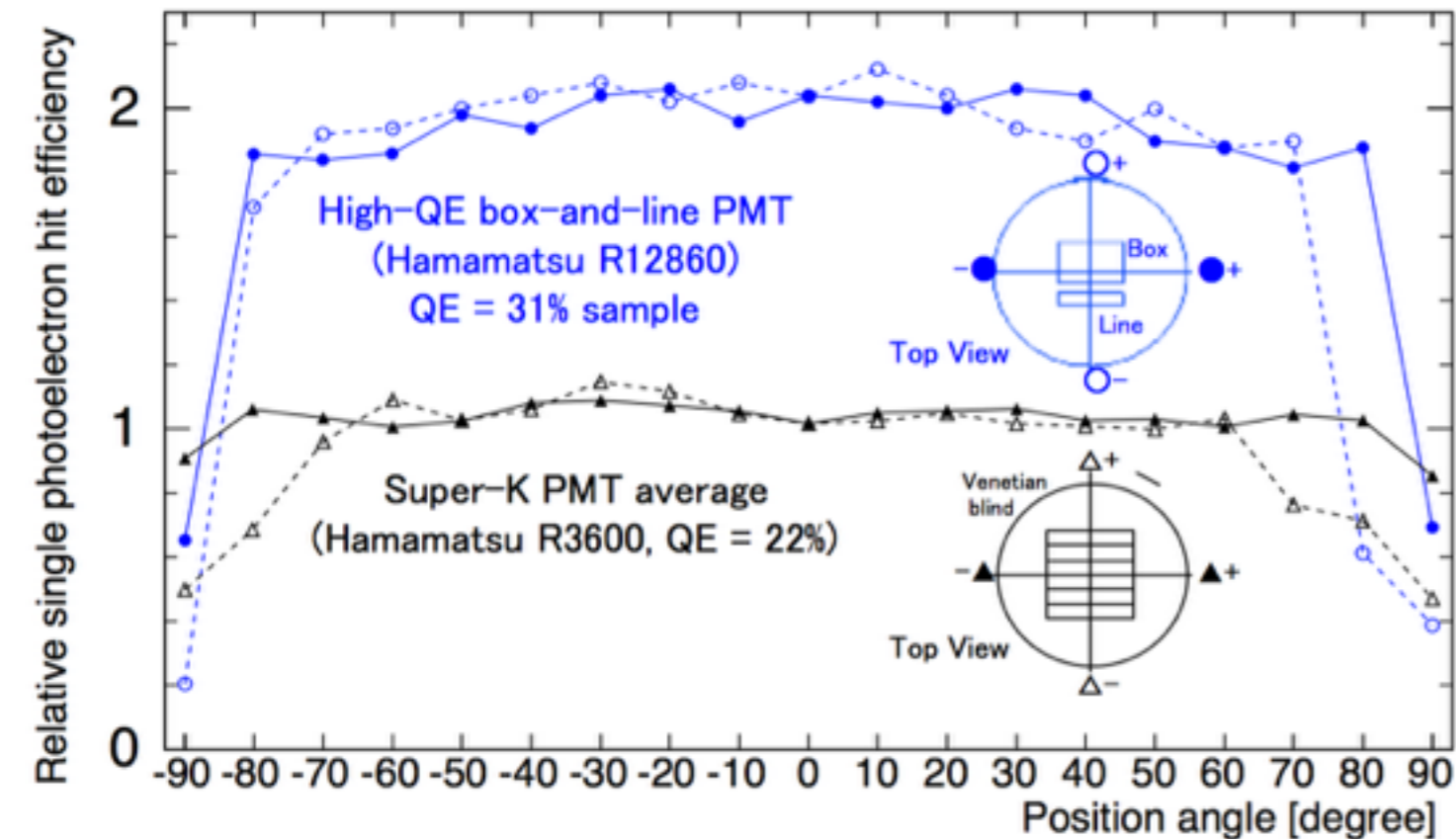
Box and Line
dynode



Avalanche diode

Photo Sensors

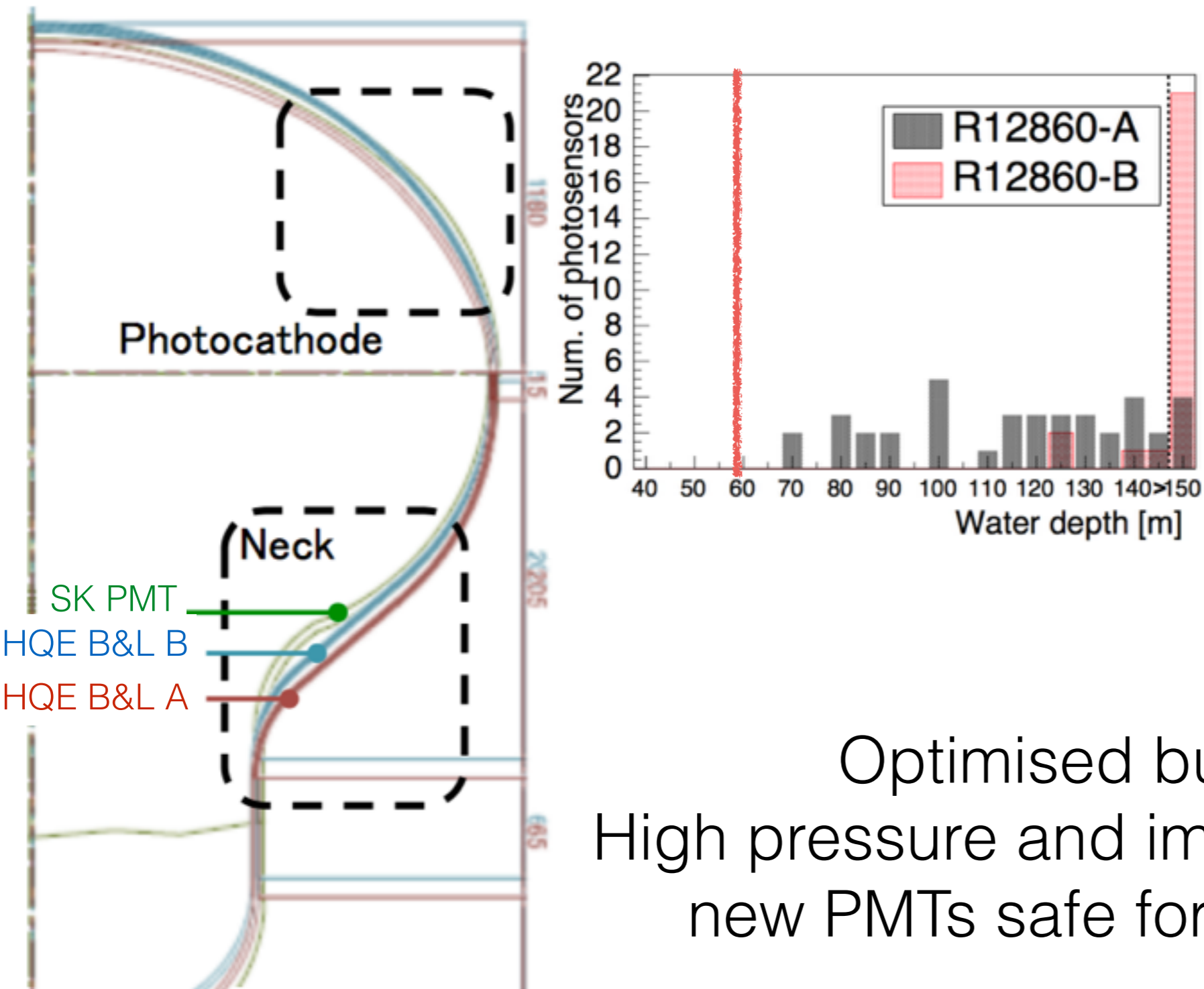
WARWICK



2x improvement in photon detection efficiency

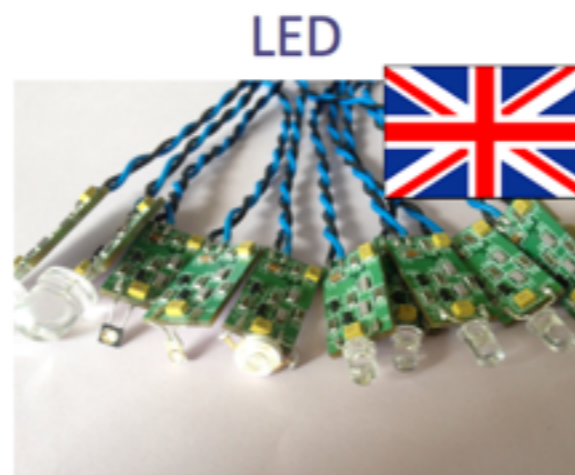
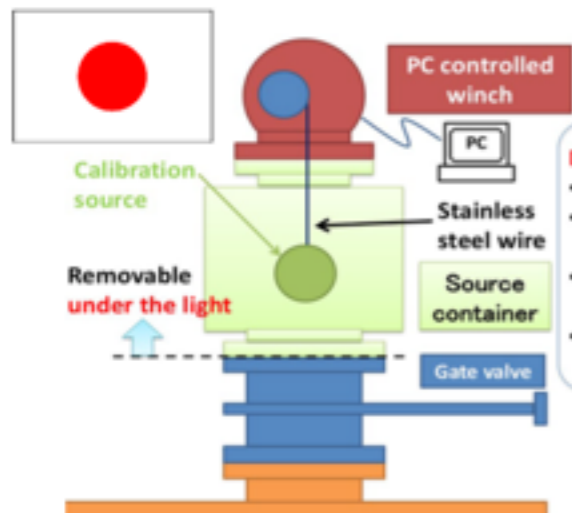
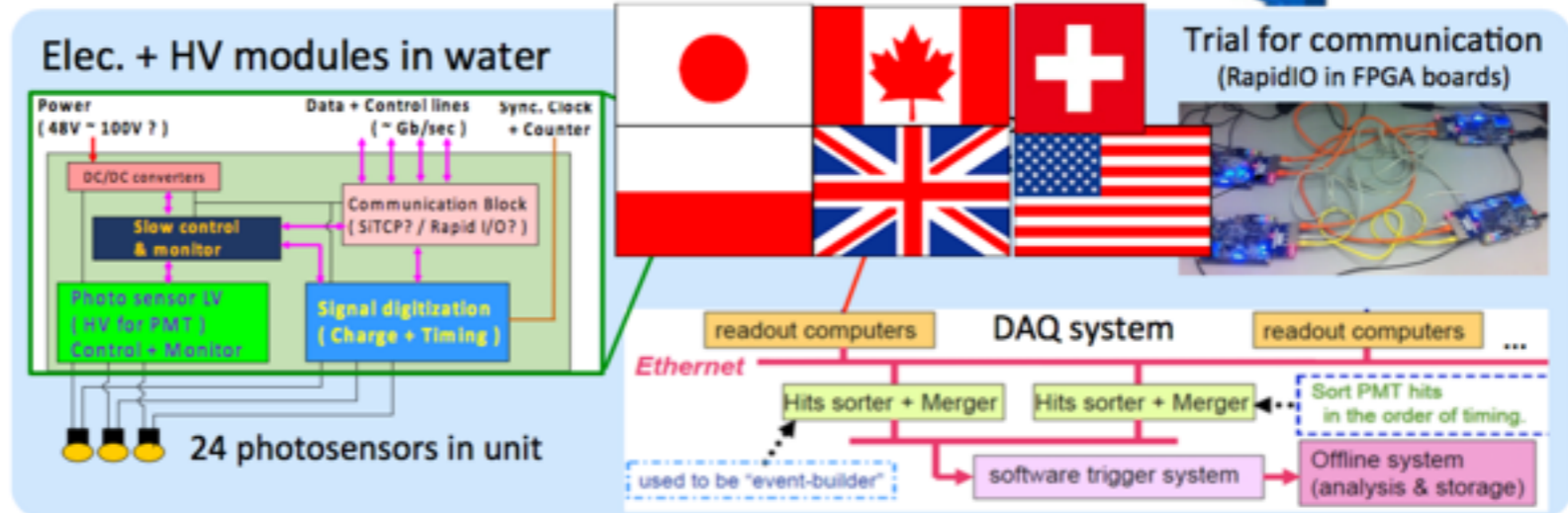
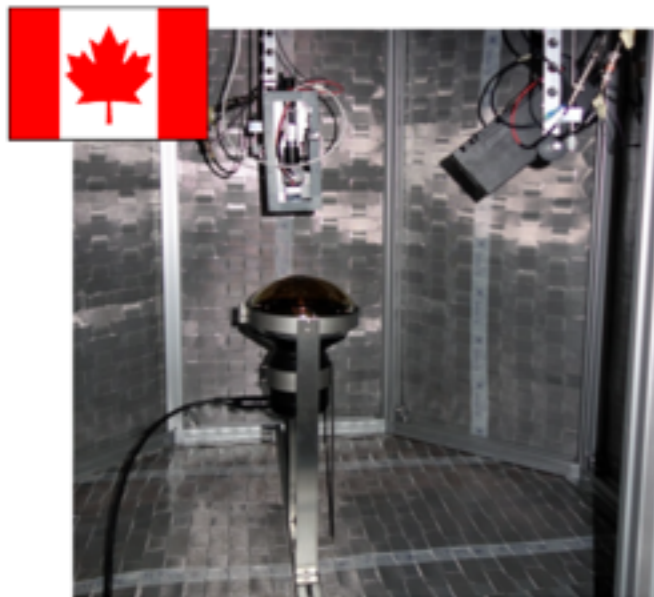
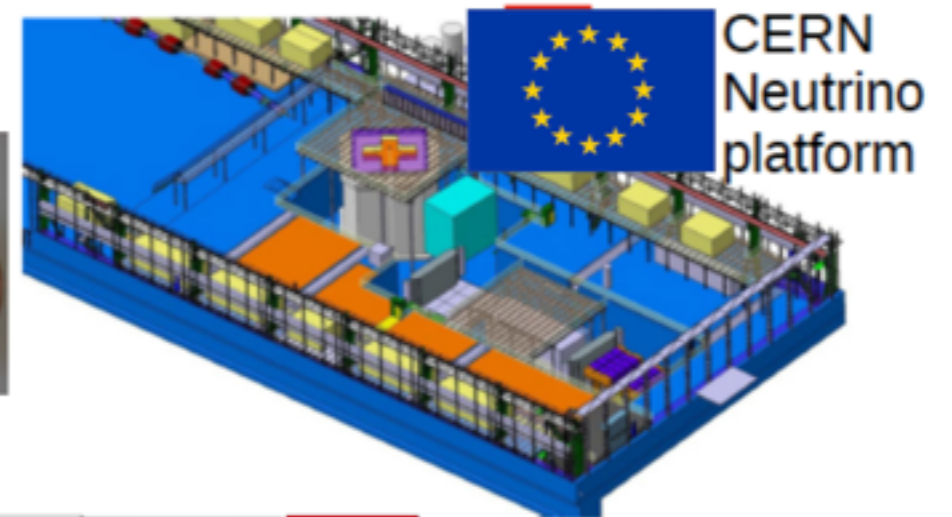
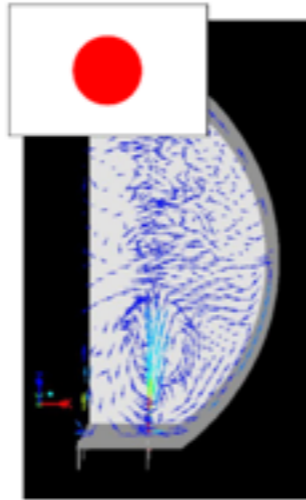
Better timing and charge resolution

Photo Sensors

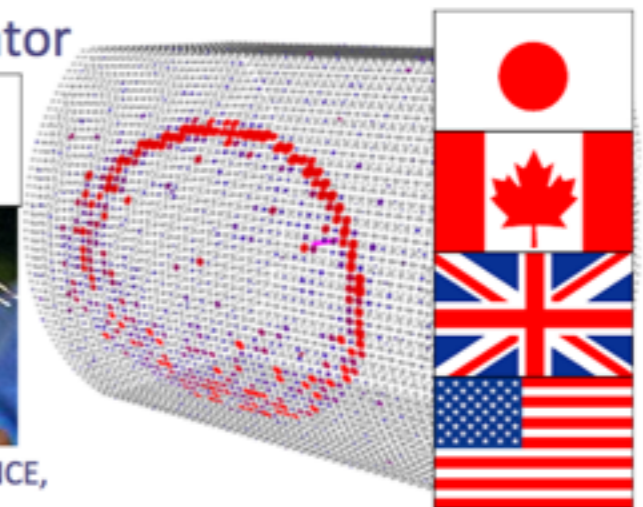
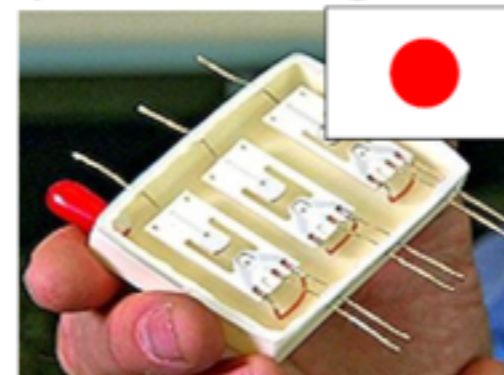


Optimised bulb design
High pressure and implosion tests show
new PMTs safe for use in HK tank

Worldwide R&D



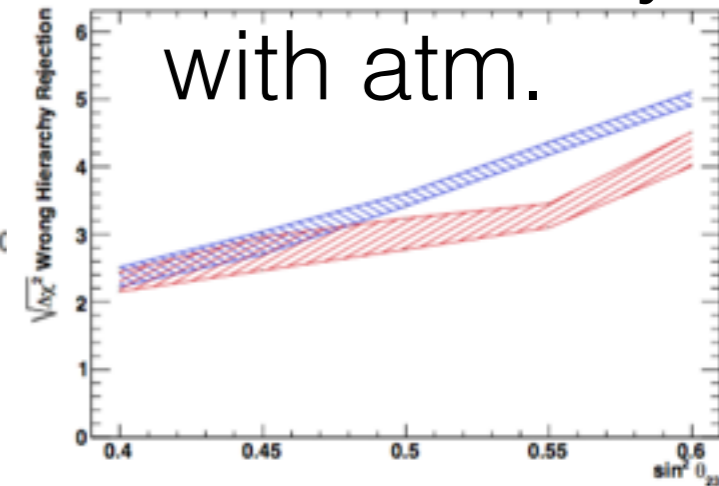
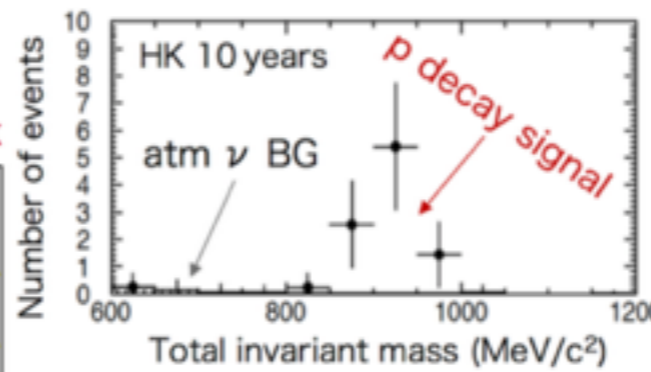
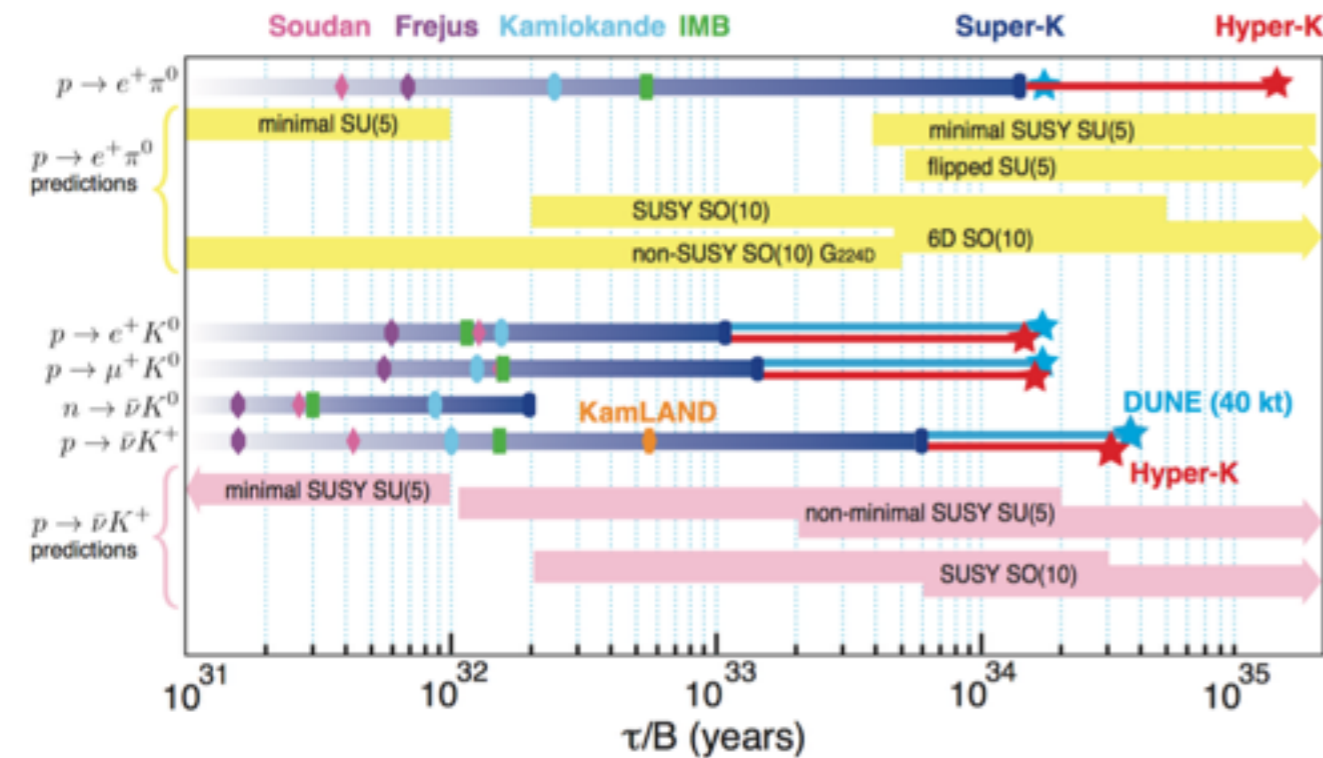
Compact neutron generator



Lots of Physics with Hyper-K

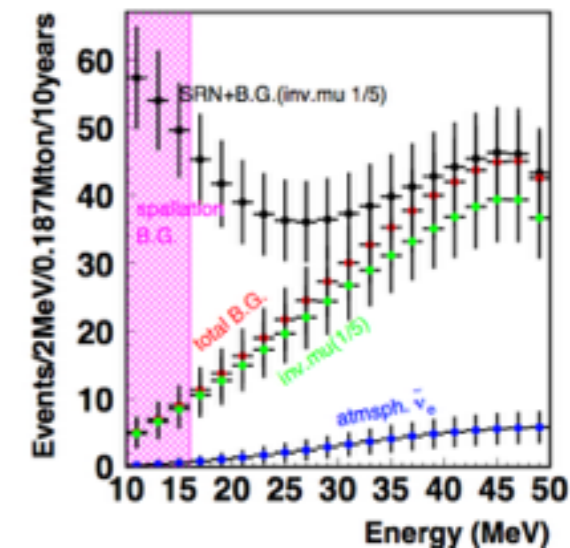
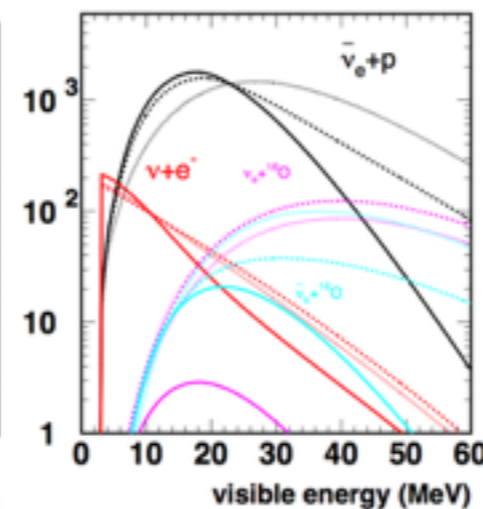
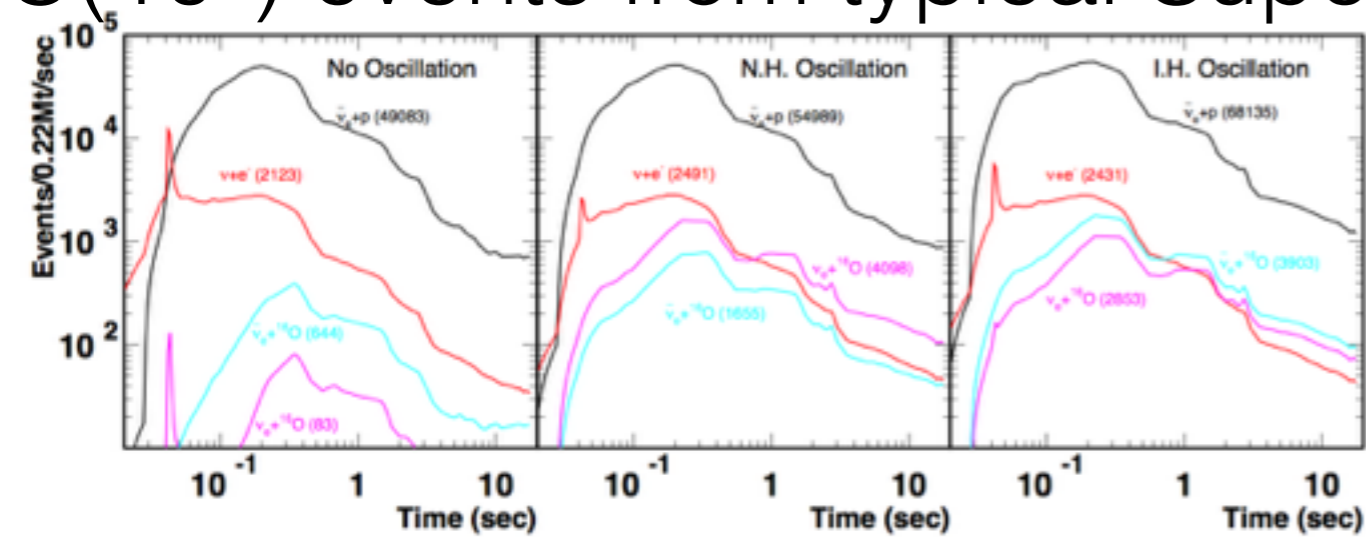
Proton Decay

Mass hierarchy with atm.



$O(10^5)$ events from typical Supernova @ 10 kpc

SRN



Neutrino Oscillations

Weak flavour eigenstates \neq Mass eigenstates

Neutrinos produced and detected in their weak flavour states

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = U_{\text{MNS}} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

Unitary PMNS mixing matrix
parameterised with 3 angles
and **CP violating phase**

$$\theta_{ij}, \delta_{\text{CP}}$$

Relative phase difference between due to mass difference, Δm^2

Appearance probability:

$$P_{\mu \rightarrow e} \approx \sin^2 \theta_{23} \cdot \sin^2 2\theta_{13} \cdot \sin^2 \left(\frac{\Delta m^2 L}{4E_\nu} \right)$$

+ higher order terms involving δ_{CP}

Neutrino Oscillations

T2K



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J-PARC-chan
lives in Tokai-mura, Naka-gun, Ibaraki, Japan.

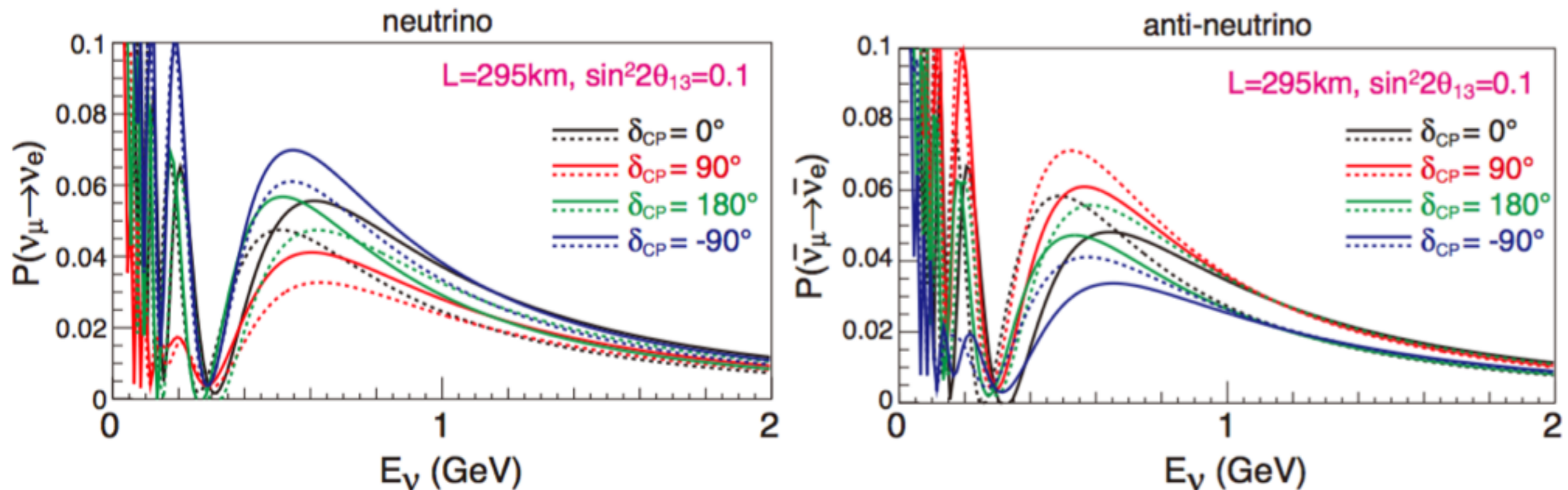


Super-Kamiokande-chan
lives in Kamioka-cho, Hida-city, Gifu, Japan.



Neutrino Oscillations

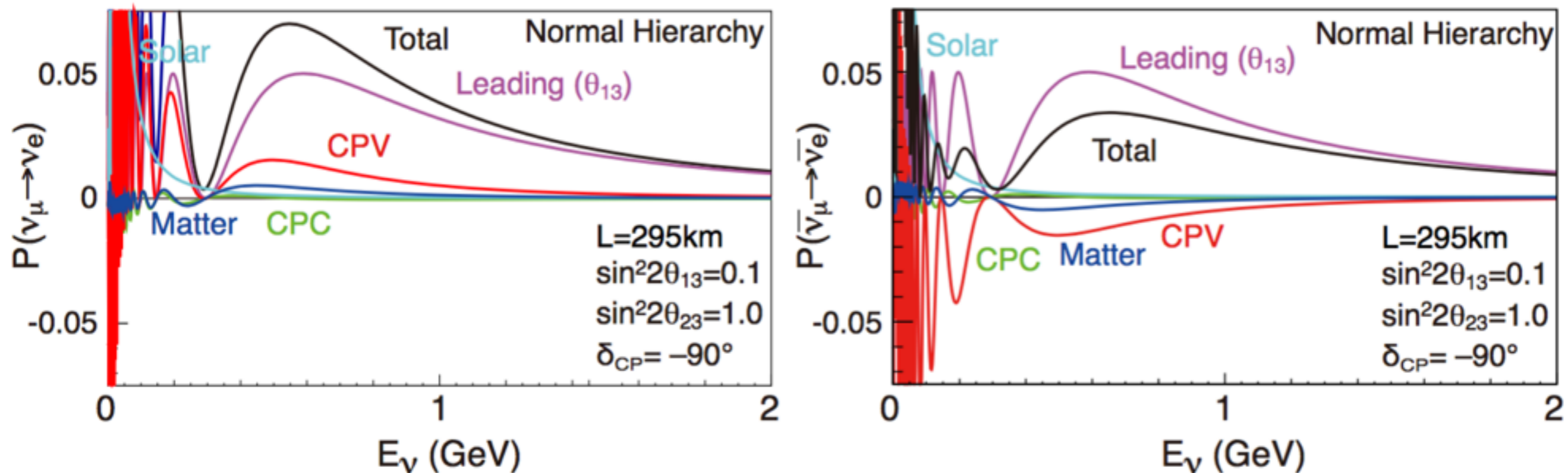
Typically perform experiment at fixed L with wide range of E



CP violation $\sim 20\%$ effect at 1st oscillation maximum
Much larger effect at 2nd oscillation maximum

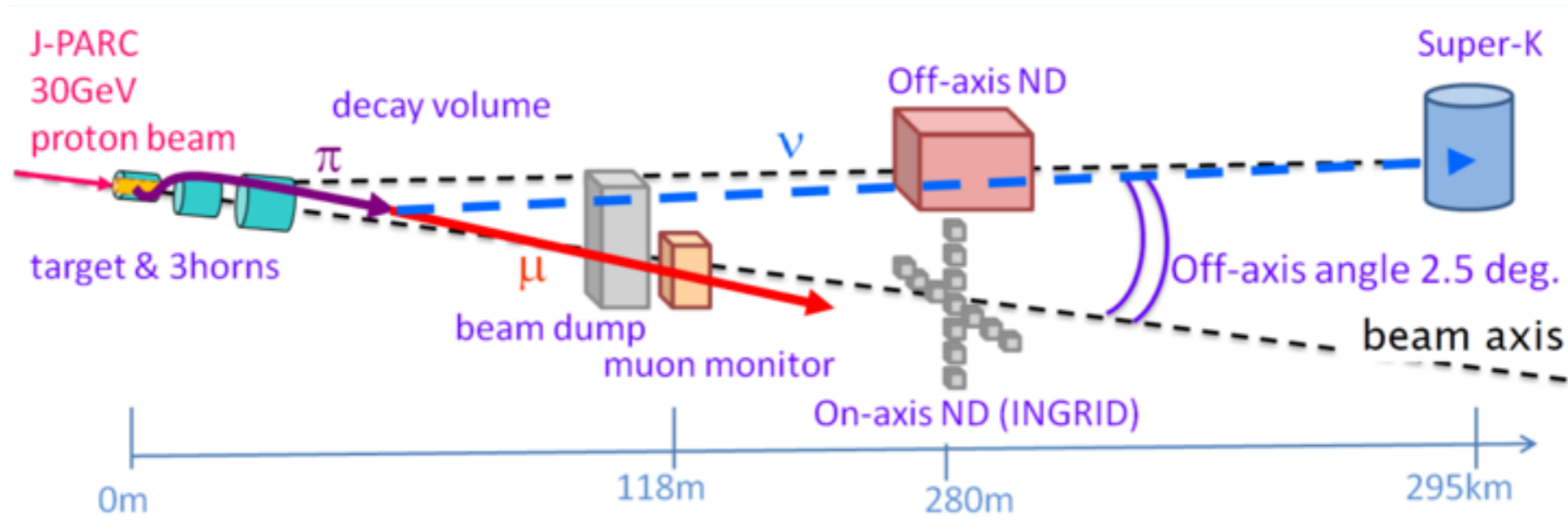
Neutrino Oscillations

Typically perform experiment at fixed L with wide range of E



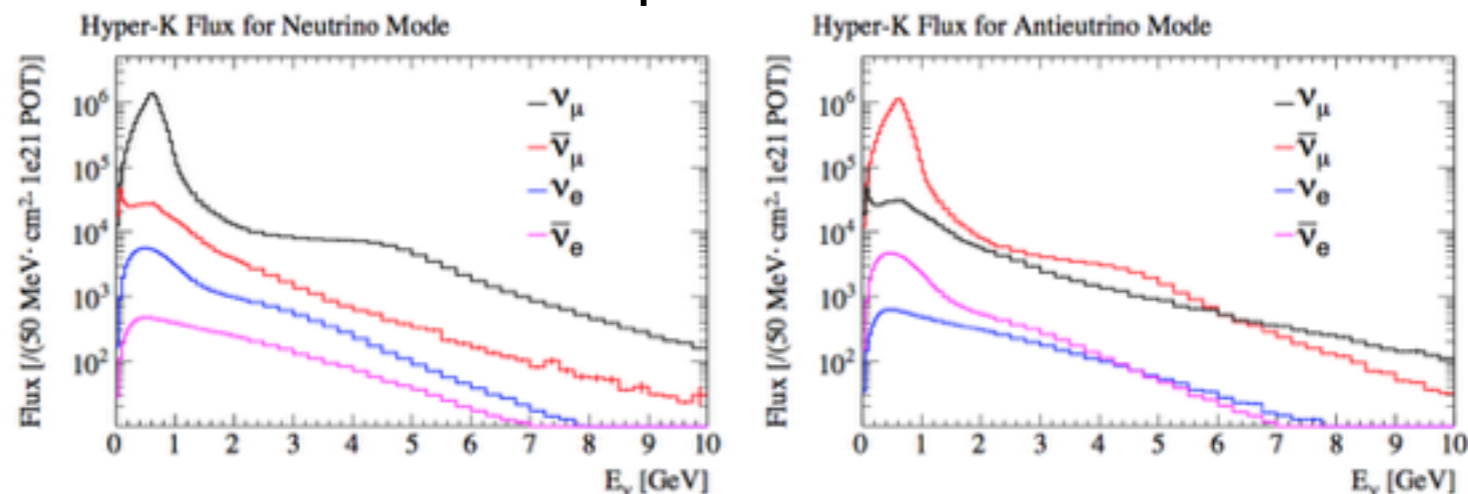
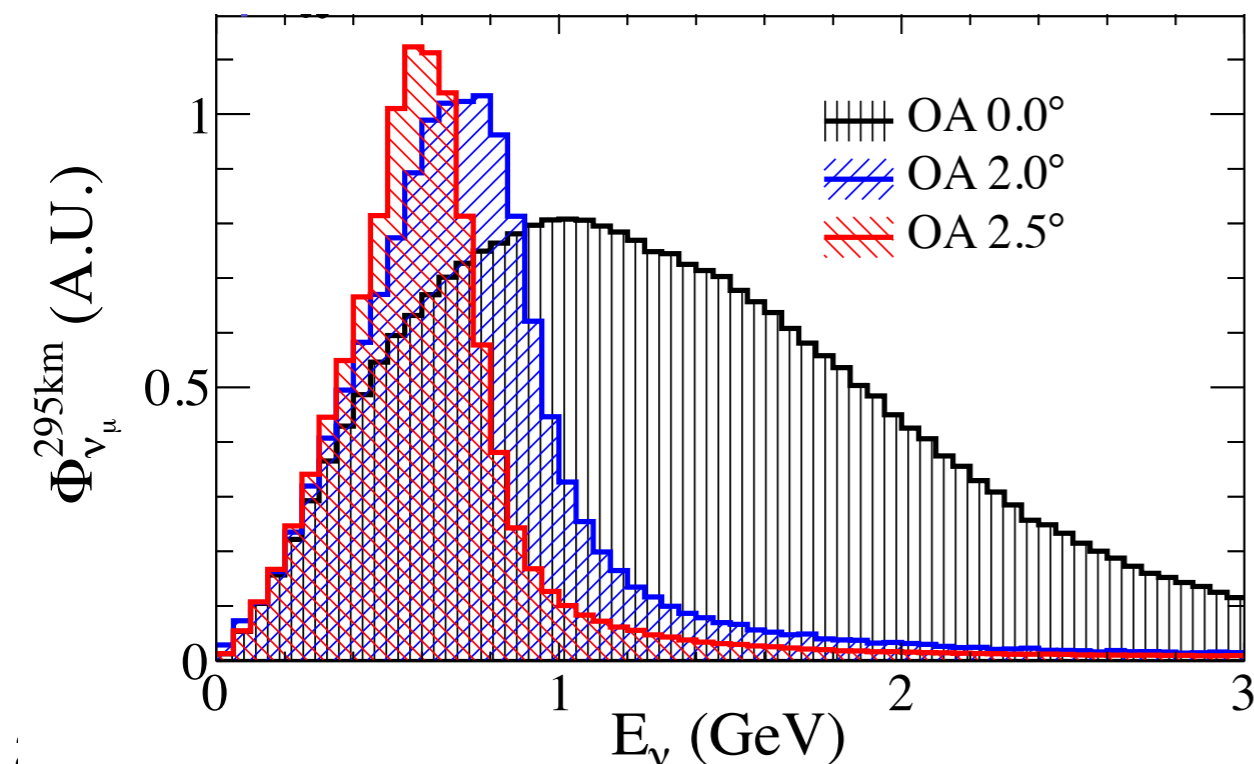
CP violation $\sim 20\%$ effect at 1st oscillation maximum
Much larger effect at 2nd oscillation maximum

T2K / Hyper-K Flux



Narrow band beam off-axis

Flavour composition



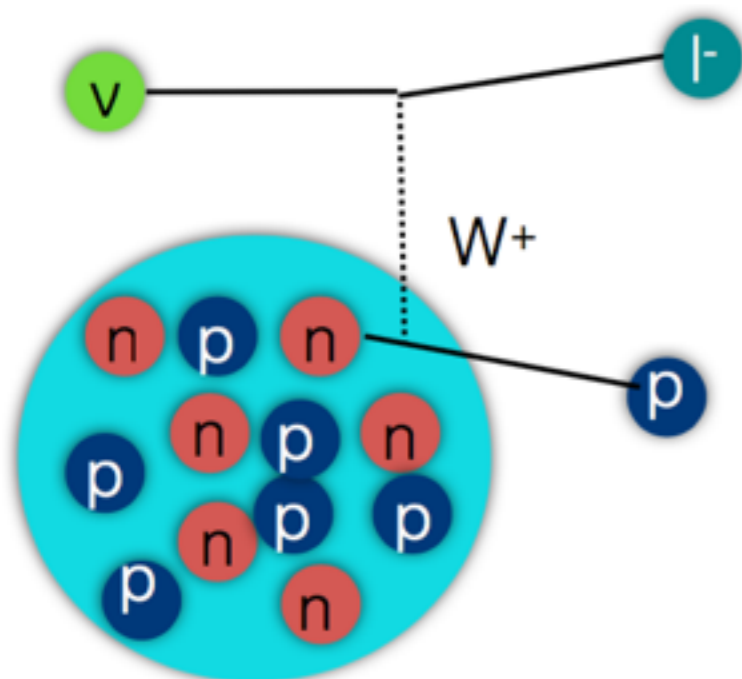
nu-mode: $\sim 94\% \nu_\mu$
 anti-nu mode: $\sim 92\% \bar{\nu}_\mu$
 (for $E < 1.25$ GeV)

Neutrino Energy Measurement

Protons usually below Cherenkov threshold
Neutrons can be counted but no energy measurement

For quasi-elastic interactions neutrino energy can be reconstructed from lepton kinematics

$$E_{\nu}^{\text{rec}} = \frac{m_p^2 - (m_n - E_b)^2 - m_e^2 + 2(m_n - E_b)E_e}{2(m_n - E_b - E_e + p_e \cos \theta_e)}$$



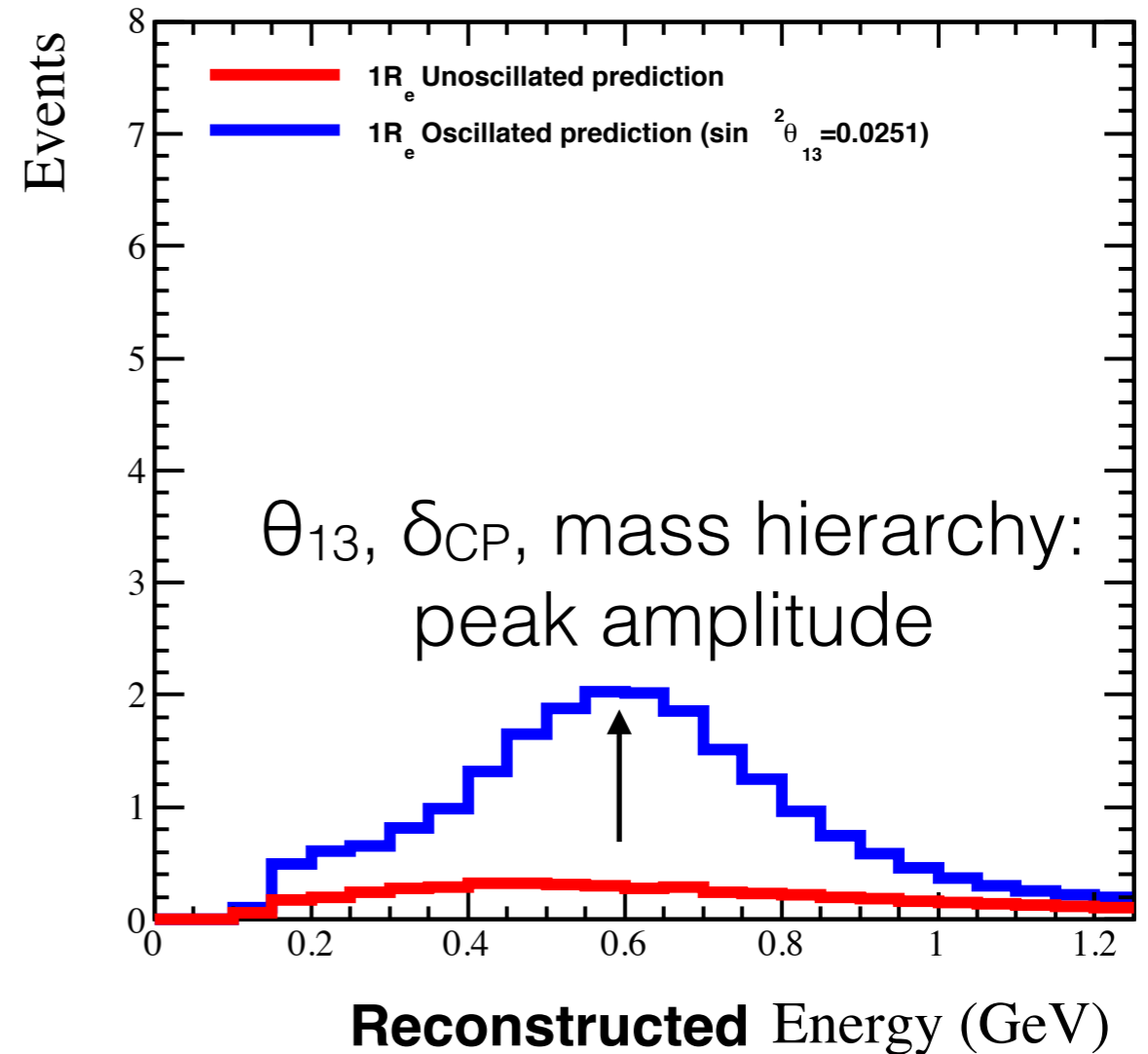
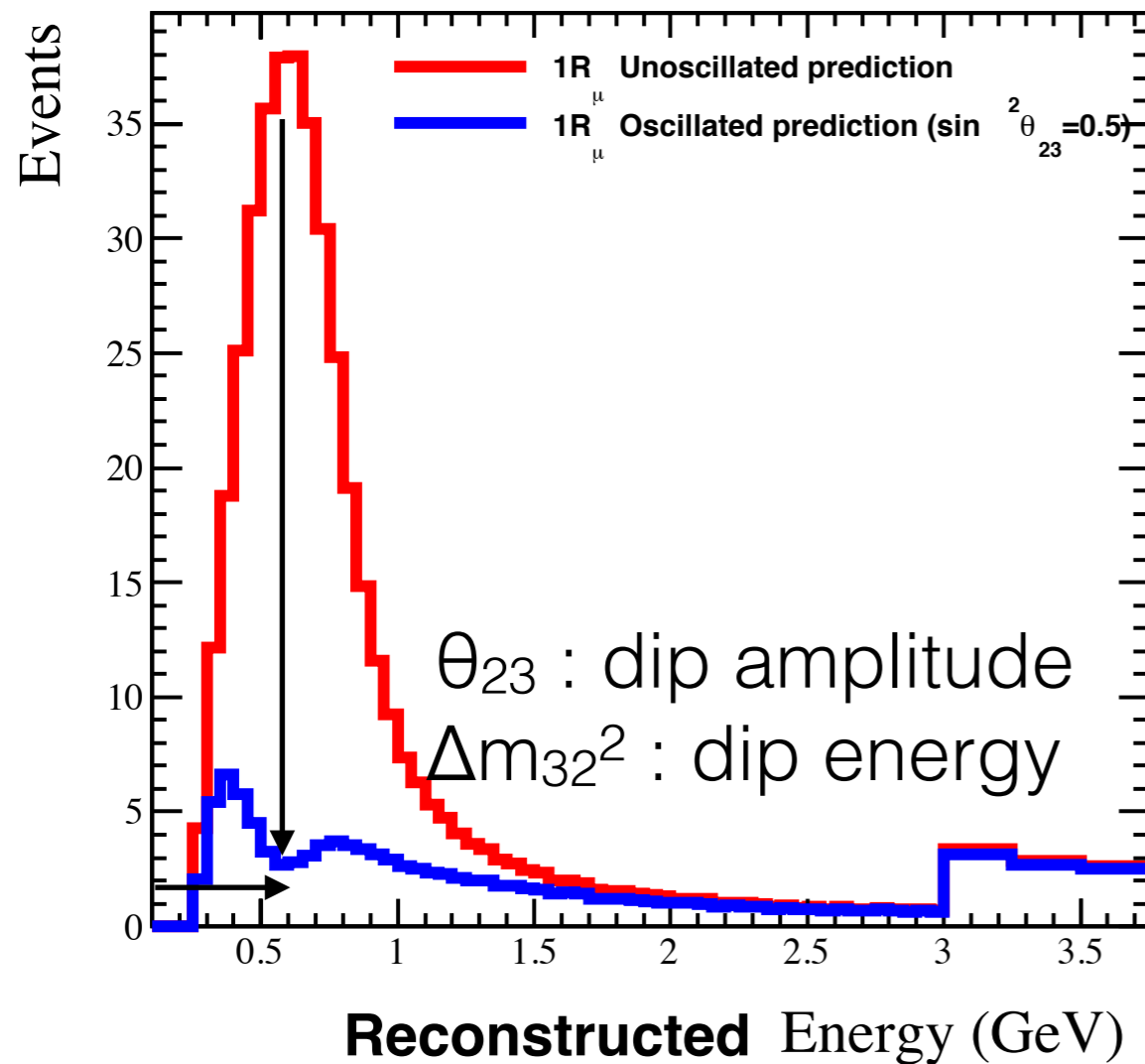
Background from inelastic scattering
where energy is mis-measured

Interaction is on bound state
Nuclear effects are important

What we actually measure:

ν_μ disappearance

ν_e appearance



Measurement precision limited by:

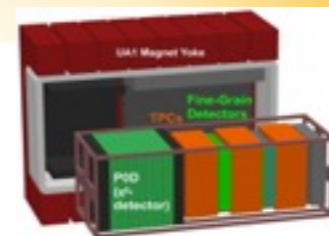
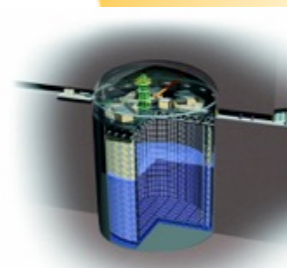
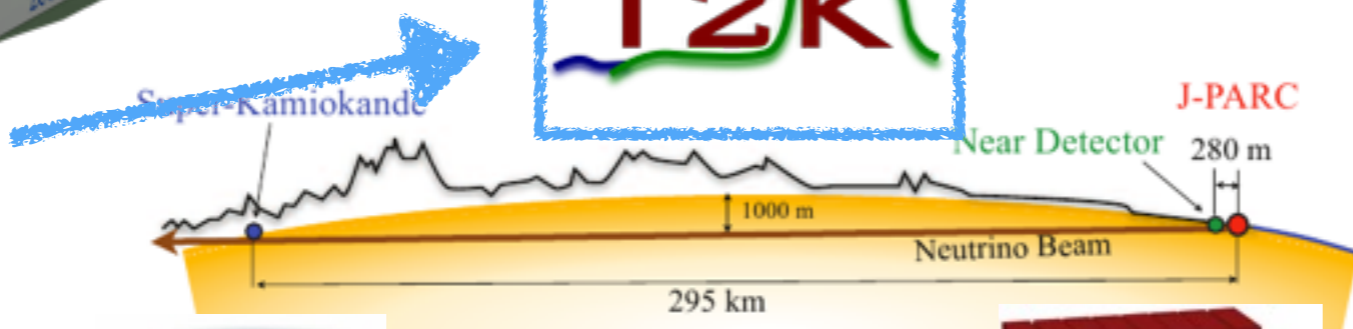
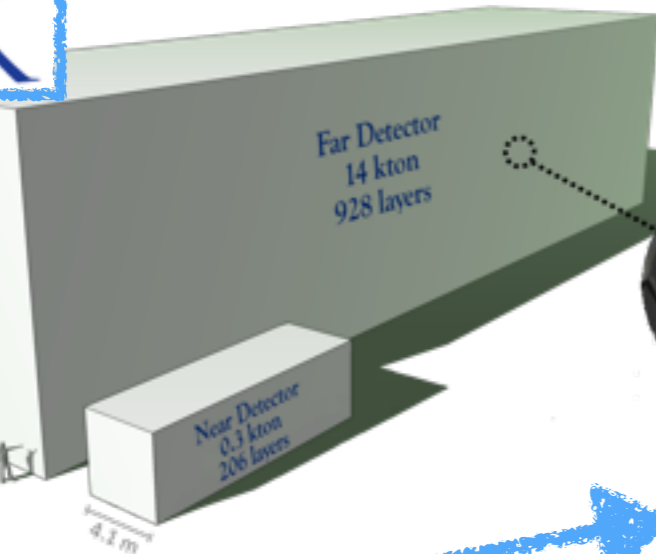
- Statistics
- Neutrino energy reconstruction
- Knowledge of unoscillated spectrum and background contamination

Accelerator based Neutrino Oscillation Experiments

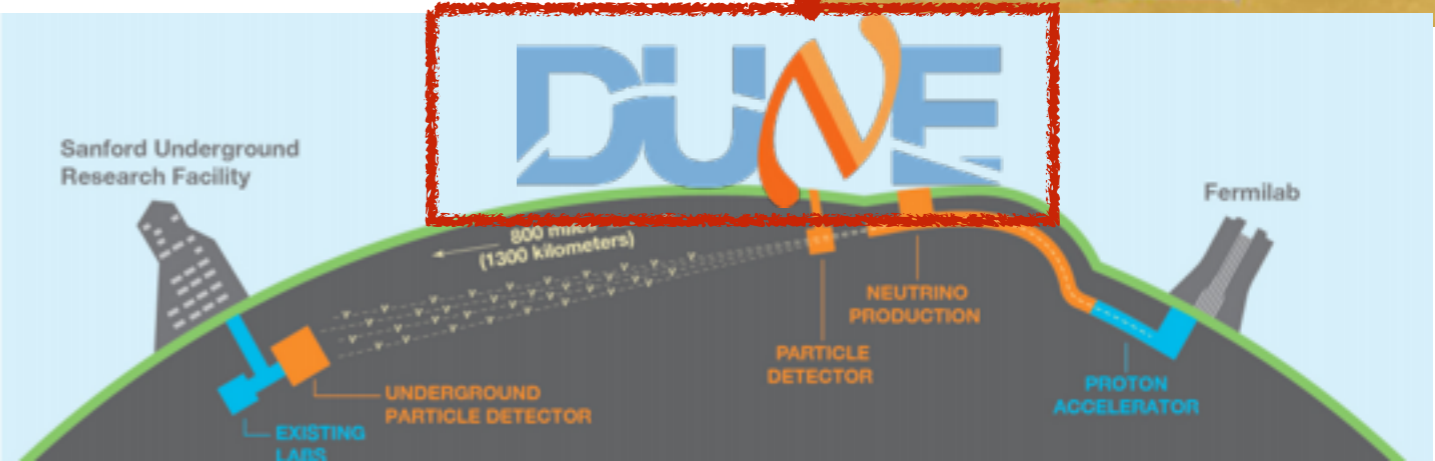
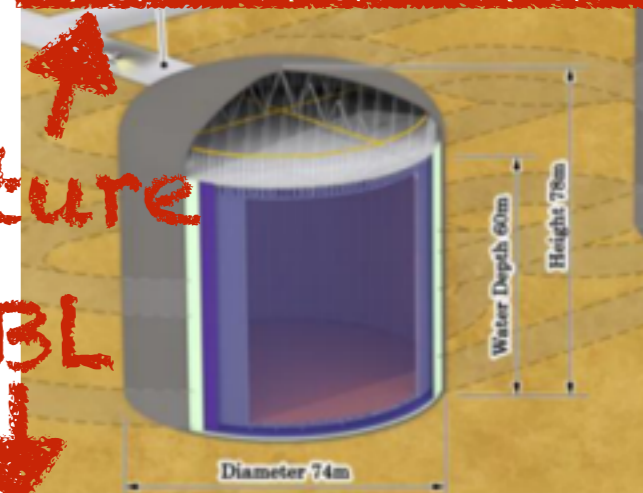
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THE UNIVERSITY OF WARWICK



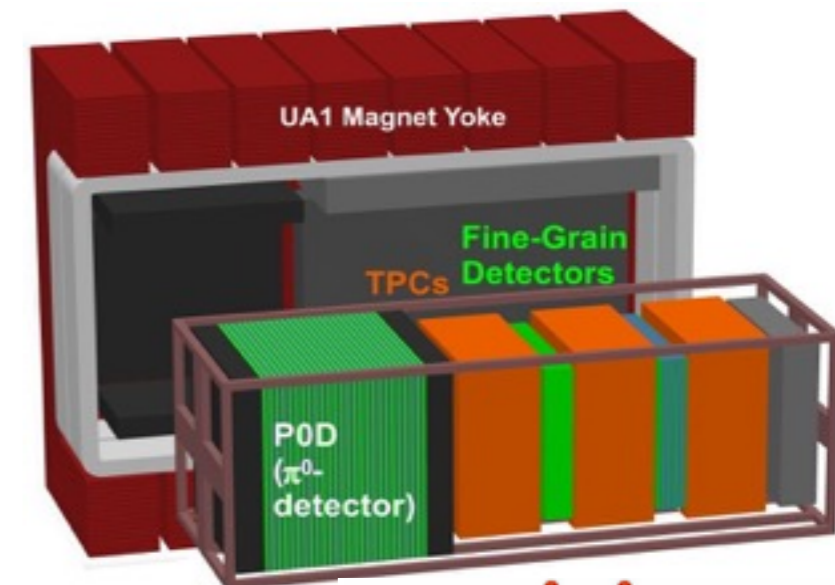
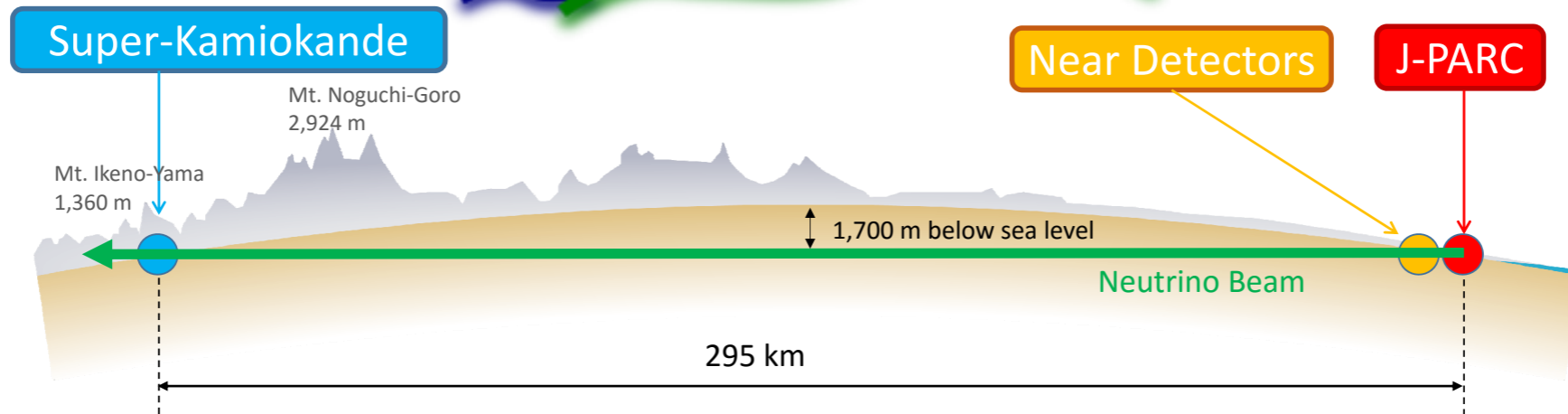
Current
LBL



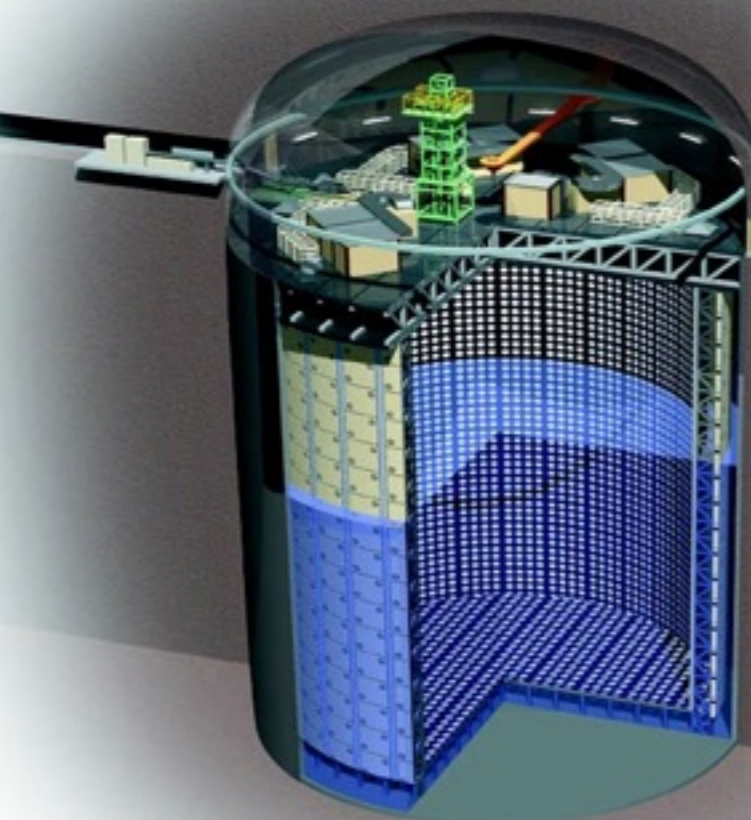
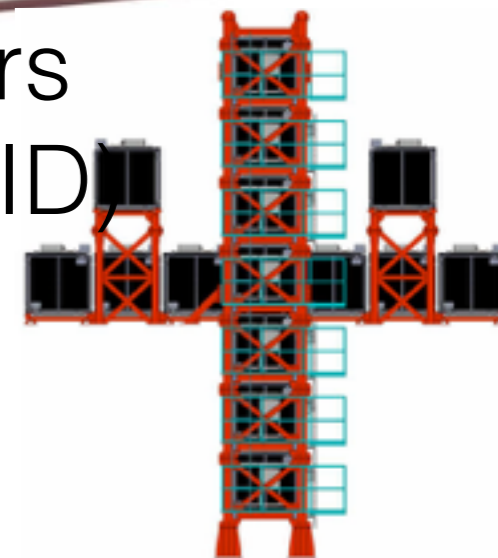
Future
LBL



T2K



Near Detectors
(ND280+INGRID)

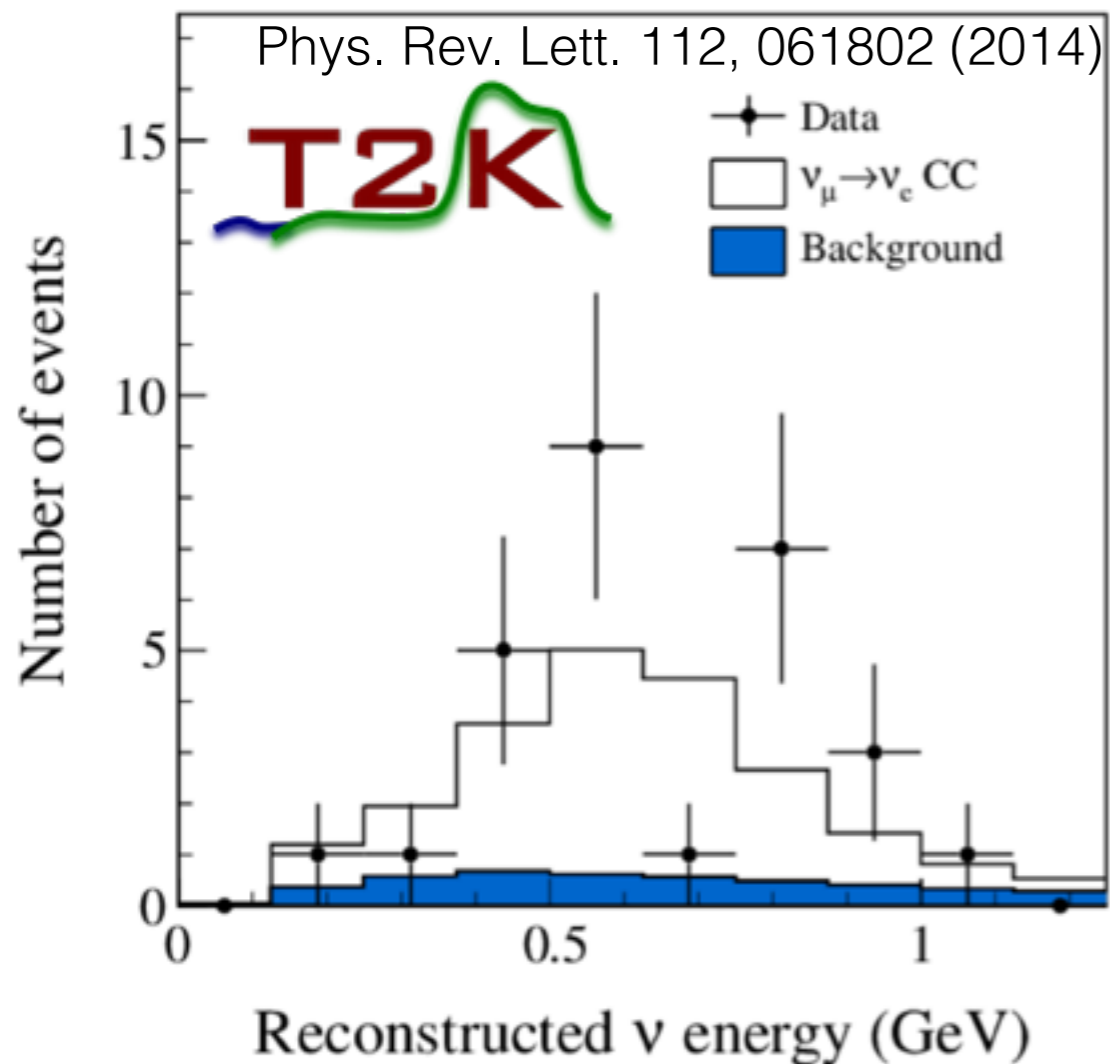


Far Detector
(Super-K)

T2K ν_e appearance

2013: ν_e appearance established → 2017: “indications” of CP violation

28 events observed (4.3 expected background)



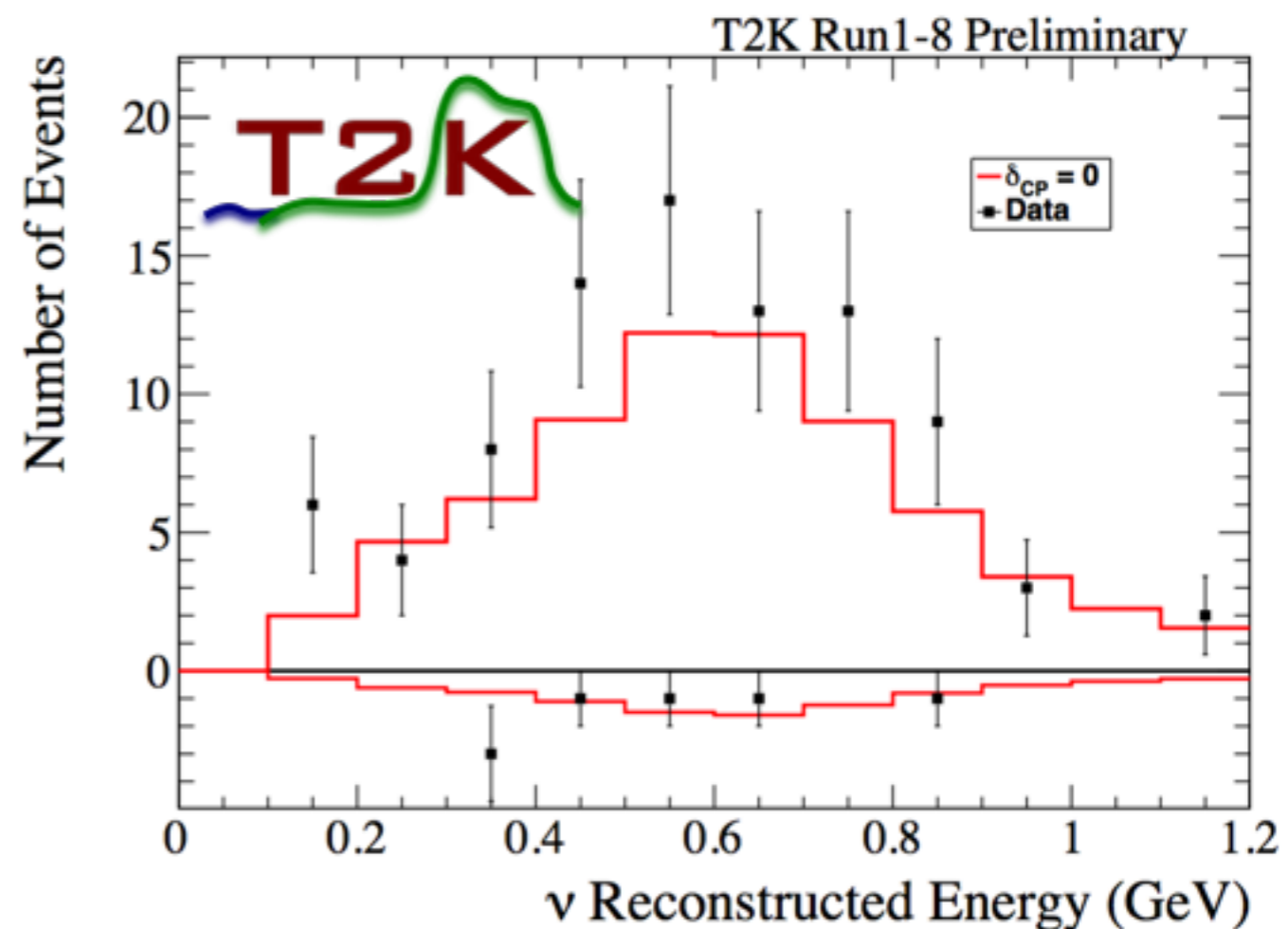
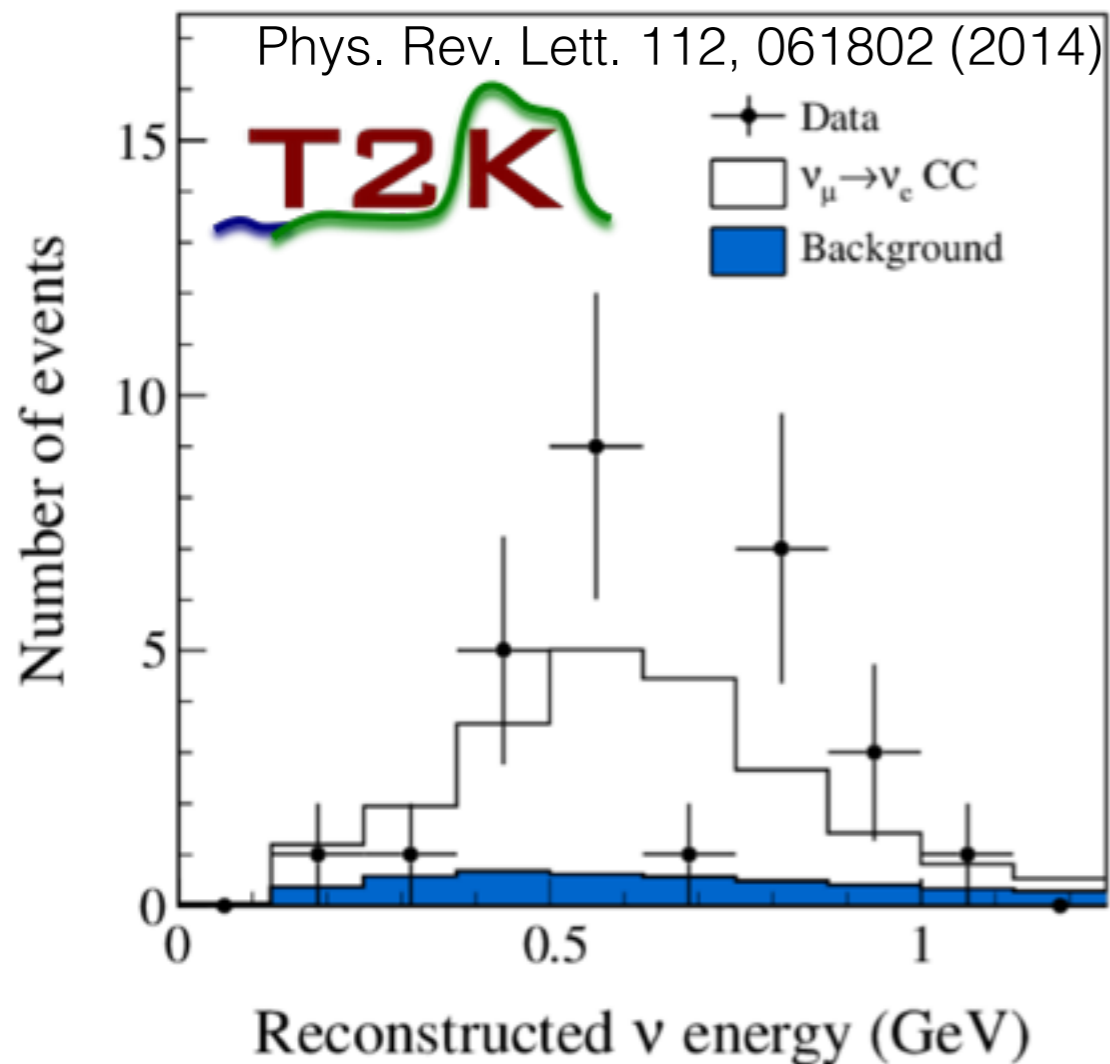
effect is large, opens the way to leptonic CP violation

δ_{CP} .

T2K ν_e appearance

2013: ν_e appearance established → 2017: “indications” of CP violation

28 events observed (4.3 expected background)



effect is large, opens the way to leptonic CP violation

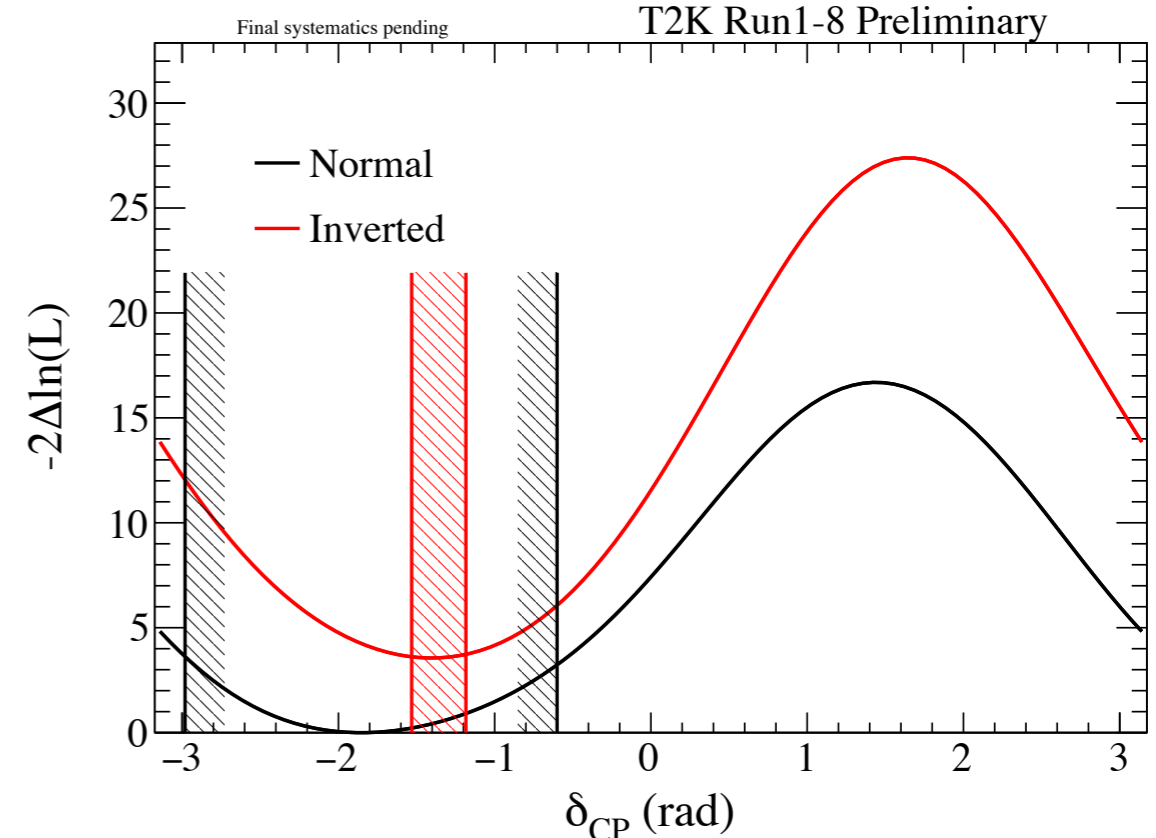
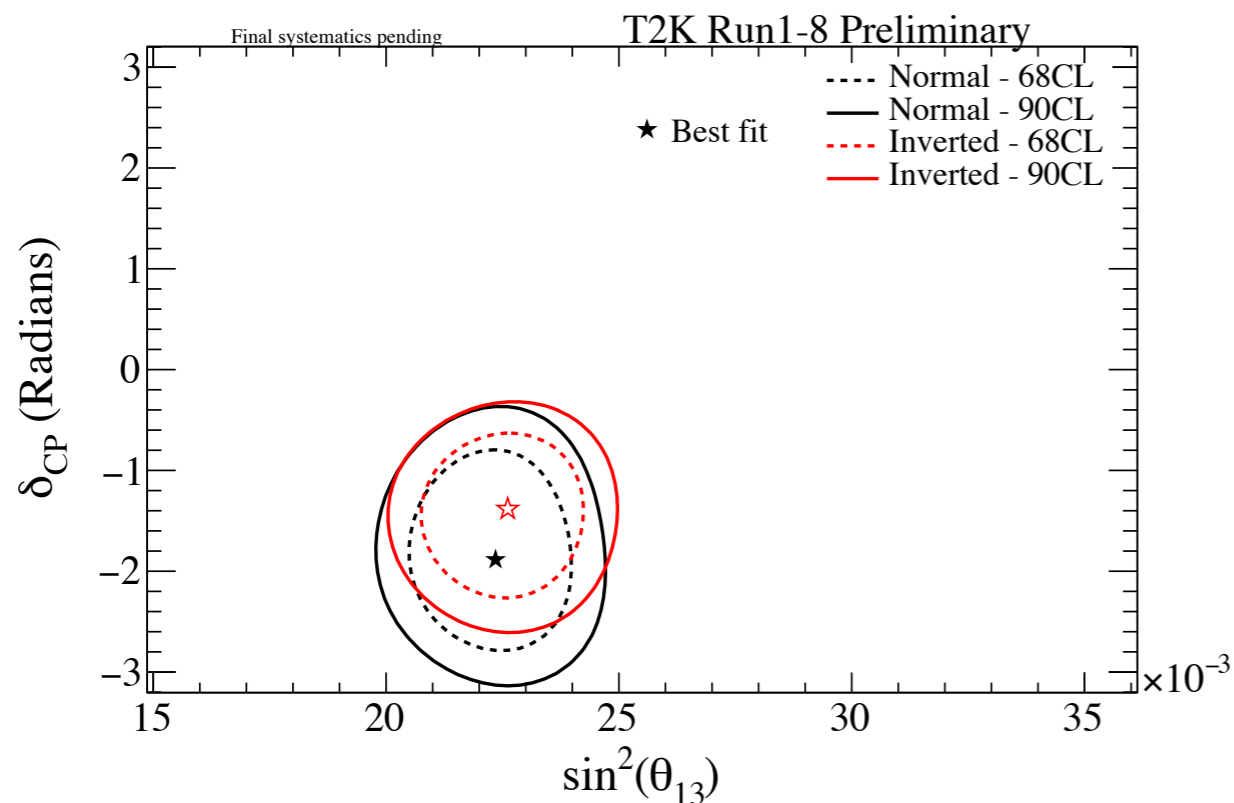
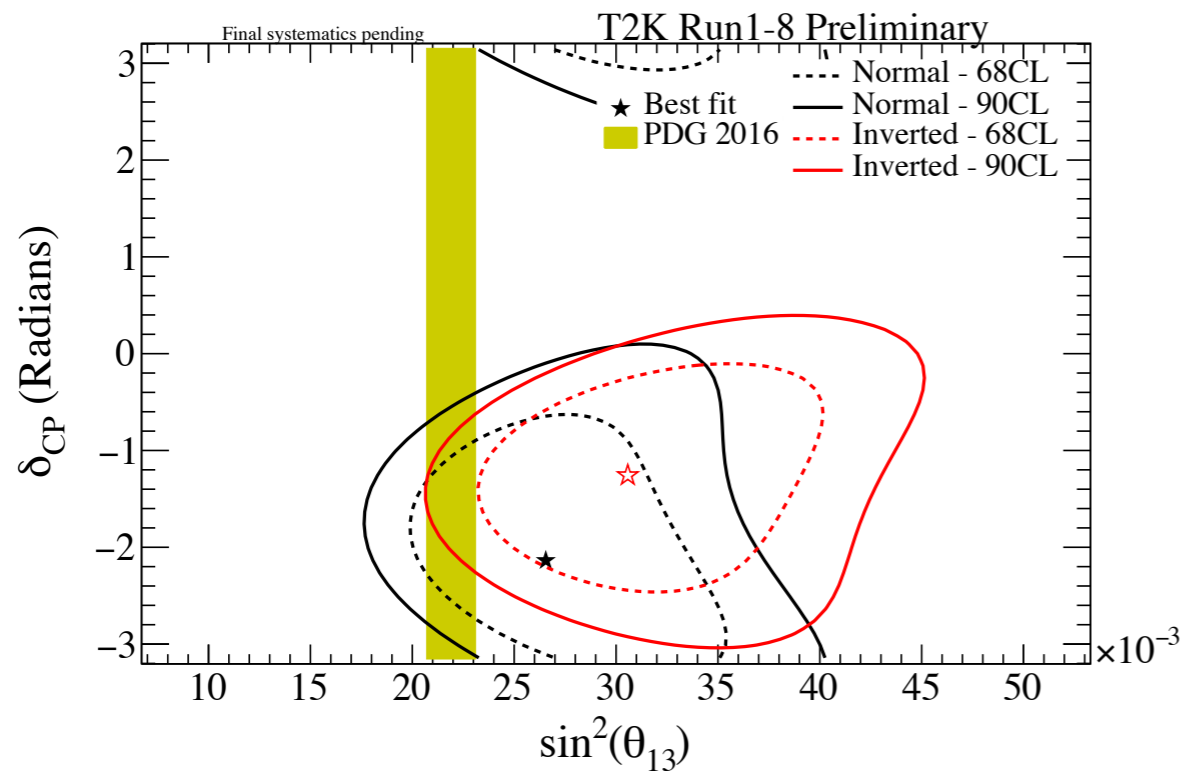
δ_{CP} .

Small ν_e excess and $\bar{\nu}_e$ deficit
Current measurement based on 74+7 events in single ring sample

First Indications of CP violation

CP conserving values
excluded at 2σ

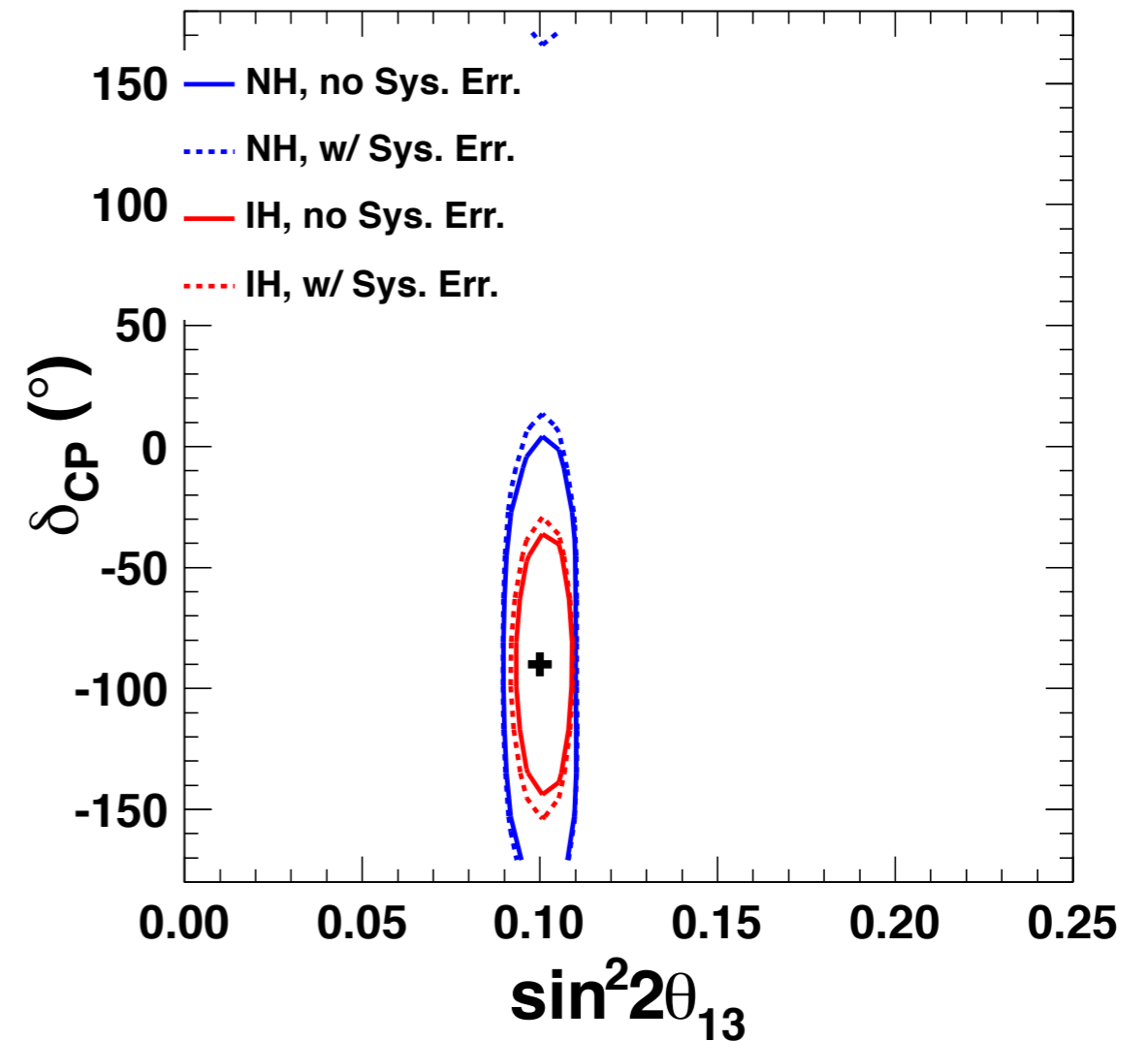
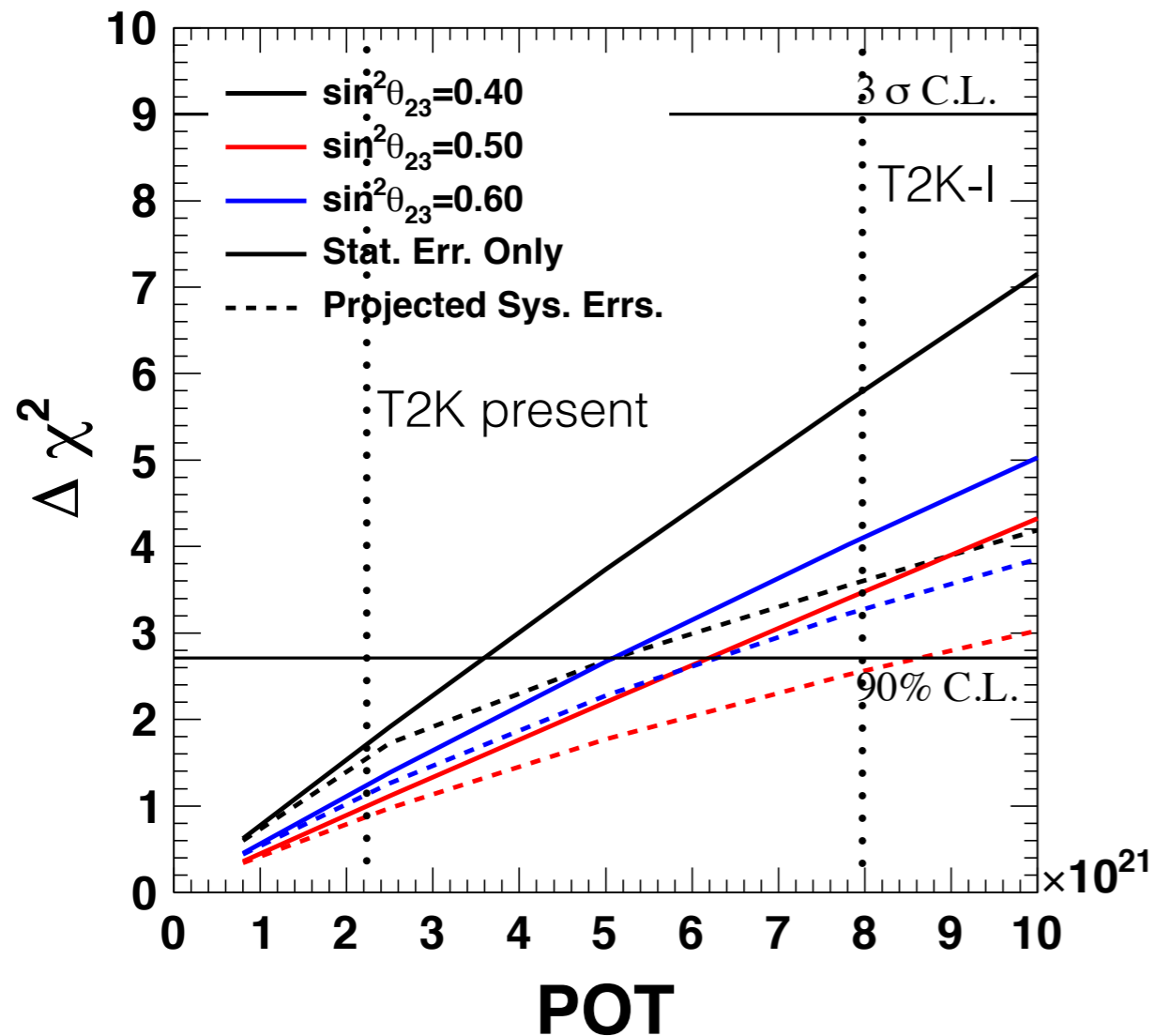
Statistically limited
Dependent on reactor $\bar{\nu}_e$
disappearance
measurement



T2K Projected Sensitivity

arXiv:1409.7469 [hep-ex]

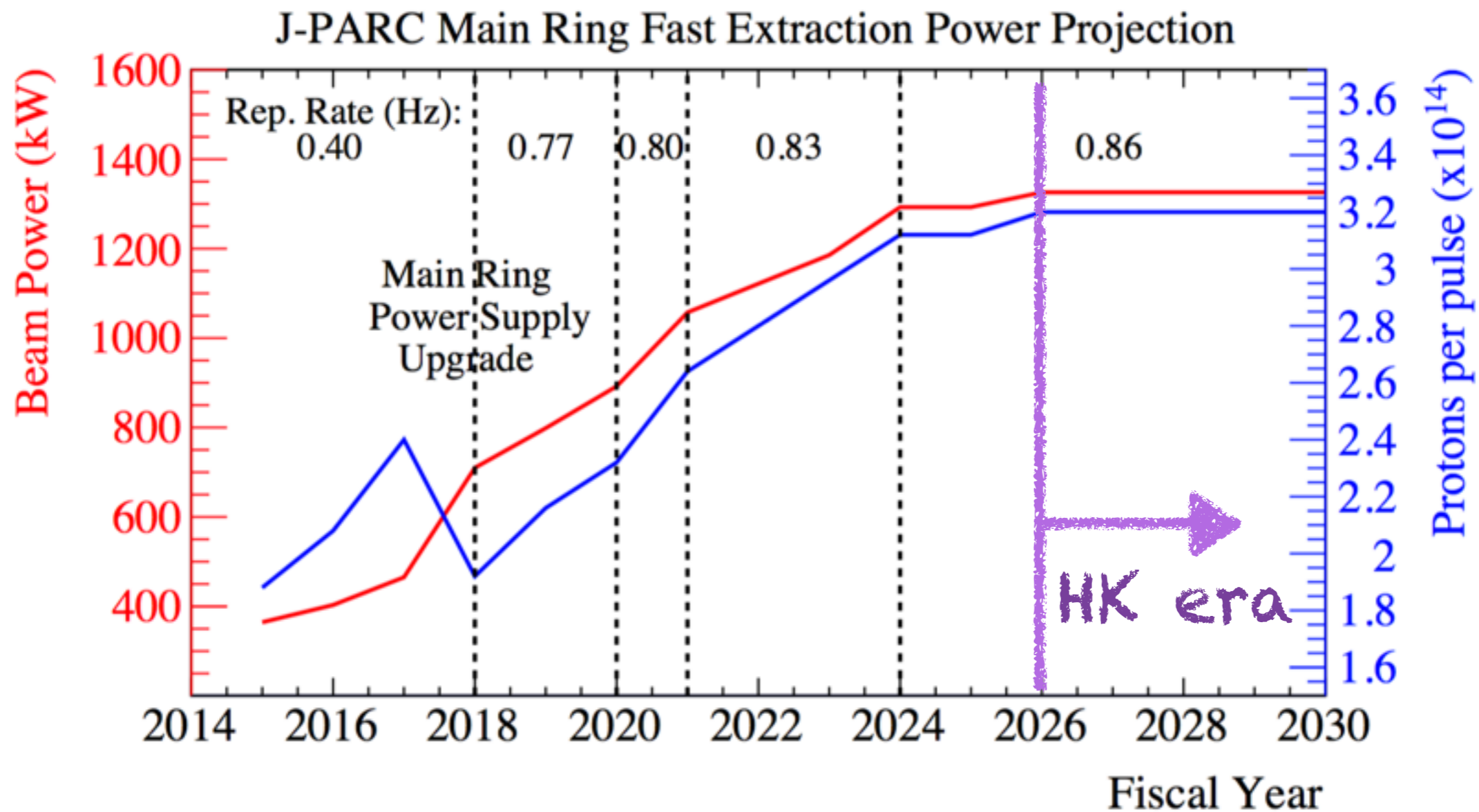
arXiv:1409.7469 [hep-ex]



$\sim 2.5\sigma$ projected significance if *maximal CP violation*.

to firmly establish CP violation we will need **Hyper-K!**

J-PARC Beam Upgrades



Current: ~470 kW

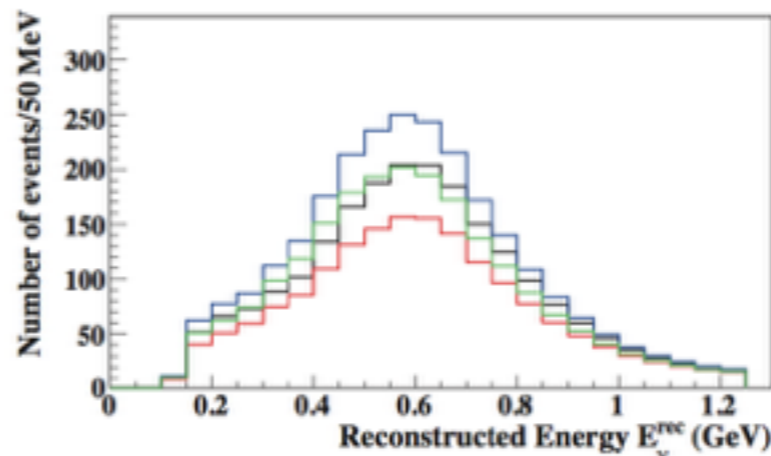
Short-term: 750 kW after 2018 long shutdown

Goal: 1.3 MW operation at HK operation

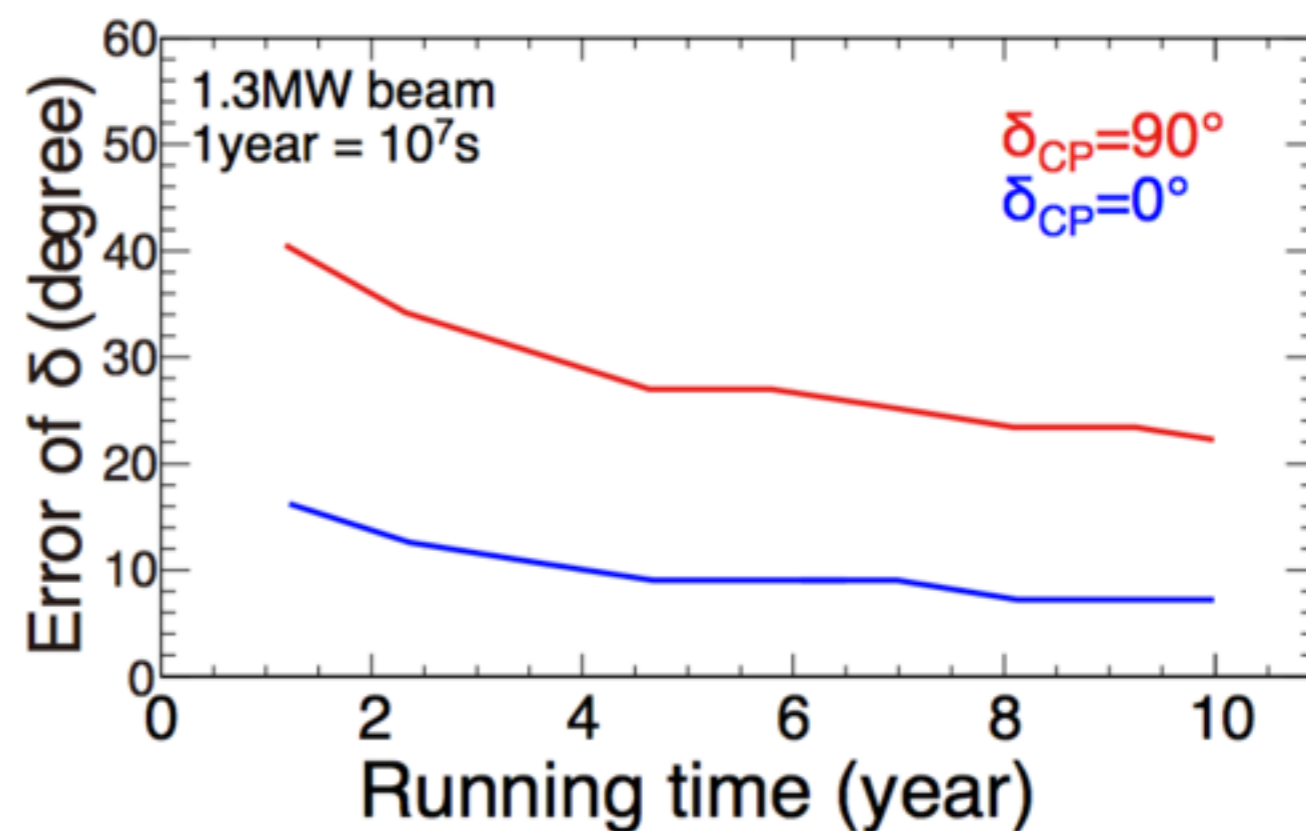
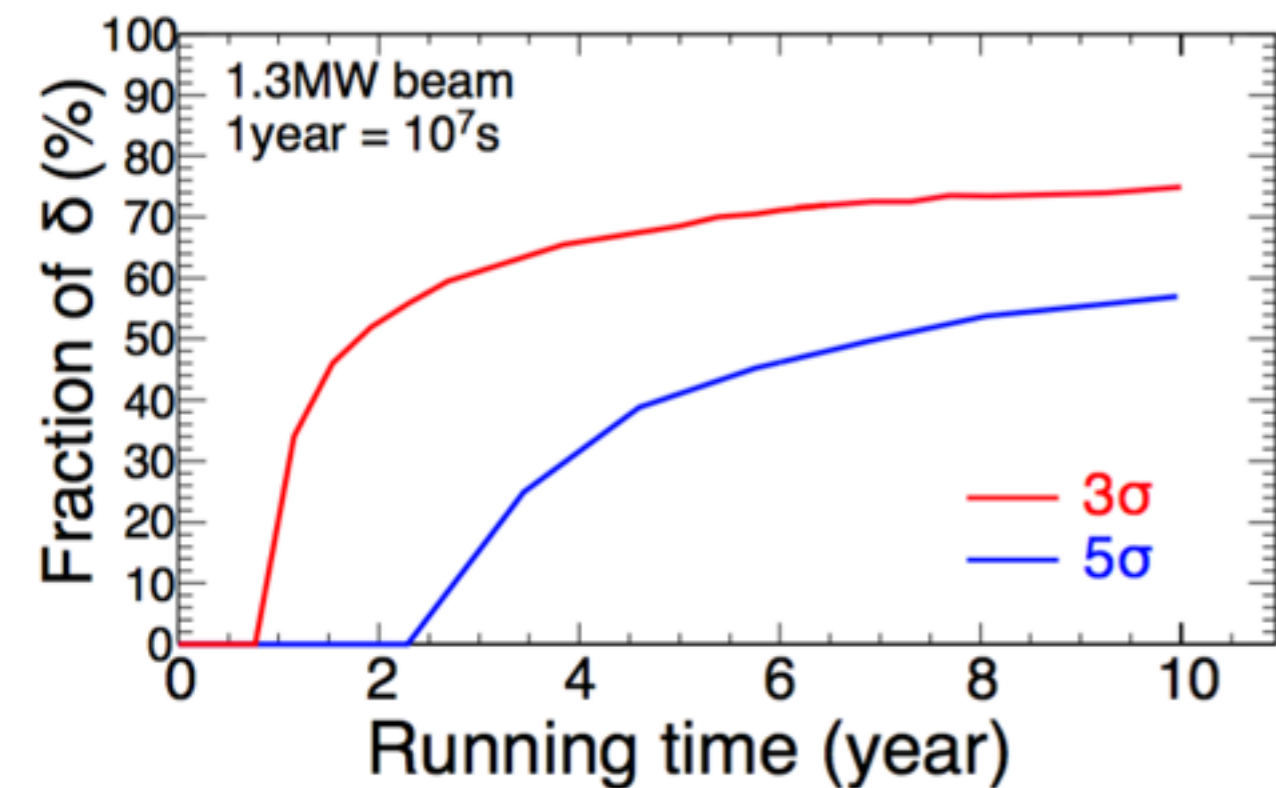
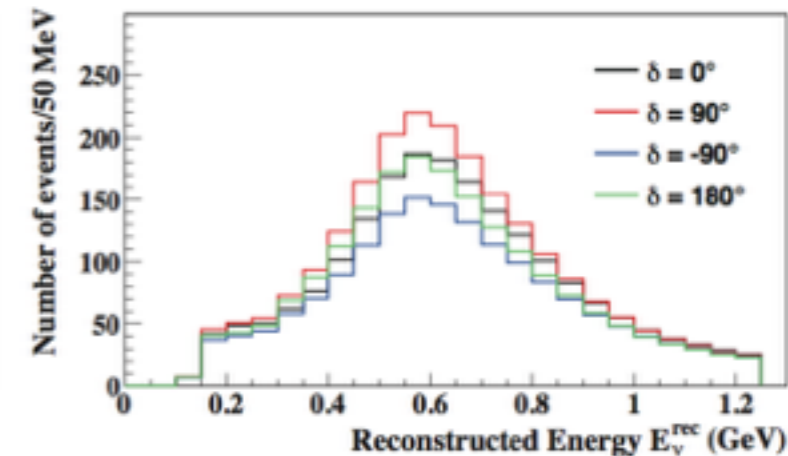
Hyper-K Projected Sensitivity

10 years x 1 tank x 1.3 MW
 $\nu_e \sim 2058$, $\bar{\nu}_e \sim 1906$ events

Neutrino mode: appearance



Antineutrino mode: appearance



Assuming 3-4% systematic uncertainty (cf T2K present $\sim 6\%$)

Statistics

Experiment	$\nu_e + \bar{\nu}_e$	$1/\sqrt{N}$	Ref.
T2K (current)	74 + 7	12% + 40%	2.2×10 ²¹ POT
NOvA (current)	33	17%	FERMILAB-PUB-17-065-ND
NOvA (projected)	110 + 50	10% + 14%	arXiv:1409.7469 [hep-ex]
T2K-I (projected)	150 + 50	8% + 14%	7.8×10 ²¹ POT, arXiv:1409.7469 [hep-ex]
T2K-II	470 + 130	5% + 9%	20×10 ²¹ POT, arXiv1607.08004 [hep-ex]
Hyper-K	2058 + 1906	2% + 2%	10 yrs 1-tank 2017 Design Report TBR
DUNE	1200 + 350	3% + 5%	3.5+3.5 yrs x 40kt @ 1.07 MW arXiv:1512.06148 [physics.ins-det]

Current appearance measurements stats dominate

$O(10^3) \nu_e$ at future experiments → demands ~2% systematics

$O(10^4) \nu_\mu$ → need systematics as good as we can get!

T2K Systematic Uncertainties

ND280 constraint
13% → 3%

Error Source	μ sample [%]		e sample [%]	
	ν	$\bar{\nu}$	ν	$\bar{\nu}$
SK Detector	1.9	1.6	3.0	4.2
SK FSI+SI+PN	2.2	2.0	2.9	2.5
ND280 Constraint (Flux + Cross Section)	3.3	2.7	3.2	2.9
$\sigma(\nu_e)/\sigma(\nu_\mu)$	-	-	2.6	1.5
NC 1γ	-	-	1.1	2.6
NC other	0.3	0.1	0.1	0.3
Total Systematic	4.4	3.8	6.3	6.4
Statistical	6.5	12	12	40

T2K preliminary (final systematics pending)

Total systematic uncertainty
~4 - 6%

Smaller than stats. uncertainty
(for now!)

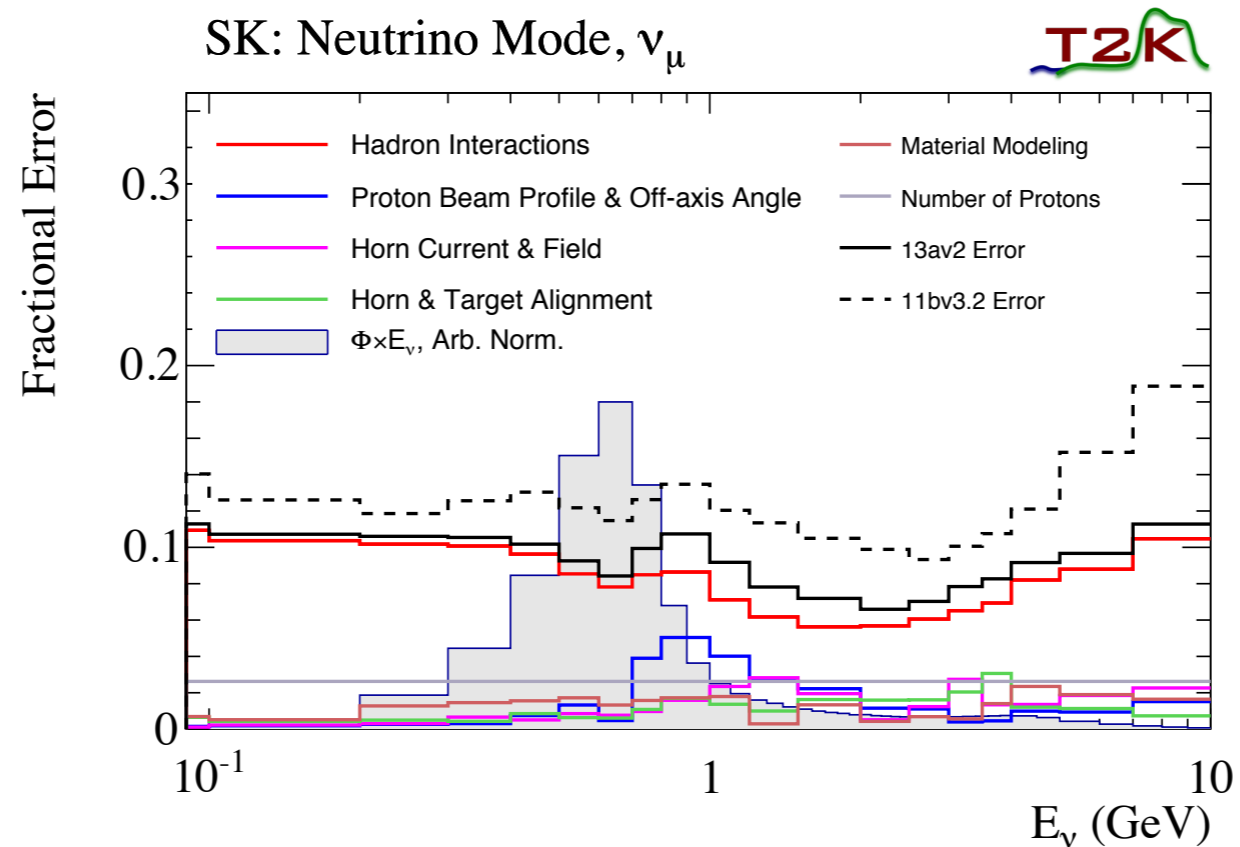
Pion Final State
Interactions (FSI) and
Secondary Interactions
(SI) modelling important

Theoretical uncertainty

ν_e to ν_μ

Difficult to constrain with
near detector

Flux Uncertainties



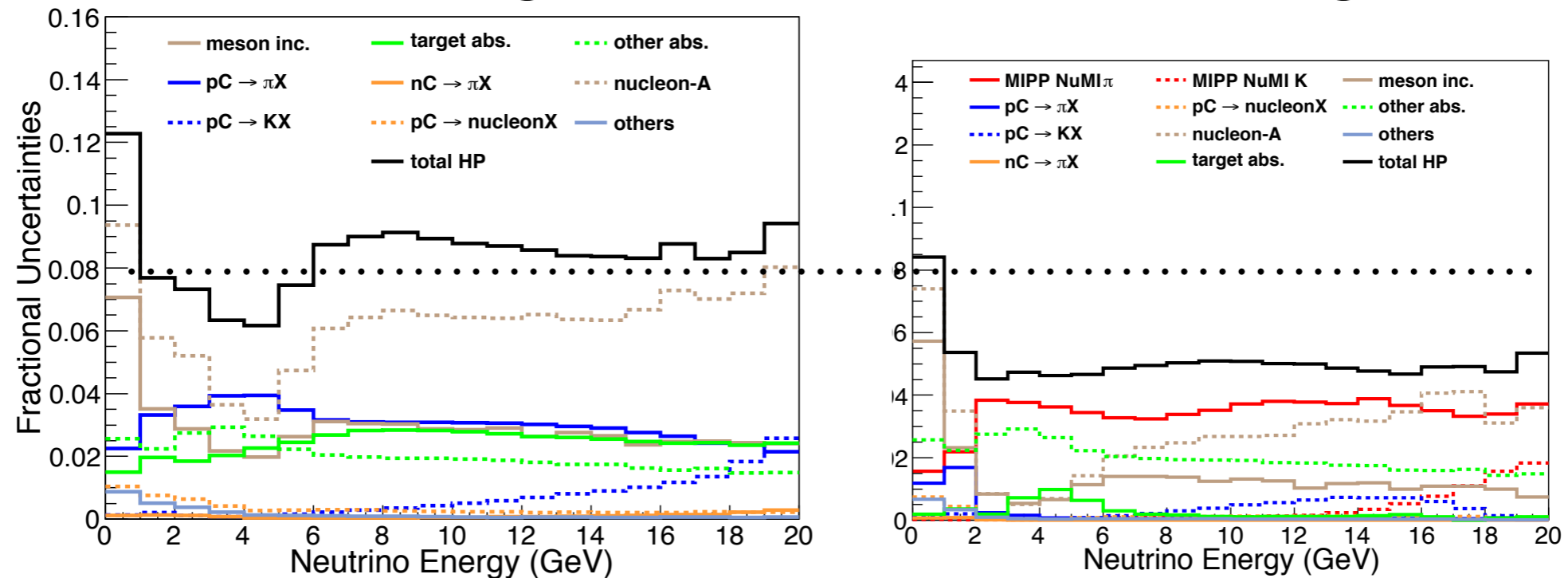
T2K \sim 8-12% (based on thin target tuning)

Dominated by hadron interaction modelling

Alignment/focussing uncertainties are also important
(especially for near to far extrapolation)

Flux Uncertainties

Thin Target \longrightarrow Thick Target

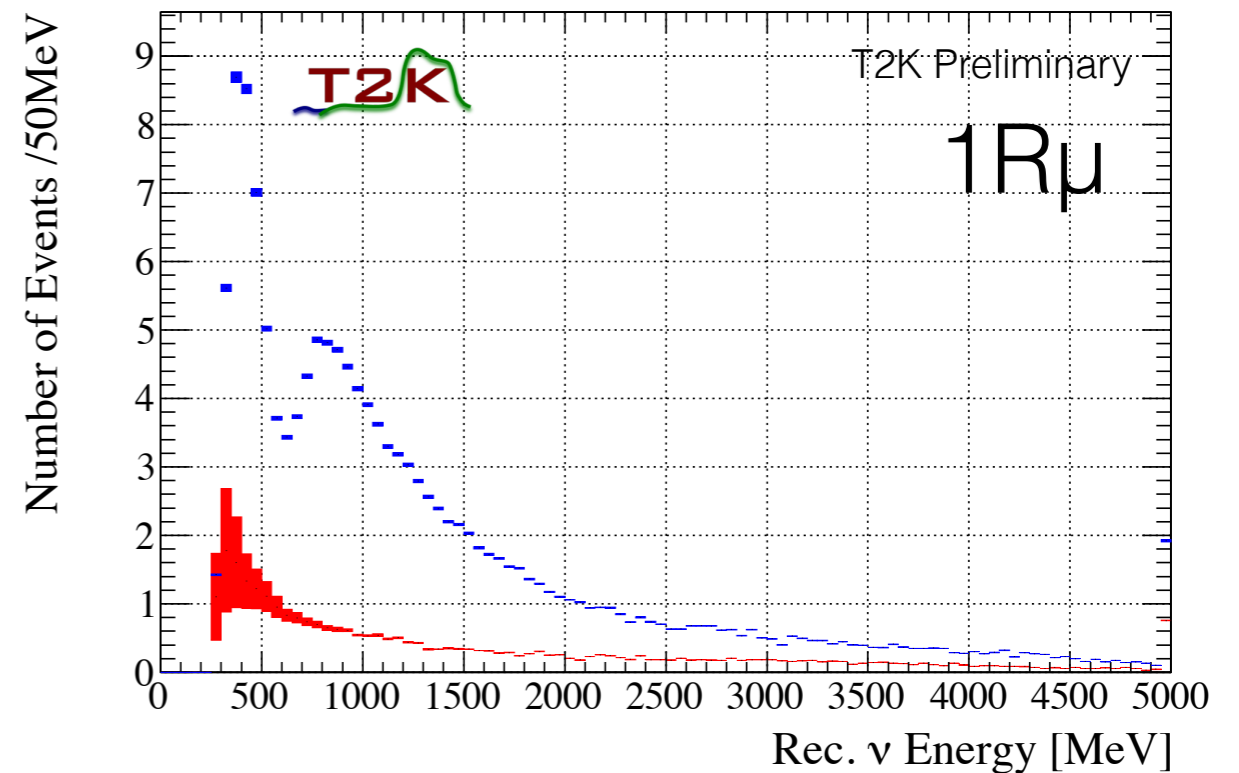
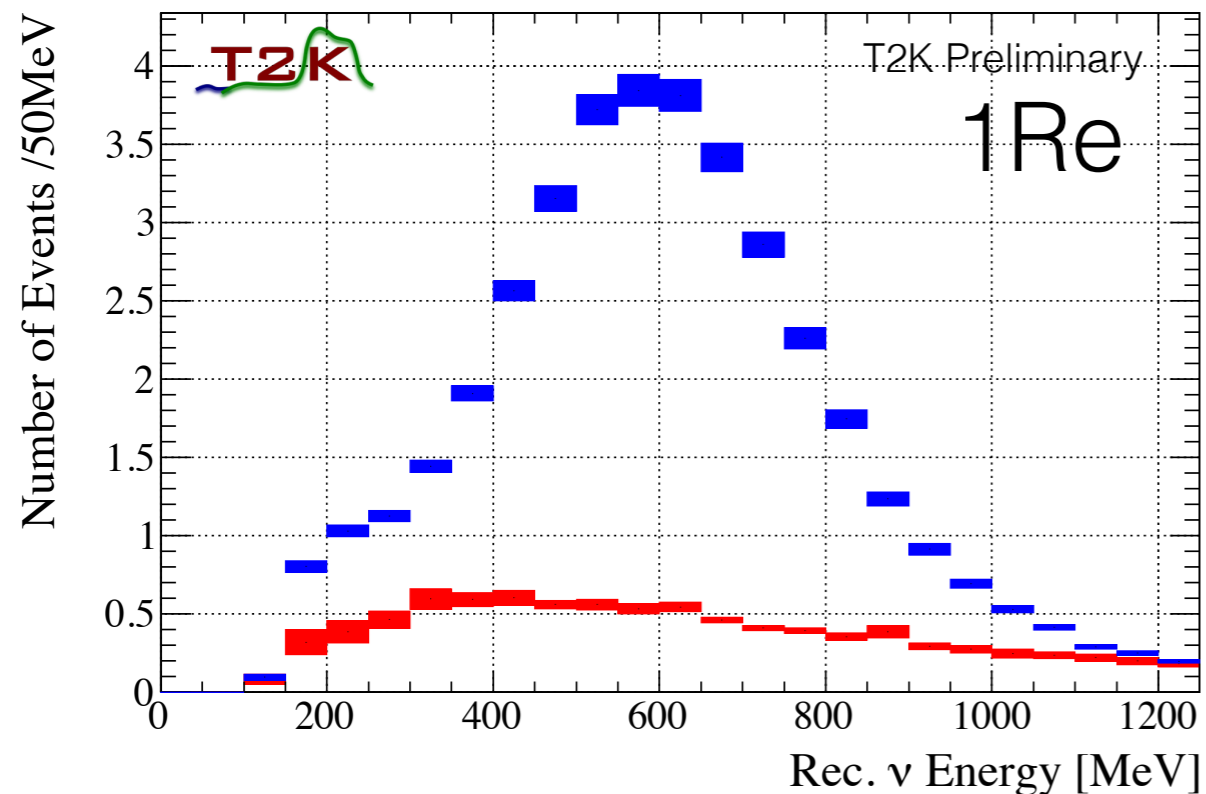


MINERvA Low E NuMI Flux Uncertainties, Phys. Rev. D 95, 039903 (2017)

Significant reductions from thick/replica target

If high power beam requires different target material/geometry
new dedicated hadron production measurements will be
necessary

Detector Modelling Uncertainties



SK detector response evaluated with data-MC
comparisons in atmospheric sample

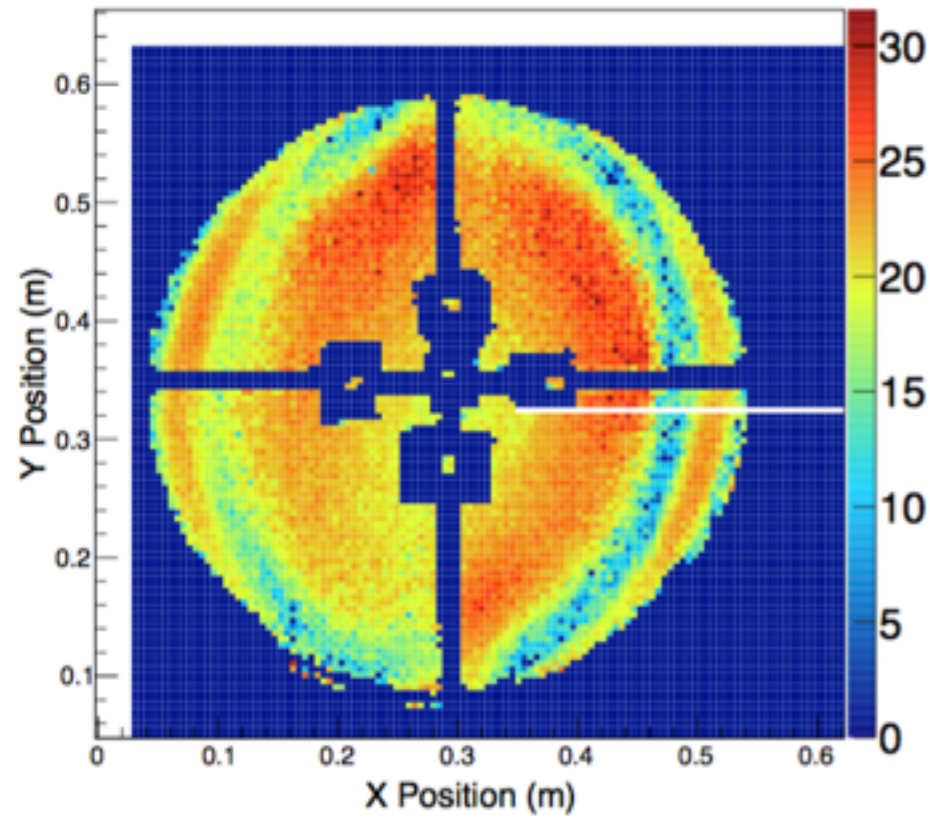
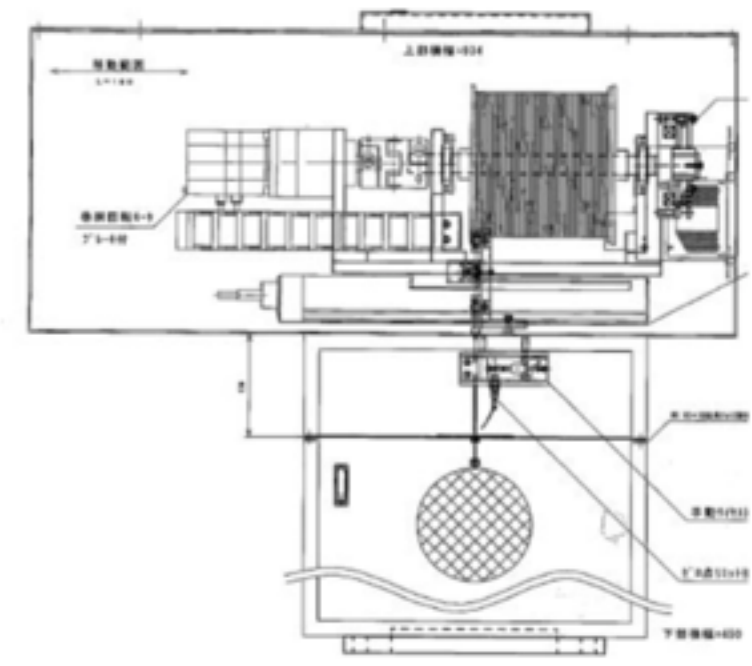
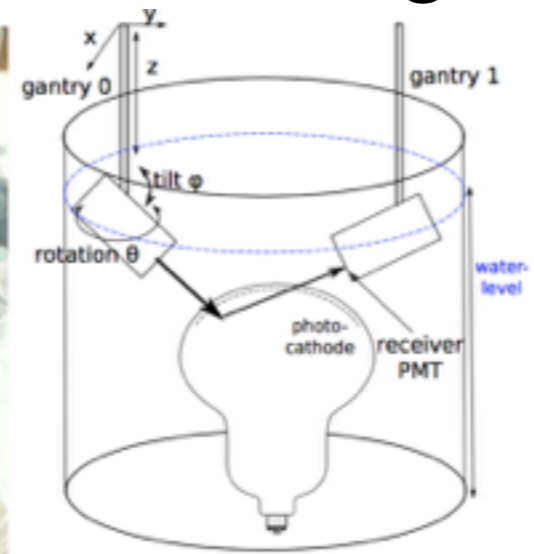
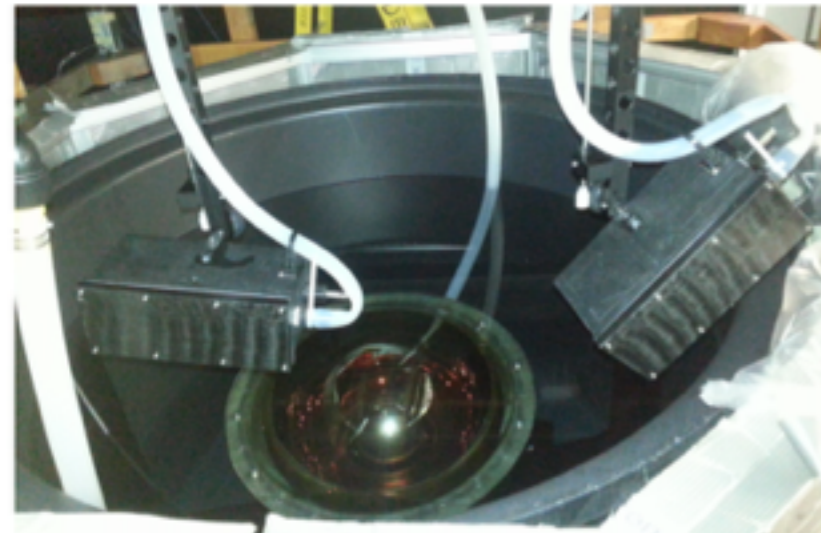
May be limited by control sample statistics

Possible to move toward bottom-up detector systematic
uncertainty

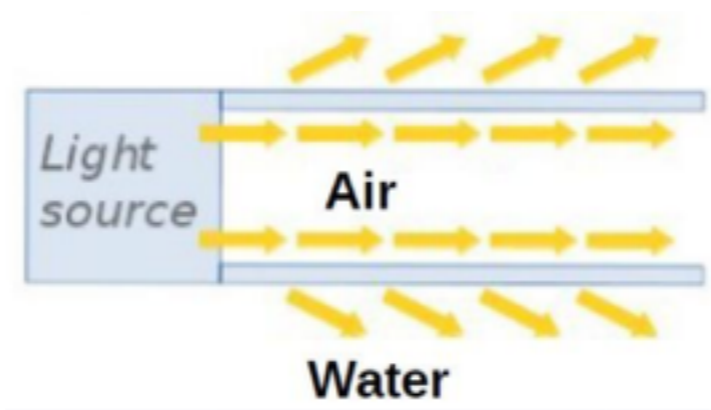
Calibration

Precise PMT response testing

Automated source deployment



Fake muon source

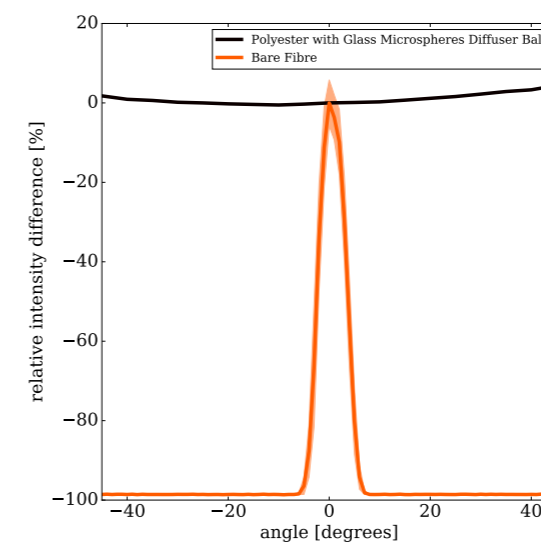
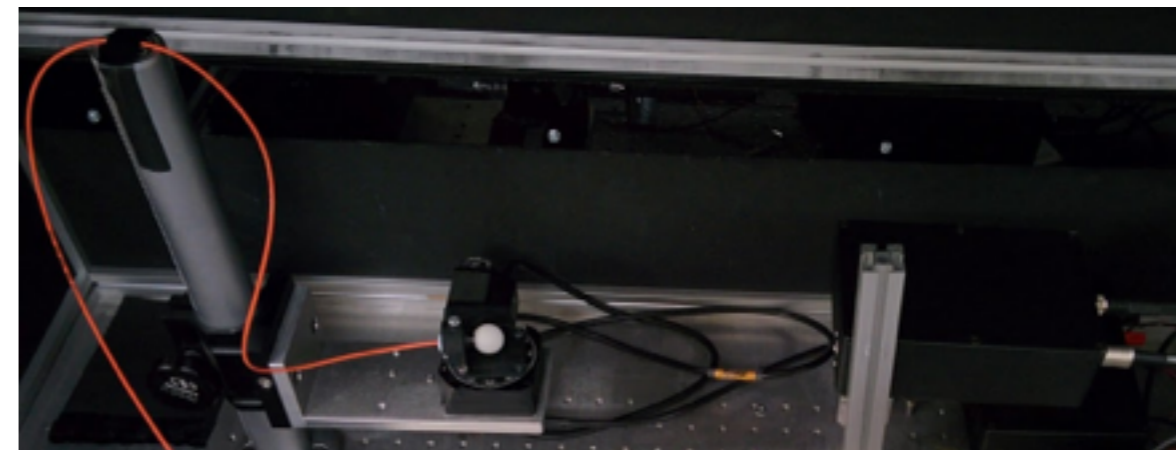
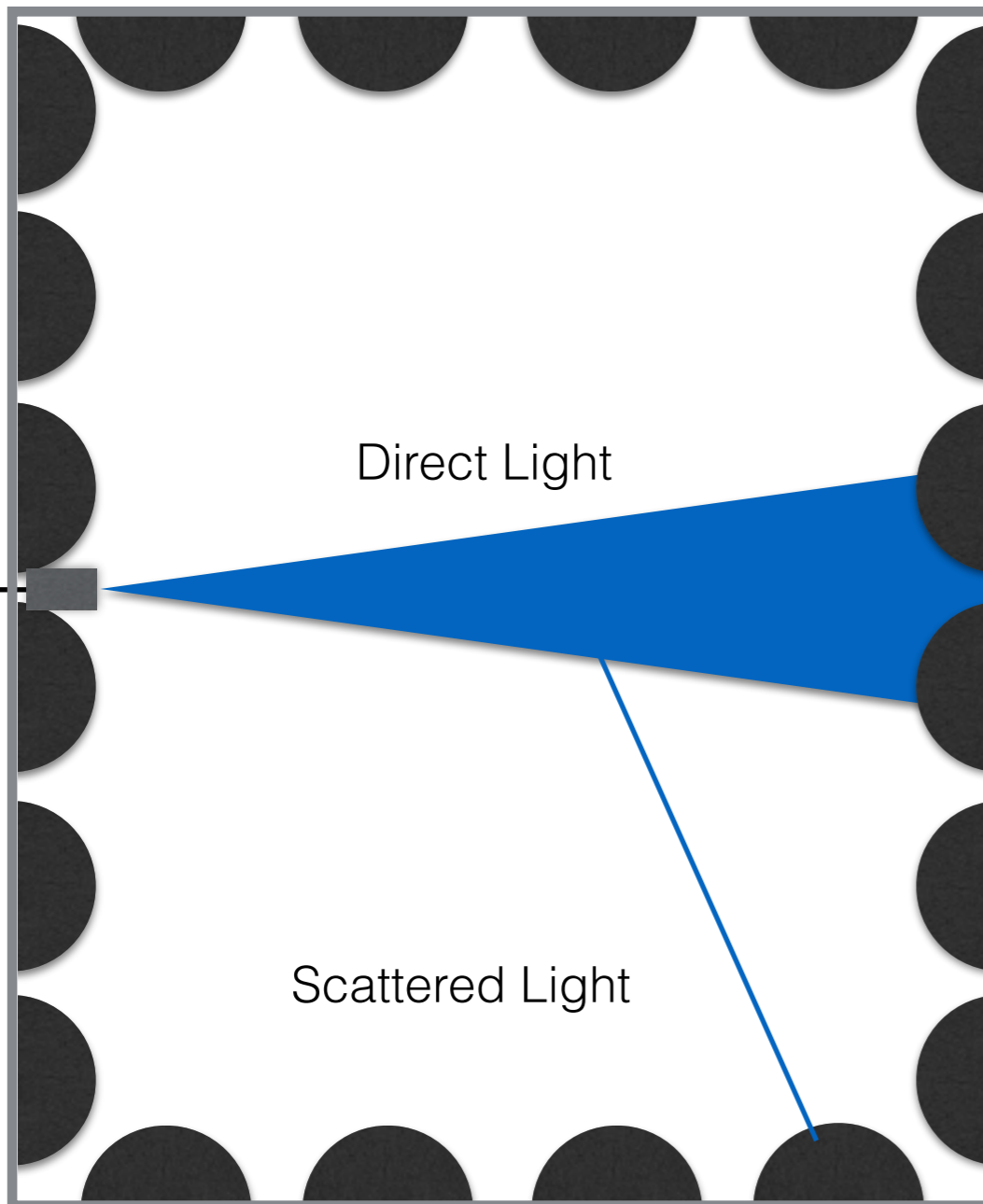


“Neutrino” Neutron Generator



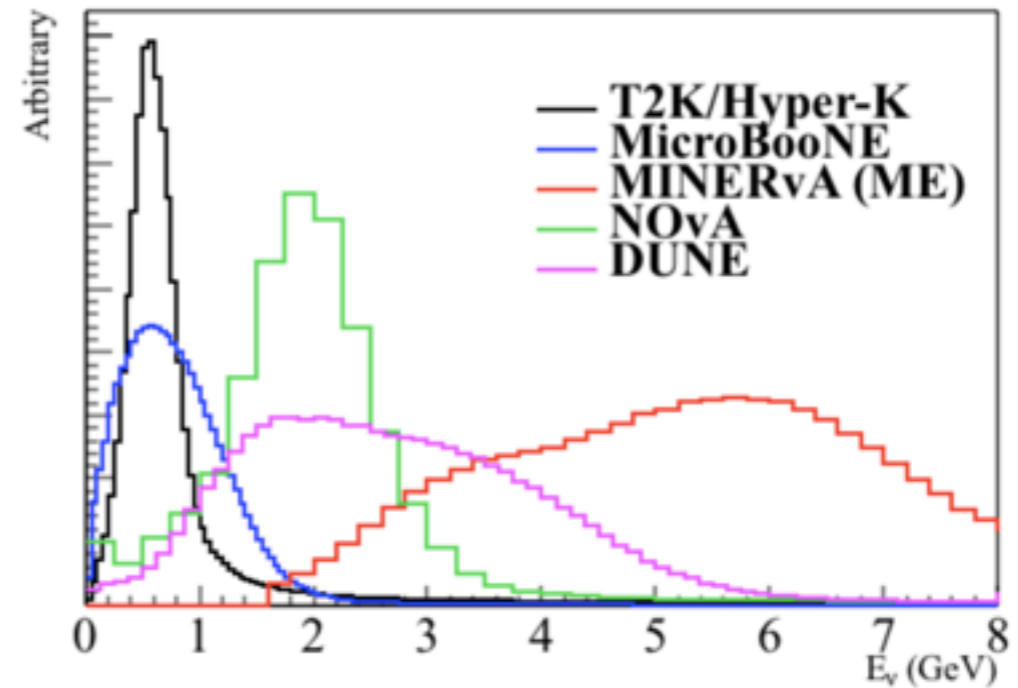
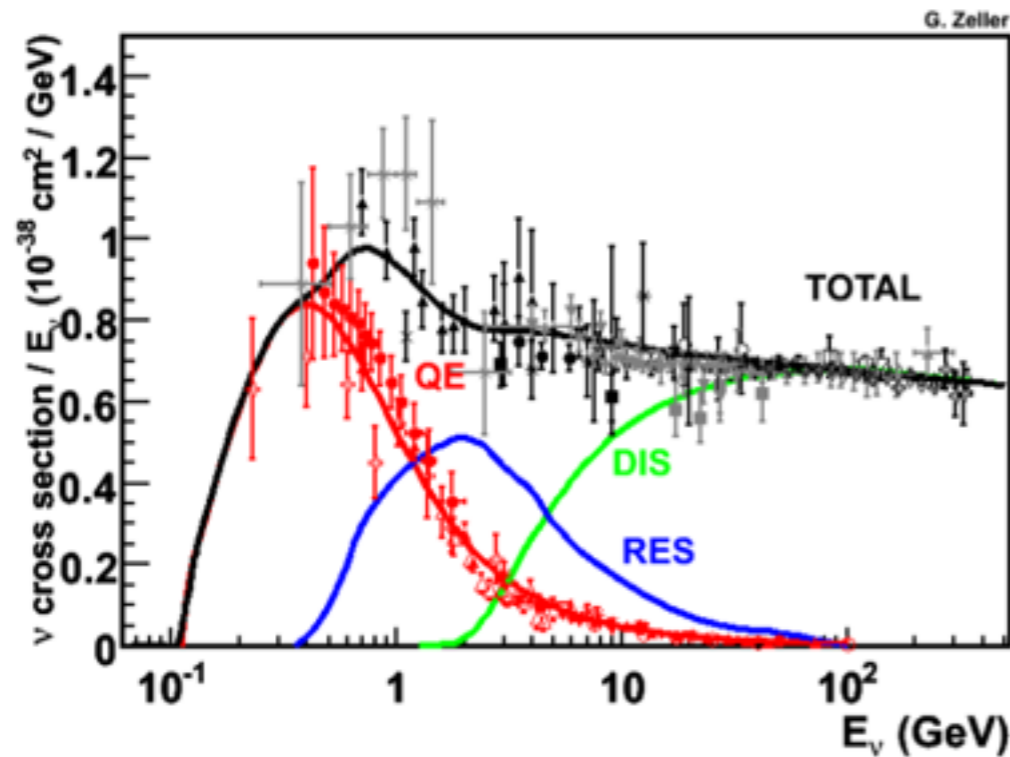
Calibration

R&D for new optical calibration system in progress



Using Super-K 2018 shutdown for direct testing of newly developed calibration systems for Hyper-K

Neutrino Interaction Model Uncertainties



Wide range of processes need to be simulated
Require both lepton and hadronic side of the interaction
Nuclear effects important in the relevant energy regime

Experiments rely on MC generators
for $E_{\text{visible}} \rightarrow E_{\nu}$ extrapolation

Model parameter uncertainties from fits to external datasets

Sometimes parameter error must be inflated or ad-hoc parameters to account
for discrepancies between model and data or known flaws in the model

T2K Cross-Section Model

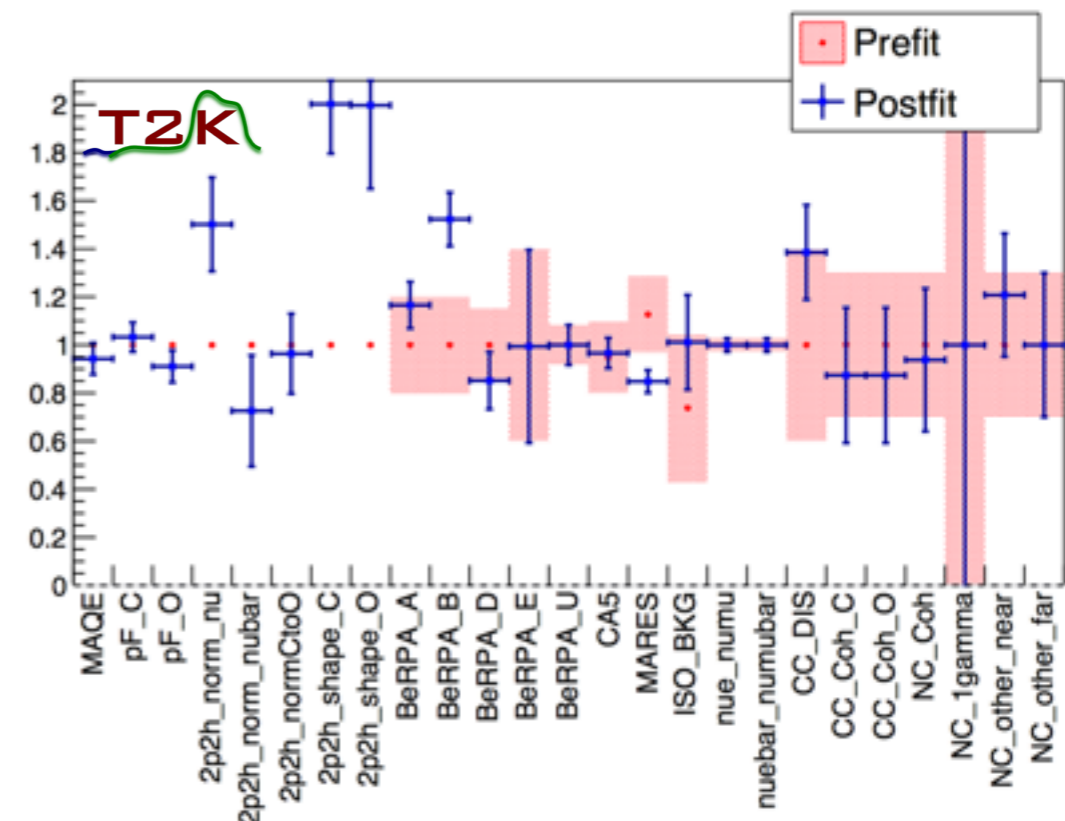
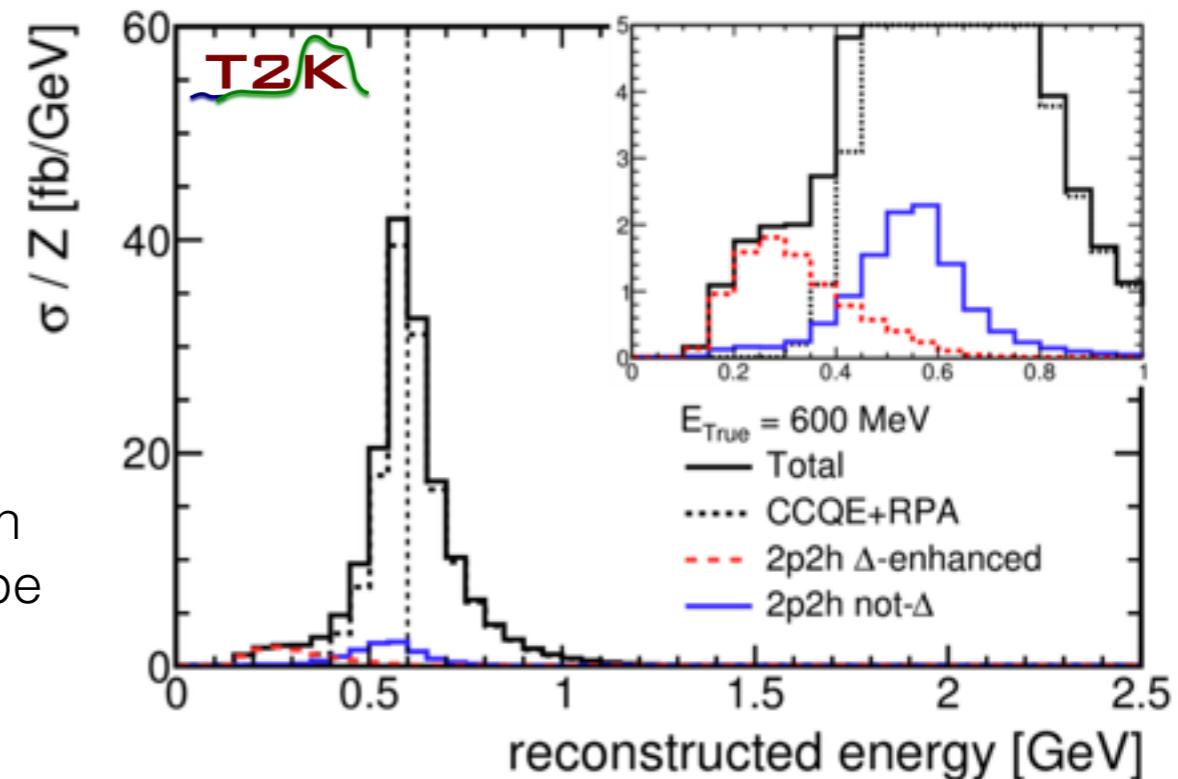
Implemented in NEUT MC generator

Quasi-elastic scattering most important process at T2K energies

- Valencia 2p-2h model Phys. Rev. C83 (2011) 045501
- Long-range effects with Random Phase Approximation
- Parameters introduced to vary normalisation and shape
- Relativistic Fermi Gas (RFG) nuclear model
- Uncertainties from RFG \leftrightarrow Local Fermi Gas
- Final state interactions with cascade model

No priors on most CCQE parameters
Constraint from near detector

Impact of alternative models not implemented in oscillation analysis evaluated with fake data studies

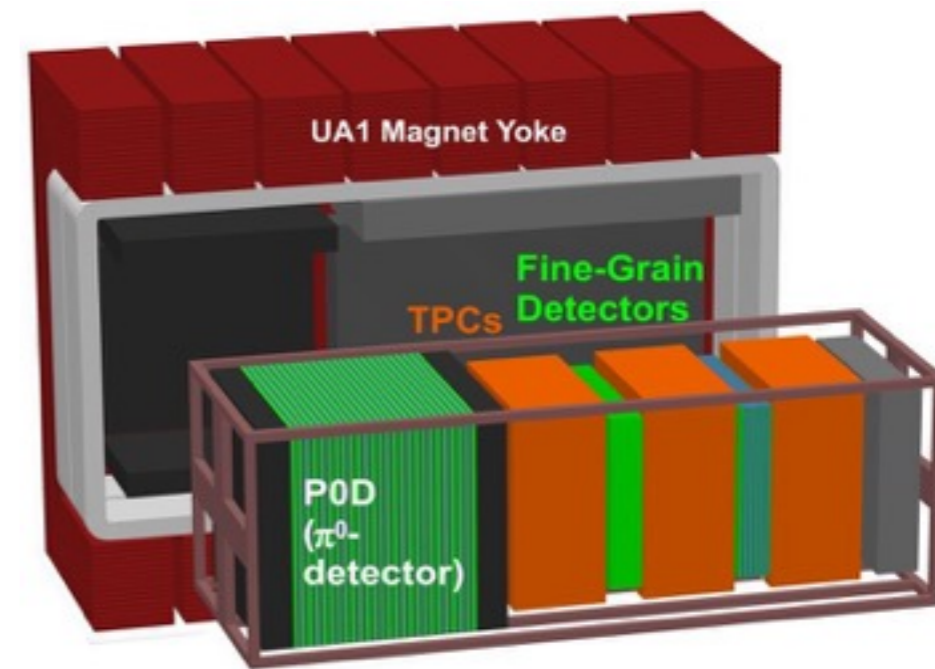


Near Detector Development

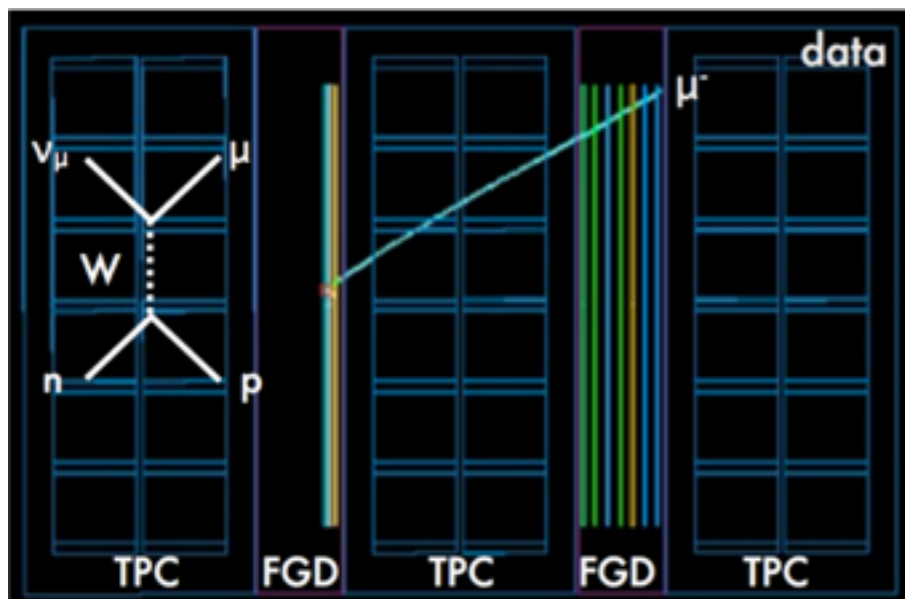
Carbon and Oxygen target materials

Acceptance differs from far detector

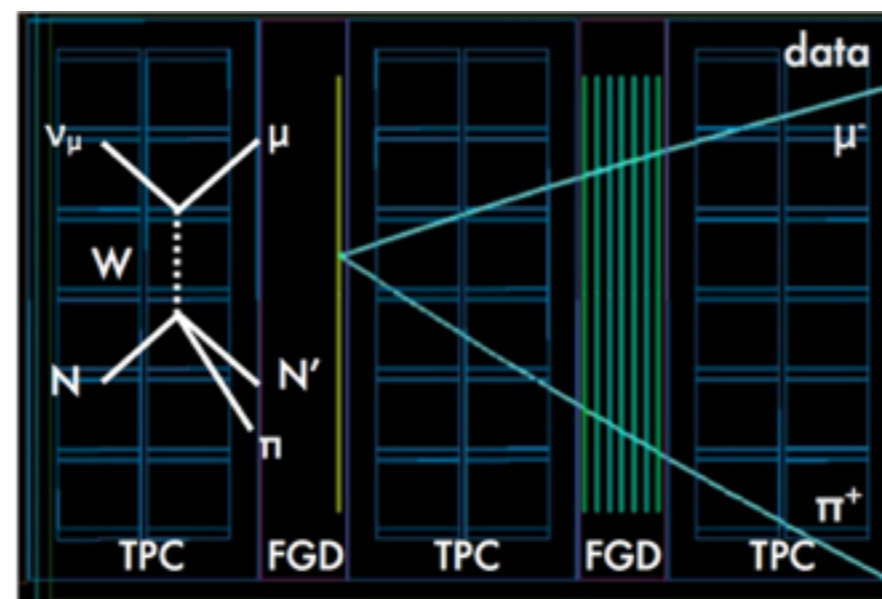
Magnetic field for sign selection



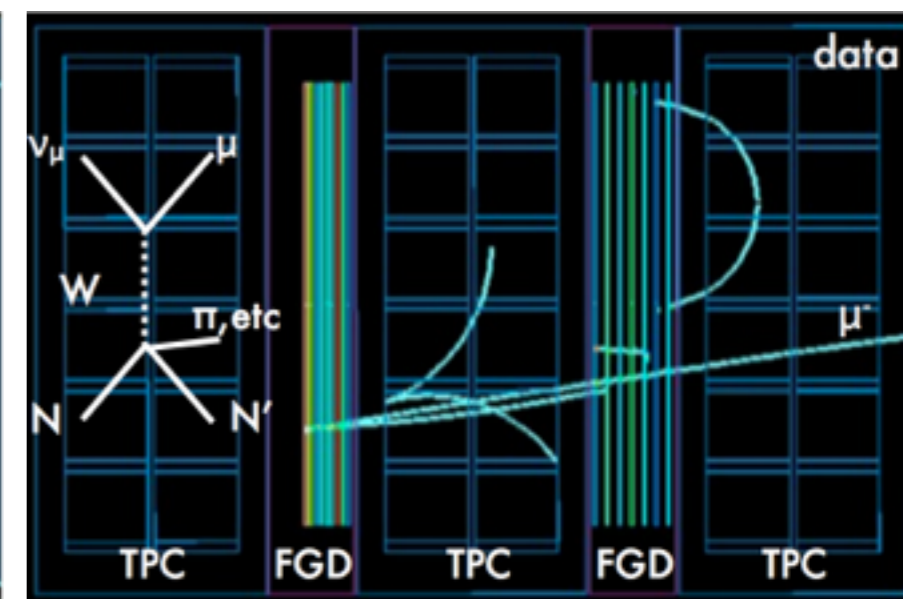
Near Detector (ND280)



CC $1\mu + 0\pi + X$



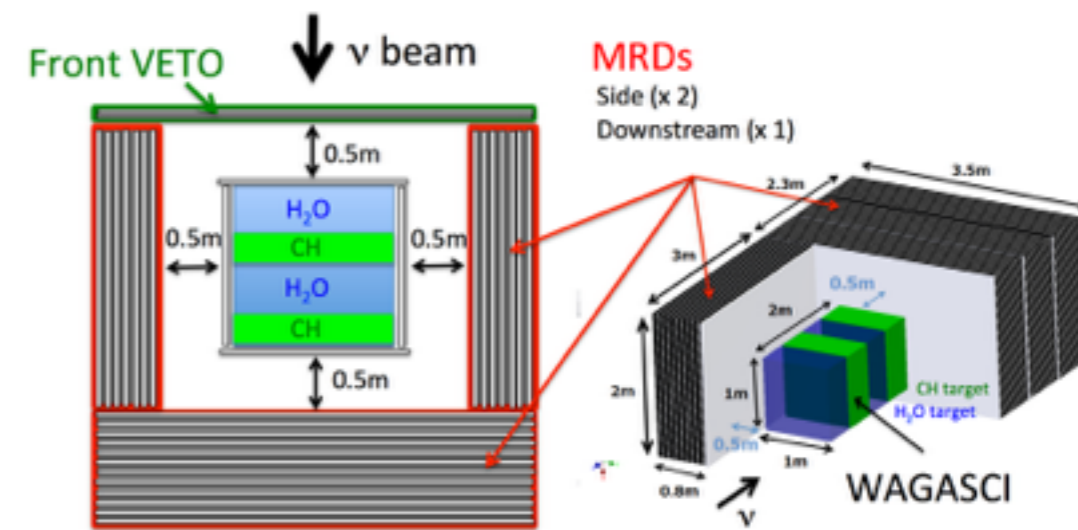
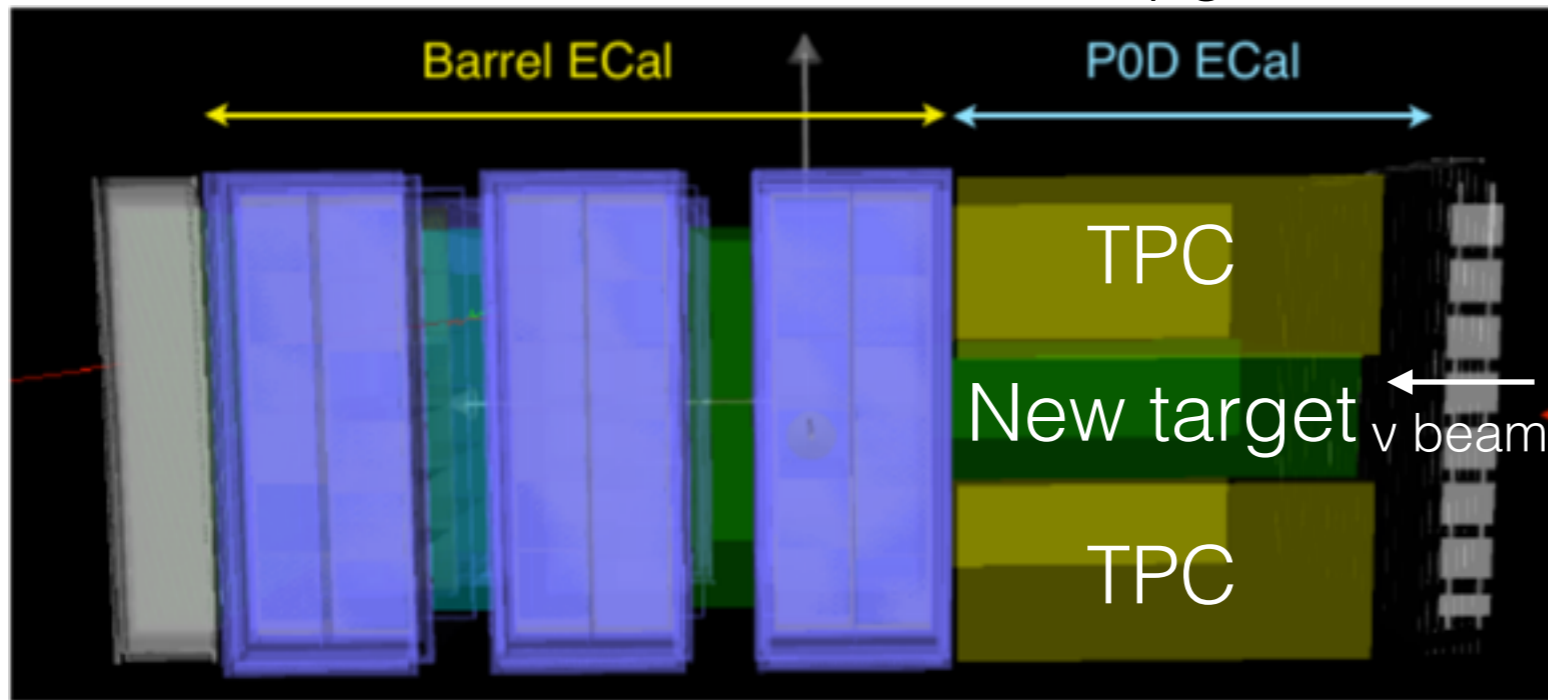
CC $1\mu + 1\pi^+ + X$



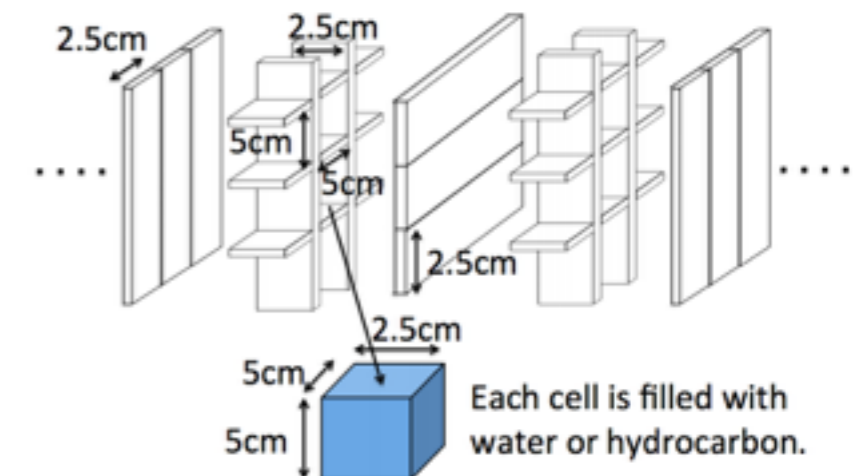
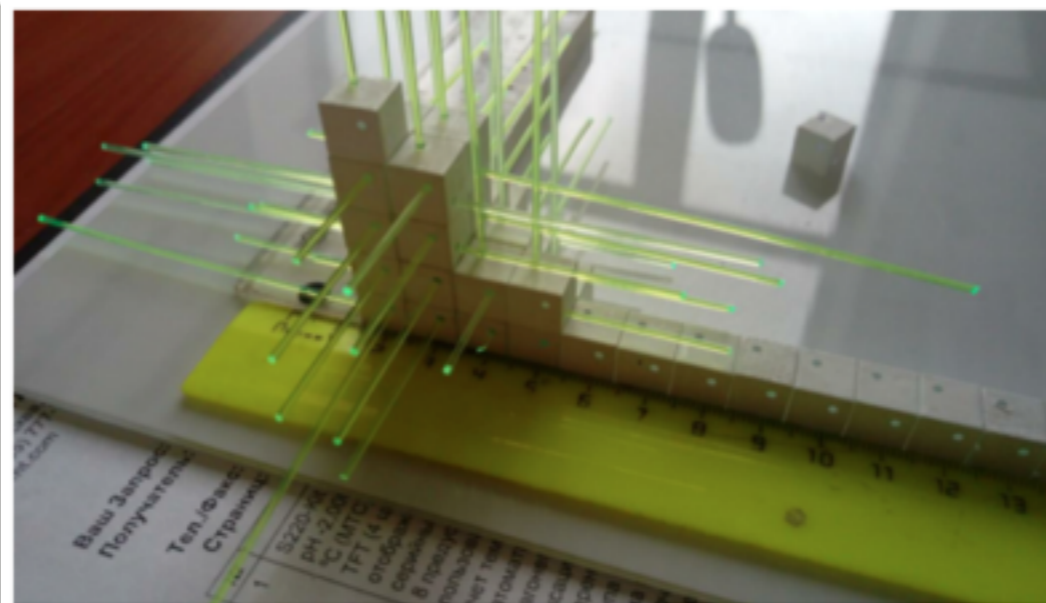
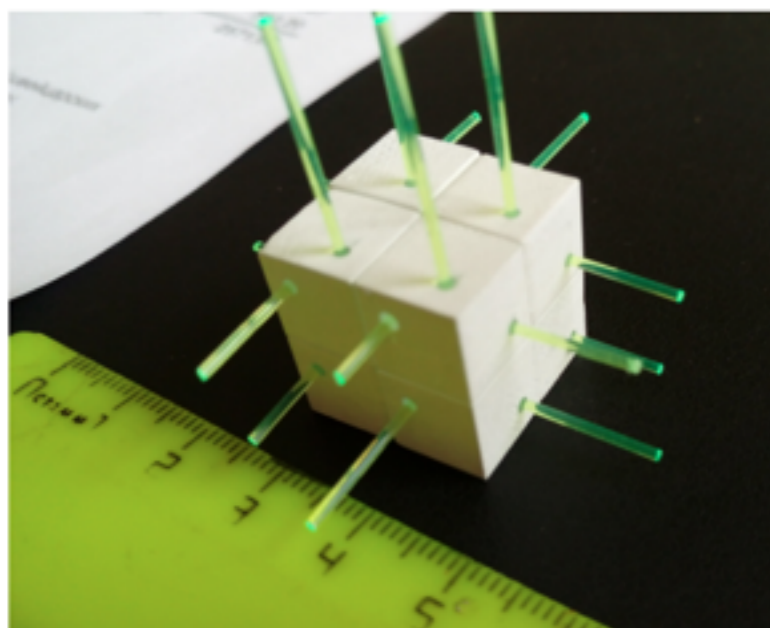
CC other

Near Detector Development

Planned ND280 Near Detector Upgrade

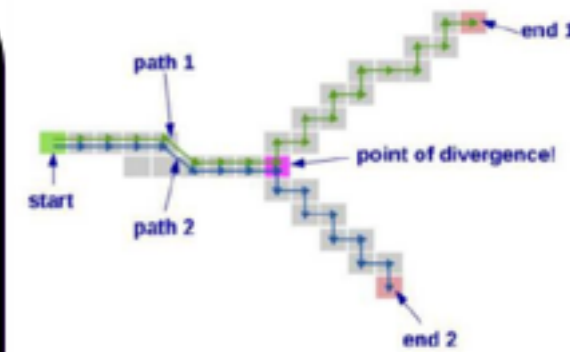
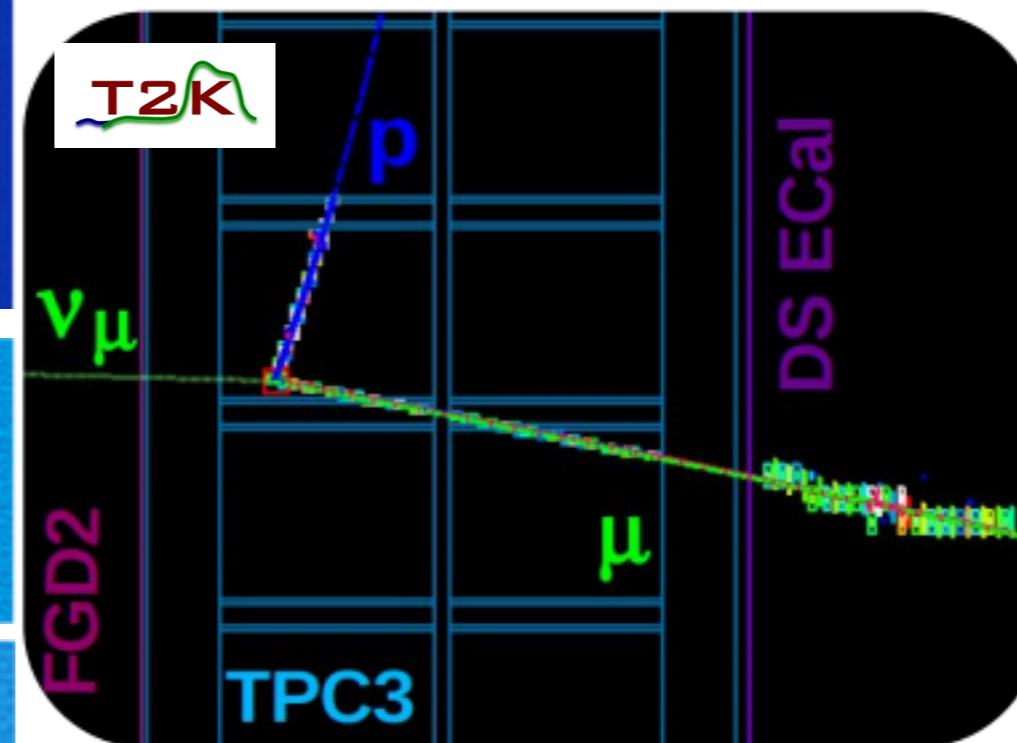
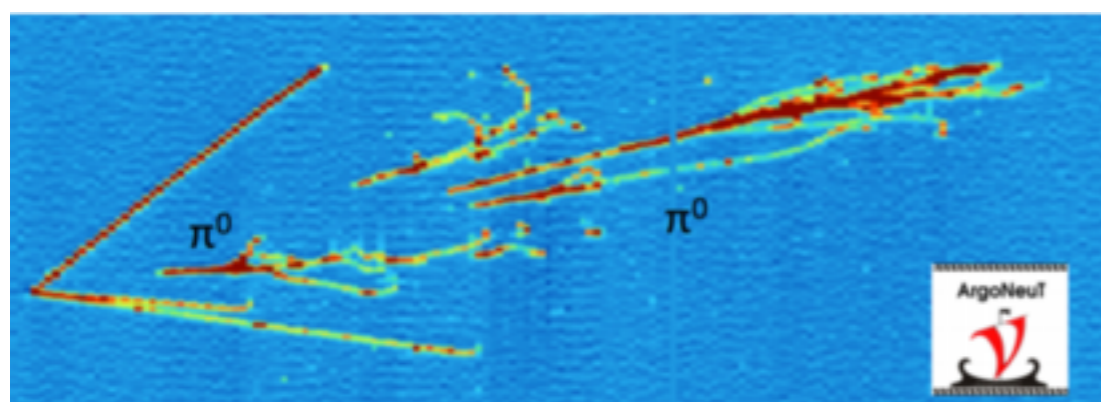
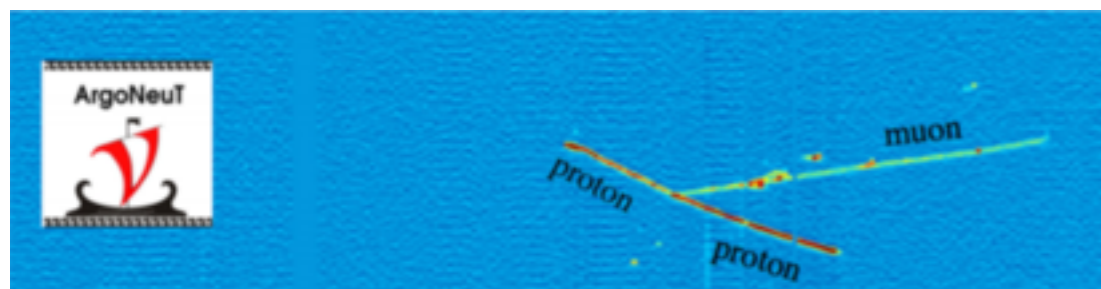
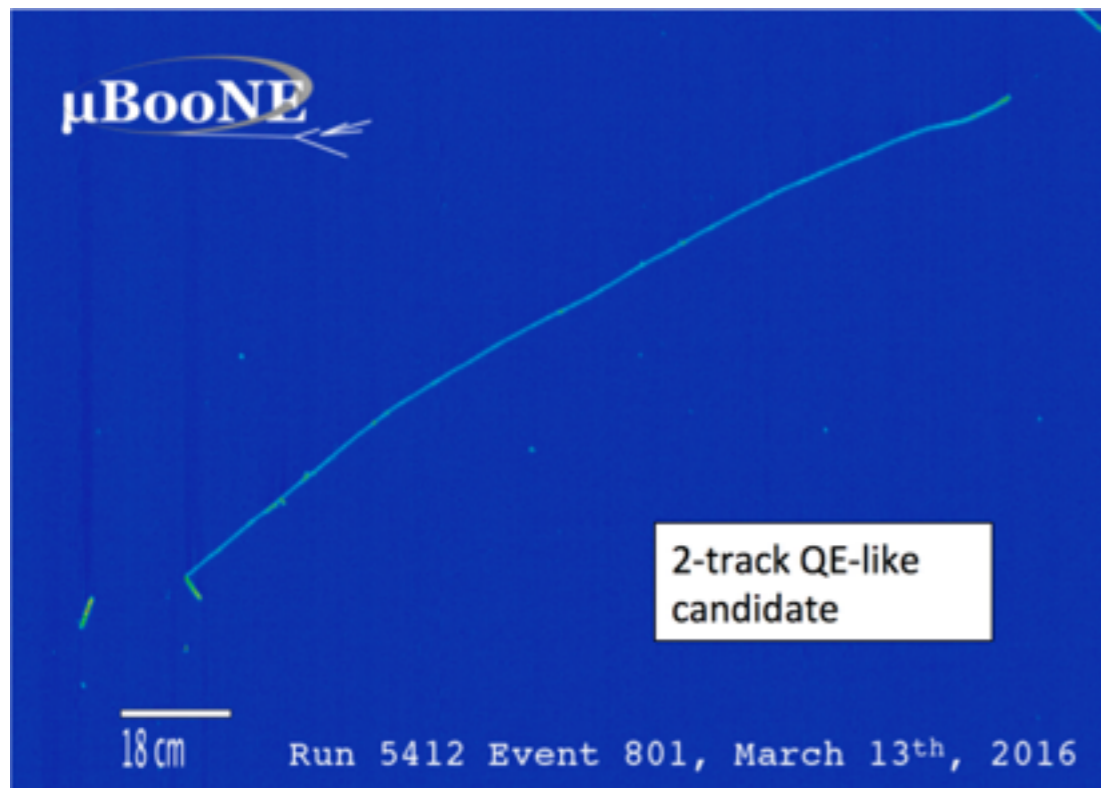


Near detector upgrades for T2K-II and T2HK era
New target with increased angular acceptance



Near Detector Development

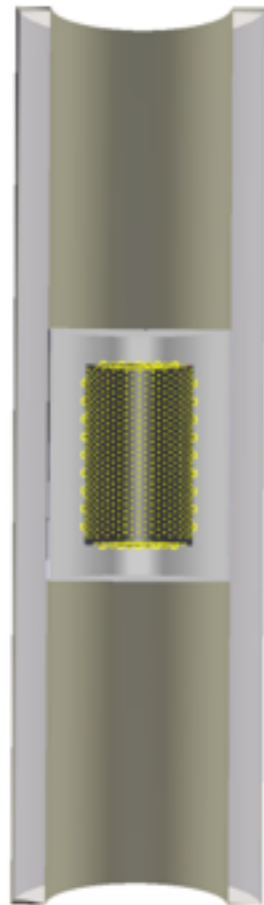
TPC measurements precisely image ν -nucleus interaction vertex
 → better constraints on models



Ultra-low thresholds with gaseous TPC

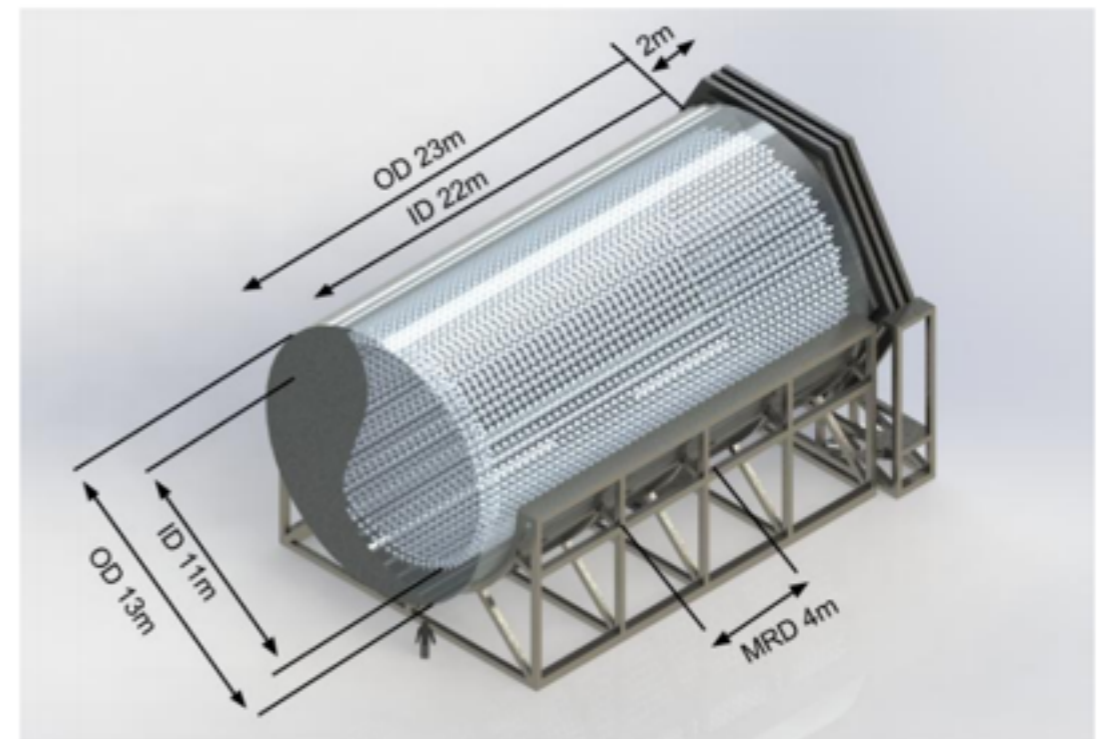
E61 Experiment

Two competing collaborations



nuPRISM

“Water elevator”
Measure $\int \sigma(E)\phi(E)dE$
as a function of theta
[arXiv:1412.3086]



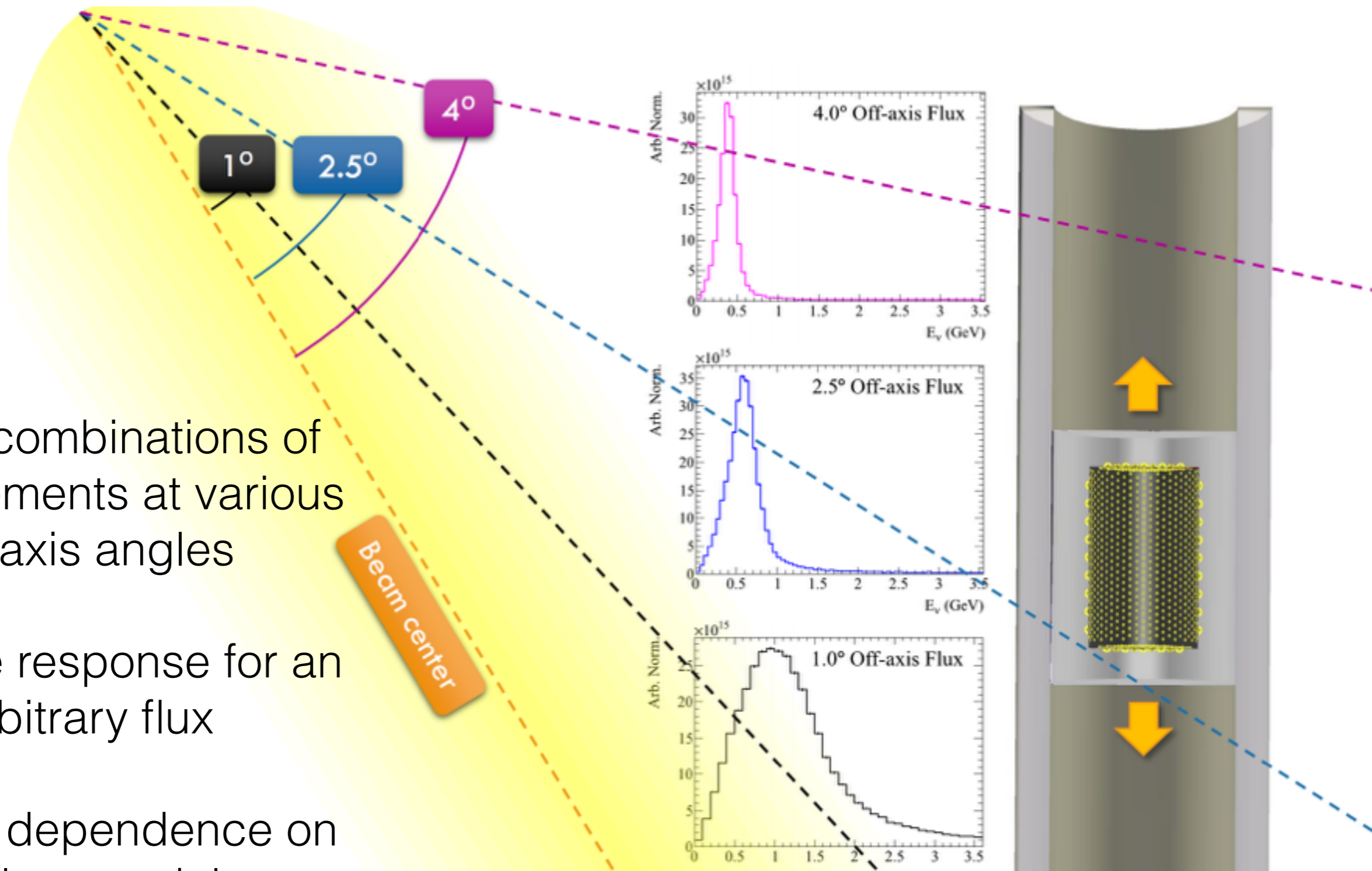
TITUS

same off-axis angle far detector
Gd, muon range detector
[arXiv:1606.08114]

Merged into a single collaboration:

E61 Experiment

E61 Experiment



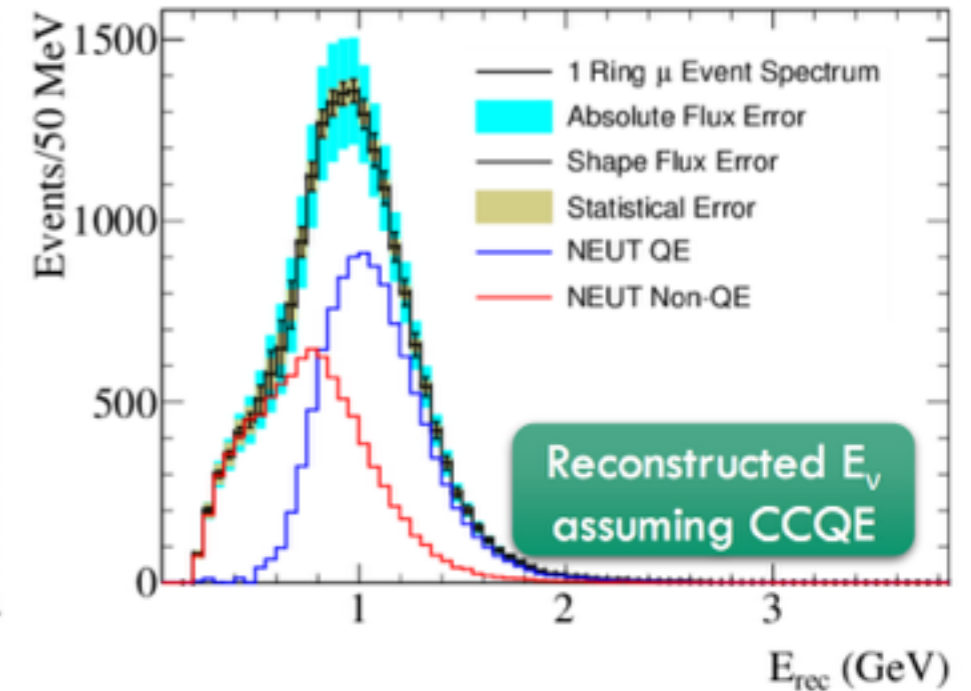
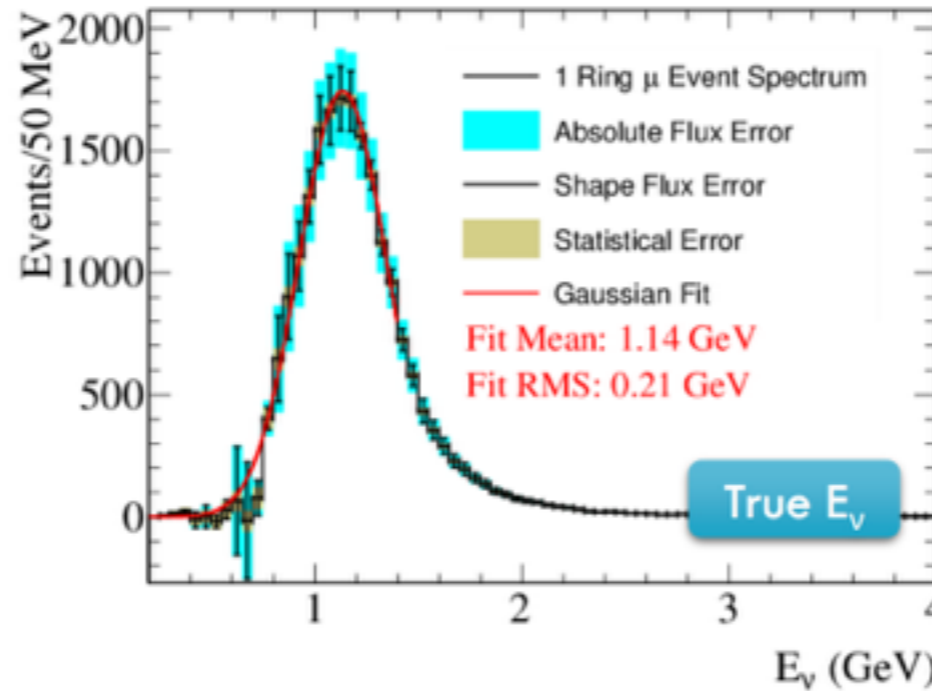
Linear combinations of measurements at various off-axis angles

Measure response for an arbitrary flux

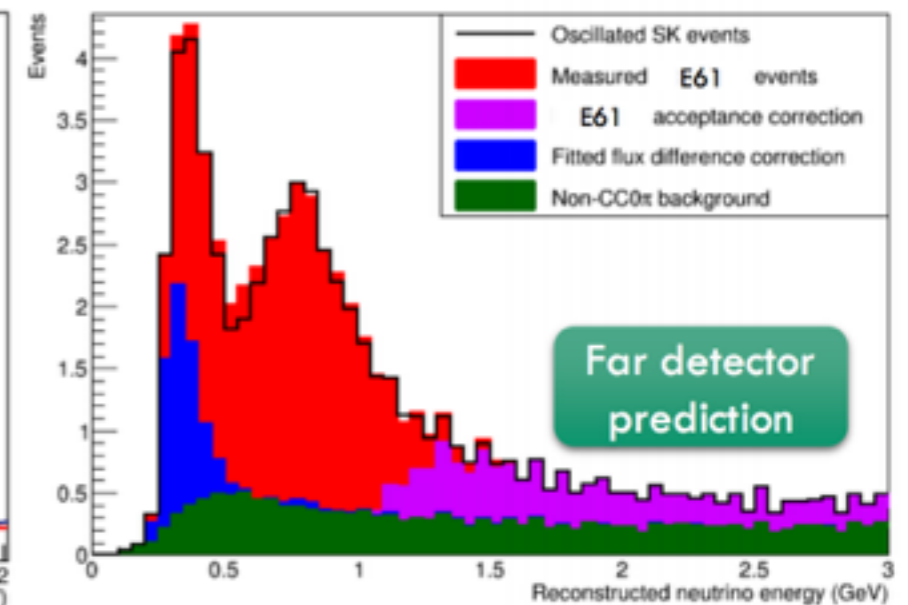
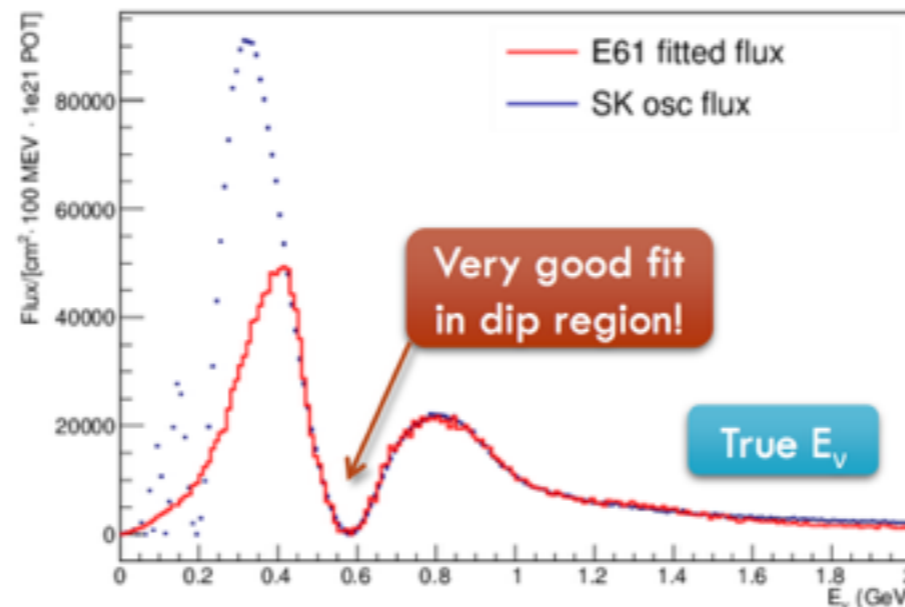
Reduce dependence on nuclear models

E61 Experiment

Pseudo-monochromatic beams



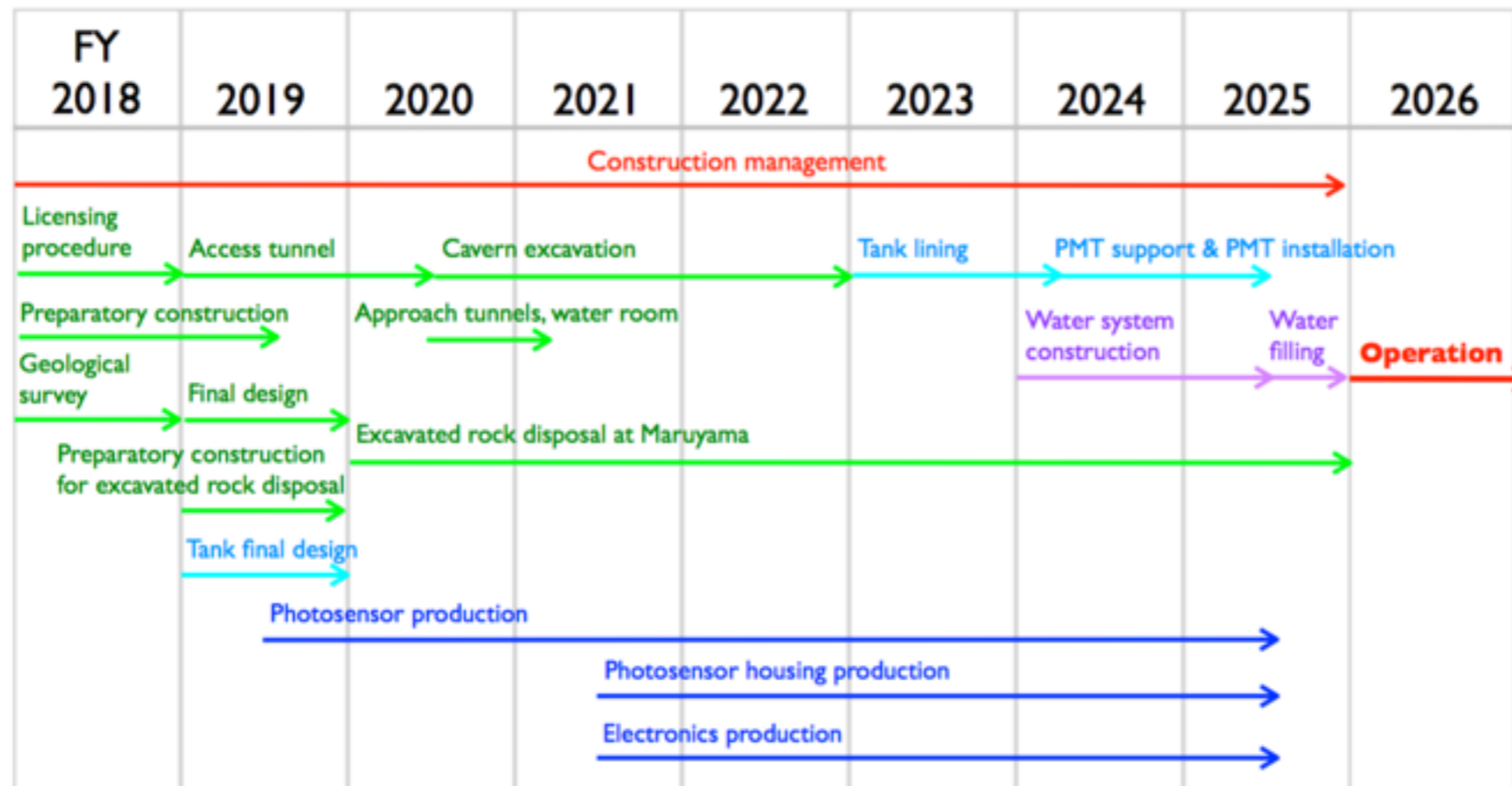
Far detector prediction for oscillated flux



Project Timeline

HK selected in “Master Plan” of Science Council in Japan
 HK selected as highest-priority large-scale projects MEXT
 Roadmap 2017

Funding request in progress
 Construction: 2018, Operation: 2026



Summary

Hyper-K well placed to build on the huge success of Super-K experiment

Capable of world leading measurements in neutrino oscillations, nucleon decay, neutrino astrophysics

Aim to start construction 2018 for operation in 2026

References:

T2HKK White Paper, arXiv:1611.06118 [hep-ex]

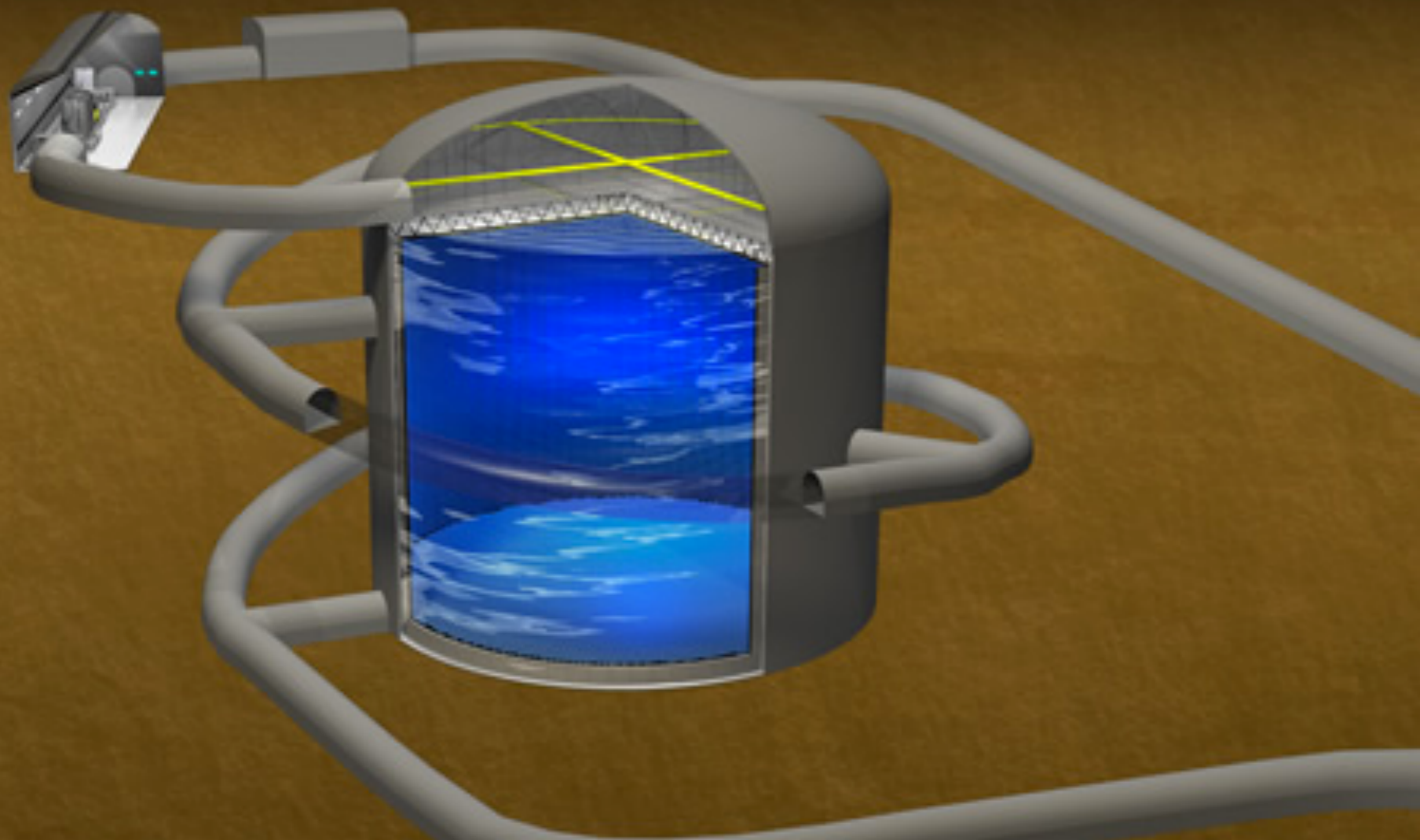
HK Design Report, KEK Preprint 2016-21

HK Physics Sensitivity, PTEP (2015) 053C02



Hyper-k

David Hadley, University of Warwick



Backup



Physics at Hyper-K

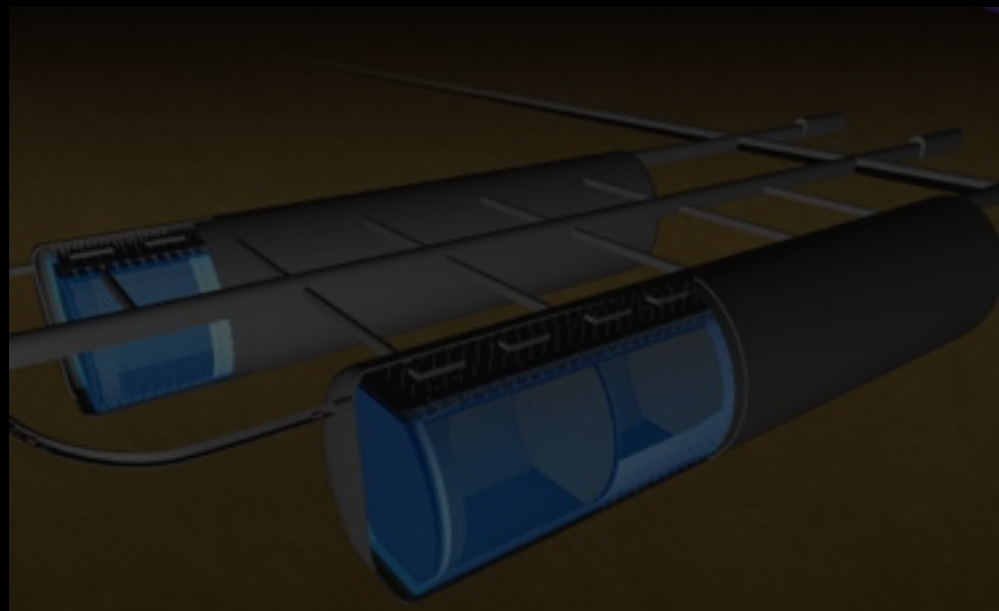
Proton Decay

$$p \rightarrow e^+ + \pi^0$$

$> 1.3 \times 10^{35}$ years 90% CL

$$p \rightarrow \bar{\nu} + K^+$$

$> 3.2 \times 10^{34}$ years 90% CL



Neutrinos

Solar

200 solar ν per day
Indirect dark matter search

Supernova

SN $\sim 200,000$ @ 10kPC

SN $\sim 30-50$ @ M31

Accelerator

Leptonic CP violation

(see following slides)

Mass Hierarchy determination

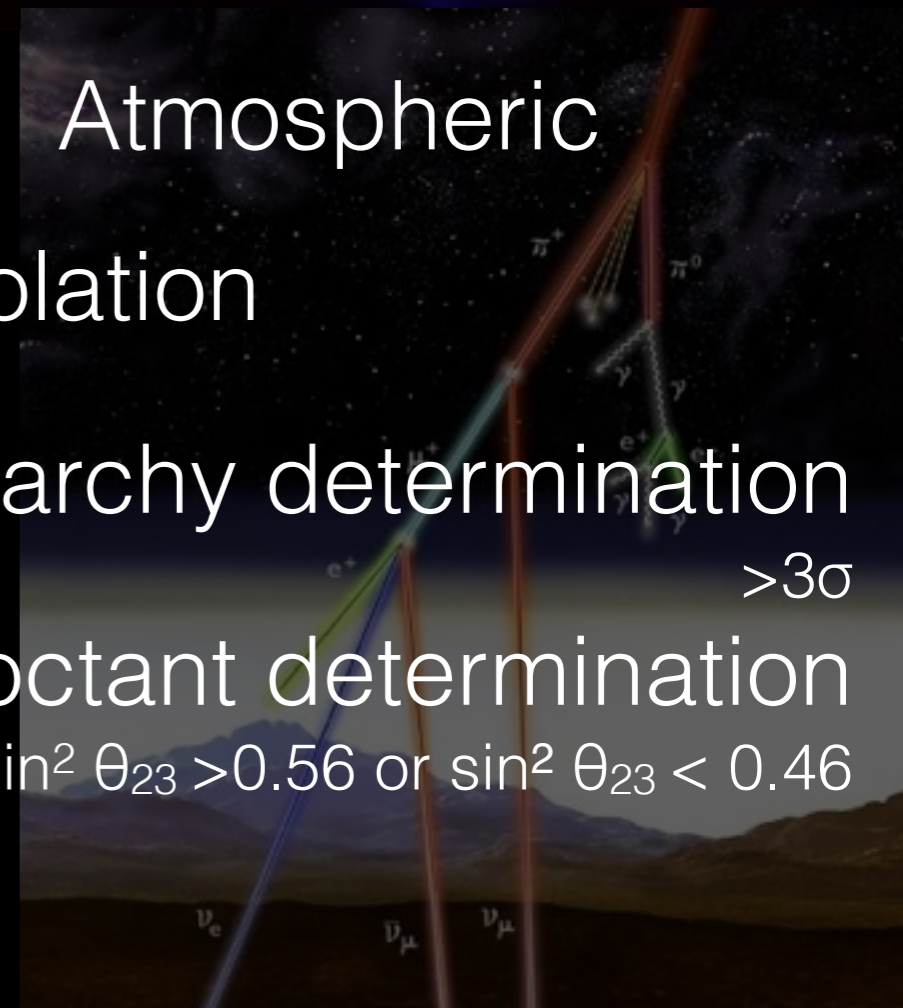
$> 3\sigma$

θ_{23} octant determination

3σ for $\sin^2 \theta_{23} > 0.56$ or $\sin^2 \theta_{23} < 0.46$

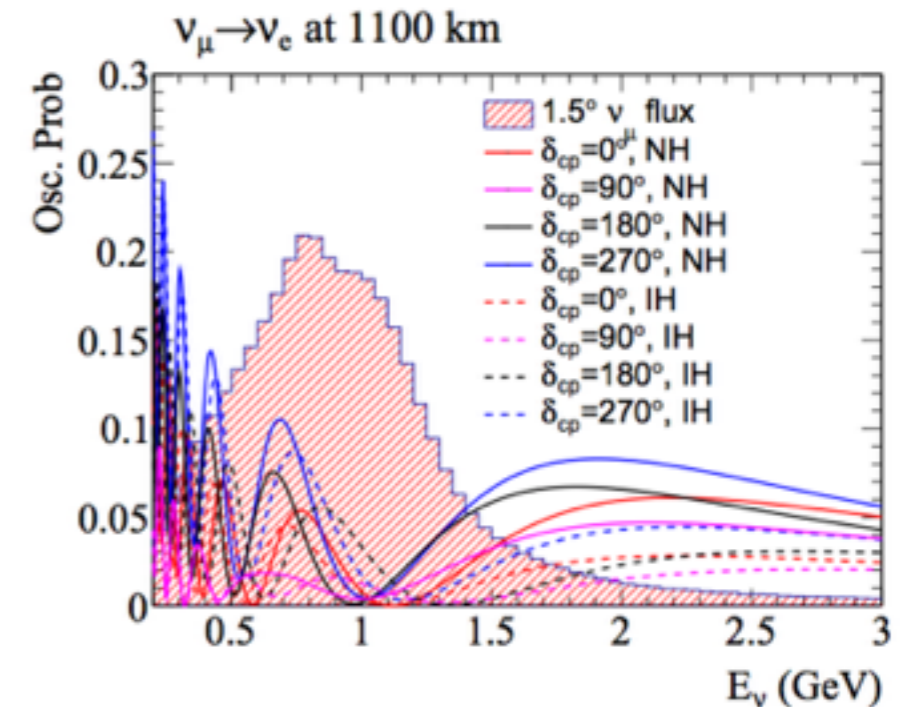
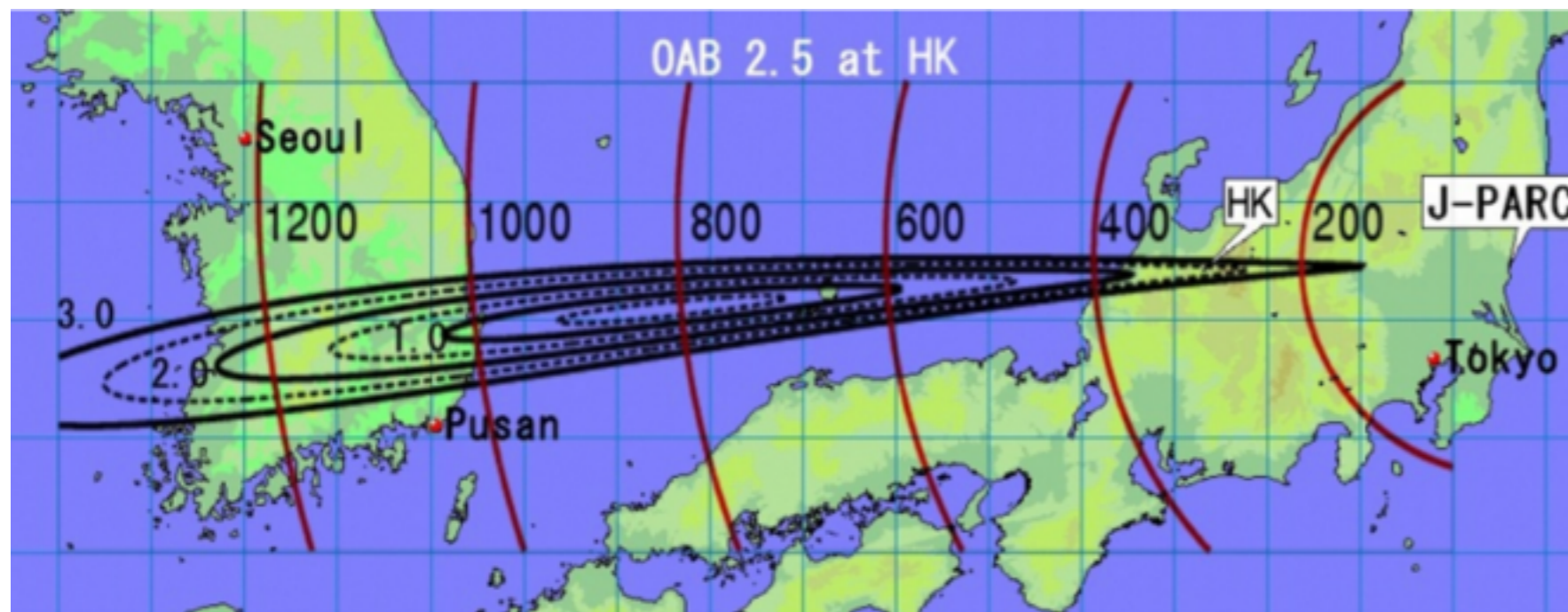


Atmospheric



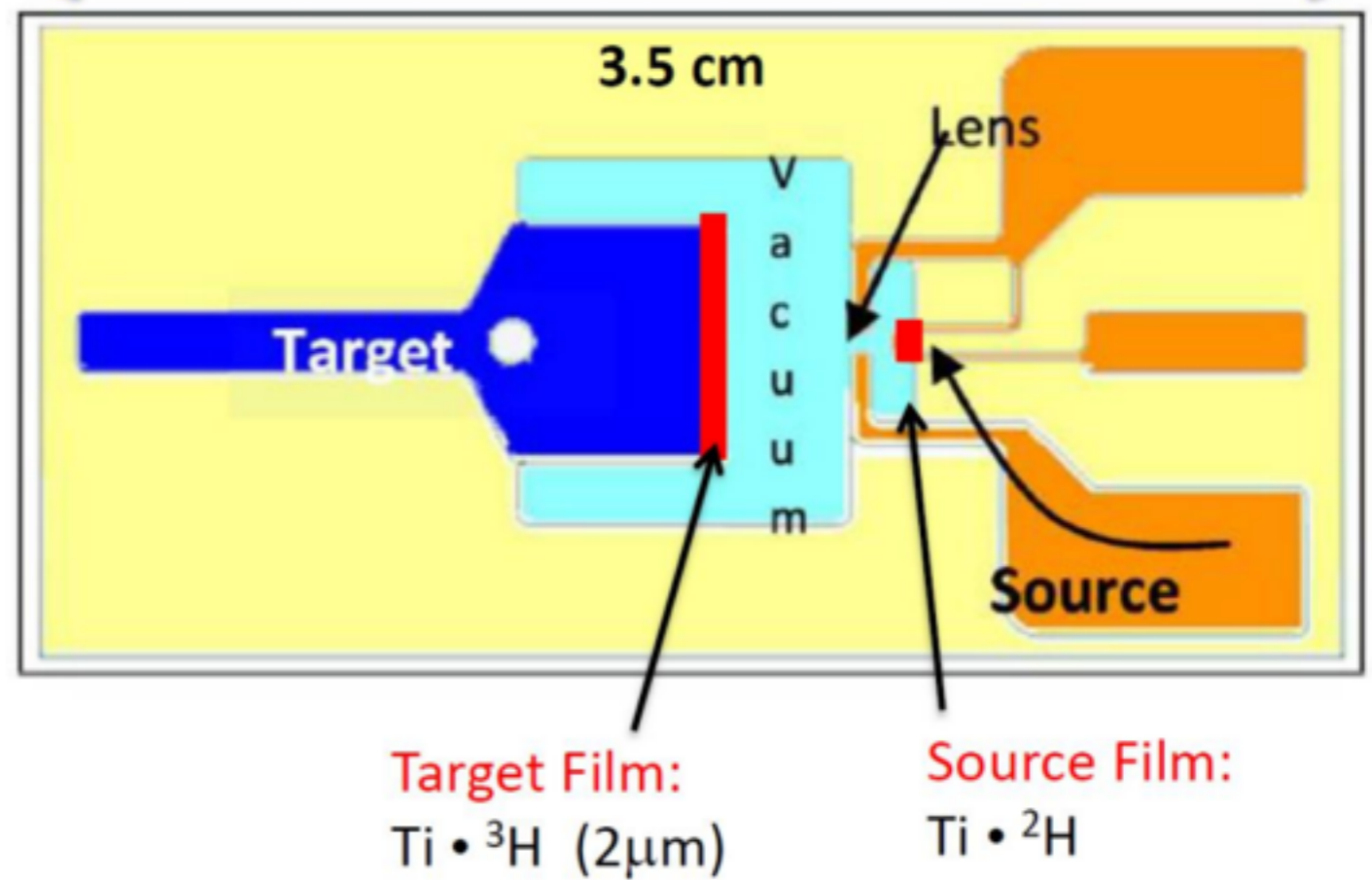
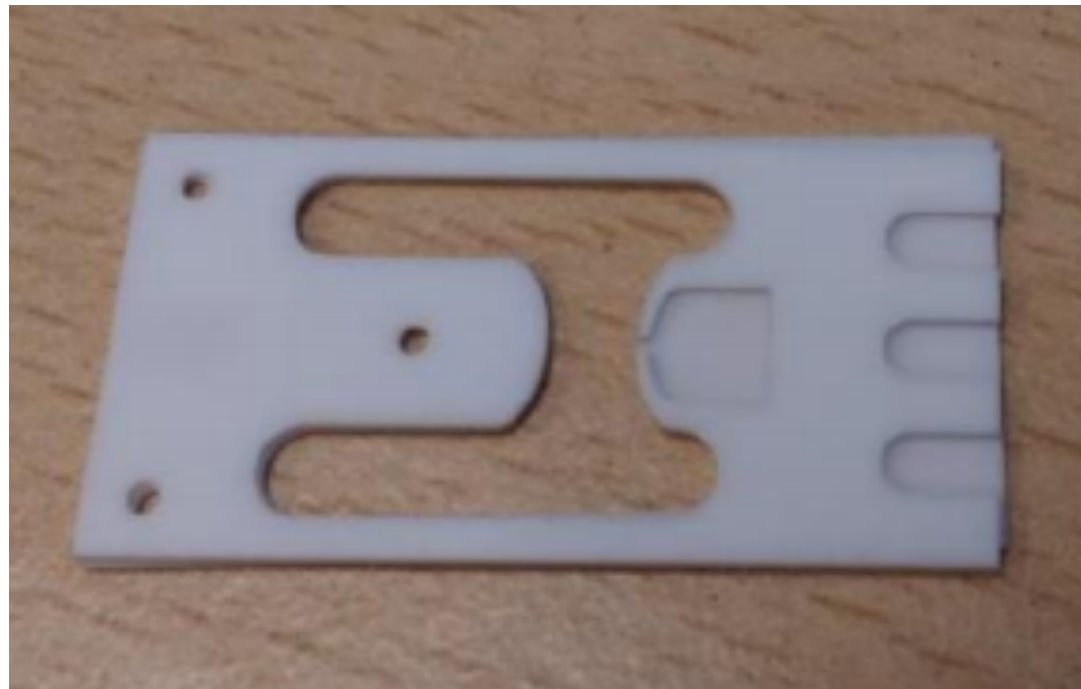
Broad physics programme.

Korean Tank



Stronger CP effect at the second oscillation maximum

A second tank in Korea would be able to measure this effect

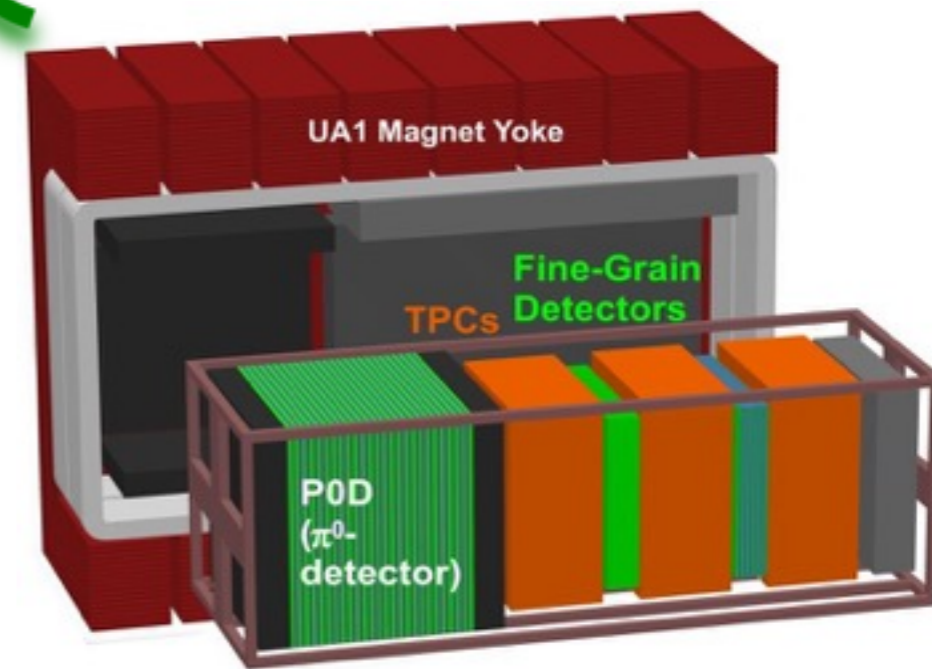


T2K

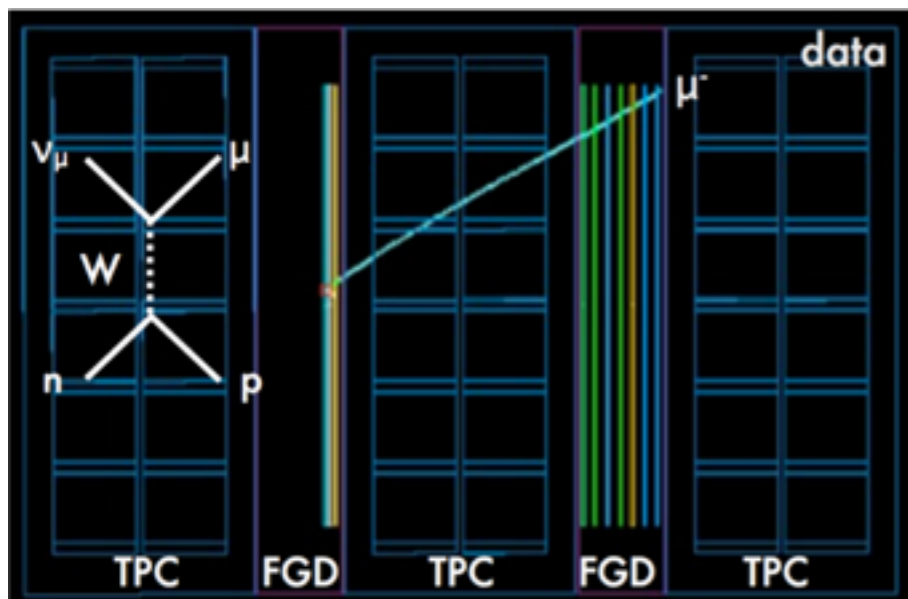
Carbon and Oxygen target materials

Acceptance differs from far detector

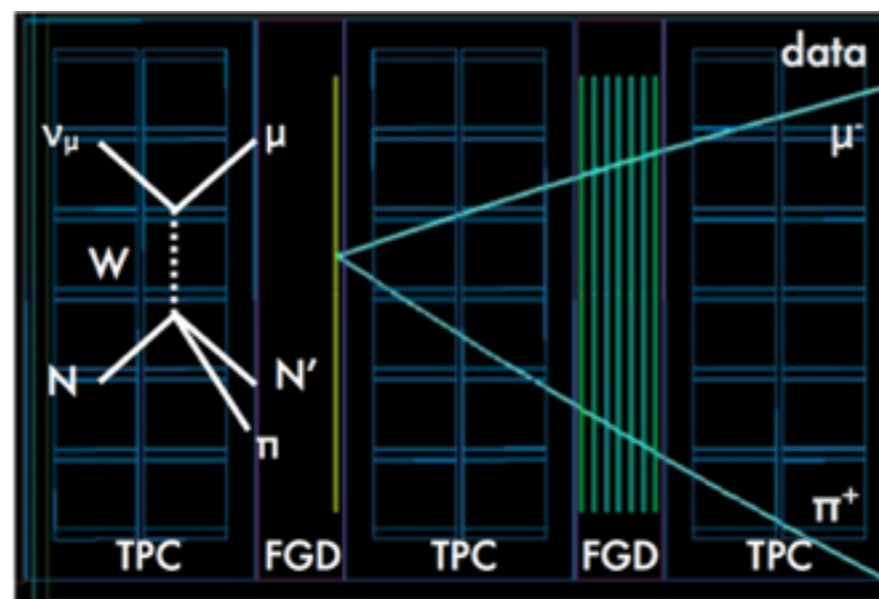
Magnetic field for sign selection



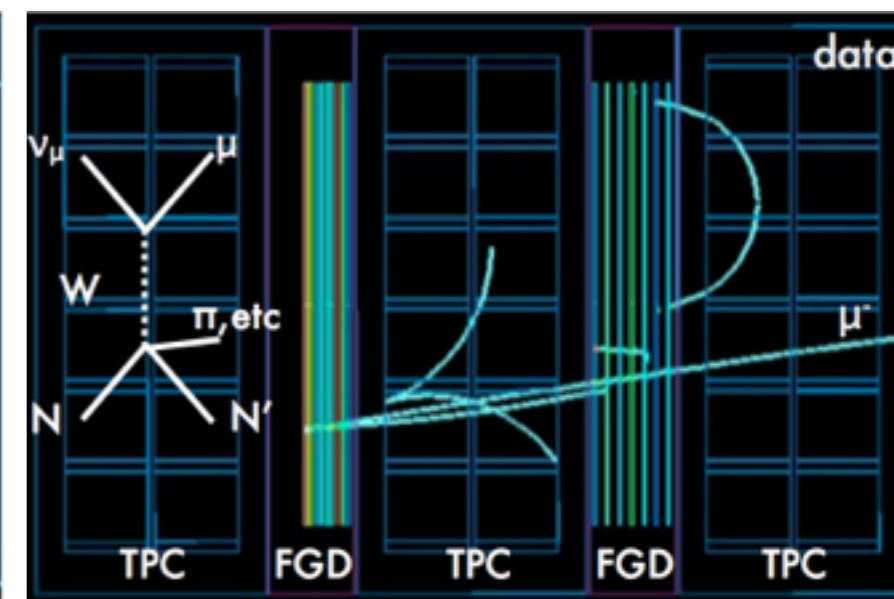
Near Detector (ND280)



CC $1\mu + 0\pi + X$



CC $1\mu + 1\pi^+ + X$



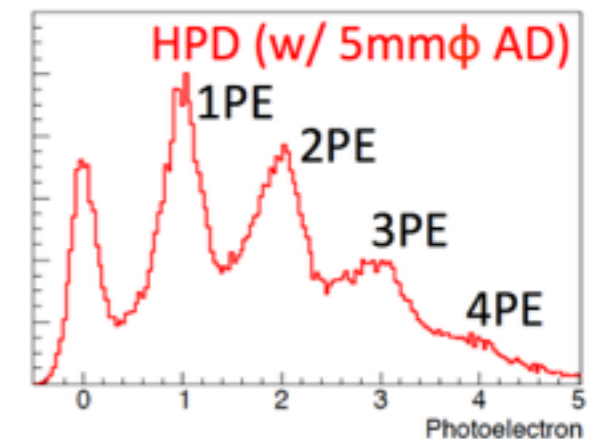
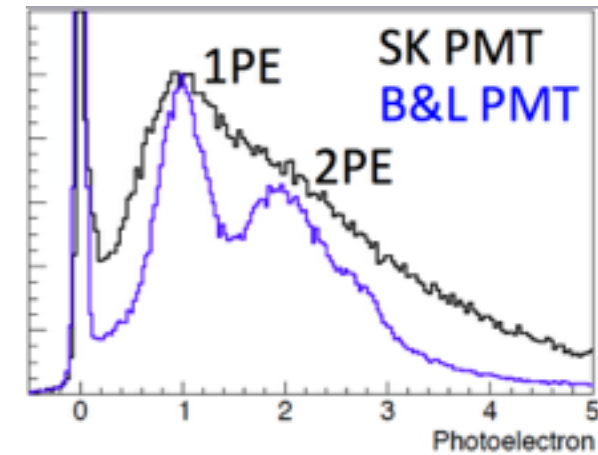
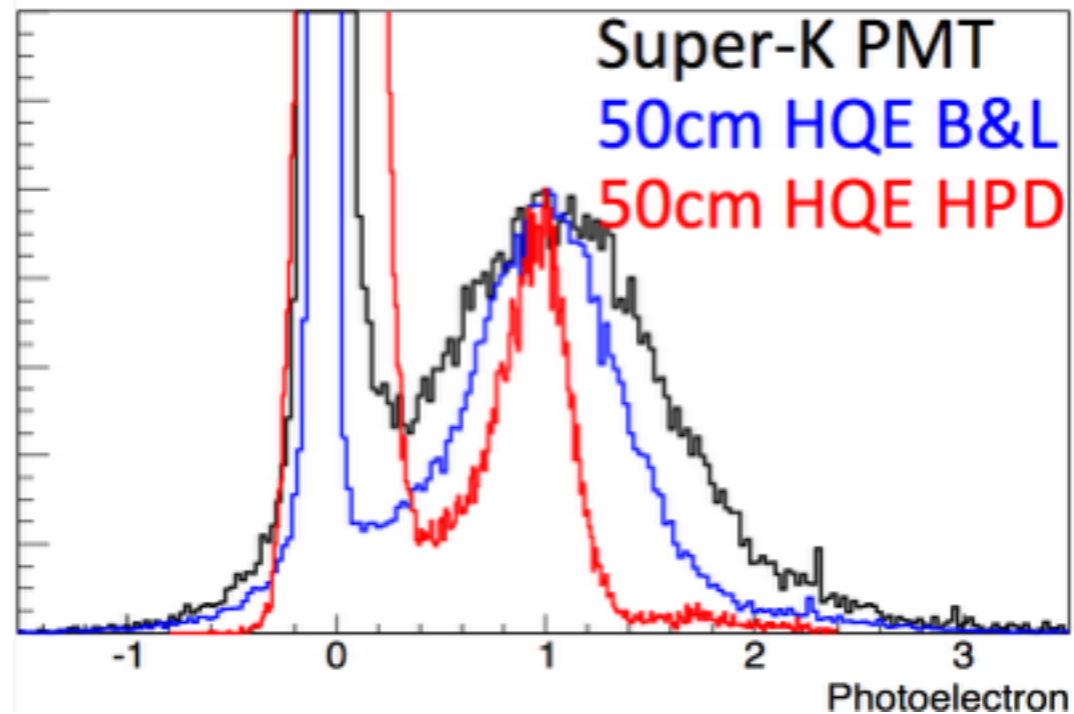
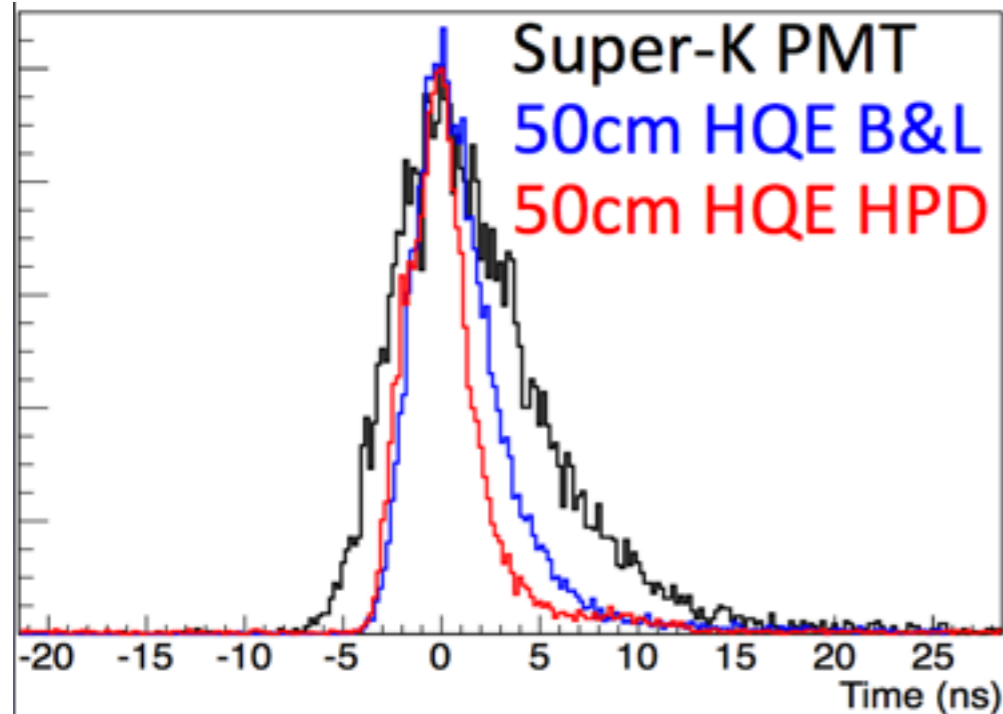
CC other

Photo Sensors

Time Resolution

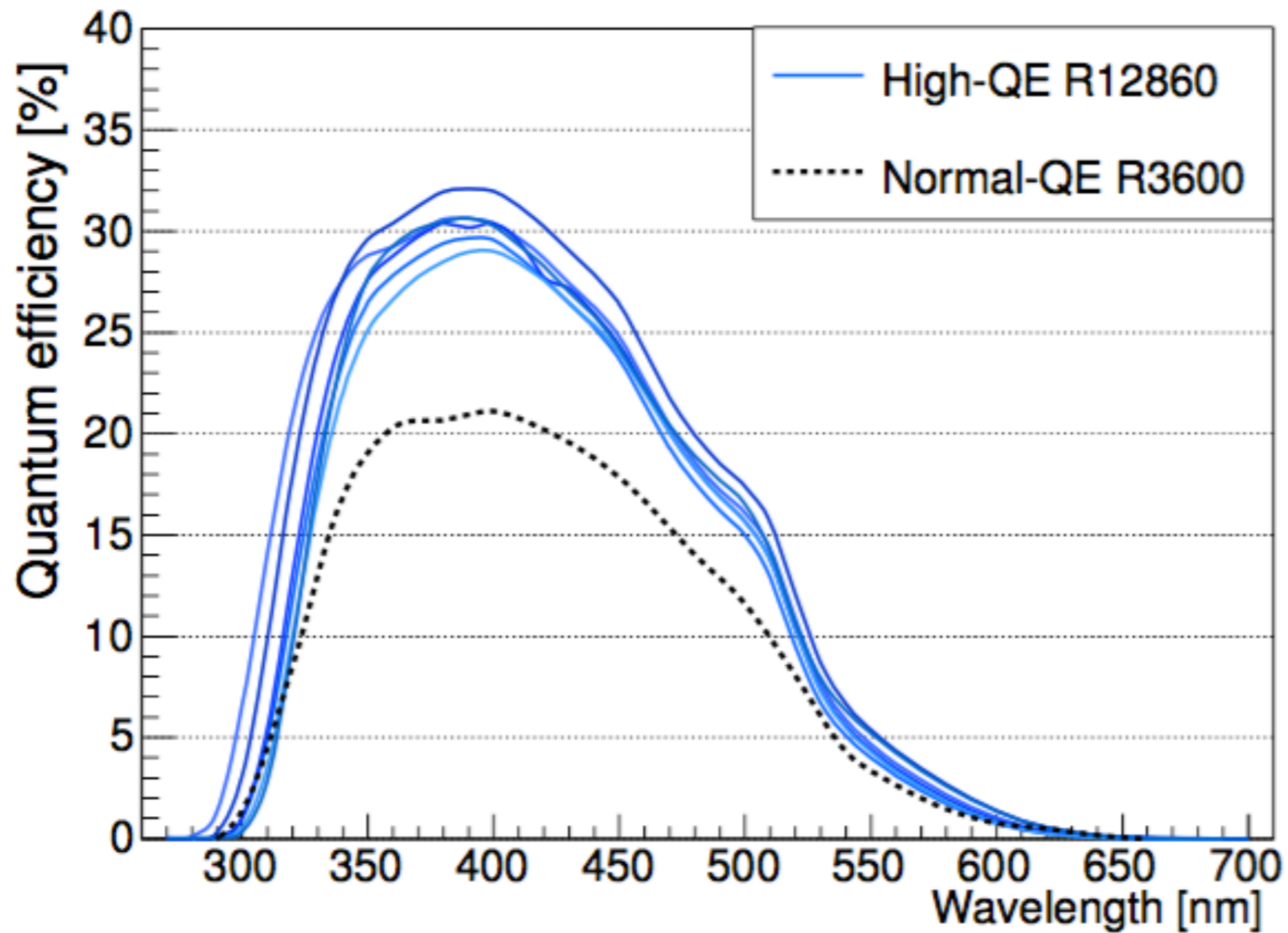
1p.e. charge distribution

Multi-p.e. charge distribution



	SK PMT	B&L PMT	50cm HPD (20cm)
1PE T resolution σ (ns)	2.1	1.1	1.4 (1.1)
FWHM (ns)	7.3	4.1	3.4 (3.3)
1PE Q resolution σ /mean	53%	35%	16% (12%)
Peak-to-Valley ratio	2.2	4.3	3.9 (5.2)

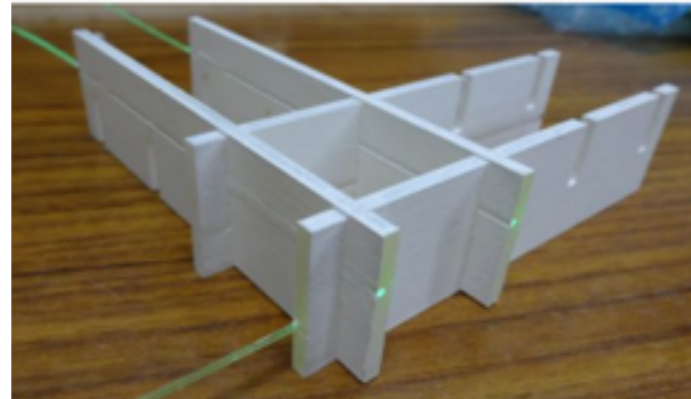
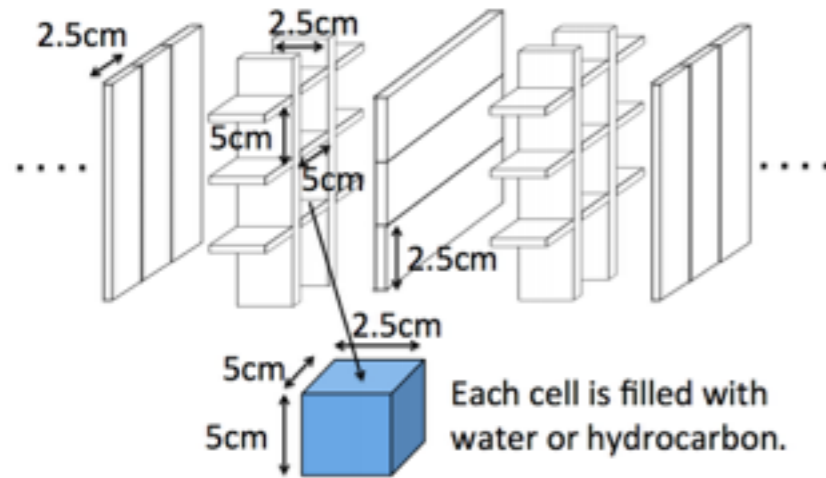
Photo Sensors



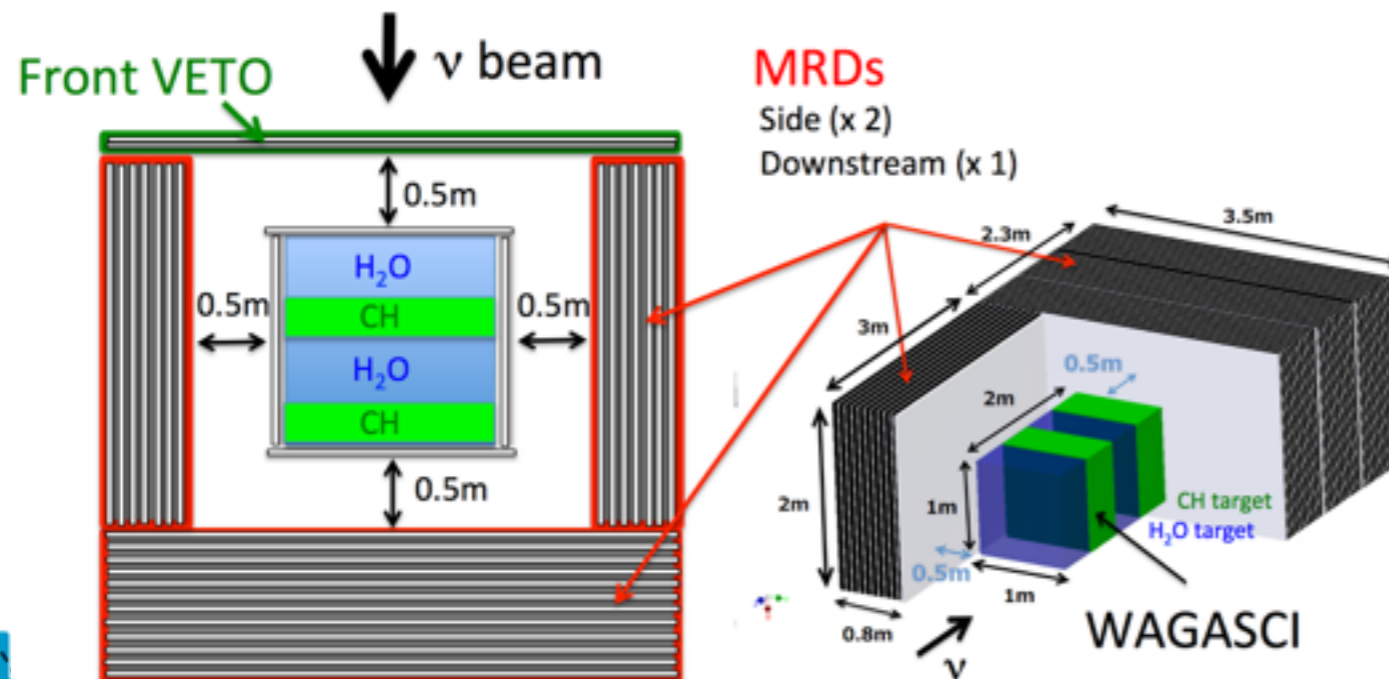
Near Detector Development

New/Upgraded Detectors in the Existing ND280 Complex

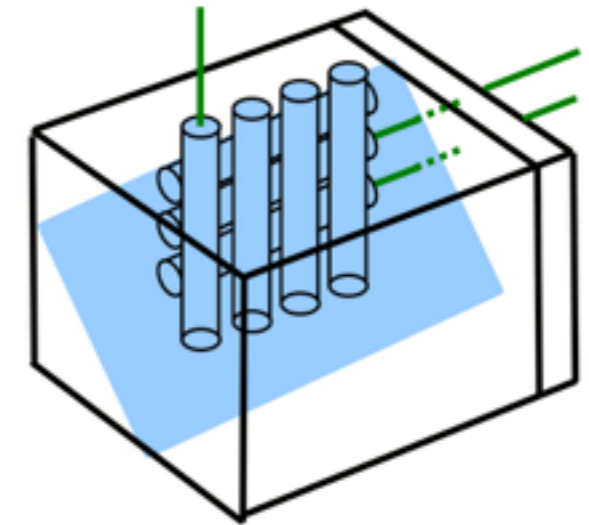
WAGASHI



Water dominated target
4π acceptance



Water based
liquid scintillator



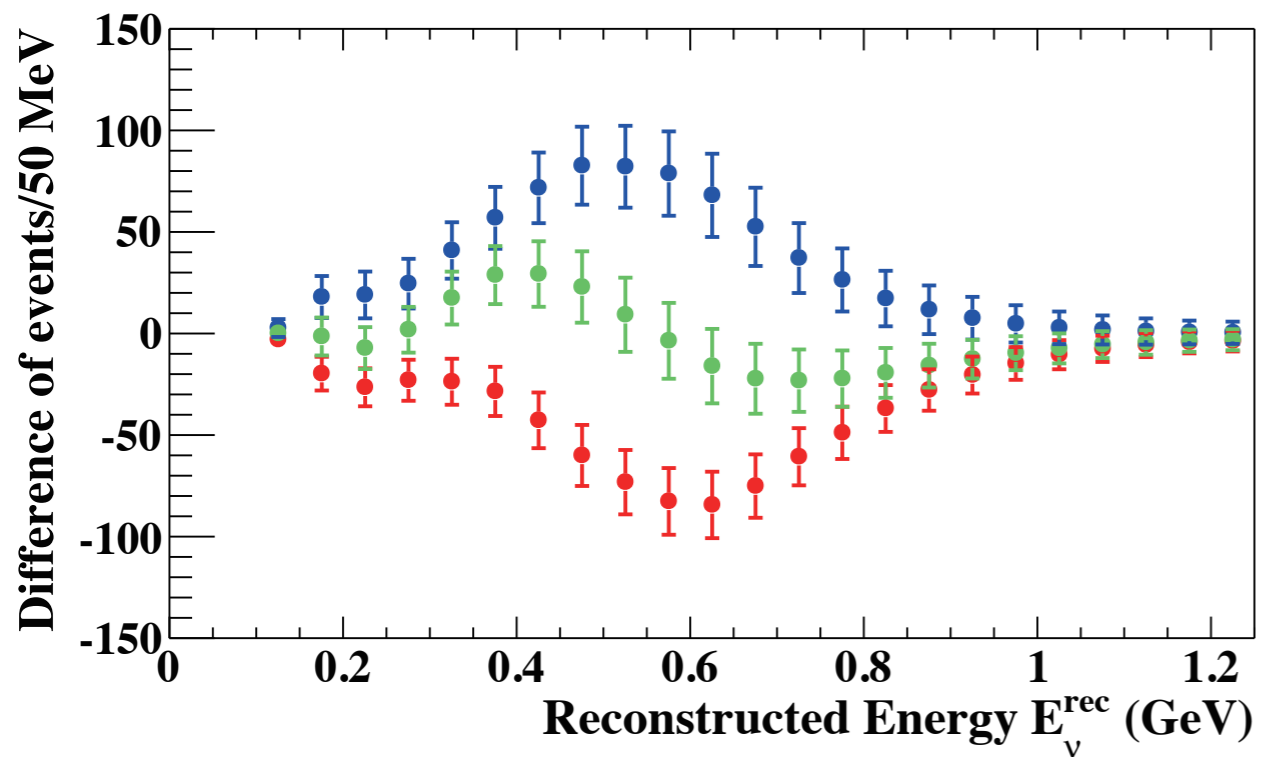
An alternative approach is to improve knowledge of neutrino-nucleus interactions

e.g. High Pressure Gas TPC



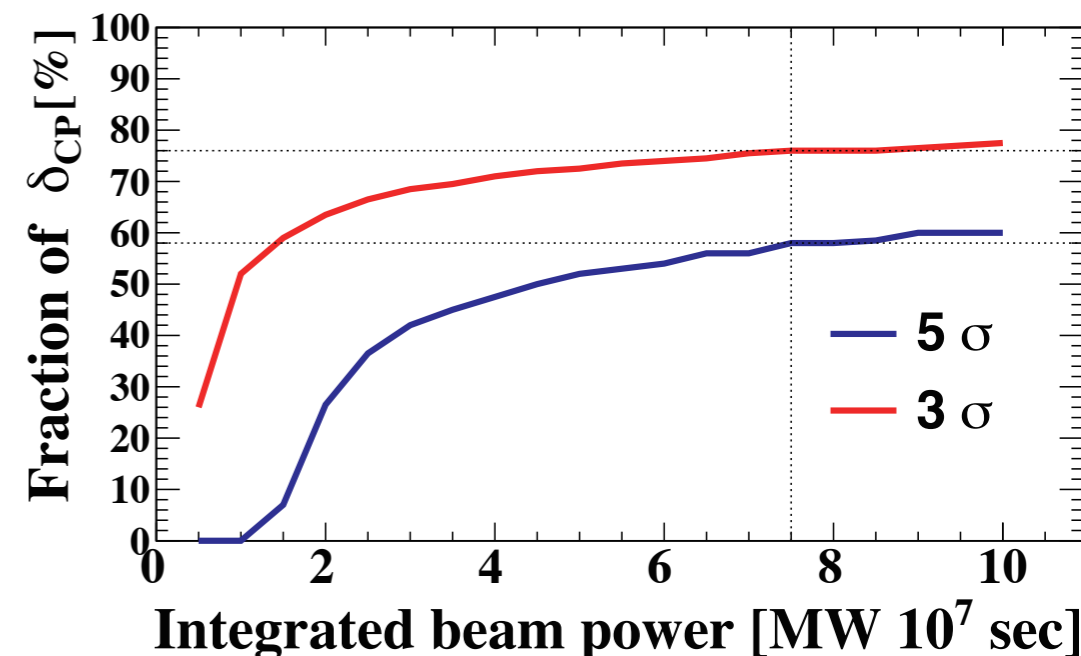
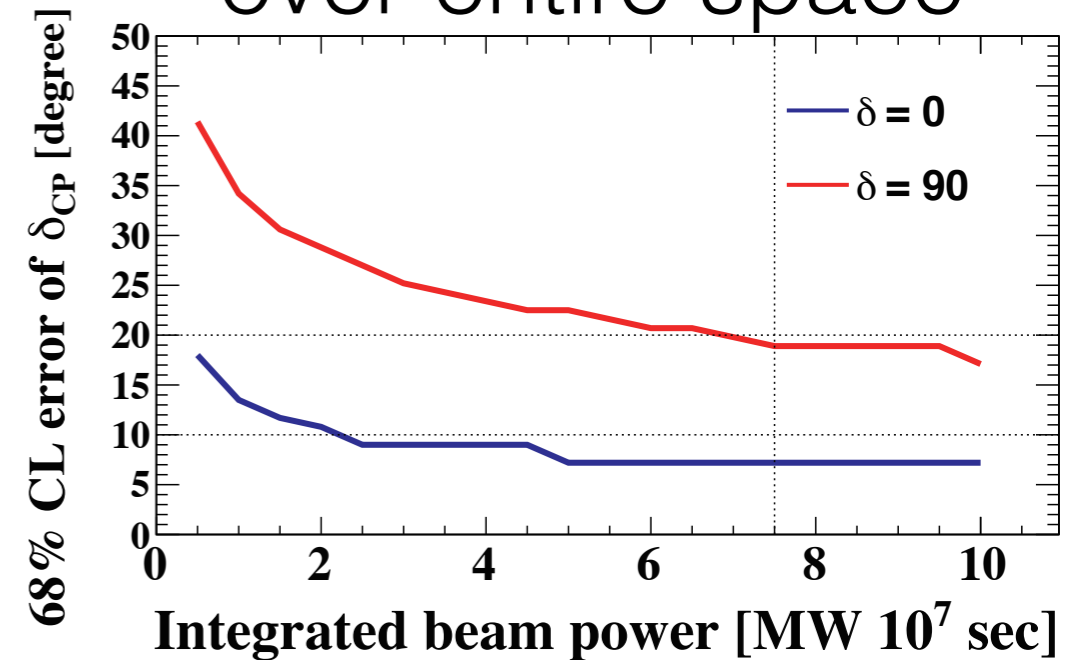
Leptonic CP Violation

Measure δ_{CP} by comparing data with beam in ν -mode with anti- ν mode

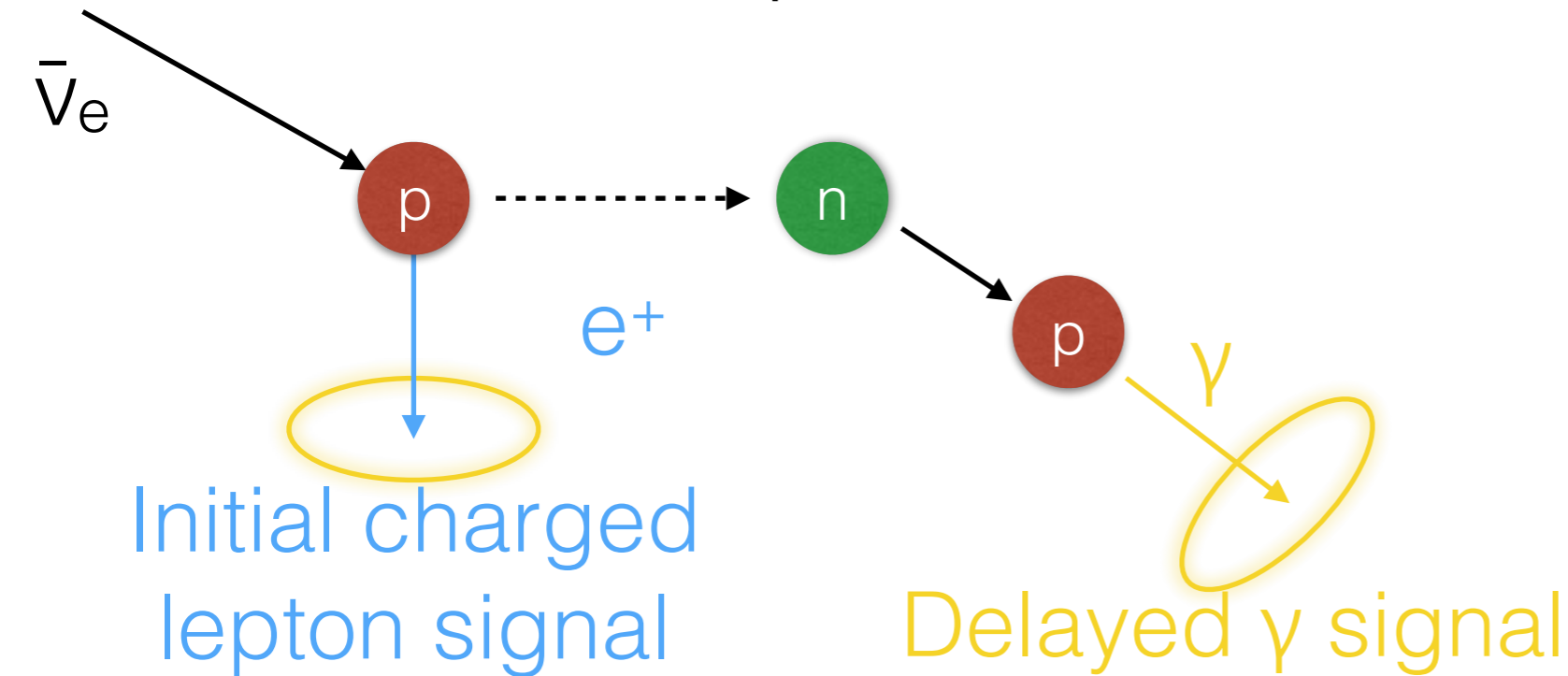
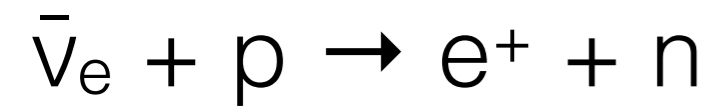


CP violation can be established at 3σ (5σ) for 76% (58%) of δ_{CP} space.

δ_{CP} measured to $< 20^\circ$ over entire space



Neutron Capture on Hydrogen



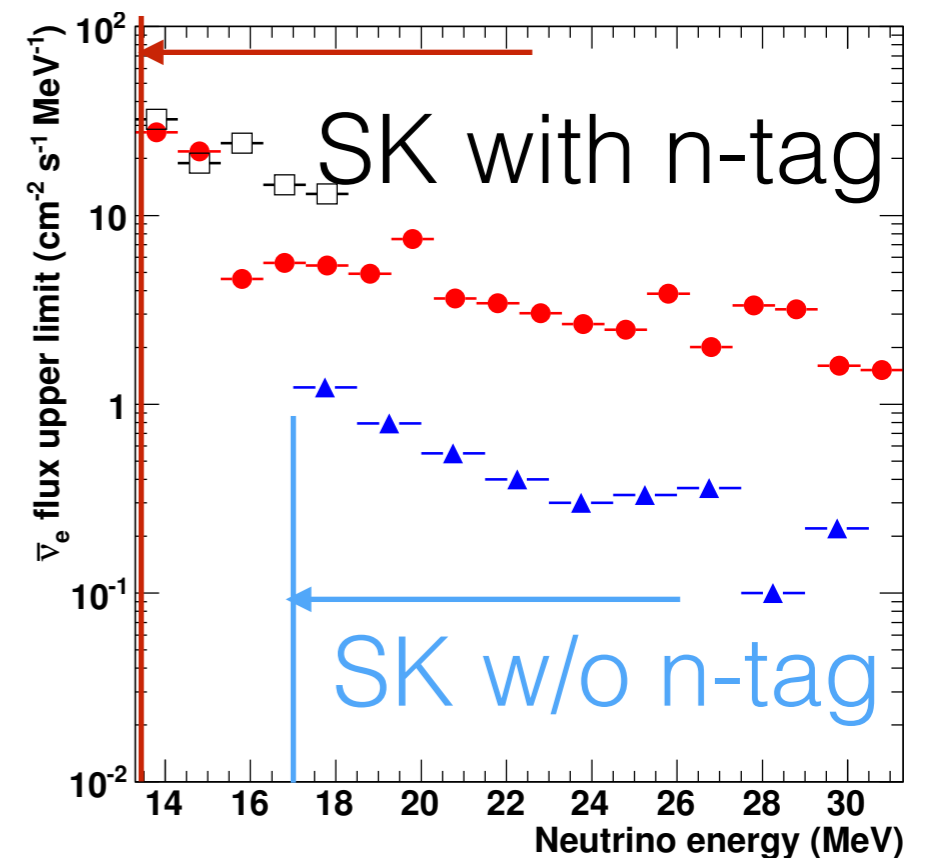
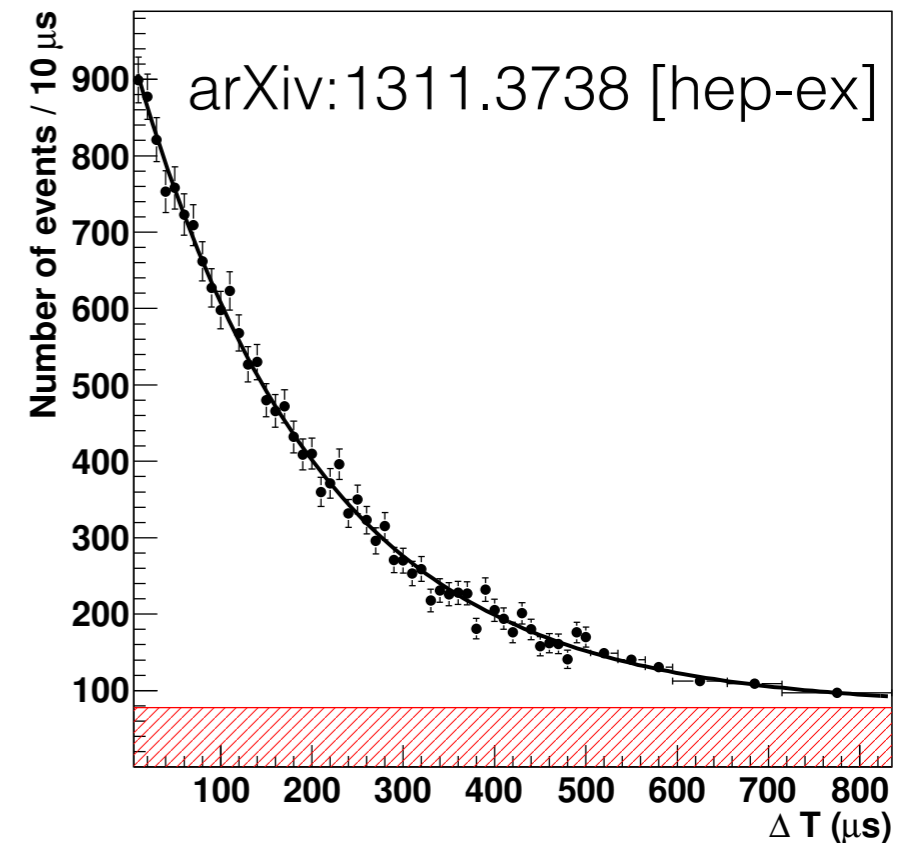
200 μs capture time

$E_\gamma = 2.2 \text{ MeV}$

Low light yield

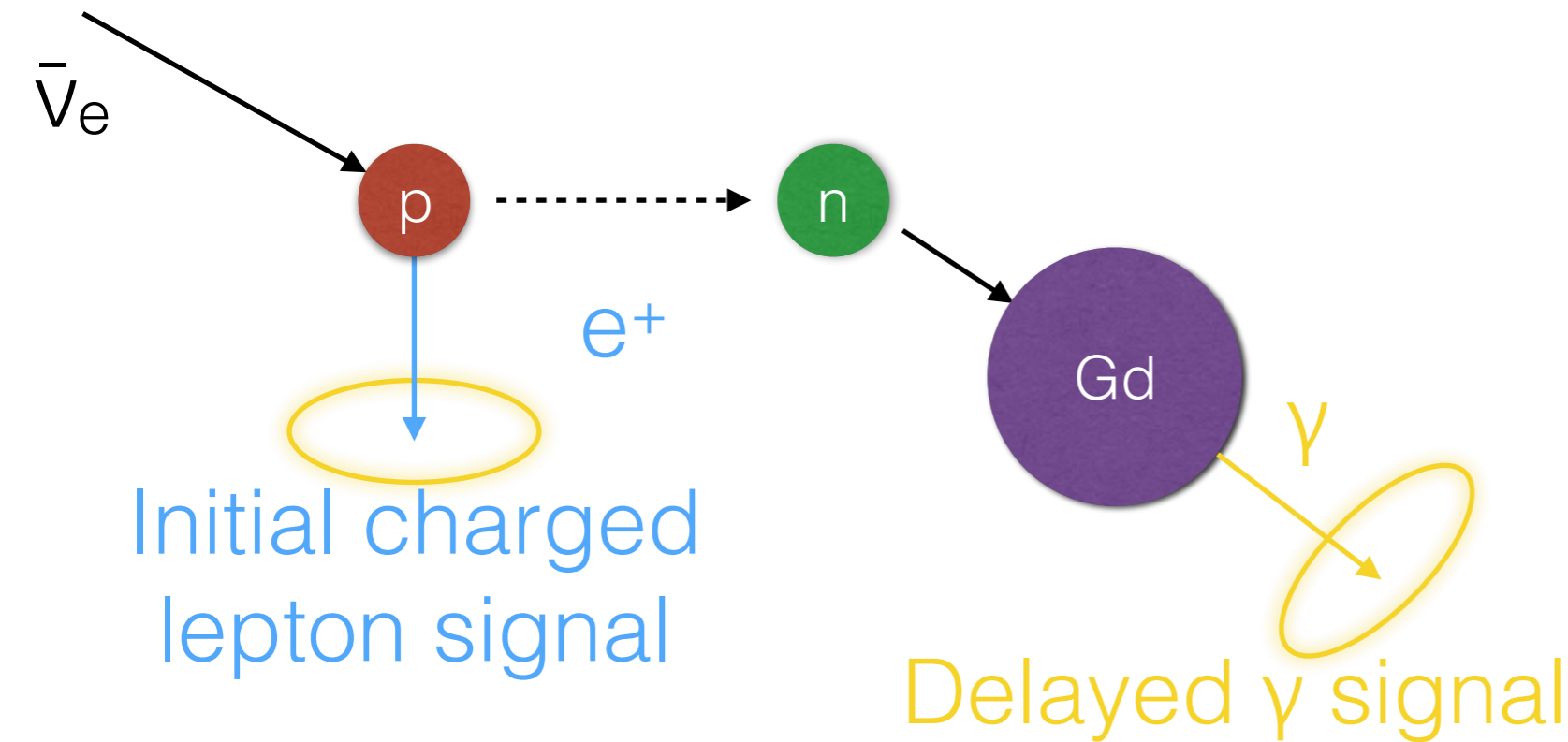
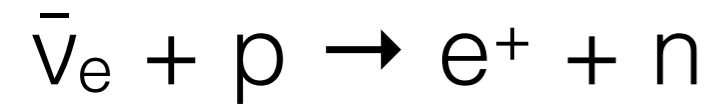
Close to or below trigger threshold

Low detection efficiency ($\sim 18\%$)



Neutron Capture on Gadolinium

arXiv:0811.0735 [hep-ex]

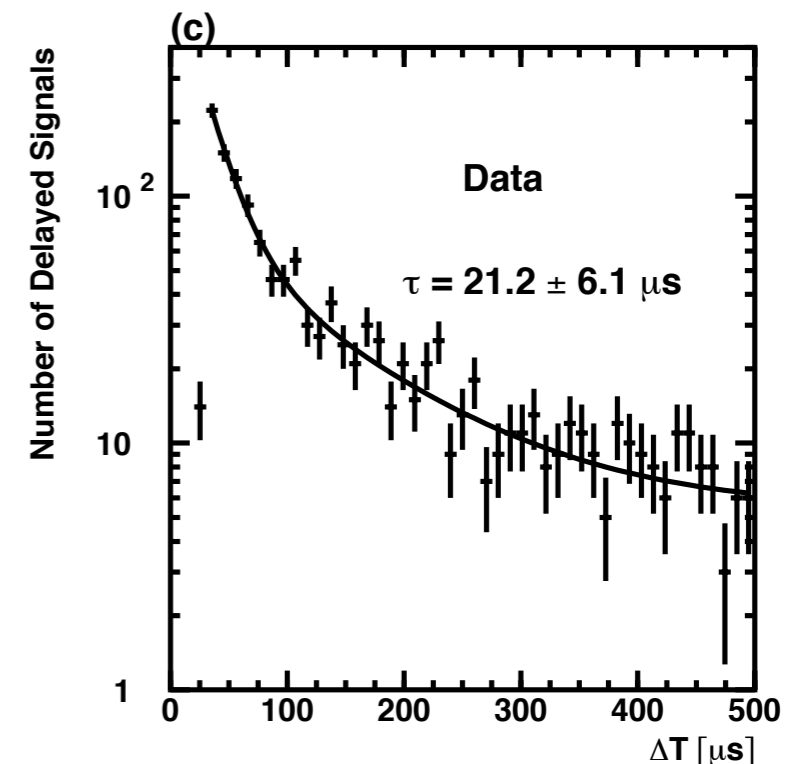
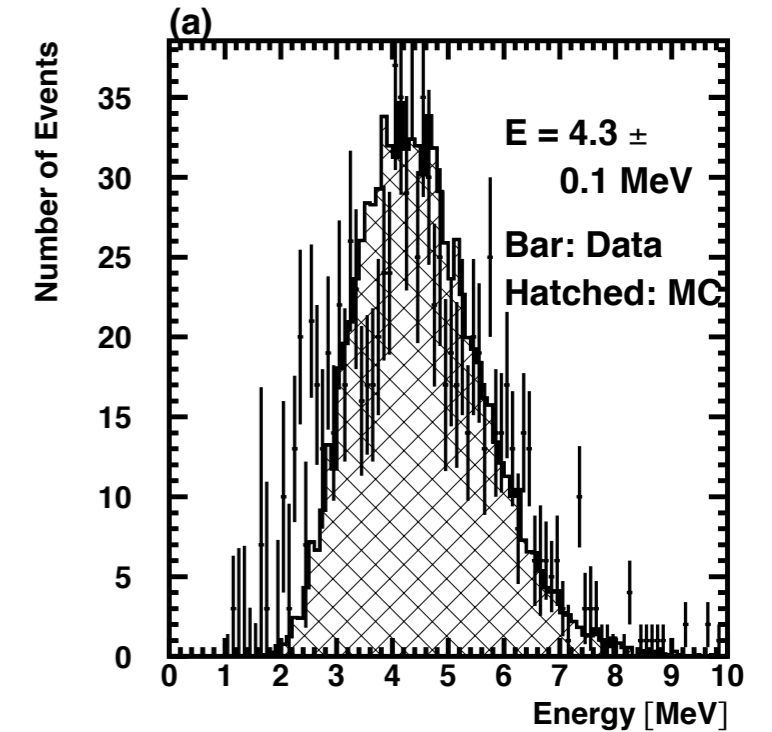


20 μs capture time

$E_\gamma \sim 8 \text{ MeV}$ cascade ($\sim 4 \text{ MeV}$ visible)

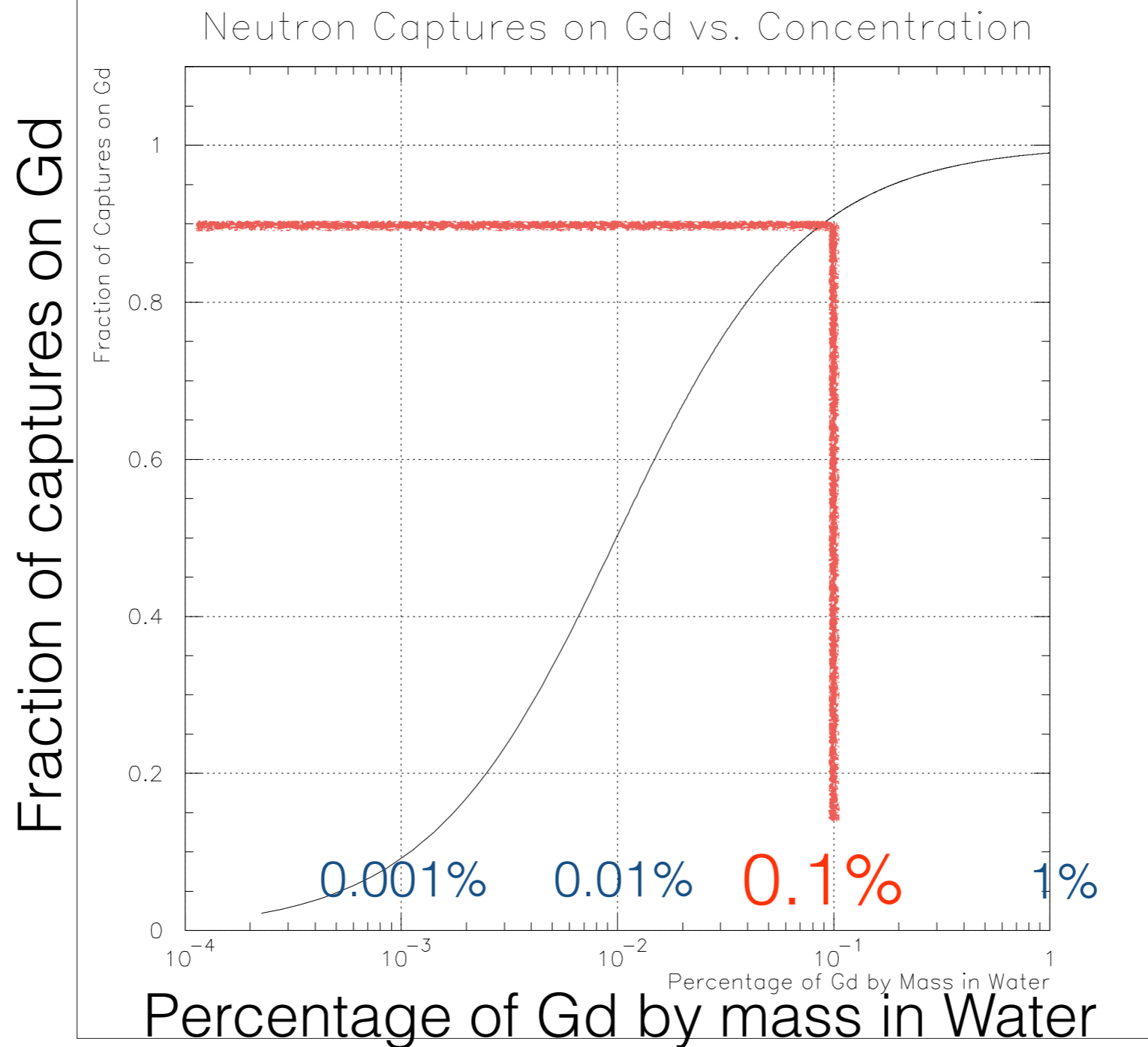
Fast capture time (small ΔT window)

Higher energy γ signal



Neutron Capture on Gadolinium

Cross section for neutron capture: Gd (49,700 b), H (0.3 b)



0.1% Gd fraction gives 90% neutrons captured on Gd.

Applications: Supernova Relic Neutrinos

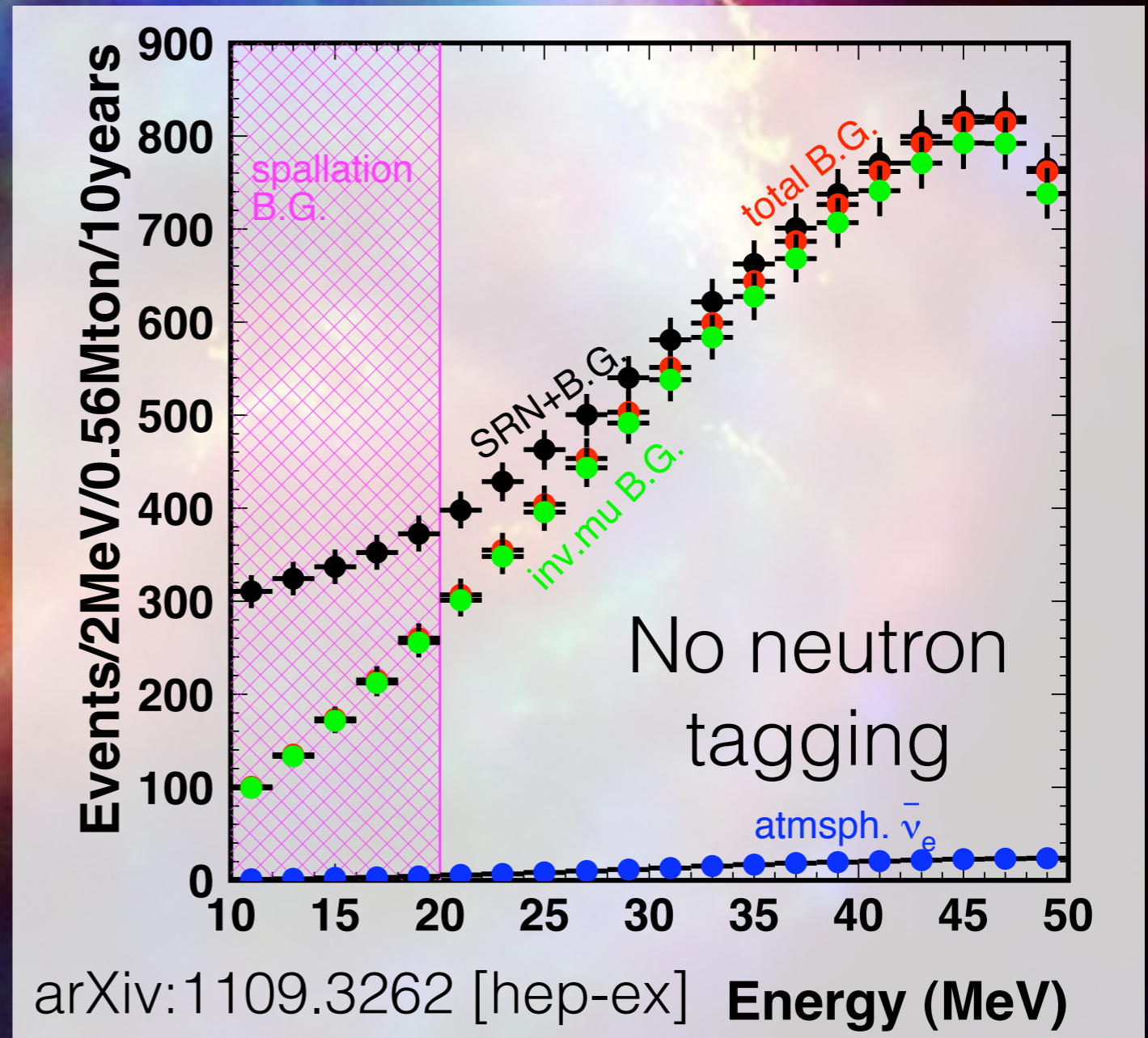
A low energy example

Directly observable local supernova are all too rare

Alternative is to measure diffuse supernova background DSNB/SRN

Very low rate

Large backgrounds



Applications: Supernova Relic Neutrinos

A low energy example

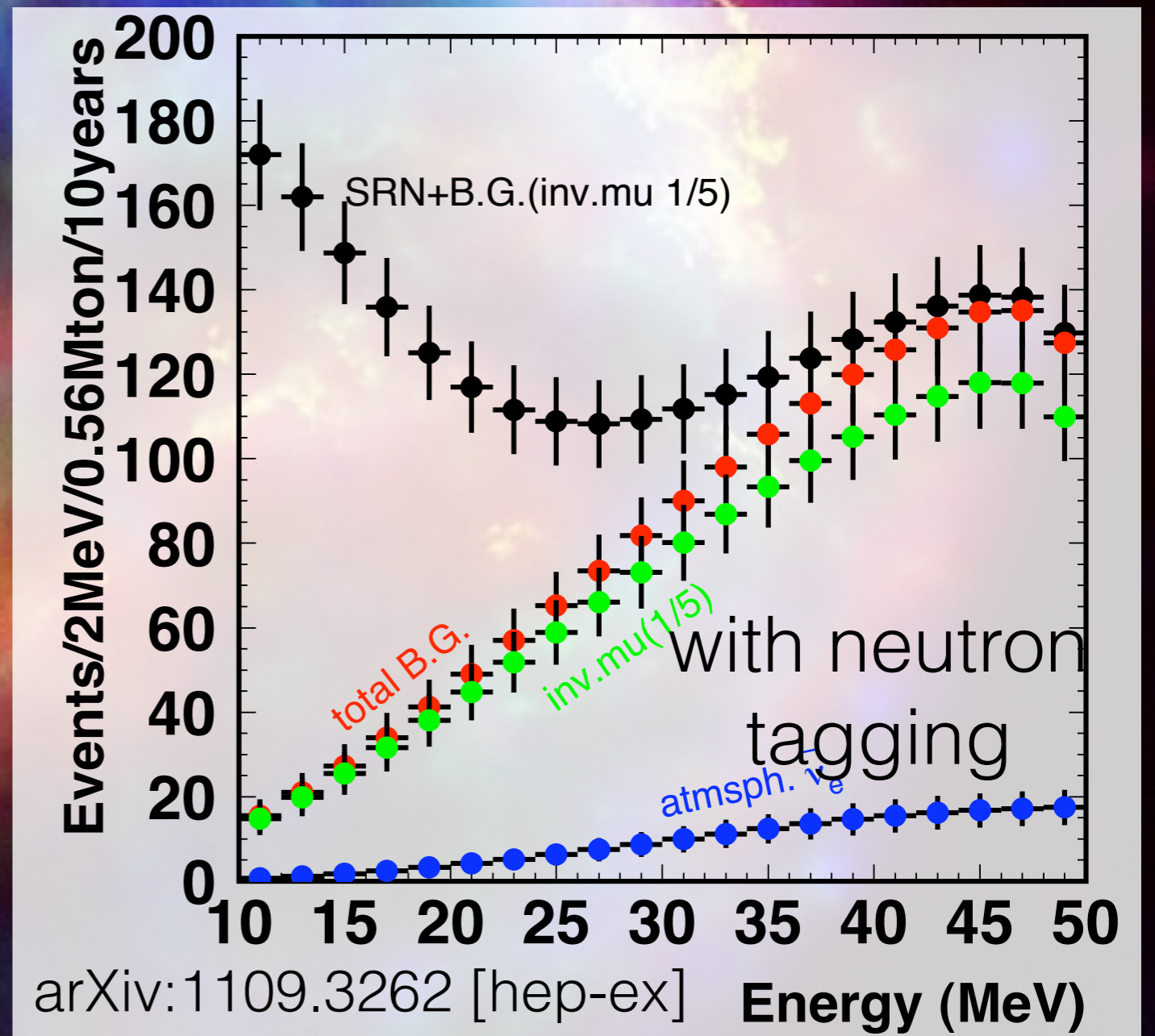
Directly observable local supernova are all too rare

Alternative is to measure diffuse supernova background DSNB/SRN

Very low rate

Large backgrounds

Removed by requiring coincidence with neutron



A few clean events per year in SK
~100s per year in HK

Tank Parameters

	KAM	SK	HK-1TankHD
Depth	1,000 m	1,000 m	650 m
Dimensions of water tank			
diameter	15.6 m ϕ	39 m ϕ	74 m ϕ
height	16 m	42 m	60 m
Total volume	4.5 kton	50 kton	258 kton
Fiducial volume	0.68 kton	22.5 kton	187 kton
Outer detector thickness	~ 1.5 m	~ 2 m	1 \sim 2 m
Number of PMTs			
inner detector (ID)	948 (50 cm ϕ)	11,129 (50 cm ϕ)	40,000 (50 cm ϕ)
outer detector (OD)	123 (50 cm ϕ)	1,885 (20 cm ϕ)	6,700 (20 cm ϕ)
Photo-sensitive coverage	20%	40%	40%
Single-photon detection efficiency of ID PMT	unknown	12%	24%
Single-photon timing resolution of ID PMT	~ 4 nsec	2-3 nsec	1 nsec

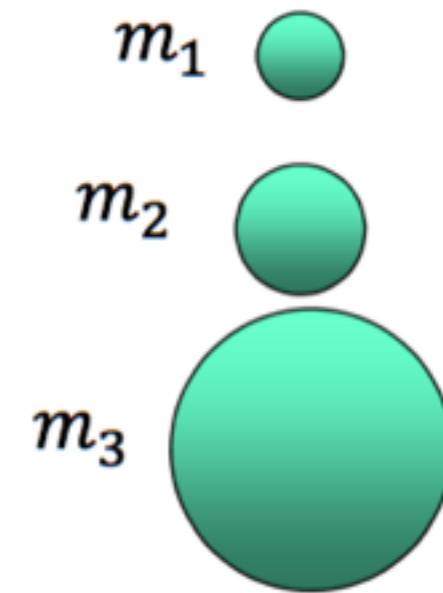
Three Flavor Mixing in Lepton Sector

Weak eigenstates



$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = U_{\text{MNS}} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

mass eigenstates



$$U_{\text{PMNS}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & +c_{23} & +s_{23} \\ 0 & -s_{23} & +c_{23} \end{pmatrix} \begin{pmatrix} +c_{13} & 0 & +s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & +c_{13} \end{pmatrix} \begin{pmatrix} +c_{12} & +s_{12} & 0 \\ -s_{12} & +c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$(c_{ij} = \cos \theta_{ij}, s_{ij} = \sin \theta_{ij})$$

$$\theta_{12}, \theta_{23}, \theta_{13}, \delta,$$

$$\Delta m_{21}^2, \Delta m_{32}^2, \Delta m_{31}^2$$

$$*\Delta m_{ij}^2 = m_i^2 - m_j^2$$

Out of three Δm^2 's, number of free parameters is two. ($\Delta m_{31}^2 = \Delta m_{21}^2 + \Delta m_{32}^2$)

ν_μ disappearance probability

$\theta_{13}=0$ case

$$P_{\mu \rightarrow x} \approx 1 - \sin^2 2\theta_{23} \cdot \sin^2 \left(\frac{\Delta m_{32}^2 L}{4E_\nu} \right)$$

For non-zero θ_{13}

$$P_{\mu \rightarrow x} \approx 1 - \left(\cos^4 \theta_{13} \cdot \sin^2 2\theta_{23} + \sin^2 \theta_{23} \cdot \sin^2 2\theta_{13} \right) \sin^2 \left(\frac{\Delta m^2 L}{4E_\nu} \right)$$
$$\Delta m^2 \approx \Delta m_{32}^2 \approx \Delta m_{31}^2$$

Maximal disappearance occurs at $\sin^2 \theta_{23} = \frac{1}{2\cos^2 \theta_{13}} = 0.513$

more on ν_μ disappearance

- ν_μ disappearance probability in vacuum

$$\begin{aligned}
 P(\nu_\mu \rightarrow \nu_\mu) = & 1 - (c_{13}^4 \sin^2 2\theta_{23} + s_{23}^2 \sin^2 2\theta_{13}) \sin^2 \Delta_{atm} \\
 & + \left\{ c_{13}^2 (c_{12}^2 - s_{13}^2 s_{23}^2) \sin^2 2\theta_{23} + s_{12}^2 s_{23}^2 \sin^2 2\theta_{13} - c_{13} \sin 2\theta_{13} \sin 2\theta_{23} \sin 2\theta_{12} \cos \delta \right\} \\
 & \times \left\{ \frac{1}{2} \sin 2\Delta_{solar} \sin 2\Delta_{atm} + 2 \sin^2 \Delta_{solar} \sin^2 \Delta_{atm} \right\} \\
 & - \left\{ \sin^2 2\theta_{12} (c_{23}^2 - s_{13}^2 s_{23}^2)^2 + s_{13}^2 \sin^2 2\theta_{23} (1 - c_\delta^2 \sin^2 2\theta_{12}) \right. \\
 & + 2s_{13} \sin 2\theta_{12} \cos 2\theta_{12} \sin \theta_{23} \cos 2\theta_{23} c_\delta \\
 & - \frac{1}{2} c_{13} \sin 2\theta_{13} \sin 2\theta_{23} \sin 2\theta_{12} \cos \delta s_{23}^2 s_{12}^2 \\
 & \left. + \sin^2 2\theta_{23} c_{13}^2 (c_{12}^2 - s_{13}^2 s_{12}^2) + s_{13}^2 s_{23}^2 \sin^2 2\theta_{13} \right\} \times \sin^2 \Delta_{solar}
 \end{aligned} \tag{26}$$

s_{ij}	=	$\sin \theta_{ij}$
c_{ij}	=	$\cos \theta_{ij}$
c_δ	=	$\cos \delta$
Δ_{atm}	=	$\frac{\Delta m_{13}^2 L}{4 E_\nu}$
Δ_{solar}	=	$\frac{\Delta m_{21}^2 L}{4 E_{\nu u}}$

T2K: $L = 295$ km, E_ν peaks at ~ 0.6 GeV $\rightarrow \sin^2 \Delta_{solar} \sim 0, \sin 2\Delta_{atm} \sim 0$

$$P(\nu_\mu \rightarrow \nu_\mu) \sim 1 - \left(\underbrace{\cos^4 \theta_{13} \cdot \sin^2 2\theta_{23}}_{\text{Leading-term}} + \underbrace{\sin^2 2\theta_{13} \cdot \sin^2 \theta_{23}}_{\text{Next-to-leading}} \right) \cdot \sin^2 \frac{\Delta m_{31}^2 \cdot L}{4E}$$

ν_μ disapp. probability depends on $\sin^2 2\theta_{13} \cdot \sin^2 \theta_{23}$ to second order
 \rightarrow Can be used in combination with known $\sin^2 2\theta_{13}$ to resolve the θ_{23} octant

ν_e appearance probability

Leading term only

$$P_{\mu \rightarrow e} \approx \sin^2 \theta_{23} \cdot \sin^2 2\theta_{13} \cdot \sin^2 \left(\frac{\Delta m^2 L}{4E_\nu} \right)$$

$$\Delta m^2 \approx \Delta m_{32}^2 \approx \Delta m_{31}^2$$

ν_e appearance probability (exact formula in vacuum)

$$\begin{aligned}
 P(\nu_\mu \rightarrow \nu_e) = & \boxed{4c_{13}^2 s_{13}^2 s_{23}^2 \sin^2 \Delta_{31}} \quad \text{Leading term} \quad \boxed{\theta_{13}} \\
 & + 8c_{13}^2 s_{12} s_{13} s_{23} (c_{12} c_{23} \cos \delta - s_{12} s_{13} s_{23}) \cos \Delta_{32} \sin \Delta_{31} \sin \Delta_{21} \quad \boxed{\text{CPC}} \\
 & - \boxed{8c_{13}^2 c_{12} c_{23} s_{12} s_{13} s_{23} \sin \delta \sin \Delta_{32} \sin \Delta_{31} \sin \Delta_{21}} \quad \boxed{\text{CPV}} \\
 & + 4s_{12}^2 c_{13}^2 (c_{12}^2 c_{23}^2 + s_{12}^2 s_{23}^2 s_{13}^2 - 2c_{12} c_{23} s_{12} s_{23} s_{13} \cos \delta) \sin^2 \Delta_{21} \quad \boxed{\text{Solar}}
 \end{aligned}$$

$$c_{ij} = \cos \theta_{ij}, s_{ij} = \sin \theta_{ij}$$

$$\Delta_{ij} = \Delta m_{ij}^2 \frac{L}{4E_\nu}$$

replace δ by $-\delta$ for $P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e)$

**CP violating term introduced by
interference among three-flavor mixing**

ν_e appearance probability with 1st order matter effect

$$\begin{aligned}
 P(\nu_\mu \rightarrow \nu_e) \approx & 4c_{13}^2 s_{13}^2 s_{23}^2 \sin^2 \Delta_{31} \left(1 + \frac{2a}{\Delta m_{31}^2} (1 - 2s_{13}^2) \right) && \text{Leading including matter effect} \\
 & + 8c_{13}^2 s_{12} s_{13} s_{23} (c_{12} c_{23} \cos \delta - s_{12} s_{13} s_{23}) \cos \Delta_{32} \sin \Delta_{31} \sin \Delta_{21} && \text{CP conserving} \\
 & - 8c_{13}^2 c_{12} c_{23} s_{12} s_{13} s_{23} \sin \delta \sin \Delta_{32} \sin \Delta_{31} \sin \Delta_{21} && \text{CP violating} \\
 & + 4s_{12}^2 c_{13}^2 (c_{12}^2 c_{23}^2 + s_{12}^2 s_{23}^2 s_{13}^2 - 2c_{12} c_{23} s_{12} s_{23} s_{13} \cos \delta) \sin^2 \Delta_{21} && \text{Solar} \\
 & - 8c_{13}^2 s_{13}^2 s_{23}^2 (1 - 2s_{13}^2) \frac{aL}{4E} \cos \Delta_{32} \sin \Delta_{31} && \text{Matter effect (small)}
 \end{aligned}$$

$$c_{ij} = \cos \theta_{ij}, s_{ij} = \sin \theta_{ij}$$

$$\Delta_{ij} = \Delta m_{ij}^2 \frac{L}{4E_\nu}$$

$$a \equiv 2\sqrt{2}G_F n_e E = 7.56 \times 10^{-5} \text{eV}^2 \frac{\rho}{\text{gcm}^{-3}} \frac{E}{\text{GeV}}$$

replace δ by $-\delta$ and a by $-a$ for $P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e)$

ν_e appearance probability

approximation at around oscillation maximum

$$P(\nu_\mu \rightarrow \nu_e) \approx \sin^2 \theta_{23} \sin^2 2\theta_{13} \sin^2 \left(\frac{\Delta m_{32}^2 L}{4E_\nu} \right) \left(1 + \frac{2a}{\Delta m_{31}^2} (1 - 2\sin^2 \theta_{13}) \right)$$

Leading including matter effect

$$- \sin 2\theta_{12} \sin 2\theta_{23} \sin 2\theta_{13} \cos \theta_{13} \sin \delta \sin^2 \left(\frac{\Delta m_{32}^2 L}{4E_\nu} \right) \sin \left(\frac{\Delta m_{21}^2 L}{4E_\nu} \right)$$

CP violating

replace δ by $-\delta$ and a by $-a$ for $P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e)$

$$a \equiv 2\sqrt{2}G_F n_e E = 7.56 \times 10^{-5} \text{eV}^2 \frac{\rho}{\text{gcm}^{-3}} \frac{E}{\text{GeV}}$$

