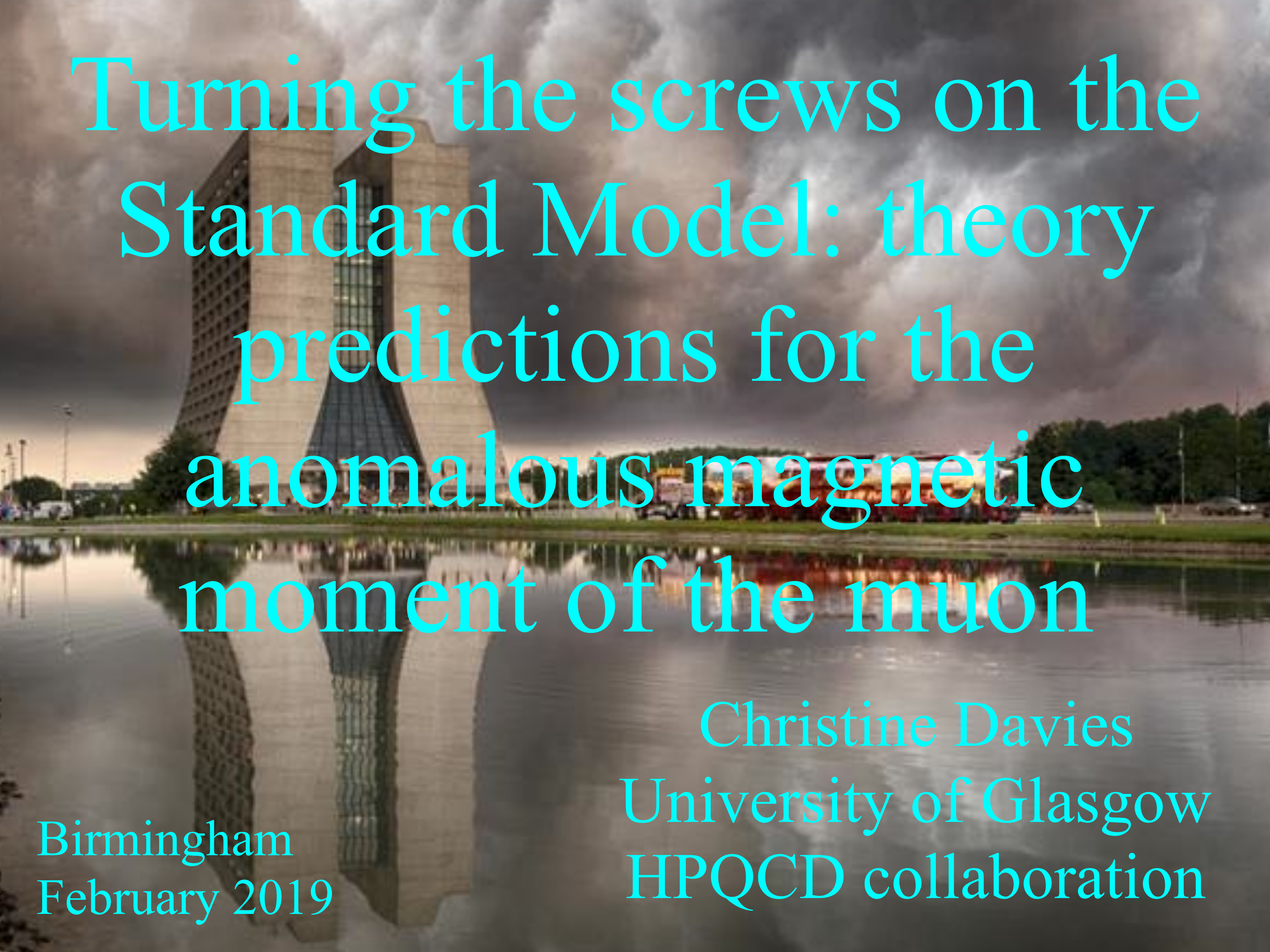


Turning the screws on the Standard Model: theory predictions for the anomalous magnetic moment of the muon

Christine Davies
University of Glasgow
HPQCD collaboration

Birmingham
February 2019

Outline

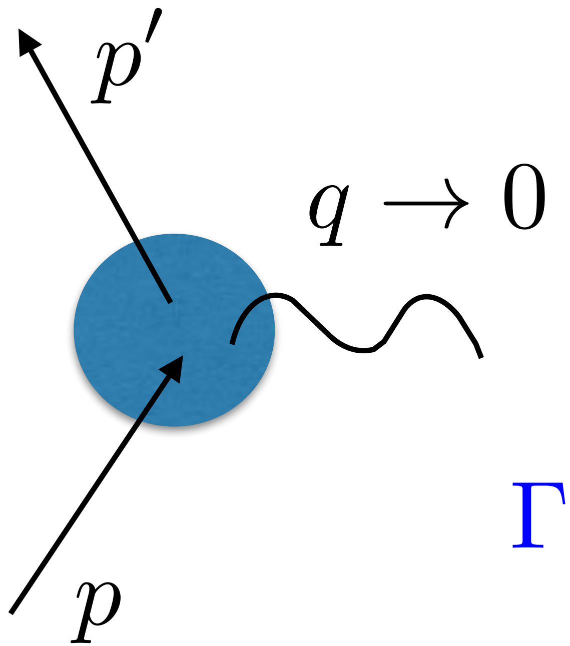
1) Introduction : what is the anomalous magnetic moment (a_μ) of the muon and how is it determined (so accurately) in experiment? (recap)

2) Theory calculations in the Standard Model: QED/EW perturbation theory

3) Pinning down QCD effects, using experimental data and using Lattice QCD calculations.

4) Conclusions and prospects

e, μ, τ have electric charge and spin



Interaction with an external em field has a magnetic component:

$$-ie\overline{u}(p')\Gamma^\mu(p,p')u(p)A_\mu(q)$$

$$\Gamma^\mu(p,p') = \gamma^\mu F_1(q^2) + \frac{i\sigma^{\mu\nu}q_\nu}{2m}F_2(q^2)$$

Electric field interaction (charge consvn): $F_1(0) = 1$

Magnetic field intn, equiv. to scattering from potential :

$$V(x) = -\langle\vec{\mu}\rangle \cdot \vec{B}(x)$$

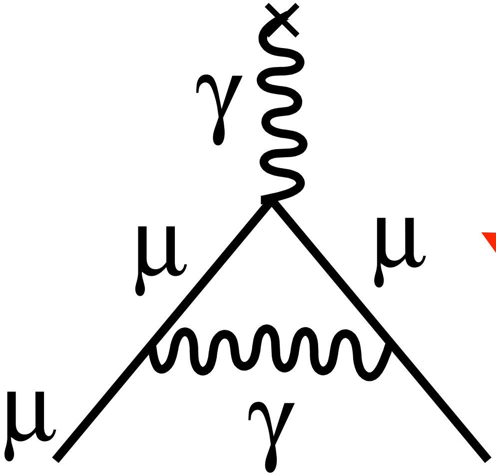
$$\vec{\mu} = \frac{e}{m}[F_1(0) + F_2(0)]\frac{\vec{\sigma}}{2} \equiv g\left(\frac{e}{2m}\right)\vec{S}$$

$$g = 2 + 2F_2(0)$$

Peskin + Schroeder

Anomalous magnetic moment

$$a_{e,\mu,\tau} = \frac{g-2}{2} = F_2(0)$$

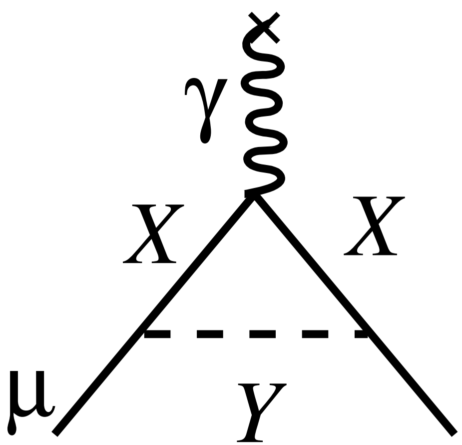


LO contribn is lepton mass independent

Schwinger 1948

$$\frac{\alpha}{2\pi} = 0.00116\ldots$$

many higher order pieces .....



New physics could appear in loops

$$\delta a_\ell^{\text{new physics}} \propto \frac{m_\ell^2}{m_X^2}$$

1 TeV?

flavour,CP-conserving
chirality flipping

Motivates study of μ rather than e

$\approx 10^{-8}$ $\approx 10^{-13}$

CURRENT STATUS

$$a_\mu^{\rm expt} = 11659209.1(6.3) \times 10^{-10}$$

$$a_\mu^{SM} = 11659182.0(3.6) \times 10^{-10}$$

Keshavarzi et al, 1802.02995

tantalising 3.7σ discrepancy! details to follow …

$$a_e^{\rm expt} = 11596521.807(3) \times 10^{-10}$$

$$a_e^{\rm SM} = 11596521.816(8) \times 10^{-10}$$

→

higher accuracy small-scale experiments possible (Penning trap) but discrepancies will be tiny …

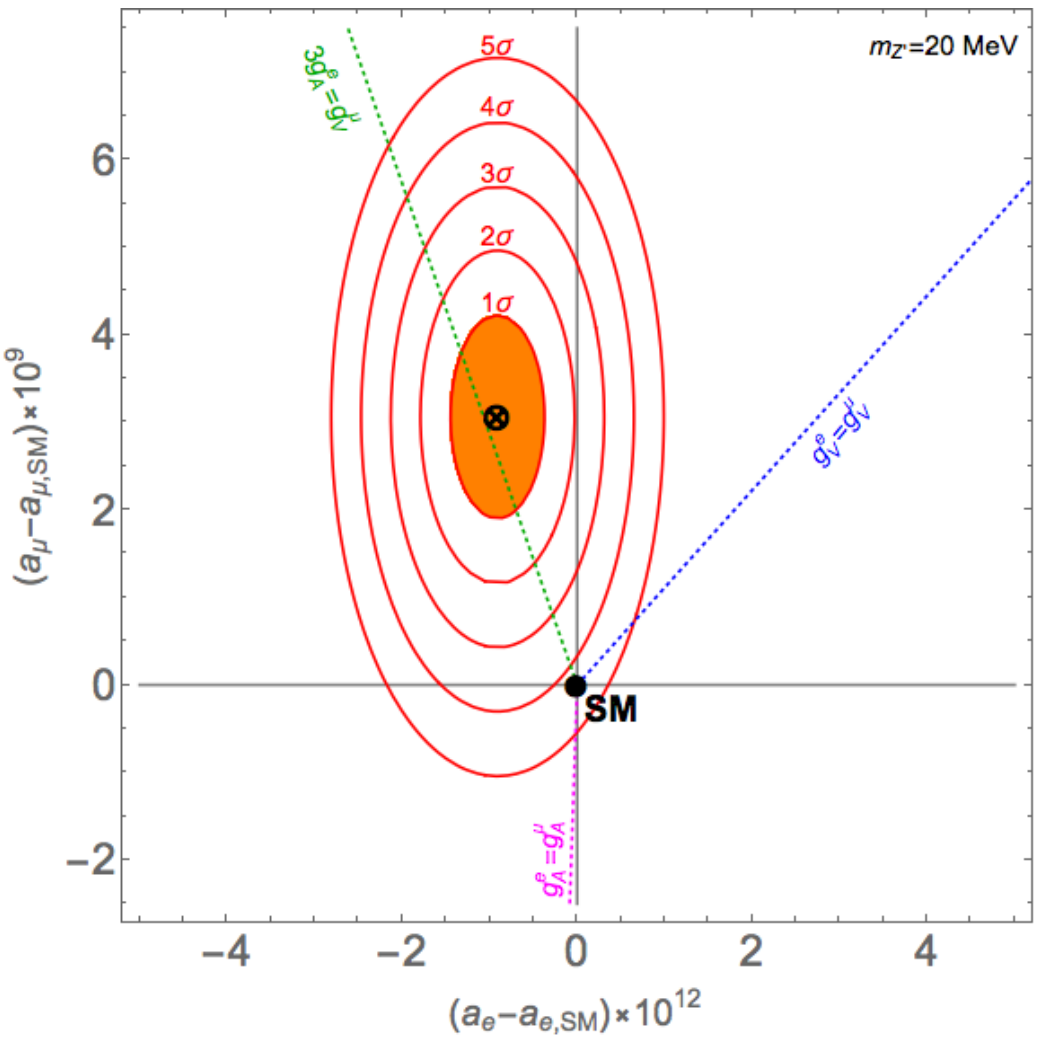
τ very hard since decays in 0.3picoseconds ….

$\delta a_\tau = 5 \times 10^{-2}$ (LEP) $e^+e^- \to e^+e^-\tau^+\tau^-$

New determination of α (2018) : Mueller et al (h/M_{Cs})

Now

$$\Delta a_e^{SM} \equiv a_e^{expt} - a_e^{SM} = -87(36) \times 10^{-14}$$



2.4σ ‘tension’ and opposite sign to discrepancy for μ potentially adds excitement to the story!

Davoudiasl+Marciano, 1806.10252

Aoyama, Kinoshita and Nio, 1712.06060 for QED calc.

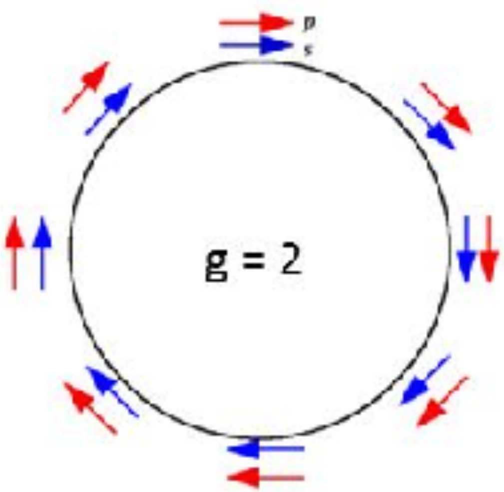
Accurate experimental results + theory calculations needed

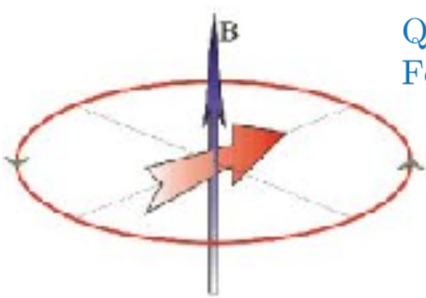
spin 0

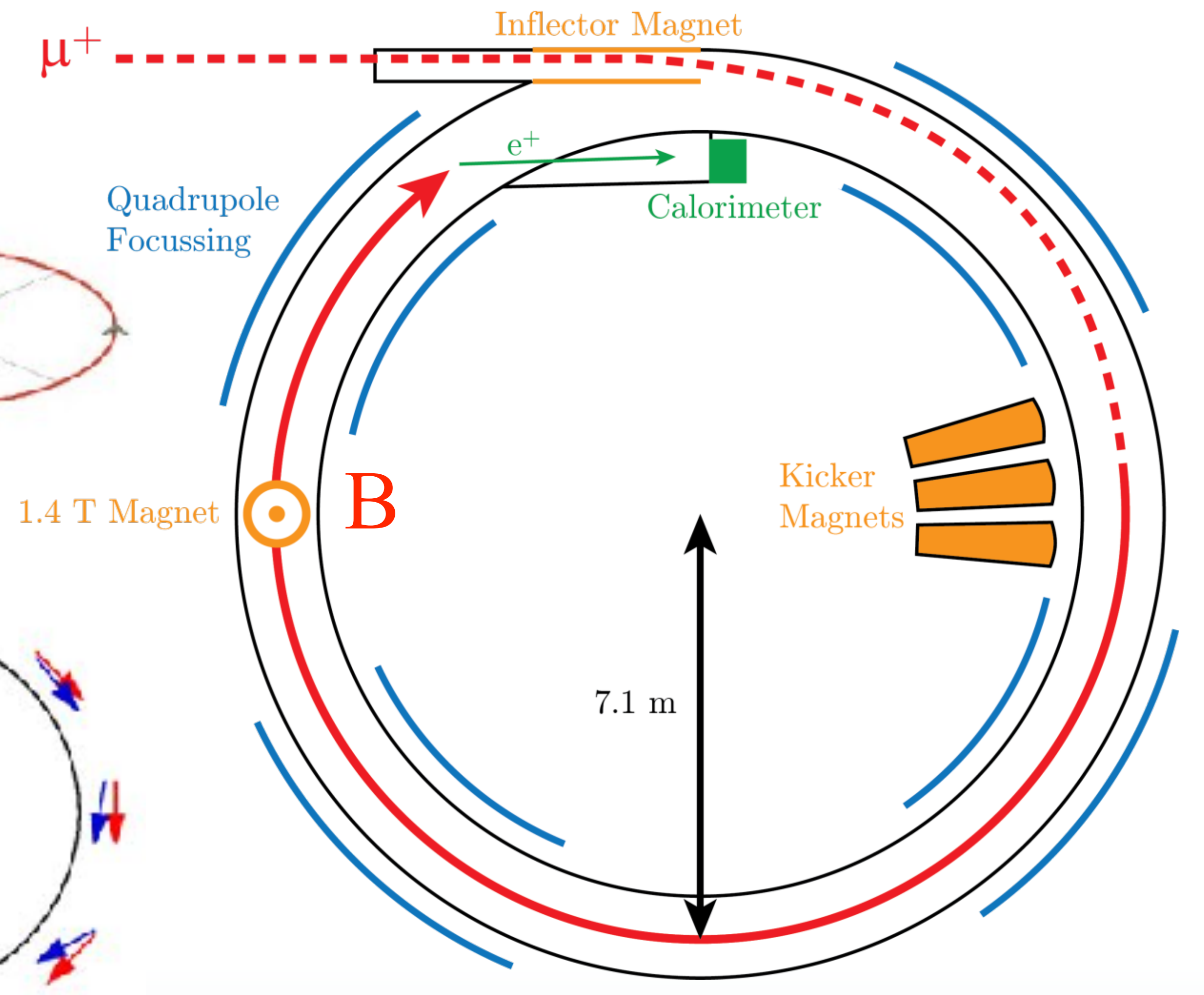
$p \to \pi^+ \to \nu_\mu + \mu^+$

both helicity -1 in π rest frame
so get polarised μ beam pulse

B field perpendicular to ring, μ spin precesses

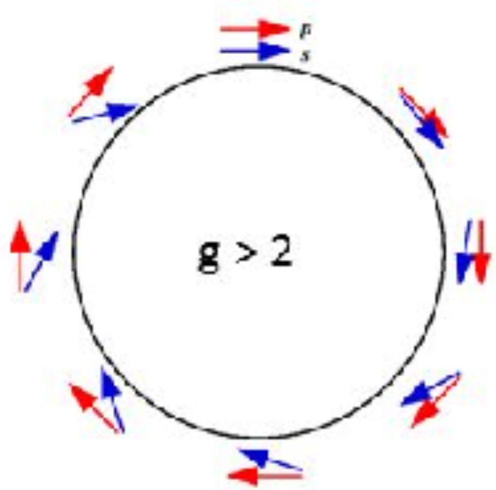






measure
frequency
difference

$\omega_S - \omega_C$



$$\vec{\omega}_a = \vec{\omega}_S - \vec{\omega}_C = -\frac{Qe}{m}\left[a_\mu \vec{B} + \left(a_\mu - \left(\frac{m}{p}\right)^2\right)\frac{\vec{\beta}\times\vec{E}}{c}\right] + ..$$

$Q = \pm 1,\ \mu^\pm$

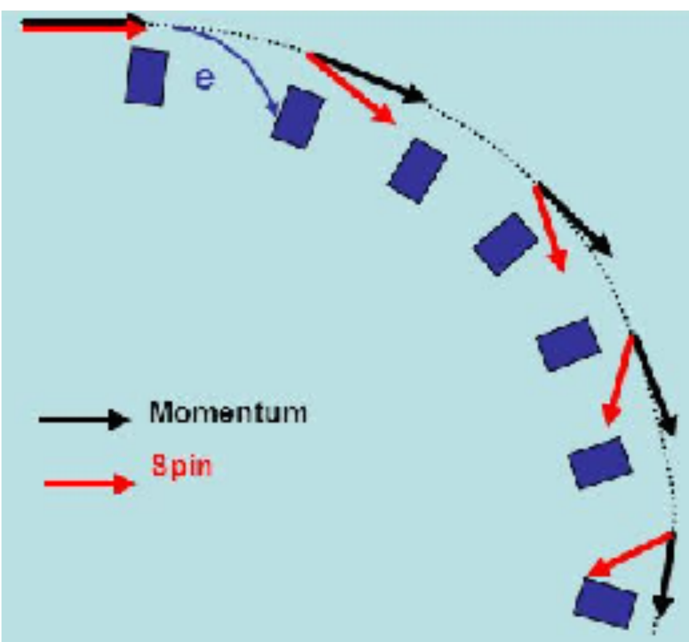
need uniform stable B, measure to sub-ppm with NMR probes calibrated using g_p

directly gives a_μ

electric field term vanishes at 'magic momentum'

$$p = 3.094\,\mathrm{GeV}/c$$

from possible EDM $\propto \vec{\beta}\times\vec{B}$



measure spin direction from e produced in weak decay

$$\mu^+ \to e^+ + \nu_e + \overline{\nu}_\mu$$

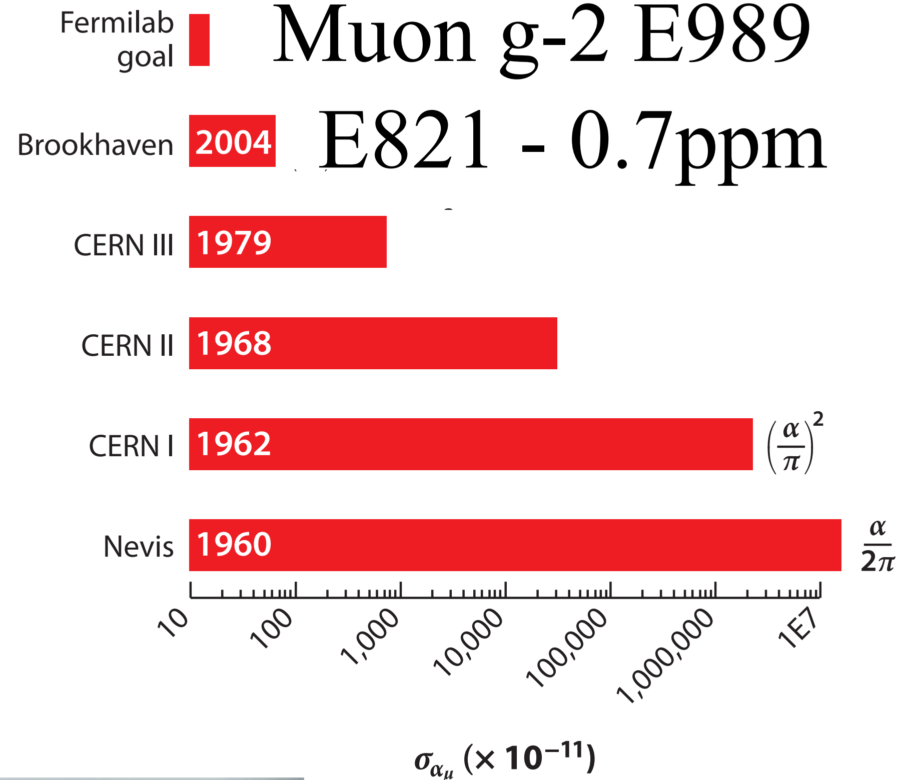
direction of highest energy e correlated with μ spin so N_e oscillates at $\omega_S - \omega_C$

Status of experiment

Muon g-2 E989

2013: E821 ring moved to Fermilab

E821 - 0.7ppm



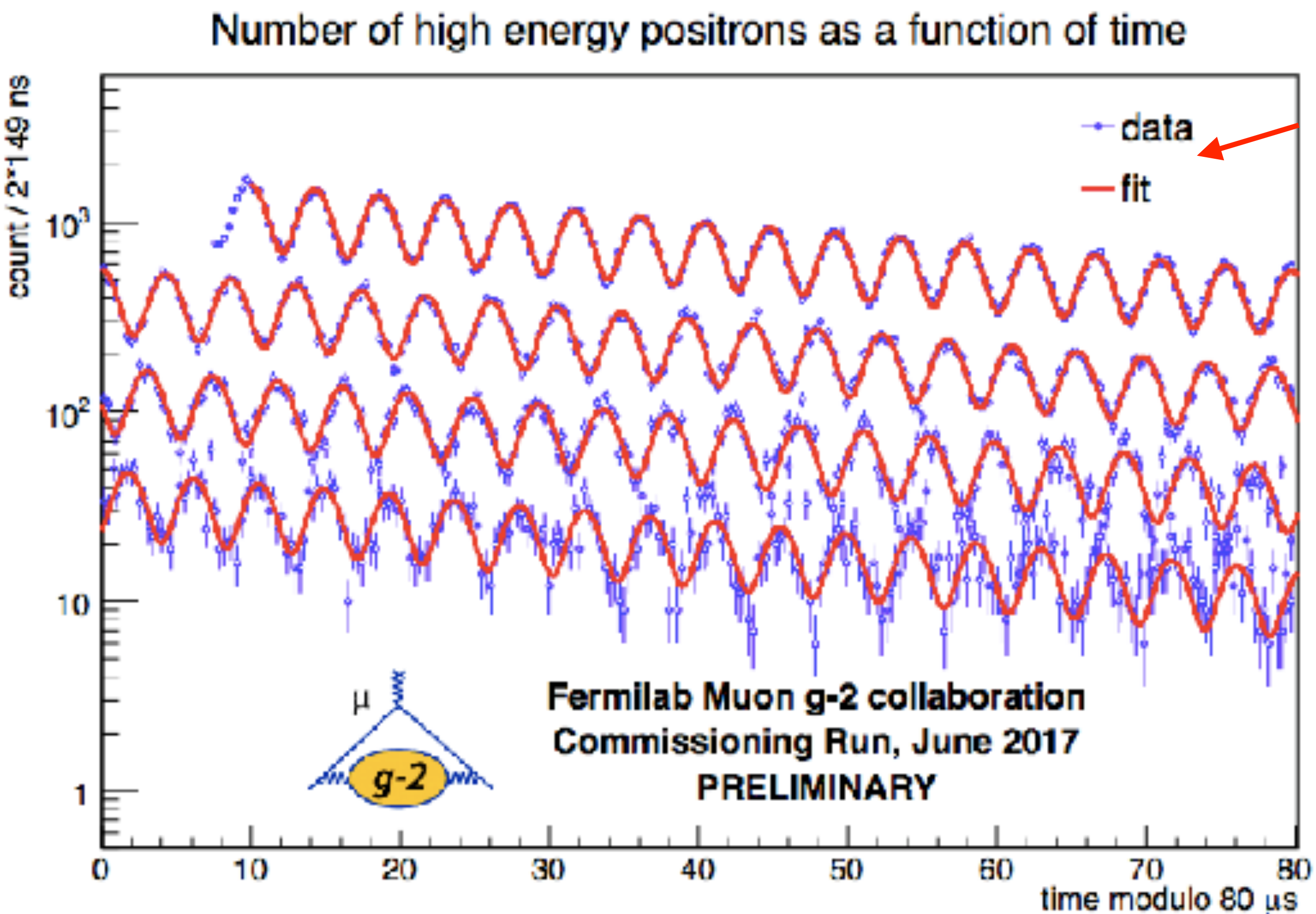
becomes E989

Involvement from Germany, Italy, UK

Aim: Much higher statistics with cleaner injection to ring, more uniform B field + temp. control : 0.15ppm i.e $\delta a_\mu = 2 \times 10^{-10}$

Muon g-2 now running at Fermilab

Run 2018 for 1-3 x E821, first results summer 2019



$\gamma\tau_\mu = 60 \times 10^{-6} s$

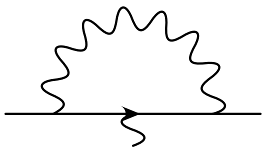
2017 commissioning run: 0.001% of final stats

$$N_e(t) = N_0 e^{-t/\gamma t_\mu} \times [1 + A\cos(\omega_a t + \phi)]$$

J-PARC future plan: slow μ in 1m ring - no need for ‘magic momentum’

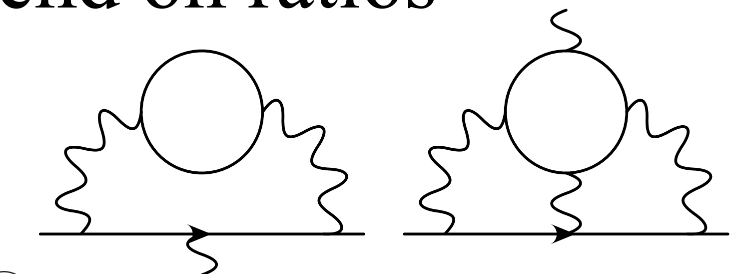
Accurate experimental results + theory calculations needed

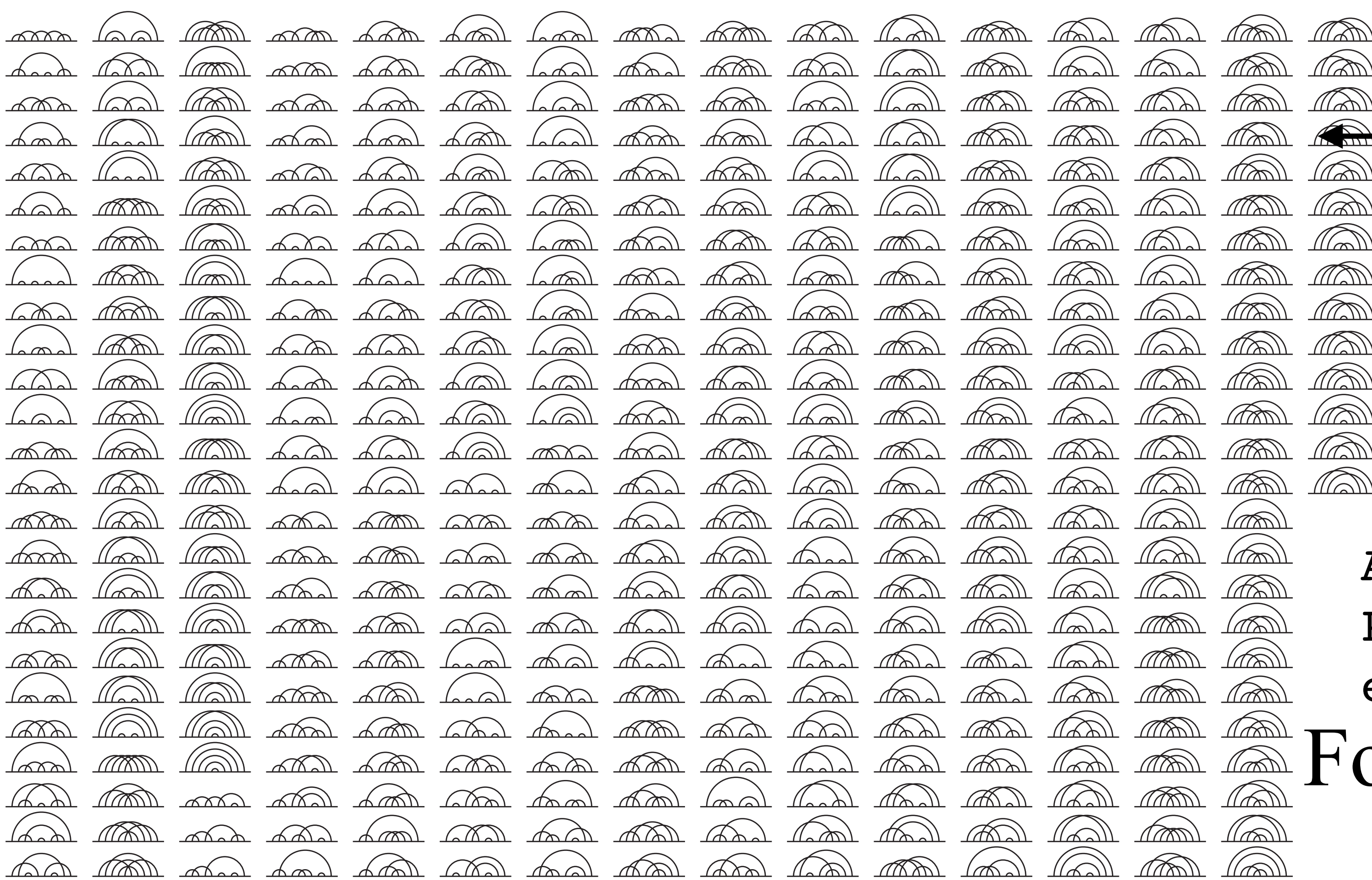
QED corrections dominate - calculate in Perturbation theory



$$0.5\frac{\alpha}{\pi}$$

higher orders depend on ratios of lepton masses:





subset of diagrams at α^5

integration challenging- use VEGAS

Aoyama, Kinoshita et al PRD91:033006(2015), err:PRD96:019901(2017)

For α use a_e or Rb/Cs

<0.5ppb

$$a_\mu^{\rm QED} = \frac{\alpha}{2\pi} + 0.765\,857\,425(17)\left(\frac{\alpha}{\pi}\right)^2 + 24.050\,509\,96(32)\left(\frac{\alpha}{\pi}\right)^3 + 130.879\,6(6\,3)\left(\frac{\alpha}{\pi}\right)^4 + 753.3(1.0)\left(\frac{\alpha}{\pi}\right)^5 + \cdots$$

Hoecker+
Marciano
RPP 2017

$$a_\mu^{\rm QED} = 0.00116 + 0.00000413 \ldots + 0.000000301 + 0.00000000381 + 0.000000000509 + \ldots$$

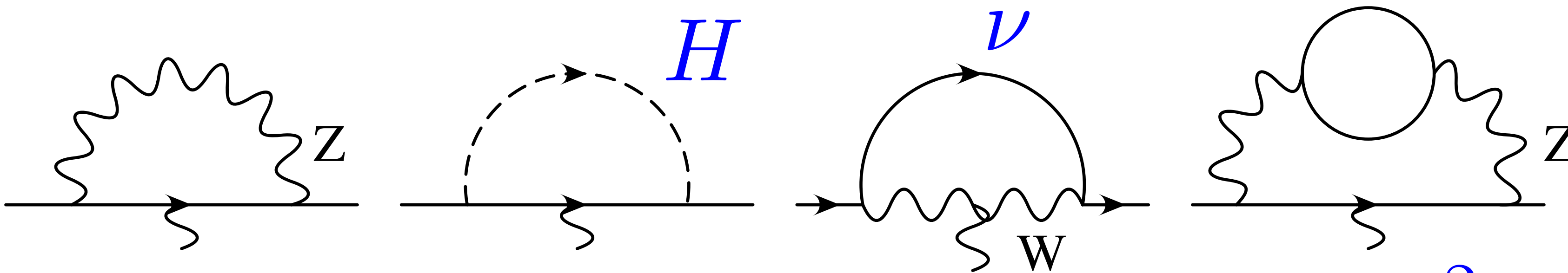
using Rb α

$$= 11{,}658{,}471.895(8) \times 10^{-10}$$

uncertainty from error in α but missing α^6 (light-by-light) also this size

Electroweak contributions from Z, W, H

Gnendiger et al, 1306.5546



$a_\mu^{\rm EW}$ is small - suppressed by powers of $\dfrac{m_\mu^2}{m_W^2}$

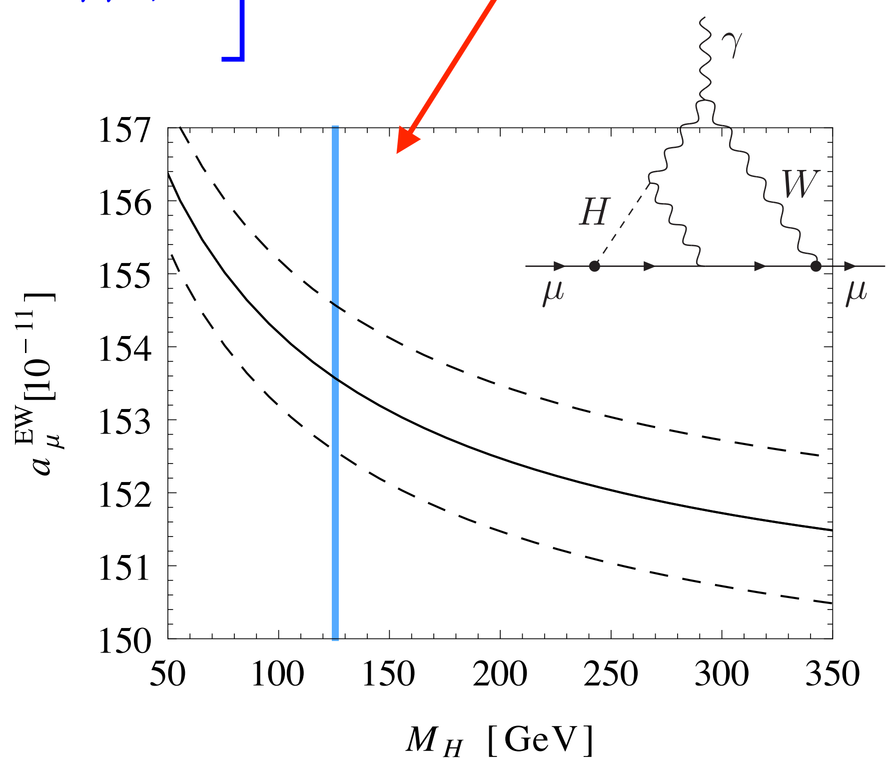
$$a_\mu^{\rm EW(1)} = \frac{G_F m_\mu^2}{\sqrt{2}\, 8\pi^2}\left[\frac{5}{3} + \frac{1}{3}(1 - 4s_W^2)^2\right]$$

$$= 19.480(1) \times 10^{-10}$$

$$a_\mu^{\rm EW(2)} = -4.12(10) \times 10^{-10}$$

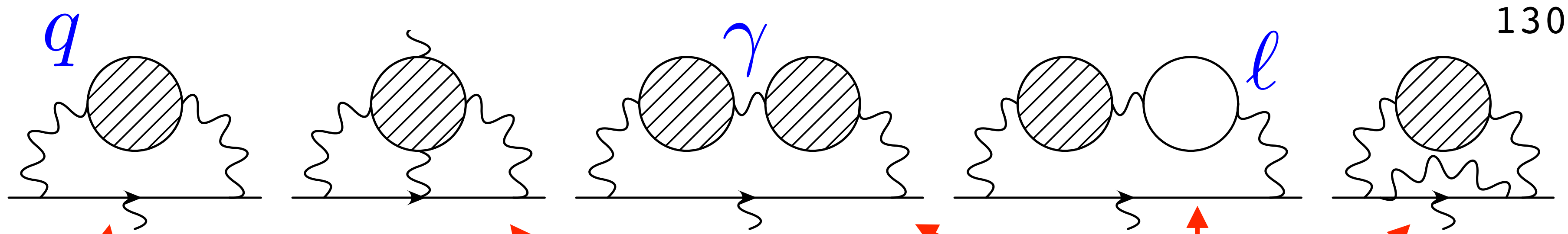
$$a_\mu^{\rm EW} = 15.36(10) \times 10^{-10}$$

H piece tiny at 1-loop; 2-loops



QCD contributions to a_μ start at α^2 , nonpert. in QCD

Blum et al, 1301.2607



LO Hadronic vacuum polarisation (HVP) dominates uncertainty in SM result

Hadronic light-by-light, not well known but small

Higher order Hadronic vacuum polarisation (HOHVP)

Since QED, EW known accurately, subtract from expt and compare QCD calculations to remainder

$$a_\mu^{E821} = 11659209.1(6.3) \times 10^{-10}$$

$$a_\mu^{\mathrm{QED}} = 11658471.895(8) \times 10^{-10} \quad a_\mu^{\mathrm{EW}} = 15.36(10) \times 10^{-10}$$

Hadronic (and other) contributions = EXPT - QED - EW

$$a_\mu^{E821} - a_\mu^{\mathrm{QED}} - a_\mu^{\mathrm{EW}} = 721.9(6.3) \times 10^{-10}$$

$$= a_\mu^{HVP} + a_\mu^{HOHVP} + a_\mu^{HLBL} + a_\mu^{new\,physics}$$

Focus on lowest order hadronic vacuum polarisation (HVP), so take:

$$a_\mu^{HLbL} = 10.5(2.6) \times 10^{-10}$$

"consensus" value will return to this

$$a_\mu^{HOHVP} = -8.85(9) \times 10^{-10}$$

NLO+NNLO

Kurz et al, 1403.6400

$$a_\mu^{\mathrm{HVP},no\,new\,physics} = 720.2(6.8) \times 10^{-10}$$

Note: much larger than a_μ^{EW}

How to calculate a_μ^{HVP} - Two approaches:

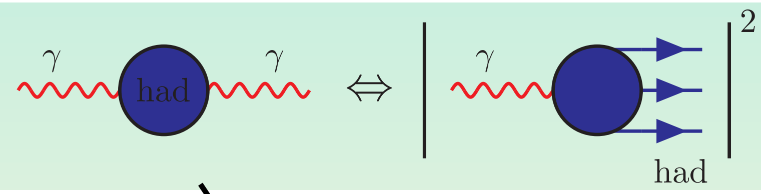
1) $\sigma(e^+e^- \rightarrow \text{hadrons})$ + dispersion relations.

2) lattice QCD - "first principles"

---

1) $\sigma(e^+e^- \rightarrow \text{hadrons})$

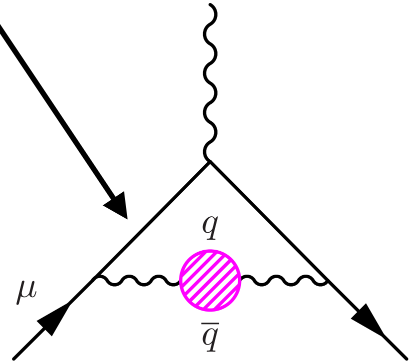
Analyticity+optical theorem



$$a_\mu^{HVP} = \frac{1}{4\pi^3}\int_{m_\pi^2}^{\infty} ds\,\sigma^0_{had}(s)K(s)$$

$\pi^0\gamma$ threshold

$e^+e^- \rightarrow \gamma^* \rightarrow hadrons$



K(s) kernel emphasises low s - integral dominated by $\rho, \pi^+\pi^-$. Use pert. QCD at high s.

$$R_{e^+e^-} = \frac{\sigma}{\sigma_{pt}}$$

σ^0 is 'bare', with running α effects removed .

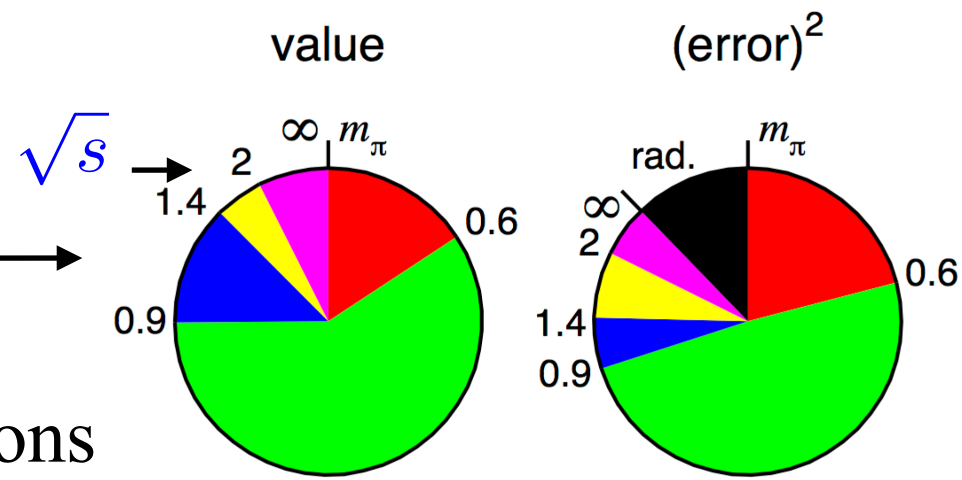
Final state em radiation IS included - γ inside hadron bubble

Need to combine multiple sets of experimental data from many hadronic channels (+ inclusive) inc. correlations

New data sets from KLOE, BESIII, SND(Novosibirsk) ..

New results

Keshavarzi, Nomura, Teubner
1802.02995 :
70% redn in uncty since 2011.
New data, more channels, correlations



KNT18 $a_\mu^{HVP} = 693.3(2.5) \times 10^{-10}$

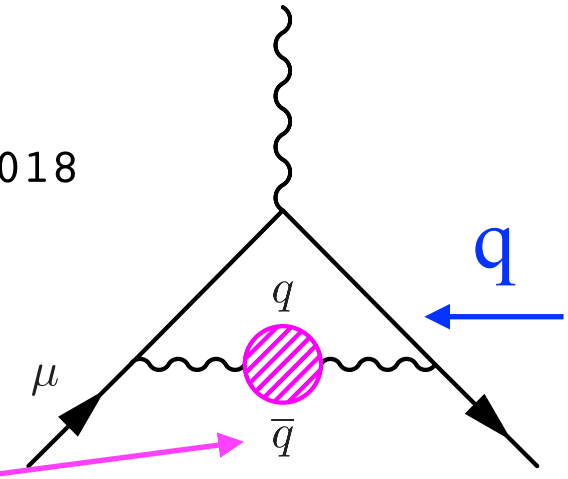
Davier et al, 1706.09436 $a_\mu^{\rm HVP} = 693.1(3.4) \times 10^{-10}$

Jegerlehner 1705.00263 $a_\mu^{\rm HVP} = 688.8(3.4) \times 10^{-10}$

agree well - 0.4% uncty 3.5σ from no new physics.

2) Lattice QCD

Blum, hep-lat/0212018



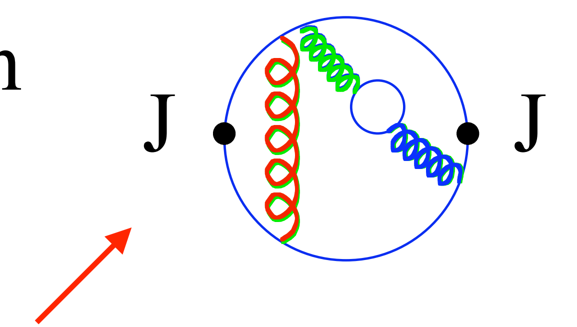
$$a_\mu^{HVP,i} = \frac{\alpha}{\pi}\int_0^\infty dq^2 f(q^2)(4\pi\alpha e_i^2)\hat{\Pi}_i(q^2)$$

'connected' contribution for flavour i

Integrate over Euclidean q^2 – $f(q^2)$ diverges at small q^2 with scale set by m_μ so $q^2 \approx 0$ dominates

Renormalised vacuum polarisation function

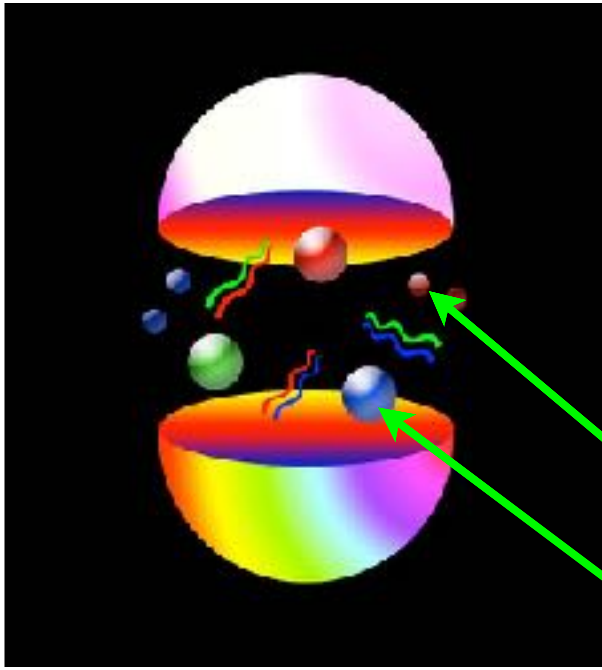
$$\hat{\Pi}(q^2) = \Pi(q^2) - \Pi(0)$$ vanishes at $q^2=0$



This is (Fourier transform of) vector meson correlators.

Can perform q^2 integral using time-moments of standard correlatorrs calculated in lattice QCD to determine meson masses.

HPQCD,1403.1778

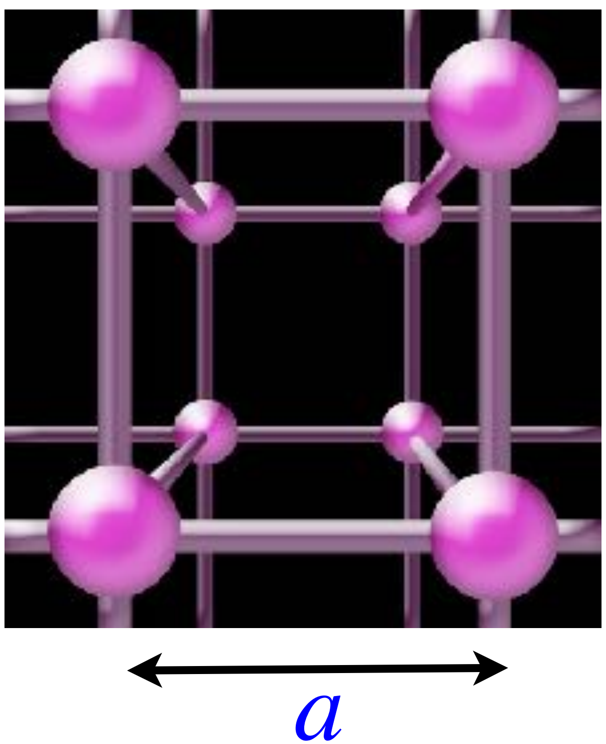


Lattice QCD: fields defined on 4-d discrete space-(Euclidean) time.

Lagrangian parameters: $\alpha_s, m_q a$

1) Generate sets of gluon fields for Monte Carlo integrn of Path Integral (inc effect of u, d, s, (c) sea quarks)

2) Calculate valence quark propagators and combine for "hadron correlators" . Average results over gluon fields. Fit for hadron masses and amplitudes



- Determine a to convert results in lattice units to physical units. Fix m_q from hadron mass

numerically extremely challenging

- cost increases as $a \to 0, m_{u/d} \to$ phys and with statistics, volume.

Using Darwin@Cambridge,

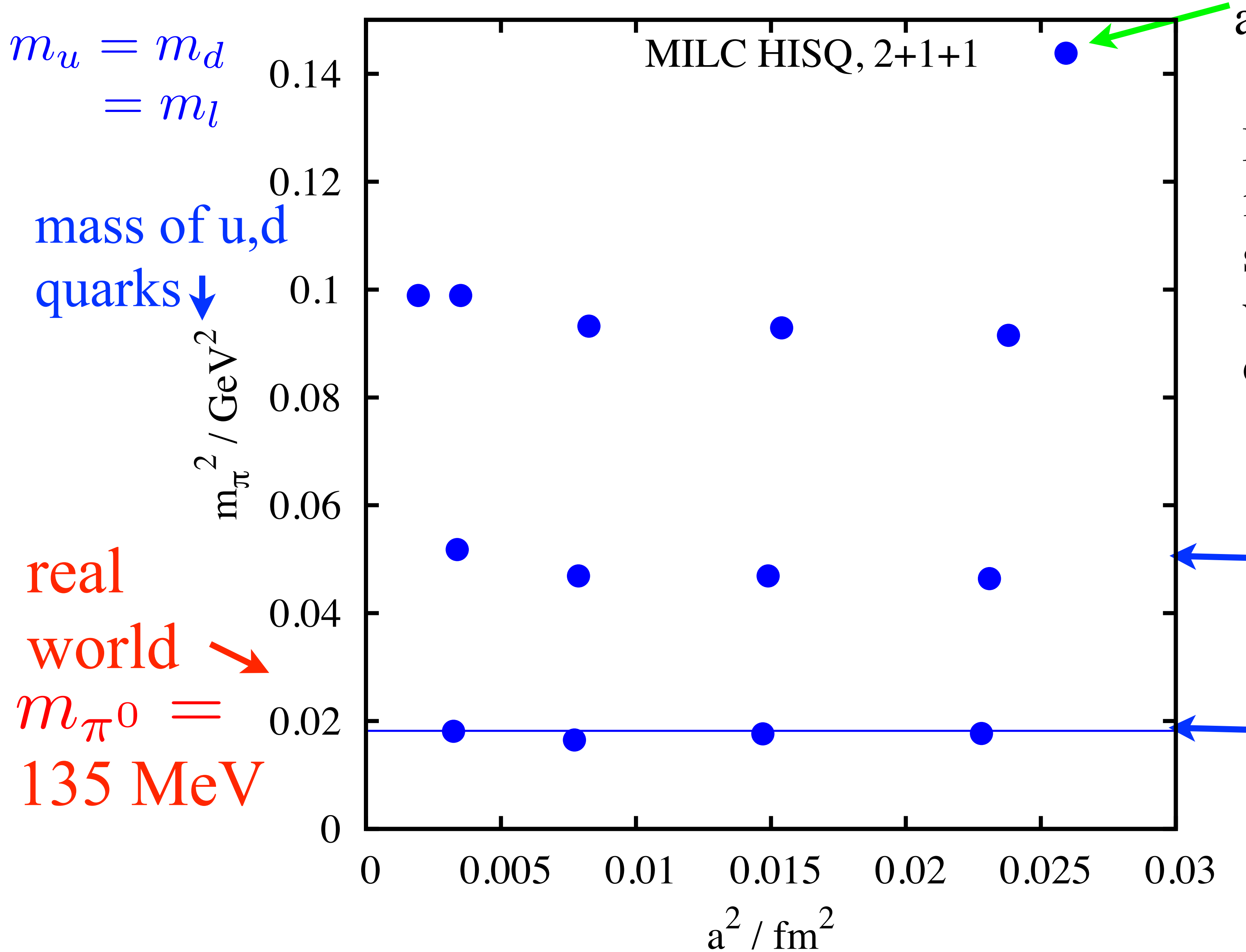
DiRAC

www.dirac.ac.uk

Allows us to calculate quark propagators rapidly and store them for flexible re-use.

Inversion of 10^7 x 10^7 sparse matrix solves the Dirac equation for the quark propagator on a given gluon field configuration. Must repeat thousands of times for statistical precision.

‘2nd generation’ gluon field configs generated by MILC including HPQCD’s HISQ sea quarks.
Physical u/d quark masses now possible.



HISQ = Highly improved staggered quarks - very accurate discretisation

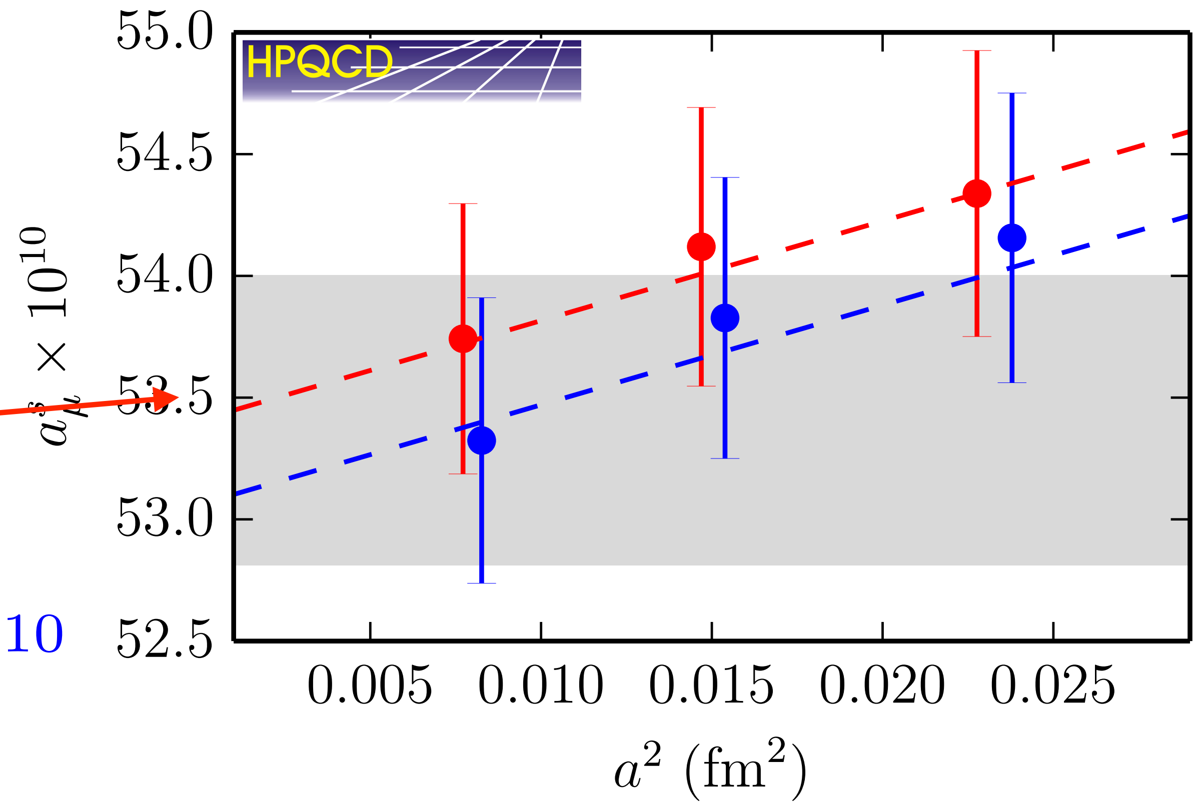
E.Follana, et al, HPQCD, hep-lat/0610092.

Volume: $m_\pi L > 3$

‘connected’ s quark contribution to a_μ^{HVP}

Chakraborty et al, HPQCD 1403.1778

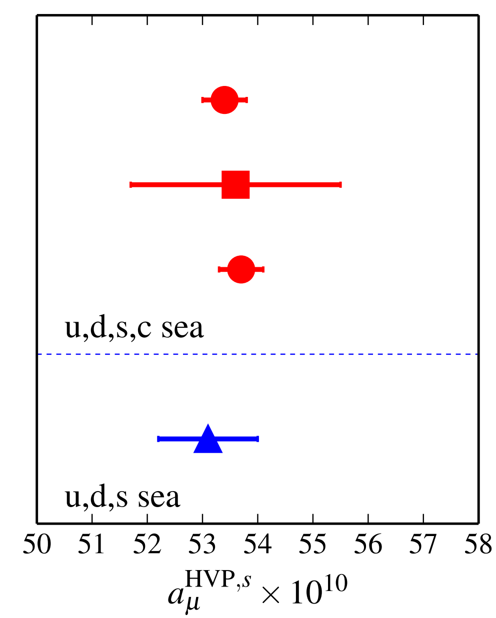
HISQ quarks on configs with u, d, s and c sea.
Local J_V - nonpert. Z_V.
multiple a (fixed by w_0),
m_l (inc. phys.), volumes.
Tune s from η_s



$$a_\mu^{HVP,s} = 53.4(4) \times 10^{-10}$$

allowing for missing QED

	a_μ^s
Uncertainty in lattice spacing (w_0, r_1):	0.4%
Uncertainty in Z_V:	0.4%
Monte Carlo statistics:	0.1%
$a^2 \to 0$ extrapolation:	0.1%
QED corrections:	0.1%
Quark mass tuning:	0.4%
Finite lattice volume:	< 0.1%
Padé approximants:	< 0.1%
Total:	0.7%

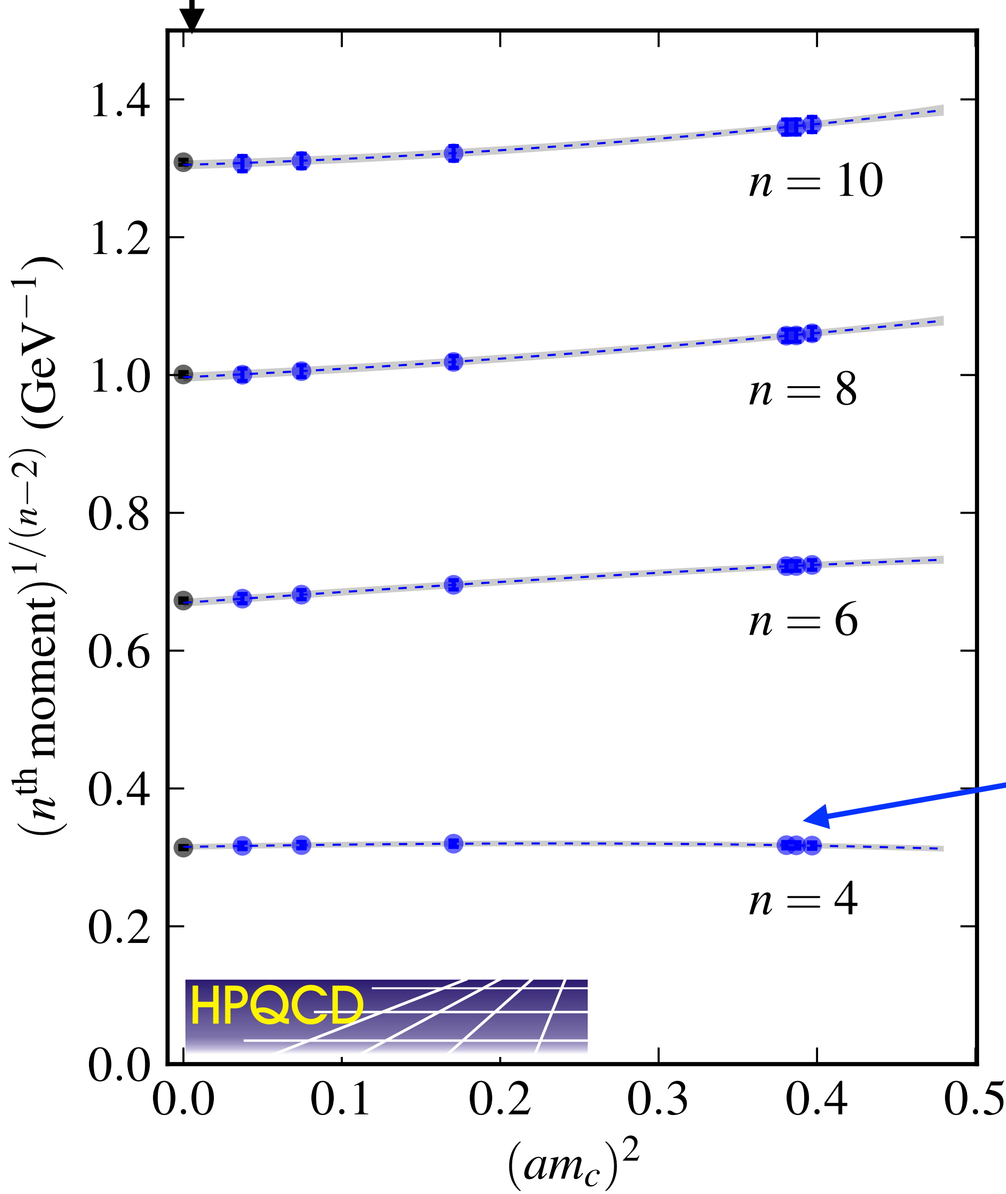


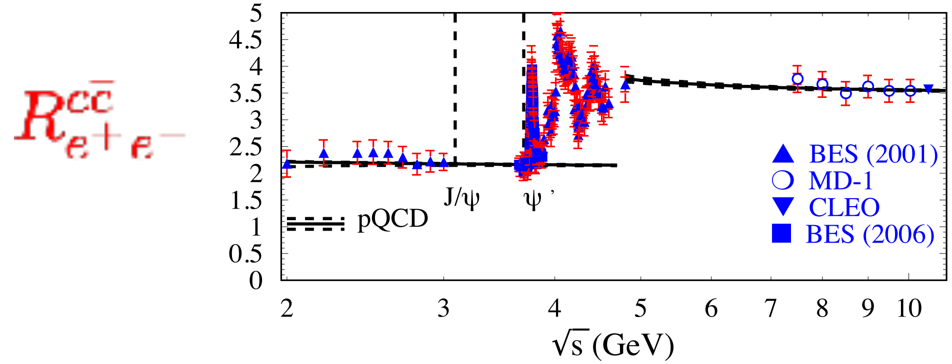
$R_{e^+e^-} <\approx 55 \times 10^{-10}$

'connected' c quark contribution to a_μ^{HVP}

HPQCD, 1208.2855, 1403.1778

$\sigma(e^+e^- \to \text{hadrons via } c\bar{c})$





For c case can directly compare lattice correlator time-moments to e+e- expt

- agree to 1.5%

Lattice QCD, gives:

$$a_\mu^{\text{HVP},c} = 14.4(4) \times 10^{-10}$$

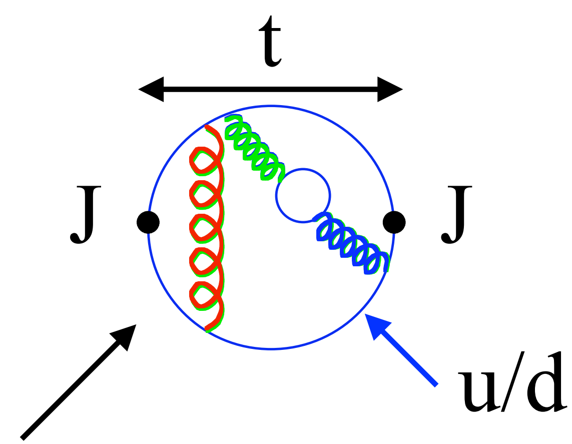
$$a_\mu^{\text{HVP},b} = 0.27(4) \times 10^{-10}$$

UP/DOWN contribution, largest and most difficult

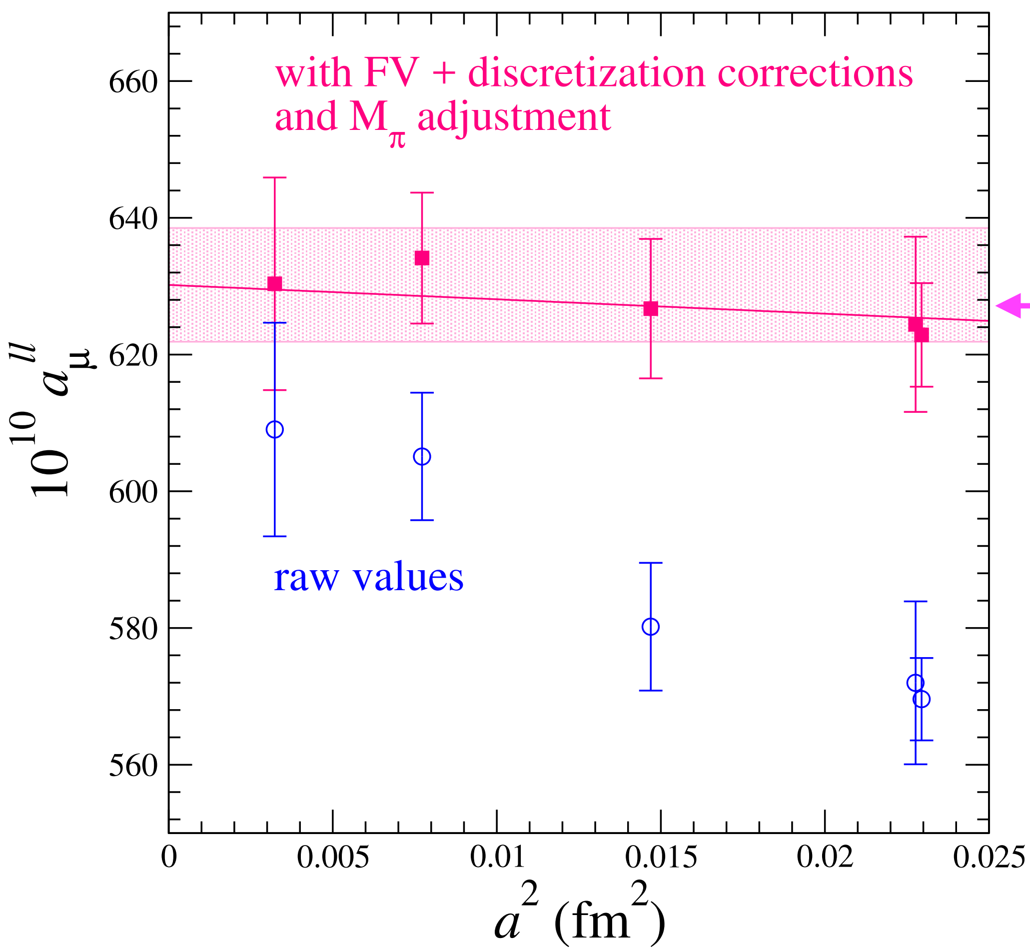
- signal/noise worse and results sensitive to u/d mass $m_u = m_d = m_l$

NEW HPQCD/Fermilab/MILC result (updating HPQCD 1601.03071)

physical $m_{u/d}$ only - high stats.



large-t correlator dominated by ρ but also has ππ - fit to constrain data



ππ mangled on coarse lattices and in finite-vol. Correct with chi.pt.

$$a_\mu^{\mathrm{HVP},u/d} = 630(8) \times 10^{-10}$$

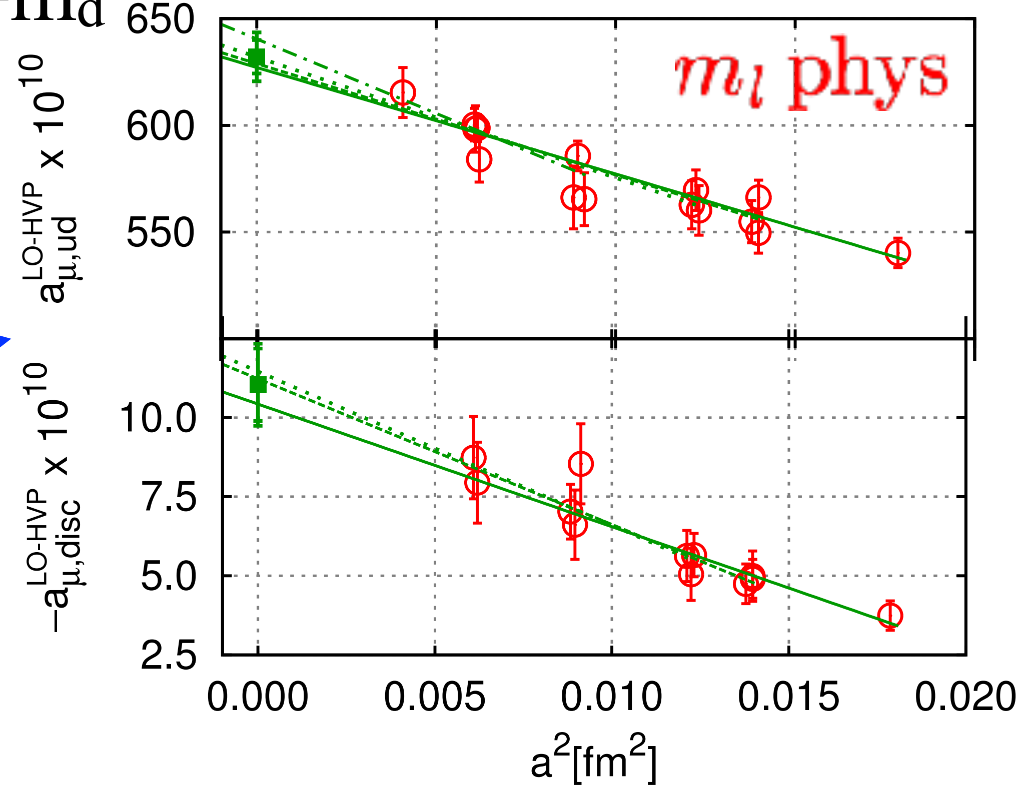
connected, m_u=m_d, no QED

$m_u \neq m_d$ gives +~1.5%

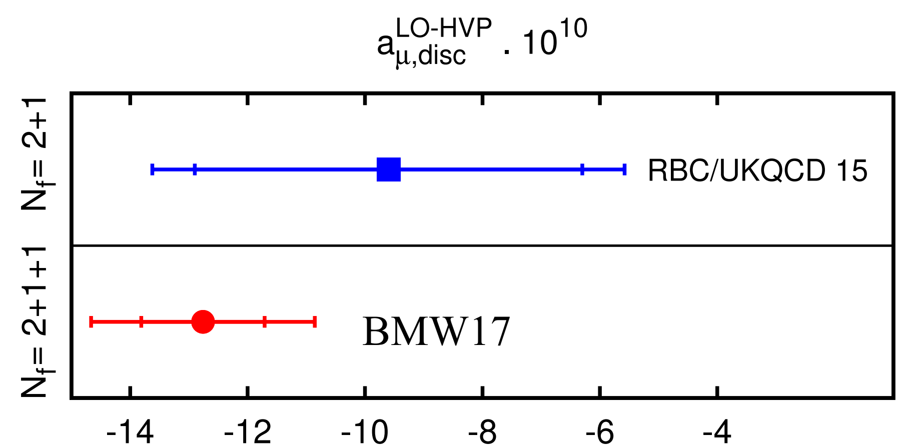
HPQCD/Fermilab/MILC:1710.11212

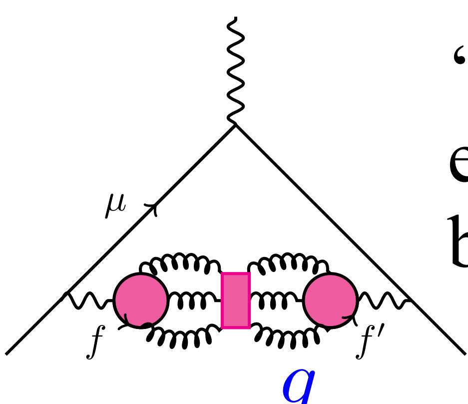
Other recent results, also $m_u = m_d$

BMW(1711.04980): ~1million correlators per point, bound $\pi^+\pi^-$ from data. Large a-dependence (handled by extrapolation, rather than correcting).



Also calculate small -ve ‘disconnected contribn’





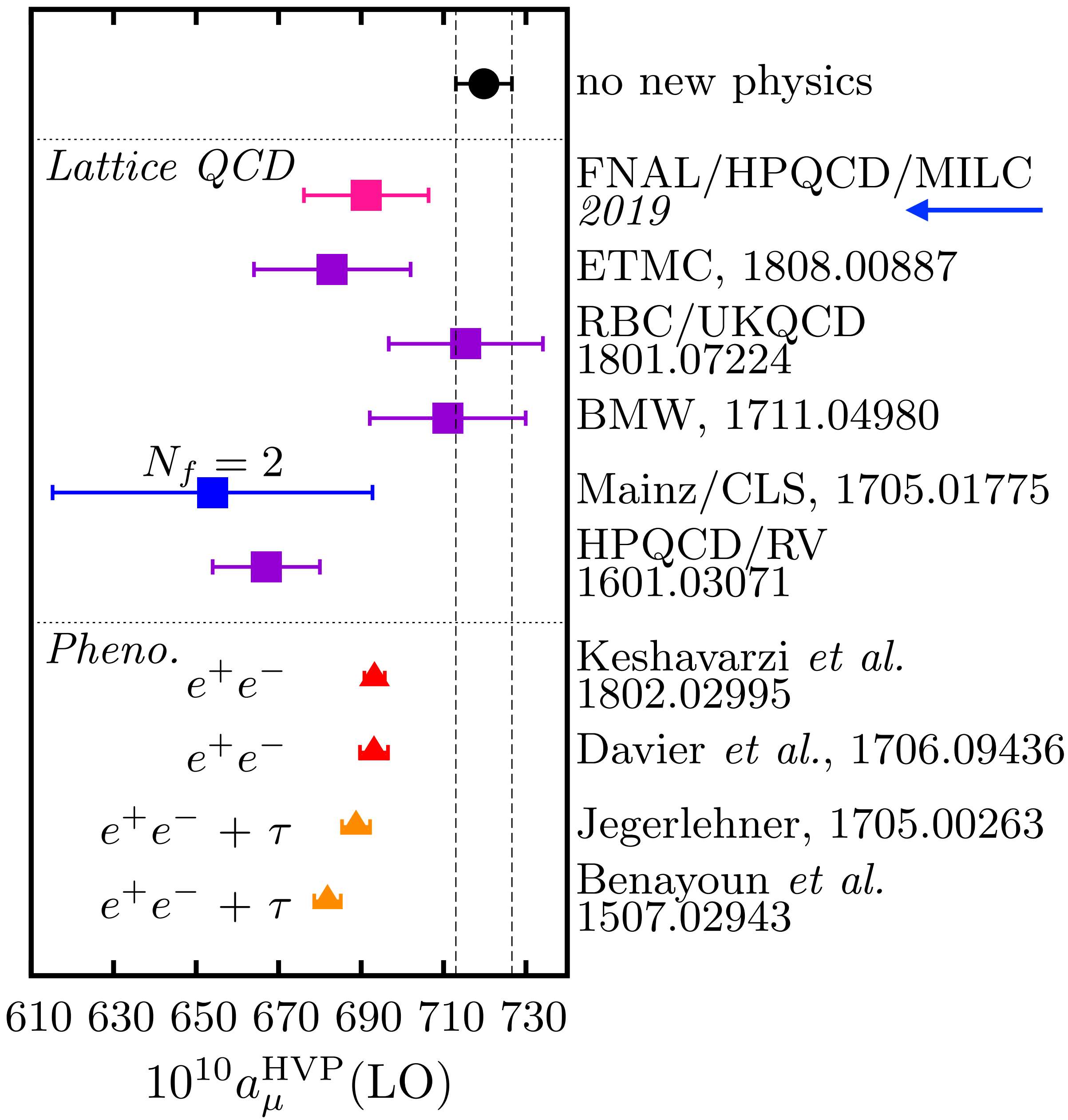
‘disc’ has u, d, s on each side, suppressed by q masses since

$$\sum_{u,d,s} Q_f = 0$$

*We find disc. contrib. sensitive to $m_u \neq m_d$

Total LO HVP contribution - compare lattice QCD and e+e-

equivalent to testing $a_\mu^{\rm expt}$ vs $a_\mu^{\rm SM}$



add u/d, s and c:

$a_\mu^{\rm HVP} = 691(15) \times 10^{-10}$

2% uncty from systs.

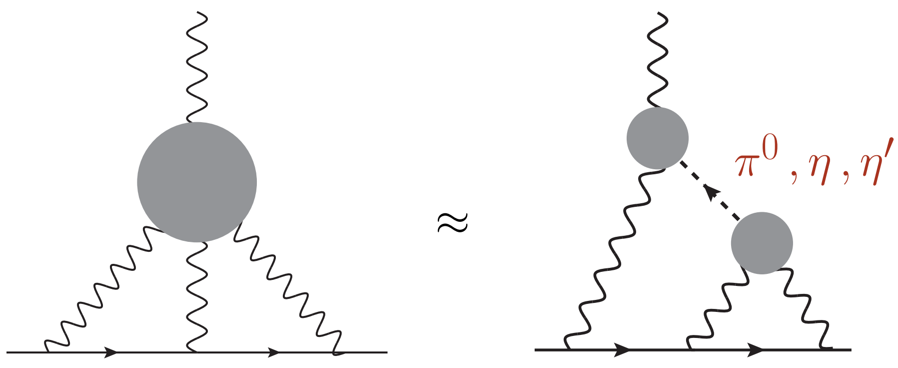
Lattice QCD future:

- QED, $m_u \neq m_d$ must be inc. fully in conn. + disc. HVP
- clarify large-t behaviour (with stats and/or $\pi\pi$)

Elephant in the room? hadronic light-by-light contribution

Not simply related to experiment, values obtained use large N_c, chiral pert. th. etc.

'Glasgow Consensus' 2009: $a_\mu^{HLbL} = 10.5(2.6) \times 10^{-10}$



μ

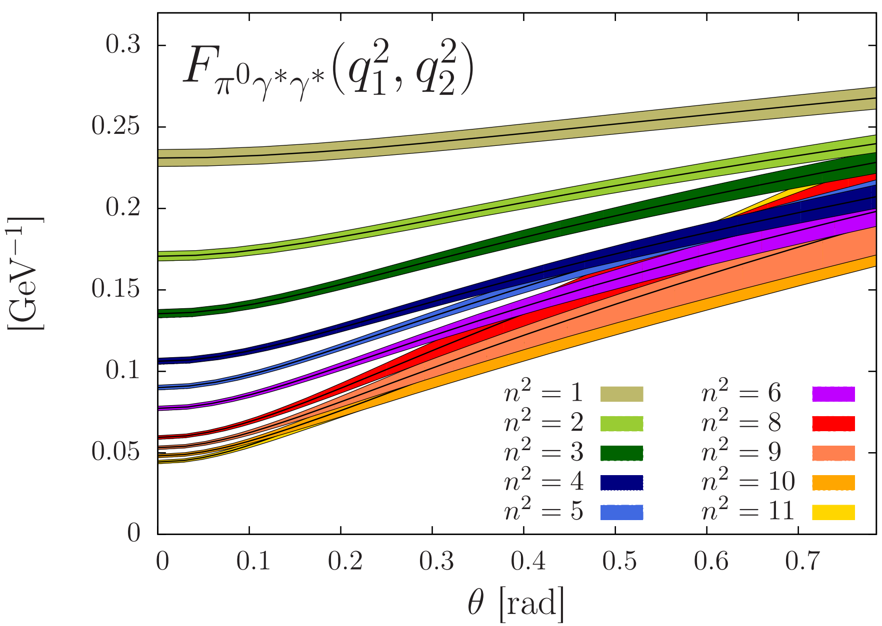
dominated by π^0 exchange :

there also OPE constraints

10% possible? with improved dispersive approaches (with imp. expt for e.g. $\pi^0 \to \gamma^*\gamma^*$

Nyffeler, 1602.03398

Colangelo et al, 1702.07347

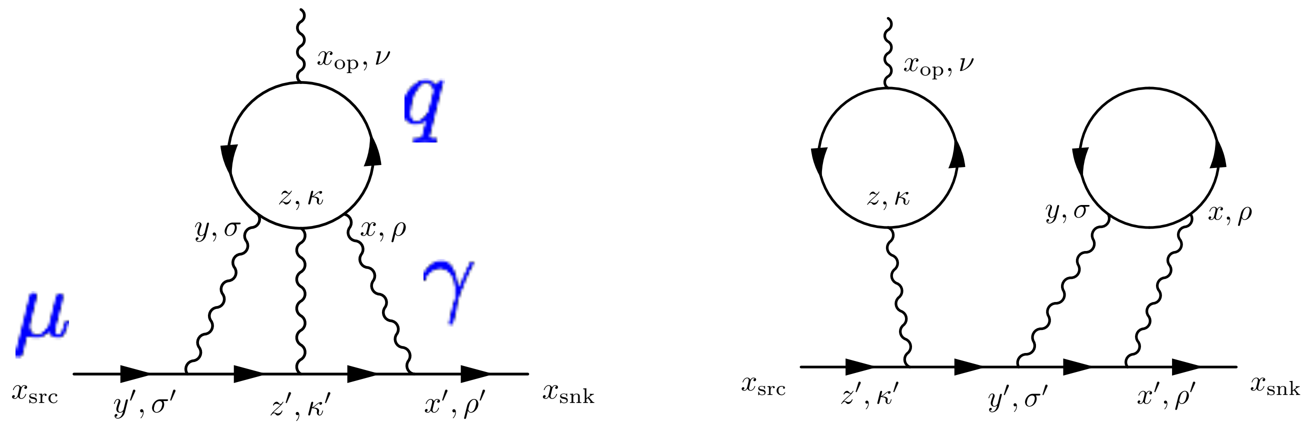


Lattice QCD calcs of $\mathcal{F}_{\pi^0\gamma^*\gamma^*}$ can test these approaches

Mainz, 1607.08174,1712.00421

Direct computation of a_μ^{HLbL} in lattice QCD

RBC 1610.04603



Note: gluons NOT shown

'connected' leading 'disconnected'

Calculate 4 quark propagators and combine with factors from muon and photon propagators, sum over points. Massless photon means that finite volume is an issue.

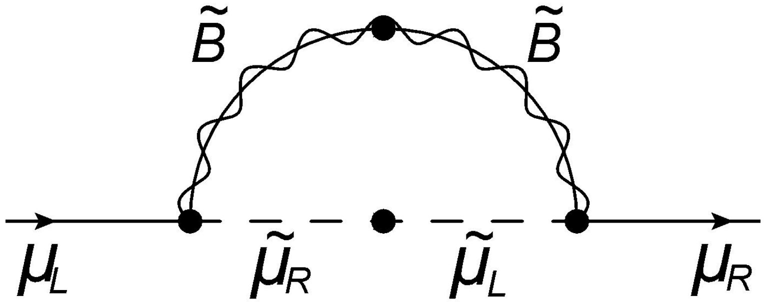
First result:
1 lattice spacing
physical m_l

$$a_\mu^{HLbL} = 5.4(1.4) \times 10^{-10}$$ ← stat. errors only

connected: 11.6 ; disc. : -6.3

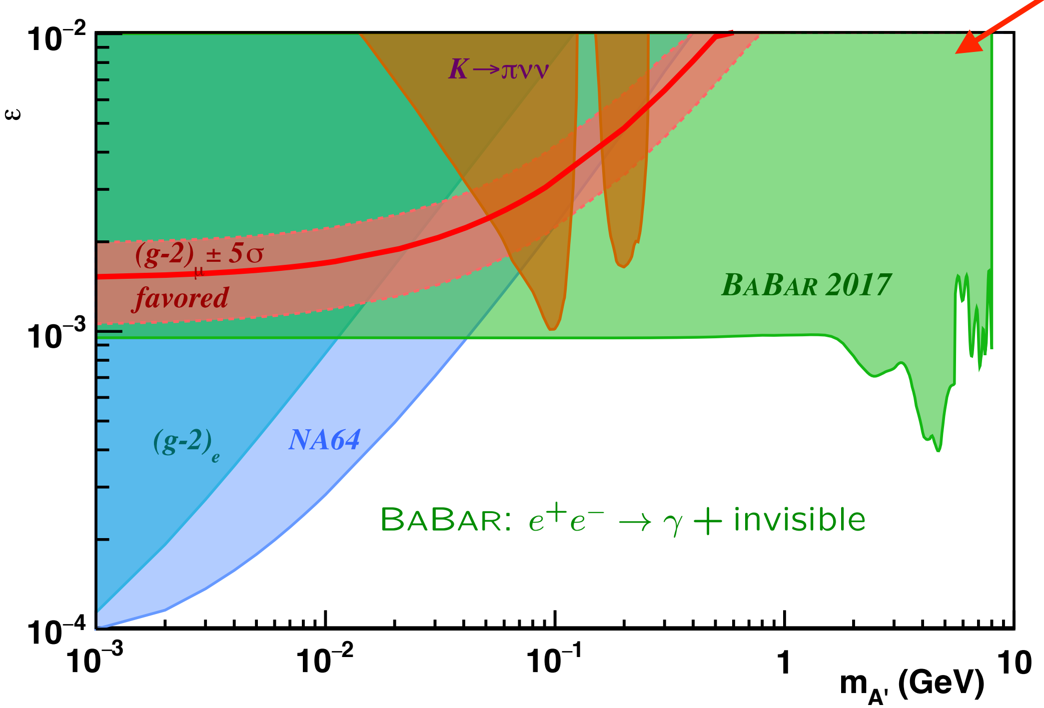
improving finite-volume systematics:
Mainz, 1711.02466; RBC 1705.01067

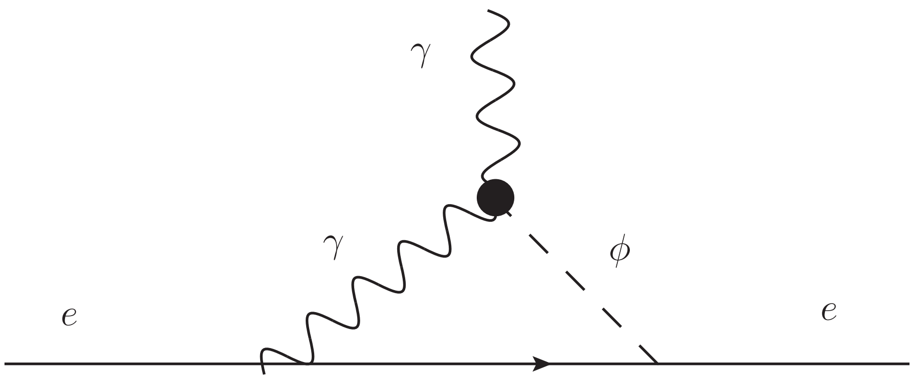
Beyond the Standard Model explanations for the discrepancy in a_μ ?



SUSY still a viable explanation
- more constrained now by LHC searches since need relatively light smuon and more fine-tuning.

simple GeV-scale 'dark photon' ruled out.





New scalar, m < 1 GeV could explain a_e and a_μ

1806.10252

Conclusion

- $a_\mu^{E821} = 11659209.1(6.3) \times 10^{-10}$
  $a_\mu^{SM} = 11659182.0(3.6) \times 10^{-10}$

disagreement $a_\mu^{\rm expt} - a_\mu^{\rm SM} = 27(7) \times 10^{-10}$

- SM uncertainty dominated by HVP.
  Methods using $R_{e^+e^-}$ have improved to 0.4%; lattice QCD results now at 2-3% - aim is <1% with QED and isospin-breaking included. A key issue is ππ .
- HLbL determination will also improve - first direct lattice QCD results now available. It seems clearly small.
- Muon g-2 @FNAL will report its first new exptl result in 2019 - final aim is to reduce uncty by factor of 4.
  If central value remains, this will be 5σ evidence for BSM