The Quest for Unification

- Understanding the universe and our place in it
- Unification of Matter and Interactions
- Realistic models & predictions
- Fundamental principles & dualities, $e.g.$ spinor–vector duality → Z'@LHC

Birmingham University, Birmingham, 1 June 2016

Unification – a constant theme in science

- 1600's – Newton: unification of terrestrial and celestial mechanics

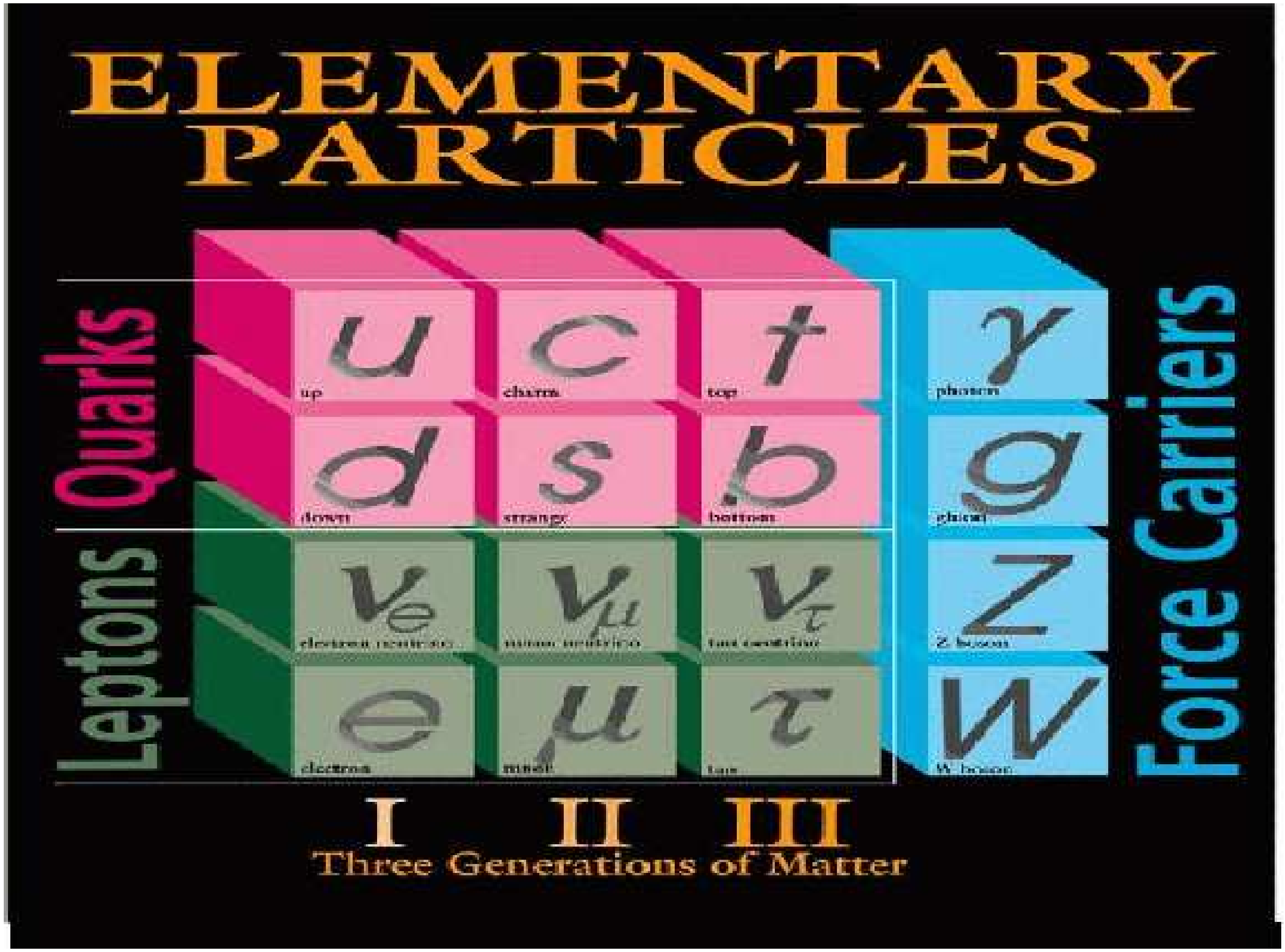
- 1800's – Maxwell: unification of electric and magnetic forces

- 1960's – Glashow, Weinberg & Salam: eletroweak unification

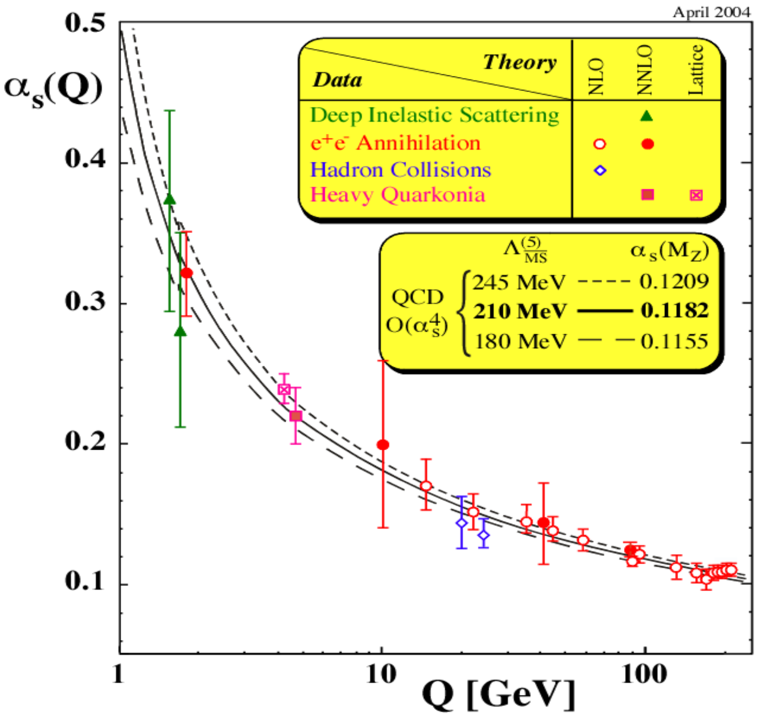
- 1970's – GUTs: strong & electroweak unification

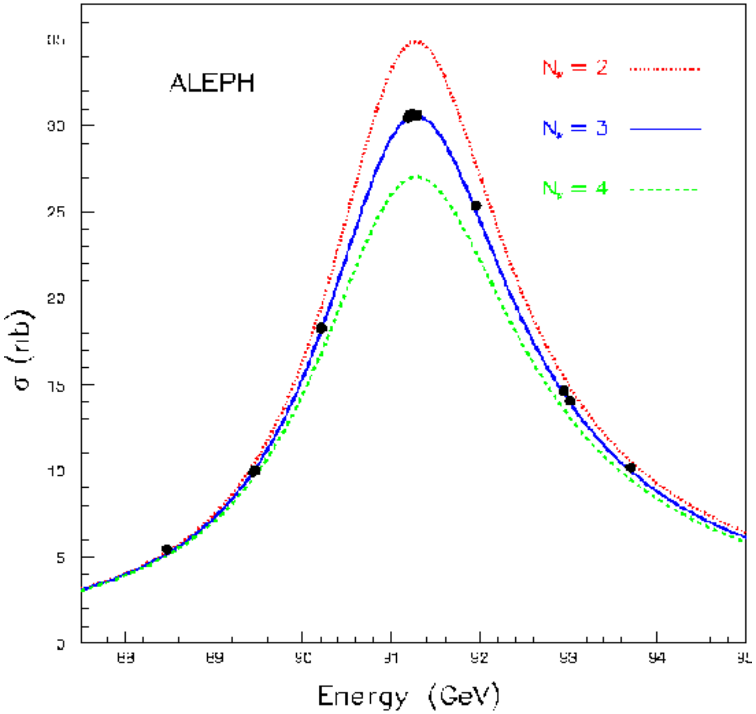
- 1980's – strings: Gauge & Gravity unification

DATA –> STANDARD MODEL



In agreement with all experimental DATA





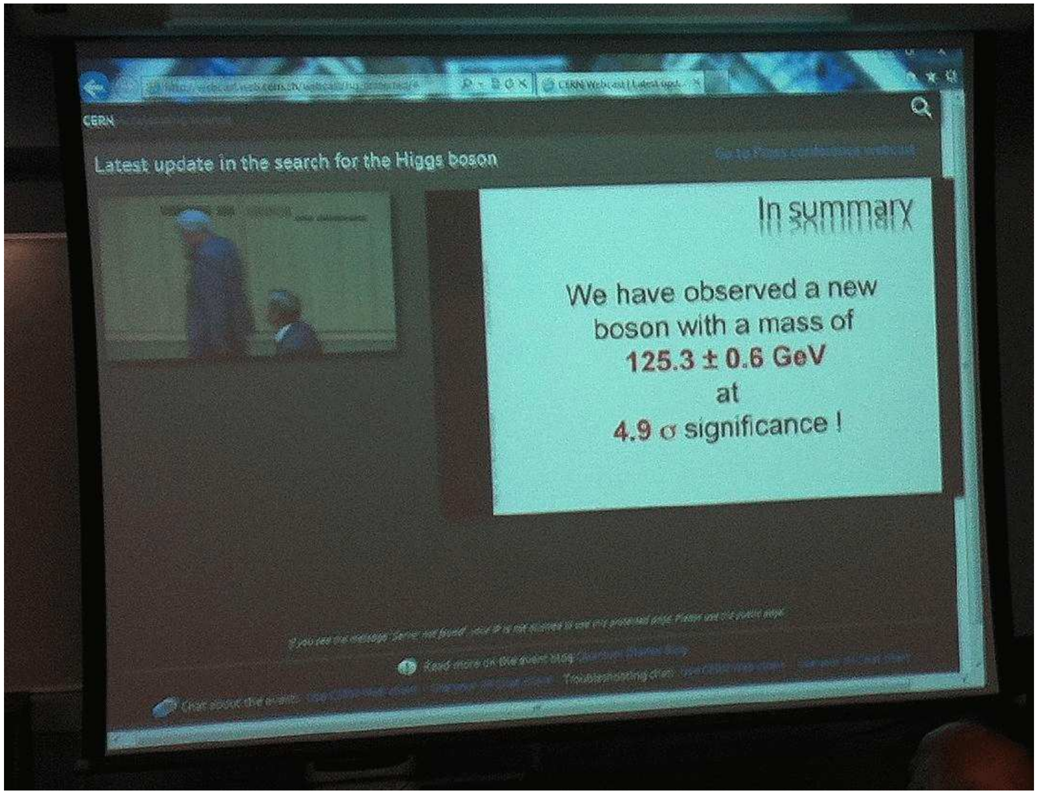
Logarithmic running of α_s

the QCD gauge coupling

Invisible decay width of the

Z gauge boson

... Before



Latest update in the search for the Higgs boson

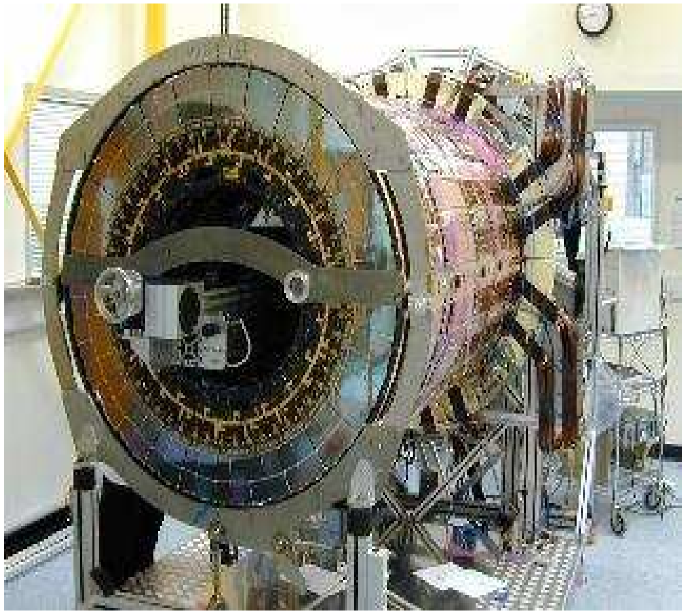
In summary

We have observed a new boson with a mass of
125.3 ± 0.6 GeV
at
4.9 σ significance !

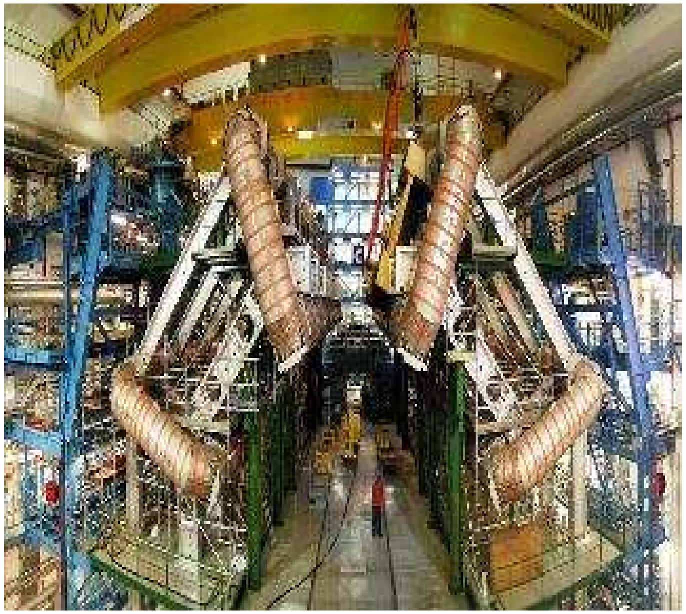
4 July 2012

After ...

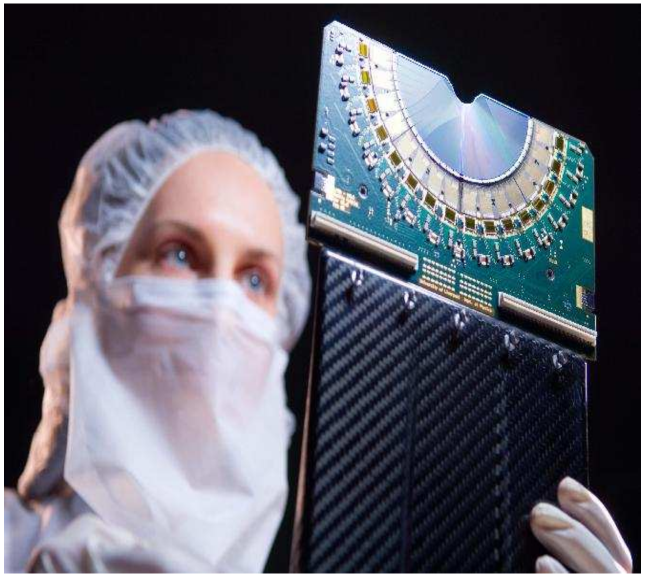
Liverpool's contributions



Silicon Central Tracker



ATLAS

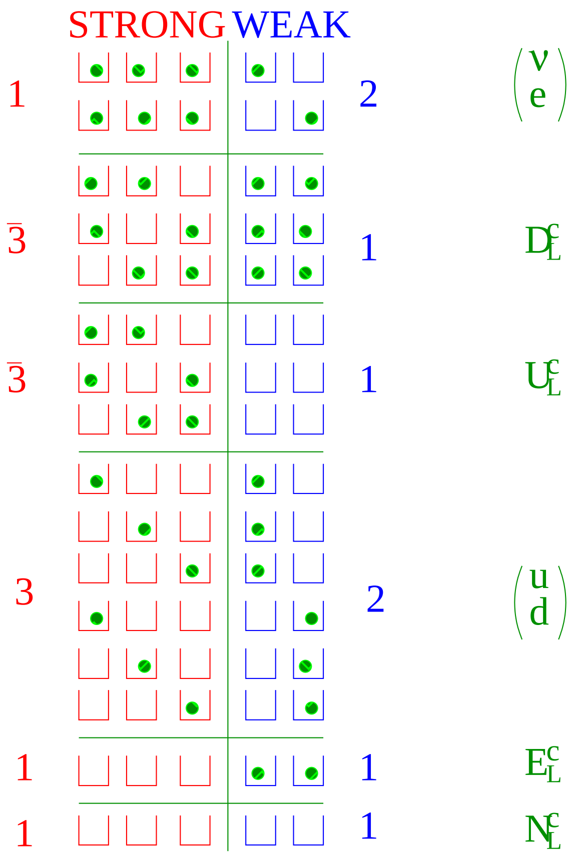


LHCb vertex detector

DATA $\longrightarrow$ STANDARD MODEL

STRONGWEAKELECTROMAGNETIC $\longrightarrow$ SO(10)

Put even # of coins in five slots 0 ; 2 ; 4



$$\bar{5} = \binom{5}{4} = \frac{5!}{4!1!}$$

$+$

$$10 = \binom{5}{2}$$

$+$

$$1 = \binom{5}{0}$$

$$\overline{}$$

16

STANDARD MODEL −> UNIFICATION

Addiotional evidence: Logarithmic running

Gauge

Matter

Higgs → supersymmetry

Addiotional evidence: Proton stability, neutrino masses

Primary guides:

3 generations

$SO(10)$ embedding

QUANTIZE GRAVITY?

QUANTUM
MECHANICS $\xcancel{\oplus}$ GRAVITY

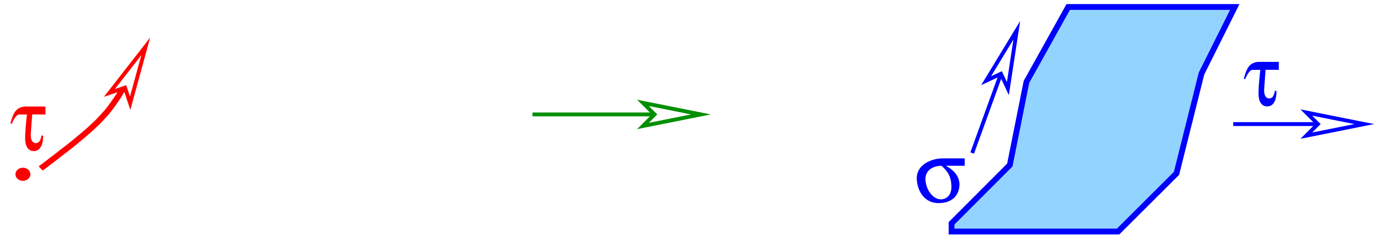
STRING

∞ finite

STRING THEORY

unification with flavour, hierarchy and gravity

ELEMENTS OF STRING THEORY



$X(\tau)$– World−line

$X(\tau,\sigma)$– World−sheet

$$\boldsymbol{S} = -T \bullet \text{Area}$$

Classically: $g^{\alpha\beta} \longrightarrow \eta^{\alpha\beta}$

Quantum : $D=26$

Eq. of motion: $$\left(\partial_\sigma^2 - \partial_\tau^2\right) X^\mu(\sigma,\tau) = 0$$

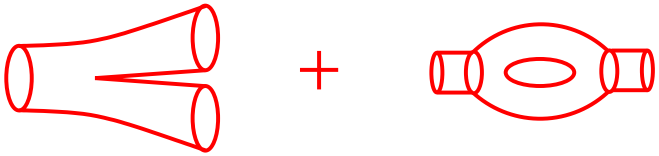
$$X^\mu(\sigma,\tau) = X_L^\mu(\sigma-\tau) + X_R^\mu(\sigma+\tau)$$

Mass levels :

$$M^2 = P^2 = -2 + 2\sum \alpha_{-n}\,\alpha_n \quad \longrightarrow \quad \text{tachyon}$$

−> Add World−sheet Fermions −> N=1 SUSY −> No tachyon

Perturbation Theory



−> Invariance under global reparameterizations

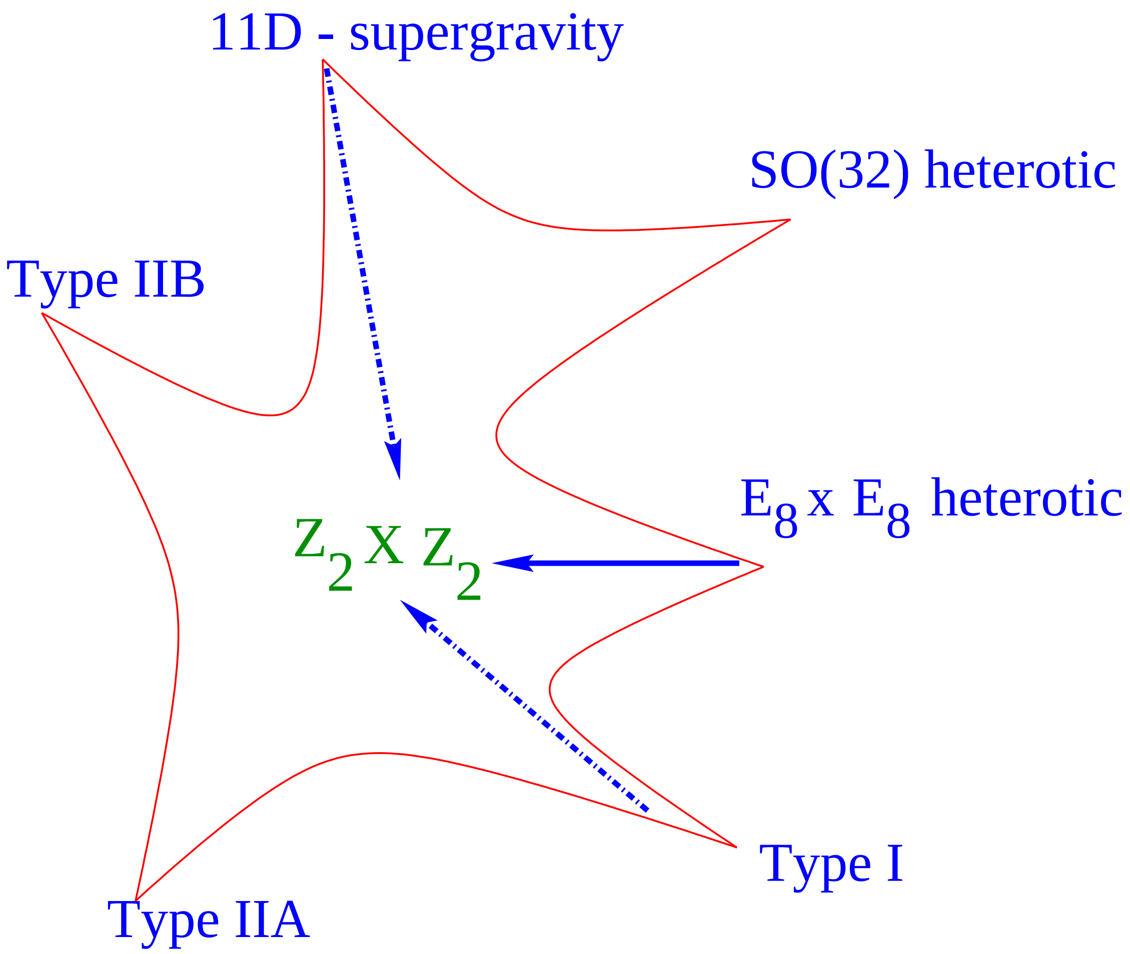
−> Consistency constraints

Heterotic string: $(X_L\,;\ \Psi_L)$ D=10

X_R D=26

Heterotic String: −> Chiral matter under SO(10)

Point, String, Membrane ....



11D - supergravity

SO(32) heterotic

Type IIB

E_8 x E_8 heterotic

Z_2 X Z_2

Type I

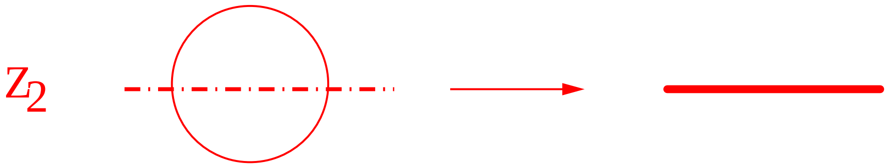
Type IIA

REALISTIC STRING MODELS :

heterotic 10D –> heterotic 4D

6D compactifications $(T^2 \times T^2 \times T^2)$

Orbifold – twists of flat 6D torus



FREE FERMIONIC MODELS –

$Z_2 \times Z_2$ Orbifold –> $U(1)_Y \in SO(10)$

$$\frac{6}{2} = 1+1+1$$

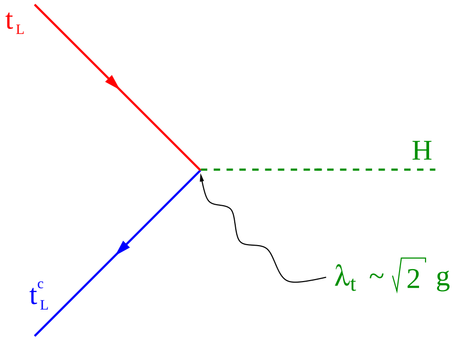
Realistic free fermionic models

‘Phenomenology of the Standard Model and string unification’

• Minimal Superstring Standard Model	NPB 335 (1990) 347 (with Nanopoulos & Yuan)
• Top quark mass $\sim$ 175–180GeV	PLB 274 (1992) 47
• Generation mass hierarchy	NPB 407 (1993) 57
• CKM mixing	NPB 416 (1994) 63 (with Halyo)
• Stringy seesaw mechanism	PLB 307 (1993) 311 (with Halyo)
• Gauge coupling unification	NPB 457 (1995) 409 (with Dienes)
• Proton stability	NPB 428 (1994) 111
• Squark degeneracy	NPB 526 (1998) 21 (with Pati)
• Moduli fixing	NPB 728 (2005) 83
• Classification & Exophobia	2003 – · · ·

(with Assel, Christodoulides, Kounnas, Nooij, Rizos & Sonmez)

Top Quark Mass Prediction



Only $\lambda_t = \langle Q t_L^c H \rangle = \sqrt{2} g$ at $N = 3$

mass of lighter quarks and leptons $\rightarrow$ nonrenormalizable terms

$$\longrightarrow \qquad \lambda_b \; = \; \lambda_\tau \; = \; 0.35 g^3 \; \sim \; \frac{1}{8} \lambda_t$$

Evolve λ_t , λ_b to low energies

$$m_t \; = \; \lambda_t v_1 \; = \; \lambda_t \frac{v_0}{\sqrt{2}} \sin\beta \qquad m_b \; = \; \lambda_b v_2 \; = \; \lambda_b \frac{v_0}{\sqrt{2}} \cos\beta$$

where $v_0 = \dfrac{2 m_W}{g_2(M_Z)} = 246 \mathrm{GeV}$ and $v_1^2 + v_2^2 = \dfrac{v_0^2}{2}$

$$m_t \; = \; \lambda_t(m_t) \, \frac{v_0}{\sqrt{2}} \, \frac{\tan\beta}{(1+\tan^2\beta)^{\frac{1}{2}}} \Longrightarrow$$

ysics Letters B 274 (1992) 47–52
orth-Holland

PHYSICS LETTERS B

Hierarchical top–bottom mass relation in a superstring derived standard-like model

Alon E. Faraggi

Center For Theoretical Physics, Texas A&M University, College Station, TX 77843-4242, USA
and Astroparticle Physics Group, Houston Advanced Research Center (HARC), The Woodlands, TX 77381, USA

Received 30 September 1991

I propose a mechanism in a class of superstring standard-like models which explains the mass hierarchy between the top and bottom quarks. At the trilinear level of the superpotential only the top quark gets a nonvanishing mass term while the bottom quarks and tau lepton mass terms are obtained from nonrenormalizable terms. I construct a model which realized this mechanism. In this model the bottom quark and tau lepton Yukawa couplings are obtained from quartic order terms. I show that $\lambda_b = \lambda_\tau \sim \frac{1}{8}\lambda_t$ at the unification scale. A naive estimate yields $m_t \sim 175$–180 GeV.

One of the unresolved puzzles of the standard model is the mass splitting between the top quark and the lighter uarks and leptons. Especially difficult to understand within the context of the standard model is the big splitting in the heaviest generation. Experimental limits [1] indicate the top mass to be above 80 GeV, while the ottom and tau lepton masses are found at 5 GeV and 1.78 GeV respectively. Possible extensions to the standard odel are grand unified theories. Although the main prediction of GUTs, proton decay, has not yet been observed, calculations of $\sin^2\theta_w$ and of the mass ratio m_b/m_τ support their validity. Recent calculations seem to upport supersymmetric GUTs versus nonsupersymmetric ones [2]. In spite of the success of SUSY GUTs in onfronting LEP data [2], an understanding of the mass splitting between the top quark and the lighter quarks nd leptons is still lacking. The next level in which such an understanding may be developed is in the context of uperstring theory [3].

Towards String Predictions

1. Low energy supersymmetry

   Specific SUSY breaking patterns $\longrightarrow$ Collider implications

2. Additional gauge bosons

   Proton Stability and low–scale Z' $\longrightarrow$ Collider signatures

   (AEF, Marco Guzzi, EPJC 75 (2015) 537)

3. Exotic matter

   In realistic string models

   Unifying gauge group $\Rightarrow$ broken by "Wilson lines".

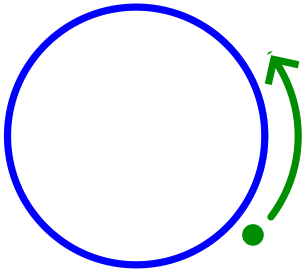
   $\Rightarrow$ non–GUT physical states.

   $\Rightarrow$ Meta-stable heavy string relics

   $\rightarrow$ Dark Mater ; UHECR candidates

T1 − COMPACTIFICATION

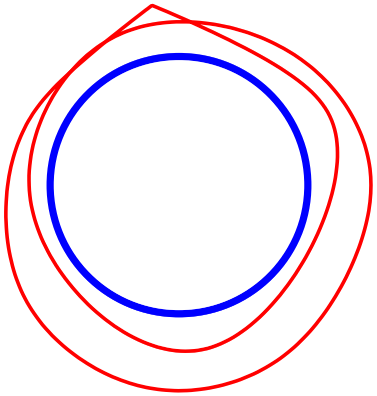
$$X \longrightarrow X \sim X + 2\pi R m$$



Point particle

$$\Psi \sim \mathrm{Exp}(i P X) \quad \Rightarrow P = \frac{m}{R}$$

String



$$P_{L,R} = \frac{m}{R} \pm \frac{n R}{\alpha'}$$

T − DUALITY

$$\text{mass}^2 = \left(\frac{n}{R}\right)^2 + \left(\frac{m\,R}{\alpha'}\right)^2$$

Invariant under

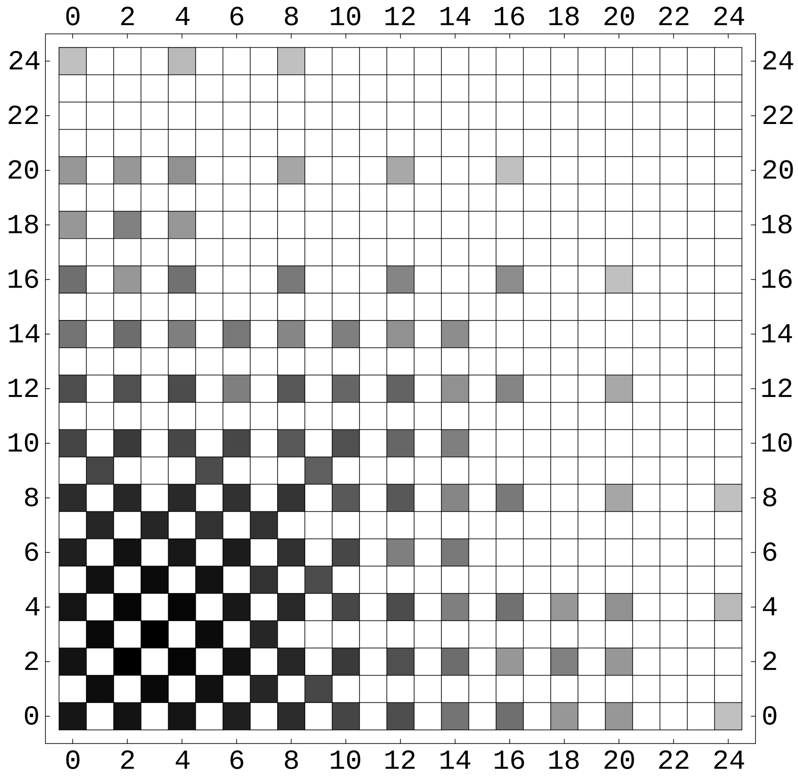
$$\frac{1}{R} \longleftrightarrow \frac{R}{\alpha'} \quad \text{with} \quad m \longleftrightarrow n$$

An exact symmetry in string perturbation theory!

Self−dual point $R = \frac{\alpha'}{R}$ = free fermionic point

Spinor–vector duality: → A new duality!

Invariance under exchange of $\#(16+\overline{16}) \;<->\; \#(10)$



Symmetric under exchange of rows and columns

$$E_6 \;: \qquad 27 = 16 + 10 + 1 \qquad \overline{27} = \overline{16} + 10 + 1$$

Self-dual: $\#(16+\overline{16}) \;=\; \#(10)$ without E_6 symmetry

$\underline{E_6 \to SO(10) \times U(1)_A} \quad \Longrightarrow \quad U(1)_A$ is anomalous!

$$\Longrightarrow \quad U(1)_A \quad \notin \quad \text{low scale} \quad U(1)_{Z'}$$

On the other hand ....(AEF, Viraf Mehta, PRD88 (2013) 025006)

$$\sin^2\theta_W(M_Z)\ ,\ \alpha_s(M_Z) \quad \Longrightarrow \quad U(1)_{Z'} \ \in \ E_6$$

light Z' heterotic–string model : AEF, John Rizos, NPB895 (2015) 233

Observable gauge group: $SO(6) \times SO(4) \times U(1)_{1,2,3}$

$U(1)_\zeta = U(1)_1 + U(1)_2 + U(1)_3 \quad \in \quad E_6$ is anomaly free

sector	field	$SU(4) \times SU(2)_L \times SU(2)_R$	$U(1)_1$	$U(1)_2$	$U(1)_3$	$U(1)_\zeta$
$S + b_1$	$\bar{F}_{1R}$	$(\bar{\mathbf{4}}, \mathbf{1}, \mathbf{2})$	$1/2$	0	0	$1/2$
$S + b_1 + e_3 + e_5$	F_{1R}	$(\mathbf{4}, \mathbf{1}, \mathbf{2})$	$1/2$	0	0	$1/2$
$S + b_2$	F_{1L}	$(\mathbf{4}, \mathbf{2}, \mathbf{1})$	0	$1/2$	0	$1/2$
$S + b_2 + e_1 + e_2 + e_5$	F_{2L}	$(\mathbf{4}, \mathbf{2}, \mathbf{1})$	0	$1/2$	0	$1/2$
$S + b_2 + e_1$	$\bar{F}_{2R}$	$(\bar{\mathbf{4}}, \mathbf{1}, \mathbf{2})$	0	$1/2$	0	$1/2$
$S + b_2 + e_2 + e_5$	$\bar{F}_{3R}$	$(\bar{\mathbf{4}}, \mathbf{1}, \mathbf{2})$	0	$1/2$	0	$1/2$
$S + b_3 + e_1 + e_2$	F_{3L}	$(\mathbf{4}, \mathbf{2}, \mathbf{1})$	0	0	$1/2$	$1/2$
$S + b_3 + e_2$	$\bar{F}_{4R}$	$(\bar{\mathbf{4}}, \mathbf{1}, \mathbf{2})$	0	0	$1/2$	$1/2$
$S + b_3 + x$	h_1	$(\mathbf{1}, \mathbf{2}, \mathbf{2})$	$-1/2$	$-1/2$	0	-1
$S + b_2 + x + e_5$	h_2	$(\mathbf{1}, \mathbf{2}, \mathbf{2})$	$-1/2$	0	$-1/2$	-1
$S + b_2 + x + e_1 + e_2$	h_3	$(\mathbf{1}, \mathbf{2}, \mathbf{2})$	$-1/2$	0	$-1/2$	-1
$S + b_3 + x + e_1$	D_4	$(\mathbf{6}, \mathbf{1}, \mathbf{1})$	$-1/2$	$-1/2$	0	-1
	χ_1^+	$(\mathbf{1}, \mathbf{1}, \mathbf{1})$	$1/2$	$1/2$	1	$+2$
	χ_1^-	$(\mathbf{1}, \mathbf{1}, \mathbf{1})$	$1/2$	$1/2$	-1	0
	$\zeta_a, a = 2, 3$	$(\mathbf{1}, \mathbf{1}, \mathbf{1})$	$1/2$	$-1/2$	0	0
	$\bar{\zeta}_a, a = 2, 3$	$(\mathbf{1}, \mathbf{1}, \mathbf{1})$	$-1/2$	$1/2$	0	0
$S + b_2 + x + e_1 + e_5$	D_5	$(\mathbf{6}, \mathbf{1}, \mathbf{1})$	$-1/2$	0	$-1/2$	-1
	χ_2^+	$(\mathbf{1}, \mathbf{1}, \mathbf{1})$	$1/2$	1	$1/2$	$+2$
	χ_2^-	$(\mathbf{1}, \mathbf{1}, \mathbf{1})$	$1/2$	-1	$1/2$	0
	$\zeta_a, a = 4, 5$	$(\mathbf{1}, \mathbf{1}, \mathbf{1})$	$1/2$	0	$-1/2$	0
	$\bar{\zeta}_a, a = 4, 5$	$(\mathbf{1}, \mathbf{1}, \mathbf{1})$	$-1/2$	0	$1/2$	0
$S + b_2 + x + e_2$	D_6	$(\mathbf{6}, \mathbf{1}, \mathbf{1})$	$-1/2$	0	$-1/2$	-1
	χ_3^+	$(\mathbf{1}, \mathbf{1}, \mathbf{1})$	$1/2$	1	$1/2$	$+2$
	χ_3^-	$(\mathbf{1}, \mathbf{1}, \mathbf{1})$	$1/2$	-1	$1/2$	0
	$\zeta_a, a = 6, 7$	$(\mathbf{1}, \mathbf{1}, \mathbf{1})$	$1/2$	0	$-1/2$	0
	$\bar{\zeta}_a, a = 6, 7$	$(\mathbf{1}, \mathbf{1}, \mathbf{1})$	$-1/2$	0	$1/2$	0
$S + b_1 + x + e_3$	$\bar{D}_6$	$(\mathbf{6}, \mathbf{1}, \mathbf{1})$	0	$1/2$	$1/2$	$+1$
	$\bar{\chi}_4^+$	$(\mathbf{1}, \mathbf{1}, \mathbf{1})$	-1	$-1/2$	$-1/2$	-2
	$\bar{\chi}_4^-$	$(\mathbf{1}, \mathbf{1}, \mathbf{1})$	1	$-1/2$	$-1/2$	0
	$\zeta_a, a = 8, 9$	$(\mathbf{1}, \mathbf{1}, \mathbf{1})$	0	$1/2$	$-1/2$	0
	$\bar{\zeta}_a, a = 8, 9$	$(\mathbf{1}, \mathbf{1}, \mathbf{1})$	0	$-1/2$	$1/2$	0
$S + b_1 + x + e_5$	D_7	$(\mathbf{6}, \mathbf{1}, \mathbf{1})$	0	$-1/2$	$-1/2$	-1
	χ_5^+	$(\mathbf{1}, \mathbf{1}, \mathbf{1})$	1	$1/2$	$1/2$	$+2$
	χ_5^-	$(\mathbf{1}, \mathbf{1}, \mathbf{1})$	-1	$1/2$	$1/2$	0
	$\zeta_a, a = 10, 11$	$(\mathbf{1}, \mathbf{1}, \mathbf{1})$	0	$1/2$	$-1/2$	0
	$\bar{\zeta}_a, a = 10, 11$	$(\mathbf{1}, \mathbf{1}, \mathbf{1})$	0	$-1/2$	$1/2$	0
$S + b_3 + x + e_2 + e_3$	ζ_1	$(\mathbf{1}, \mathbf{1}, \mathbf{1})$	$1/2$	$-1/2$	0	0
	$\bar{\zeta}_1$	$(\mathbf{1}, \mathbf{1}, \mathbf{1})$	$-1/2$	$1/2$	0	0
$S + b_1 + x + e_3 + e_4 + e_6$	ϕ_1	$(\mathbf{1}, \mathbf{1}, \mathbf{1})$	0	$1/2$	$1/2$	$+1$
	$\bar{\phi}_1$	$(\mathbf{1}, \mathbf{1}, \mathbf{1})$	0	$-1/2$	$-1/2$	-1
$S + b_1 + x + e_4 + e_5 + e_6$	ϕ_2	$(\mathbf{1}, \mathbf{1}, \mathbf{1})$	0	$1/2$	$1/2$	$+1$
	$\bar{\phi}_2$	$(\mathbf{1}, \mathbf{1}, \mathbf{1})$	0	$-1/2$	$-1/2$	-1

Table 1: *Observable twisted matter spectrum and* $SU(4) \times SU(2)_L \times SU(2)_R \times U(1)^3$ *quantum numbers.*

The chiral spectrum is self–dual under spinor–vector duality

maintains the E_6 embedding $\Rightarrow$ $U(1)_\zeta$ is anomaly free

three generations $+$ heavy and light higgs $+$ $\lambda_t \sim 1$

$$U(1)_{Z'} \;=\; \tfrac{1}{5}\,U(1)_C \;-\; \tfrac{1}{5}\,U(1)_L - U_\zeta$$

$Z' \Rightarrow$ Additional matter at the Z' breaking scale

Exotic $SO(10)$ singlets with non–standard $U(1)_\zeta$ charges

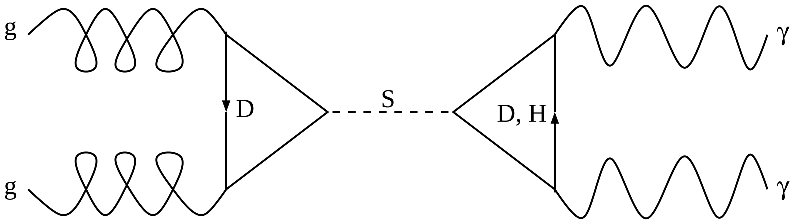
$\Rightarrow$ Natural Wilsonian dark–matter candidates

Z' at the LHC

Heavy Higgs $\langle \mathcal{N} \rangle \sim M_{\mathrm{String}}$ → high seesaw → Z'

Field	$SU(3)_C$	$\times SU(2)_L$	$U(1)_Y$	$U(1)_{Z'}$
Q_L^i	3	2	$+\frac{1}{6}$	$-\frac{2}{5}$
u_L^i	$\bar{3}$	1	$-\frac{2}{3}$	$-\frac{2}{5}$
d_L^i	$\bar{3}$	1	$+\frac{1}{3}$	$-\frac{4}{5}$
e_L^i	1	1	$+1$	$-\frac{2}{5}$
L_L^i	1	2	$-\frac{1}{2}$	$-\frac{4}{5}$
D^i	3	1	$-\frac{1}{3}$	$+\frac{4}{5}$
$\bar{D}^i$	$\bar{3}$	1	$+\frac{1}{3}$	$+\frac{6}{5}$
H^i	1	2	$-\frac{1}{2}$	$+\frac{6}{5}$
$\bar{H}^i$	1	2	$+\frac{1}{2}$	$+\frac{4}{5}$
S^i	1	1	0	-2

di–photon events



AEF and John Rizos, arXiv:1601.03604, and references therein

General Relativity: Covariance & Equivalence Principle

→ fundamental geometrical principle

Quantum Mechanics: Axiomatic formulation ... $\mathrm{P} \sim |\Psi|^2$

However Quantum + Gravity Theory

not known

Main effort: quantize GR; quantize space–time: *e.g.* superstring theory

The main successes of string theory:

1) Viable perturbative approach to quantum gravity

2) Unification of gravity, gauge & matter structures

i.e. construction of phenomenologically realistic models

→ relevant for experimental observation

State of the art: MSSM from string theory

Quantum Mechanics from an Equivalence Postulate

Adaptation of Hamilton–Jacobi theory

Hamilton's equations of motion $\dot{q} = \frac{\partial H}{\partial p} \ , \ \dot{p} = -\frac{\partial H}{\partial q}$

$$H(q,p) \longrightarrow K(Q,P) \equiv 0 \qquad \Longrightarrow \qquad \dot{Q} = \frac{\partial K}{\partial P} \equiv 0 \, , \ \dot{P} = -\frac{\partial K}{\partial Q} \equiv 0$$

The solution is the Classical Hamilton–Jacobi Equation

stationary case $\longrightarrow \frac{1}{2m}\left(\frac{\partial S_0}{\partial q}\right)^2 + V(q) - E = 0$

$(q,p) \rightarrow (Q,P)$ via canonical transformations

$q, \ p$ are independent. Solve. Then $p = \frac{\partial S}{\partial q}$

Quantum mechanics: $[\hat{q},\hat{p}] \; = \; i\hbar \;\; \rightarrow \;\; q, \; p \;\; \rightarrow$ not independent

Assume $H \;\; \rightarrow \;\; K \quad i.e. \; W(Q) \; = \; V(Q) - E \; = \; 0$ always exists

But $\quad q, \; p \quad$ not independent. $\qquad p \; = \; \frac{\partial S}{\partial q}.$

Equivalence postulate:

Consider the transformations on

$$(\; q \; , \; S_0(q) \; , \; p \; = \; \frac{\partial S_0}{\partial q}) \;\; \longrightarrow \;\; (\; \tilde{q} \; , \; \tilde{S}_0(\tilde{q}) \; , \; \tilde{p} \; = \; \frac{\partial \tilde{S}_0}{\partial \tilde{q}} \;)$$

Such that $\qquad W(q) \;\; \longrightarrow \;\; \tilde{W}(\tilde{q}) \; = \; 0 \qquad$ exist for all $W(q)$

$$\Longrightarrow \; \frac{1}{2m}\left(\frac{\partial S_0}{\partial q}\right)^2 \; + \; W(q) \; + \; Q(q) \; = \; 0 \qquad \text{QHJE}$$

$\longrightarrow$ Schrödinger equation

QHJE: Invariant under Möbius transformations in

$$\hat{R}^D = R^D \cup \{\infty\} \rightarrow \text{Compact Space}$$

Decompactification limit $\leftrightarrow$ $Q(q) \rightarrow 0$ $\leftrightarrow$ classical limit

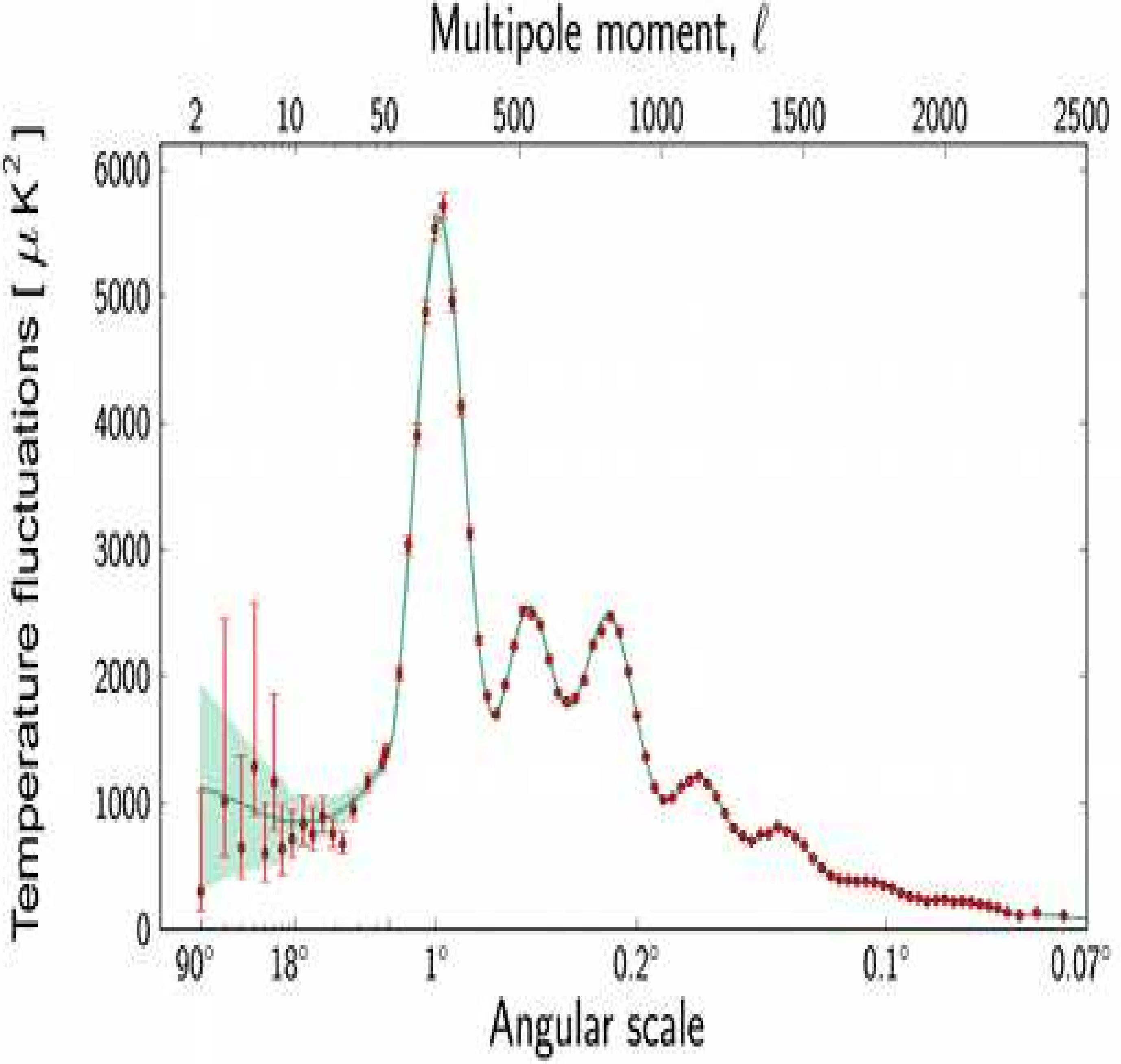
$Q(q)$ Intrinsic curvature terms of elementary particles $\neq 0$ Always

The equivalence postulate $\Longrightarrow$ $S_0 \neq const \Leftrightarrow \hbar \neq 0$

$$\Psi(q) = \frac{1}{\sqrt{S_0'}} \left(A \mathrm{e}^{+\frac{i}{\hbar} S_0} + B \mathrm{e}^{-\frac{i}{\hbar} S_0} \right)$$

$\mathrm{Re}\ell_0 = \lambda_P \rightarrow$ fundamental length scale

Phase–space duality *vs* T–duality


Multipole moment, ℓ
2
10
50
500
1000
1500
2000
2500
Temperature fluctuations [μ K^2]
6000
5000
4000
3000
2000
1000
0
90°
18°
1°
0.2°
0.1°
0.07°
Angular scale

Conclusions

- DATA $\longrightarrow$ UNIFICATION

- STRINGS $\longrightarrow$ GAUGE & GRAVITY UNIFICATION

- EXPERIMENTAL PREDICTIONS ? Z' `@` LHC? ; FCC?

- FUNDAMENTAL PRINCIPLES ?

  $e.g.$ spinor–vector duality $\to$ Physics & Geometry

  phase–space duality & the equivalence postulate of QM