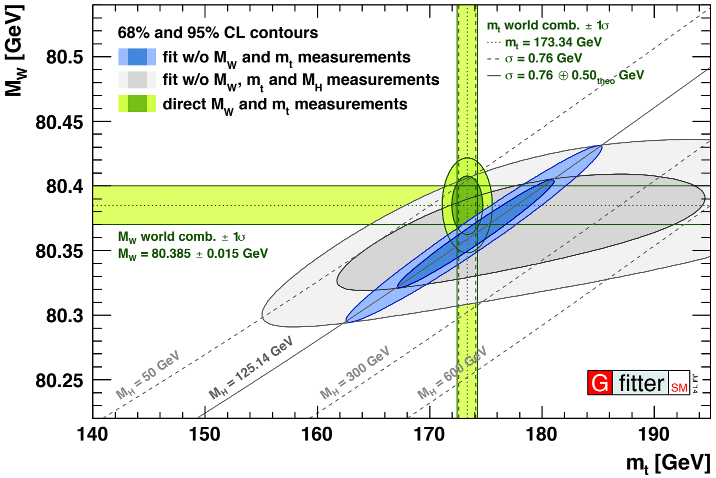
Max Baak (CERN),
on behalf of the Gfitter group (*)

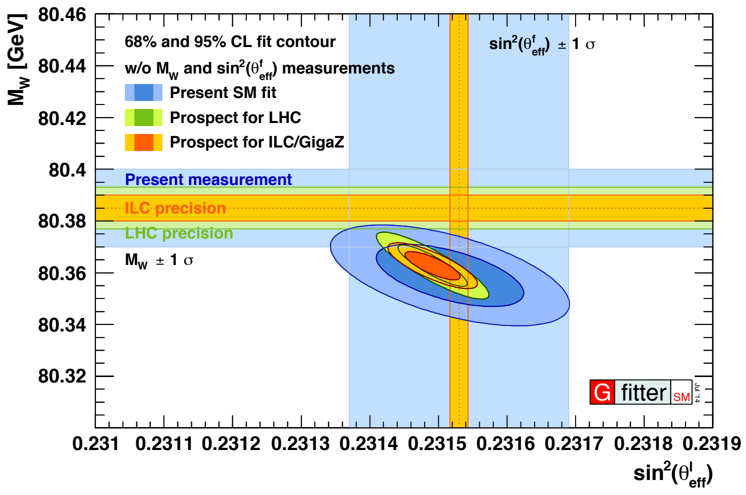
University of Birmingham / Warwick, UK
January 14th / 15th, 2015

http://cern.ch/Gfitter

EPJC 74, 3046 (2014), arXiv:1407.3792

The global electroweak fit at NNLO
Prospects for LHC and ILC





(*) M. Baak, J. Cuth, J. Haller, A. Höcker, R. Kogler, K. Mönig, M. Schott, J. Stelzer

Outline

G fitter

This presentation:

- Introduction to the Electroweak Fit
  - Inputs to the electroweak fit
    - *Full set of 2-loop calculations and theory uncertainties*
- ✓ After the Higgs: predictions for key observables
- ✓ Modified Higgs couplings
- ✓ Prospects for LHC and ILC
- Conclusion & Outlook

The Gfitter Project – Introduction

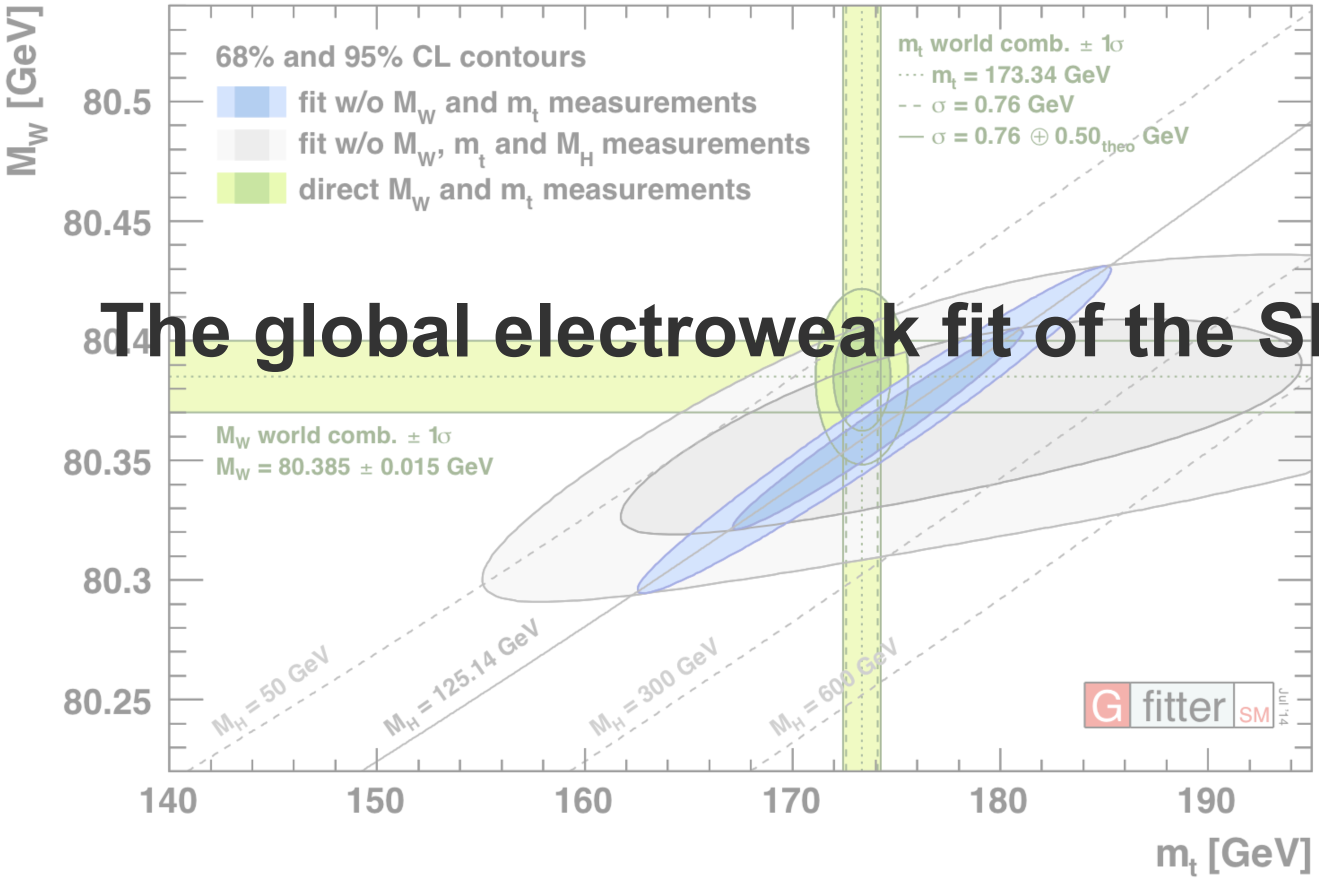
Gfitter

A Generic Fitter Project
for HEP Model Testing

- Gfitter = state-of-the-art HEP model testing tool
- Latest results always available at: http://cern.ch/Gfitter
  - *(Most) results of this presentation: EPJC 74, 3046 (2014)*

- Gfitter software and features:
  - Modular, object-oriented C++, relying on ROOT, XML, python, etc.
  - Core package with data-handling, fitting, and statistics tools
  - Independent "plug-in" physics libraries: SM, 2HDM, multiple BSM models, ...

The global electroweak fit of the SM



Idea behind electroweak fits

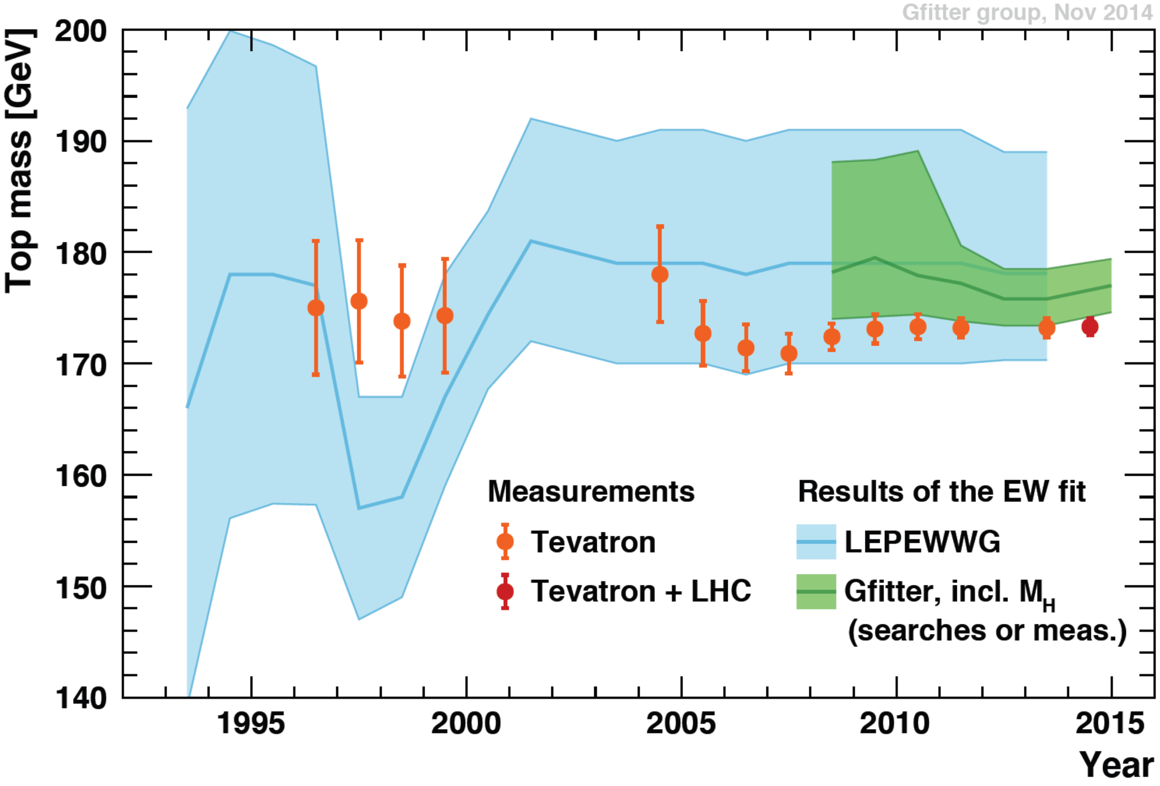
- ✓ Observables receive quantum loop corrections from ‘unseen’ virtual effects.

- ✓ If system is over-constrained, fit for unknown parameters or test the model’s self-consistency.

- ✓ If precision is better than typical loop factor ($\alpha \approx 1/137$), test the model or try to obtain info on new physics in loops.
  - For example, in the past EW fits were used to predict the Higgs mass.

Global EW fits: a long history

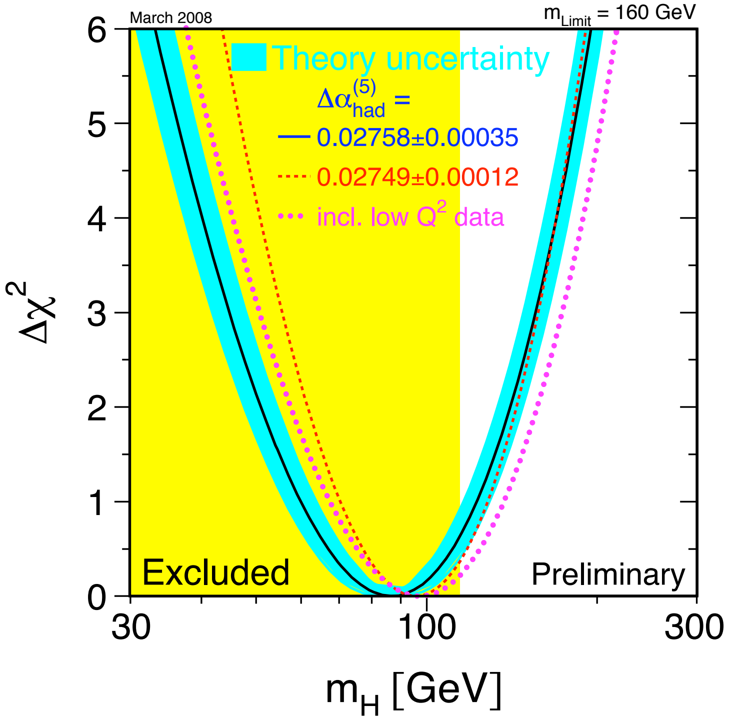
- Huge amount of pioneering work by many!
  - Needed to understand importance of loop corrections
    - Important observables (now) known at least at two-loop order, sometimes more.
  - High-precision Standard Model (SM) predictions and measurements required
    - First from LEP/SLC, then Tevatron, now LHC.



- Top mass predictions from loop effects available since ~1990.
  - Official LEPEW fit since 1993.
- The EW fits have always been able to predict the top mass correctly!

Global EW fits: many fit codes

- EW fits performed by many groups in past and present.
  - D. Bardinet al. (ZFITTER), G. Passarino et al. (TOPAZ0), LEPEW WG (M. Grünewald, et al.), J. Erler (GAP), Bayesian fit (M. Ciuchini et al.), etc …
  - Important results obtained!
- Several groups pursuing global beyond-SM fits, especially SUSY.
- Global SM fits also used at lower energies [CKM-matrix].

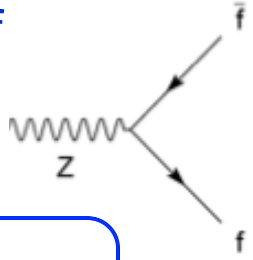


- Fits of the different groups agree very well.

- Some differences in treatment of theory errors, which just start to matter.
  - E.g. theoretical and experimental errors added linearly (= conservative) or quadratically.
    - In following: theoretical errors treated as Gaussian (quadratic addition.)

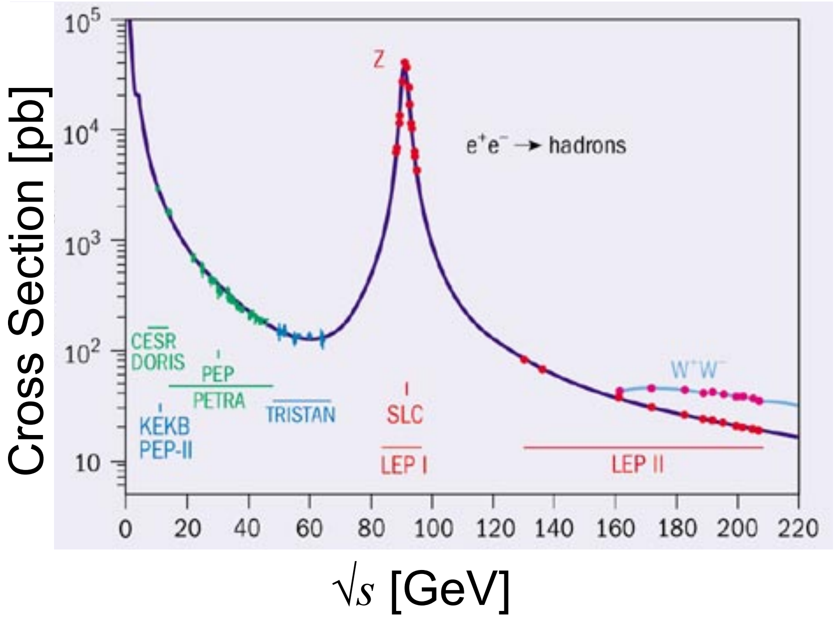
The predictive power of the SM

- As the Z boson couples to all fermions, it is ideal to measure & study both the electroweak and strong interactions.
- Tree level relations for $Z\to f\bar{f}$
  - $i\bar{f}\gamma^{\mu}\left(g_{V,f}-g_{A,f}\gamma_5\right)fZ_{\mu}$



- Prediction EWSB at tree-level:

$$\boxed{\frac{M_W^2}{M_Z^2\cos\theta_W^2}=1}$$



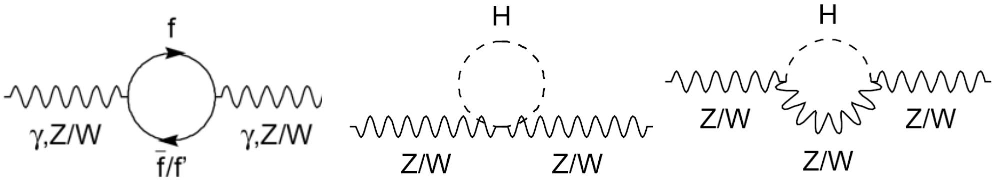
- The impact of loop corrections
  - Absorbed into EW form factors: ρ, κ, Δr
  - Effective couplings at the Z-pole
  - Quadraticly dependent on m_t, *logarithmic* dependence on M_H

$$g_{V,f}=\sqrt{\rho_Z^f}\left(I_3^f-2Q^f\sin^2\theta_{\mathrm{eff}}^f\right)$$

$$g_{A,f}=\sqrt{\rho_Z^f}I_3^f$$

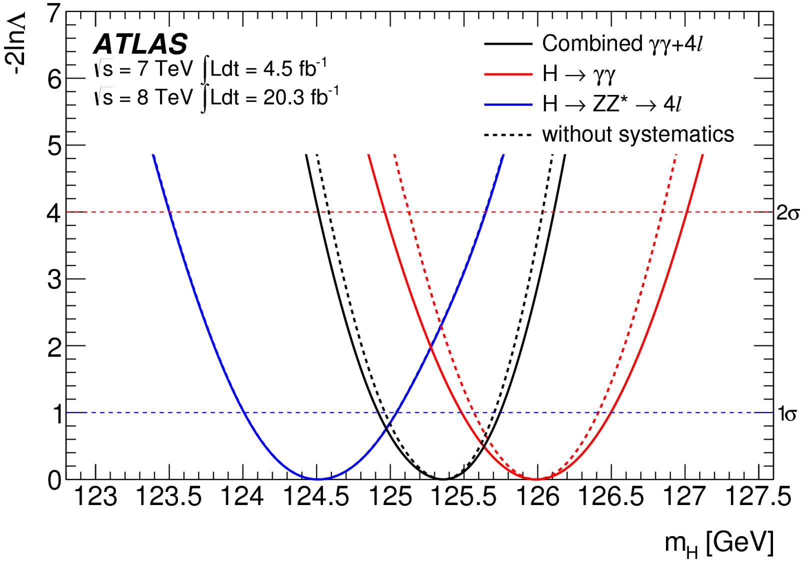
$$\sin^2\theta_{\mathrm{eff}}^f=\kappa_Z^f\sin^2\theta_W$$

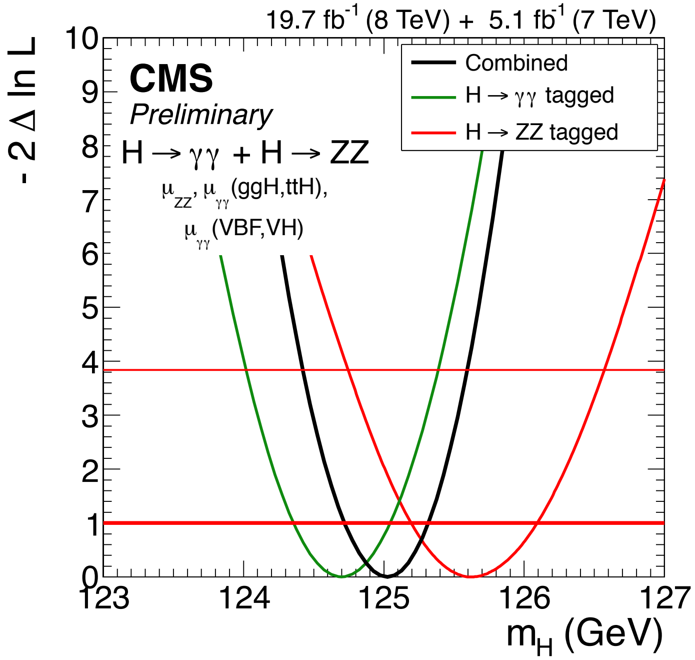
$$M_W^2=\frac{M_Z^2}{2}\left(1+\sqrt{1-\frac{\sqrt{8}\pi\alpha(1+\Delta r)}{G_F M_Z^2}}\right)$$



The SM fit with Gfitter, including the Higgs

- Discovery of Higgs-like boson at LHC
  - Cross section, production rate time branching ratios, spin, parity sofar compatible with SM Higgs boson.
- This talk: assume boson is SM Higgs.
- Use in EW fit: $M_H = 125.14 \pm 0.24$ GeV
  - ATLAS: $M_H = 125.36 \pm 0.37 \pm 0.18$ GeV
CMS: $M_H = 125.03 \pm 0.27 \pm 0.14$ GeV
[arXiv:1406.3827, CMS-PAS-HIG-14-009]
- Change in average between fully uncorrelated and fully correlated systematic uncertainties is minor: δM_H : 0.24 → 0.32 GeV
  - EW fit unaffected at this level of precision





The SM fit with Gfitter, including the Higgs

Unique situation:

- *For first time SM is fully over-constrained.*
- *And for first time electroweak observables can be unambiguously predicted at loop level.*
- *Powerful predictions of key observables now possible, much better than w/o M_H.*

Can now test for:

→ Self-consistency of SM.

→ Possible contributions from BSM models.

- Part of focus of this talk …

Measurements at the Z-pole (1/2)

- Total cross-section of $e^-e^+ \to Z \to f\bar{f}$
  - Expressed in terms of partial decay width of initial and final width:

$$\boxed{\sigma^Z_{f\bar f} = \sigma^0_{f\bar f}\frac{s\Gamma_Z^2}{(s-M_Z^2)^2+s^2\Gamma_Z^2/M_Z^2}\frac{1}{R_{\text{QED}}} \quad \text{with} \quad \sigma^0_{f\bar f}=\frac{12\pi}{M_Z^2}\frac{\Gamma_{ee}\Gamma_{f\bar f}}{\Gamma_Z^2}}$$

Corrected for QED radiation

  - Full width: $\Gamma_Z = \Gamma_{ee} + \Gamma_{\mu\mu} + \Gamma_{\tau\tau} + \Gamma_{\text{had}} + \Gamma_{\text{inv}}$
  - (Correlated set of measurements.)

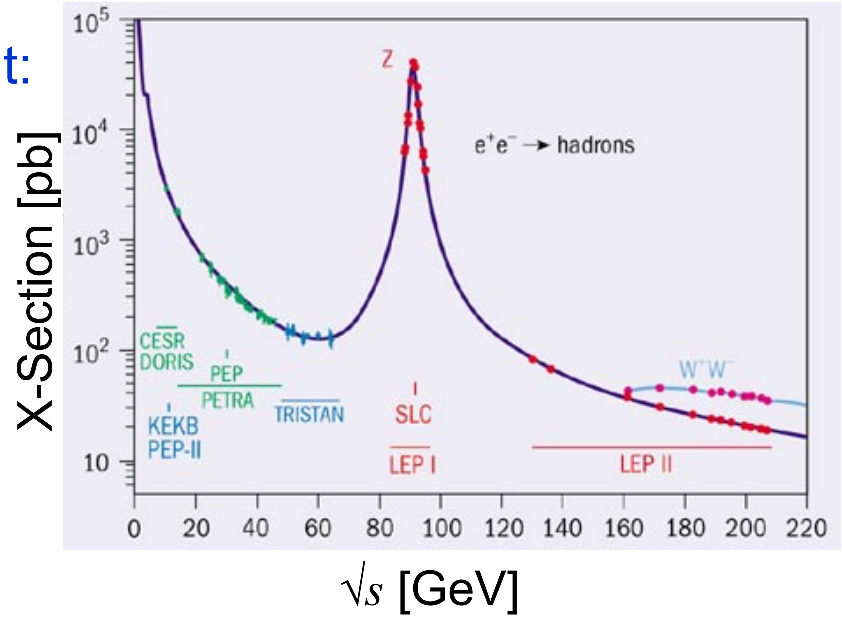
- Set of input (width) parameters to EW fit:
  - Z mass and width: $\boxed{M_Z}$, $\boxed{\Gamma_Z}$
  - Hadronic pole cross section:

$$\boxed{\sigma^0_{\text{had}}} = 12\pi/M_Z^2 \cdot \Gamma_{ee}\Gamma_{\text{had}}/\Gamma_Z^2$$

  - Three leptonic ratios (lepton univ.):

$$\boxed{R^0_\ell} = R^0_e = \Gamma_{\text{had}}/\Gamma_{ee} \; (= R^0_\mu = R^0_\tau)$$

  - Hadronic-width ratios: $\boxed{R^0_b}$, $\boxed{R^0_c}$



Measurements at the Z-pole (2/2)

Definition of Asymmetry

- Distinguish vector and axial-vector couplings of the Z

$$A_f = \frac{g_{L,f}^2 - g_{R,f}^2}{g_{L,f}^2 + g_{R,f}^2} = \frac{2 g_{V,f}\, g_{A,f}}{g_{V,f}^2 + g_{A,f}^2}$$

- Directly related to: $\boxed{\sin^2 \theta_{\text{eff}}^{f\bar{f}}} = \frac{1}{4Q_f}\left(1 + \mathcal{R}e\left(\frac{g_{V,f}}{g_{A,f}}\right)\right)$

Observables

- In case of no beam polarisation (LEP) use final state angular distribution to define *forward/backward asymmetry:*

$$A_{FB}^f = \frac{N_F^f - N_B^f}{N_F^f + N_B^f} \qquad A_{FB}^{0,f} = \frac{3}{4} A_e A_f$$

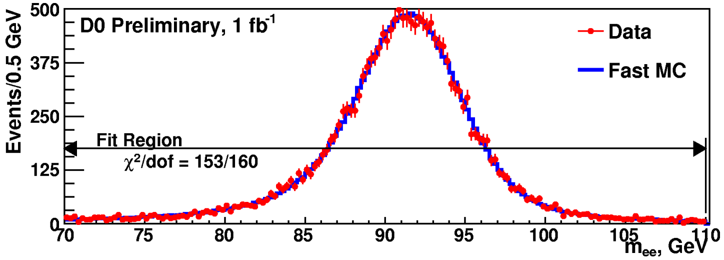
- Polarised beams (SLC), define *left/right asymmetry:*

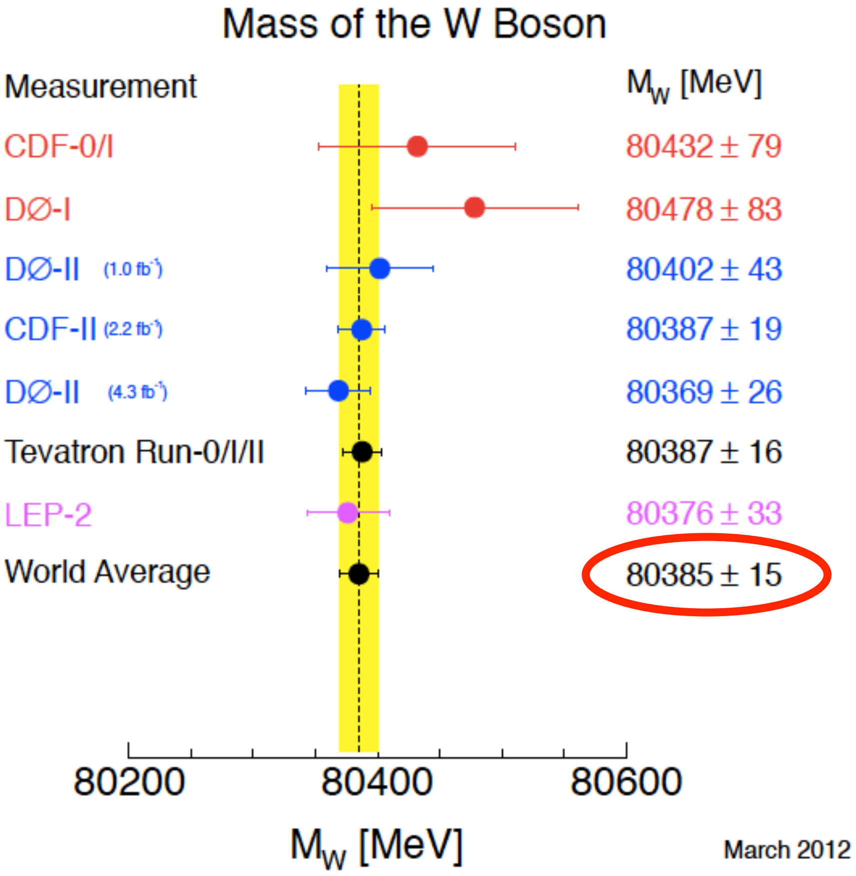
$$A_{LR}^f = \frac{N_L^f - N_R^f}{N_L^f + N_R^f} \frac{1}{\langle |P|_e \rangle} \qquad A_{LR}^0 = A_e$$

- Measurements: $\boxed{A_{FB}^{0,\ell},}\ \boxed{A_{FB}^{0,c},}\ \boxed{A_{FB}^{0,b}}\ \boxed{A_\ell,}\ \boxed{A_c,}\ \boxed{A_b}$

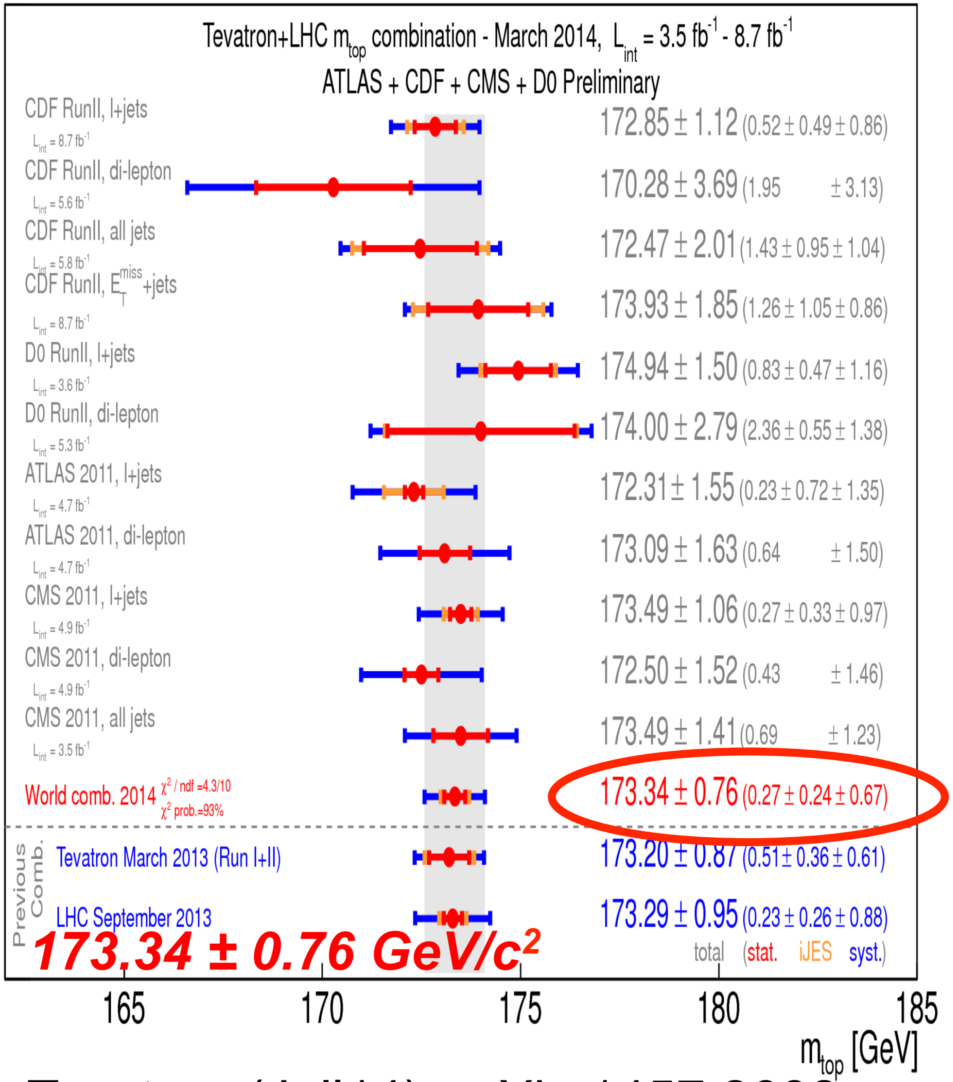
Latest averages for M_W and m_{top}

Latest Tevatron result from: arXiv:1204.0042





Top mass WA (**March 2014**): arXiv:1403.4427



173.34 ± 0.76 GeV/c²

Tevatron (Jul'14): arXiv:1457.2682

174.34 ± 0.64 GeV/c²

The electromagnetic coupling

- The EW fit requires precise knowledge of α(M_Z) – better than 1% level
  - Enters various places: hadr. radiator functions, predictions of M_W and $\sin^2\theta^f_{eff}$
- Conventionally parametrized as (α(0) = fine structure constant) :

$$\alpha(s) = \frac{\alpha(0)}{1 - \Delta\alpha(s)}$$

- Evolution with renormalization scale:

$$\Delta\alpha(s) = \Delta\alpha_{\text{lep}}(s) + \Delta\alpha^{(5)}_{\text{had}}(s) + \Delta\alpha_{\text{top}}(s)$$

The electromagnetic coupling

- The EW fit requires precise knowledge of $\alpha(M_Z)$ – better than 1% level
  - Enters various places: hadr. radiator functions, predictions of M_W and $\sin^2\theta^f_{eff}$
- Conventionally parametrized as ($\alpha(0)$ = fine structure constant) :

$$\alpha(s) = \frac{\alpha(0)}{1 - \Delta\alpha(s)}$$

- Evolution with renormalization scale:

$$\Delta\alpha(s) = \boxed{\Delta\alpha_{\rm lep}(s)} + \Delta\alpha^{(5)}_{\rm had}(s) + \boxed{\Delta\alpha_{\rm top}(s)}$$

- Leptonic term known up to *four* loops (for $q^2 \gg m_l^2$)
- Top quark contribution known up to 2 loops, *small: -0.7x10*$^{-4}$

[C.Sturm, arXiv: 1305.0581]

[M. Steinhauser, PLB 429, 158 (1998)]

The electromagnetic coupling

- The EW fit requires precise knowledge of $\alpha(M_Z)$ – better than 1% level
  - Enters various places: hadr. radiator functions, predictions of M_W and $\sin^2\theta^f_{eff}$
- Conventionally parametrized as ($\alpha(0)$ = fine structure constant) :

$$\alpha(s) = \frac{\alpha(0)}{1 - \Delta\alpha(s)}$$

- Evolution with renormalization scale:

$$\Delta\alpha(s) = \Delta\alpha_{\mathrm{lep}}(s) + \Delta\alpha^{(5)}_{\mathrm{had}}(s) + \Delta\alpha_{\mathrm{top}}(s)$$

- Hadronic contribution (from the 5 light quarks) completely dominates overall uncertainty on $\alpha(M_Z)$.

- Difficult to calculate, cannot be obtained from pQCD alone.
  - Analysis of low-energy e^+e^- data
  - Usage of pQCD if lack of data

$$\boxed{\Delta\alpha^{(5)}_{had}(M_Z) = (274.9 \pm 1.0)\cdot 10^{-4}}$$

- Similar analysis to evaluation of hadronic contribution to $(g-2)_\mu$

[M. Davier et al., Eur. Phys. J. C71, 1515 (2011)]

Theoretical inputs *at NNLO*

- Radiative corrections are important!
  - E.g. consider tree-level EW unification relation:
    - This predicts: $M_W = (79.964 \pm 0.005)$ GeV
    - Experiment: $M_W = (80.385 \pm 0.015)$ GeV

$$M_W^2\Big|_{\text{tree-level}} = \frac{M_Z^2}{2} \cdot \left(1 + \sqrt{1 - \frac{\sqrt{8}\pi\alpha}{G_F M_Z^2}}\right)$$

- Without loop corrections: shift of 400 MeV, 27σ discrepancy!

Theoretical inputs *at NNLO*

- Radiative corrections are important!
  - E.g. consider tree-level EW unification relation:
    - This predicts: $M_W = (79.964 \pm 0.005)$ GeV
    - Experiment: $M_W = (80.385 \pm 0.015)$ GeV

$$M_W^2\Big|_{\text{tree-level}} = \frac{M_Z^2}{2} \cdot \left(1 + \sqrt{1 - \frac{\sqrt{8}\pi\alpha}{G_F M_Z^2}}\right)$$

- Without loop corrections: shift of 400 MeV, 27σ discrepancy!

1. Experimental precision (<1%), better than typical loop factor (α≈1/137)
   → Requires radiative corrections at 2-loop level.

2. Before Higgs discovery: uncertainty on M_H largest uncertainty in EW fit.
   → *After:* inclusion of all relevant theoretical uncertainties.

(Part of focus of this talk …)

Theoretical inputs *at NNLO*

- Radiative corrections are important!
  - E.g. consider tree-level EW unification relation:
    - This predicts: $M_W = (79.964 \pm 0.005)$ GeV
    - Experiment: $M_W = (80.385 \pm 0.015)$ GeV

$$M_W^2\Big|_{\text{tree-level}} = \frac{M_Z^2}{2} \cdot \left(1 + \sqrt{1 - \frac{\sqrt{8}\pi\alpha}{G_F M_Z^2}}\right)$$

- Without loop corrections: shift of 400 MeV, 27σ discrepancy!

- In EW fit with Gfitter we use state-of-the-art calculations:
  - $\sin^2\theta^f_{\text{eff}}$ Effective weak mixing angle [M. Awramik et al., JHEP 11, 048 (2006), M. Awramik et al., Nucl.Phys.B813:174-187 (2009)]
    - Full two-loop + leading beyond-two-loop form factor corrections
  - M_W Mass of the W boson [M. Awramik et al., arXiv:0311148v2]
    - Full two-loop + leading beyond-two-loop + 4-loop QCD correction ⟵ **New!** [Kuhn et al., hep-hp/0504055,0605201,0606232]
  - Γ_{had} QCD Adler functions at N^3LO [P. A. Baikov et al., PRL108, 222003 (2012)]
    - N^3LO prediction of the hadronic cross section
  - Γ_i Partial Z decay widths [A. Freitas, JHEP04, 070 (2014)] ⟵ **New! full fermionic 2-loop calc.**

- *New: all EWPOs$^{(*)}$ now described at 2-loop level or better!*

Theory uncertainties from unknown H.O. terms

Most important observables:

Observable	Exp. error	Theo. error
M_W	15 MeV	4 MeV
$\sin^2\theta^l_{\rm eff}$	$1.6 \cdot 10^{-4}$	$0.5 \cdot 10^{-4}$
Γ_Z	2.3 MeV	0.5 MeV
$\sigma^0_{\rm had} = \sigma[e^+e^- \to Z \to {\rm had.}]$	37 pb	6 pb
$R^0_b = \Gamma[Z \to b\bar{b}]/\Gamma[Z \to {\rm had.}]$	$6.6 \cdot 10^{-4}$	$1.5 \cdot 10^{-4}$
m_t	0.76 GeV	$\leq \mathcal{O}(1)$ GeV

Theory uncertainties accounted for in EW fit
(w/ Gauss constraints):

New in EW fit

- Old setup: two nuisance pars for theoretical uncertainties:
  - *δM_W (4 MeV), $\delta\sin^2\theta^l_{\rm eff}$ (4.7×10^{-5})*

Newly included in EW fit setup:

- Full fermionic 2-loop corrections of partial Z decay widths (A. Freitas)
  - *6 corresponding nuisance parameters. ($\delta\Gamma_Z$ = 0.5 MeV)*
- $\Gamma_{\rm had}$ QCD Adler functions at N^{3}LO
  - *2 nuisance parameters.*
- Top quark mass: conversion from measurement to pole to MS-bar mass
  - *Agnostic value used here: **$\delta_{theo}\, m_t$ = 0.5 GeV**.* *(more later)*

Electroweak Fit – Experimental inputs

- **Latest experimental inputs:**
  - Z-pole observables: from LEP / SLC
    [ADLO+SLD, Phys. Rept. 427, 257 (2006)]
  - M_W and Γ_W from LEP/Tevatron
    [arXiv:1204.0042, arXiv:1302.3415]
  - m_{top} latest avg from Tevatron+LHC
    [arXiv:1403.4427]
  - m_c, m_b world averages (PDG)
    [PDG, J. Phys. G33,1 (2006)]
  - $\Delta\alpha_{had}^{(5)}(M_Z^2)$ including α_S dependency
    [Davier et al., EPJC 71, 1515 (2011)]
  - M_H from LHC
    [arXiv:1406.3827, CMS-PAS-HIG-14-009]

- **7 (+10) free fit parameters:**
  - M_H, M_Z, $\alpha_S(M_Z^2)$, $\Delta\alpha_{had}^{(5)}(M_Z^2)$, m_t, m_c, m_b
  - 10 theory nuisance parameters
    - e.g. δM_W (4 MeV), $\delta\sin^2\theta^l_{eff}$ (4.7×10^{-5})

M_H [GeV]$^{(\circ)}$	125.14 ± 0.24	**LHC**
M_W [GeV]	80.385 ± 0.015	**Tevatron**
Γ_W [GeV]	2.085 ± 0.042	
M_Z [GeV]	91.1875 ± 0.0021	
Γ_Z [GeV]	2.4952 ± 0.0023	
$\sigma^0_{\rm had}$ [nb]	41.540 ± 0.037	**LEP**
R^0_ℓ	20.767 ± 0.025	
$A^{0,\ell}_{\rm FB}$	0.0171 ± 0.0010	
A_ℓ $^{(\star)}$	0.1499 ± 0.0018	**SLC**
$\sin^2\theta^\ell_{\rm eff}(Q_{\rm FB})$	0.2324 ± 0.0012	
A_c	0.670 ± 0.027	**SLC**
A_b	0.923 ± 0.020	
$A^{0,c}_{\rm FB}$	0.0707 ± 0.0035	**LEP**
$A^{0,b}_{\rm FB}$	0.0992 ± 0.0016	
R^0_c	0.1721 ± 0.0030	
R^0_b	0.21629 ± 0.00066	
$\overline{m}_c$ [GeV]	$1.27^{+0.07}_{-0.11}$	
$\overline{m}_b$ [GeV]	$4.20^{+0.17}_{-0.07}$	
m_t [GeV]	173.34 ± 0.76	**Tevatron**
$\Delta\alpha^{(5)}_{\rm had}(M_Z^2)^{(\dagger\triangle)}$	2757 ± 10	**+ LHC**

Electroweak Fit – SM Fit Results

- From the Gfitter group: www.cern.ch/gfitter
- Left: full fit result
- Middle: fit excluding the row
- Right: not incl. theory errors

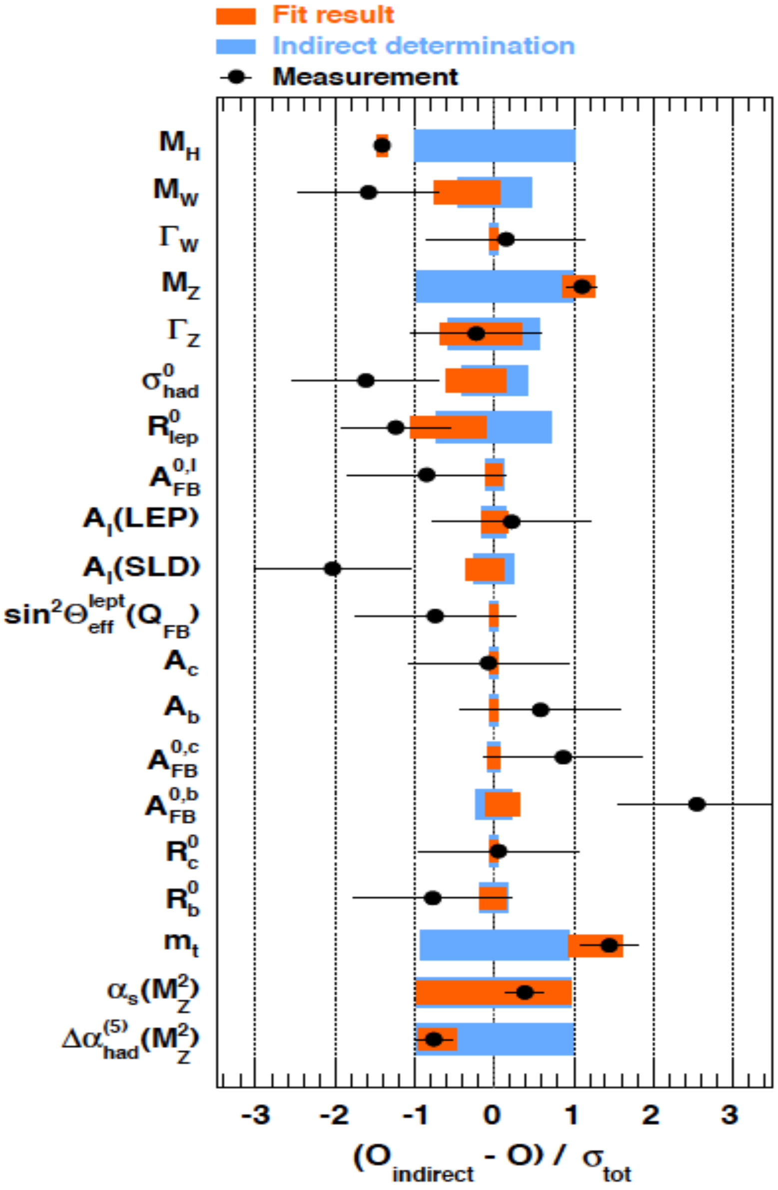
Parameter	Input value	Free in fit	Fit Result	w/o exp. input in line	w/o exp. input in line, no theo. unc
M_H [GeV]$^{(\circ)}$	125.14 ± 0.24	yes	125.14 ± 0.24	93^{+25}_{-21}	93^{+24}_{-20}
M_W [GeV]	80.385 ± 0.015	–	80.364 ± 0.007	80.358 ± 0.008	80.358 ± 0.006
Γ_W [GeV]	2.085 ± 0.042	–	2.091 ± 0.001	2.091 ± 0.001	2.091 ± 0.001
M_Z [GeV]	91.1875 ± 0.0021	yes	91.1880 ± 0.0021	91.200 ± 0.011	91.2000 ± 0.010
Γ_Z [GeV]	2.4952 ± 0.0023	–	2.4950 ± 0.0014	2.4946 ± 0.0016	2.4945 ± 0.0016
$\sigma^0_{\rm had}$ [nb]	41.540 ± 0.037	–	41.484 ± 0.015	41.475 ± 0.016	41.474 ± 0.015
R^0_ℓ	20.767 ± 0.025	–	20.743 ± 0.017	20.722 ± 0.026	20.721 ± 0.026
$A^{0,\ell}_{\rm FB}$	0.0171 ± 0.0010	–	0.01626 ± 0.0001	0.01625 ± 0.0001	0.01625 ± 0.0001
A_ℓ $^{(\star)}$	0.1499 ± 0.0018	–	0.1472 ± 0.0005	0.1472 ± 0.0005	0.1472 ± 0.0004
$\sin^2\theta^\ell_{\rm eff}(Q_{\rm FB})$	0.2324 ± 0.0012	–	0.23150 ± 0.00006	0.23149 ± 0.00007	0.23150 ± 0.00005
A_c	0.670 ± 0.027	–	0.6680 ± 0.00022	0.6680 ± 0.00022	0.6680 ± 0.00016
A_b	0.923 ± 0.020	–	0.93463 ± 0.00004	0.93463 ± 0.00004	0.93463 ± 0.00003
$A^{0,c}_{\rm FB}$	0.0707 ± 0.0035	–	0.0738 ± 0.0003	0.0738 ± 0.0003	0.0738 ± 0.0002
$A^{0,b}_{\rm FB}$	0.0992 ± 0.0016	–	0.1032 ± 0.0004	0.1034 ± 0.0004	0.1033 ± 0.0003
R^0_c	0.1721 ± 0.0030	–	$0.17226^{+0.00009}_{-0.00008}$	0.17226 ± 0.00008	0.17226 ± 0.00006
R^0_b	0.21629 ± 0.00066	–	0.21578 ± 0.00011	0.21577 ± 0.00011	0.21577 ± 0.00004
$\overline{m}_c$ [GeV]	$1.27^{+0.07}_{-0.11}$	yes	$1.27^{+0.07}_{-0.11}$	–	–
$\overline{m}_b$ [GeV]	$4.20^{+0.17}_{-0.07}$	yes	$4.20^{+0.17}_{-0.07}$	–	–
m_t [GeV]	173.34 ± 0.76	yes	$173.81 \pm 0.85^{(\bigtriangledown)}$	$177.0^{+2.3}_{-2.4}{}^{(\bigtriangledown)}$	177.0 ± 2.3
$\Delta\alpha^{(5)}_{\rm had}(M_Z^2)^{(\dagger\triangle)}$	2757 ± 10	yes	2756 ± 10	2723 ± 44	2722 ± 42
$\alpha_s(M_Z^2)$	–	yes	0.1196 ± 0.0030	0.1196 ± 0.0030	0.1196 ± 0.0028

Electroweak Fit – SM Fit Results

- Results drawn as *pull values:* → deviations to the *indirect* determinations, divided by *total error*.
- Total error: *error of direct measurement plus error from indirect determination.*

- Black: direct measurement (data)
- Orange: full fit
- Light-blue: fit excluding input from the row

- *The prediction (light blue) is often more precise than the measurement!*

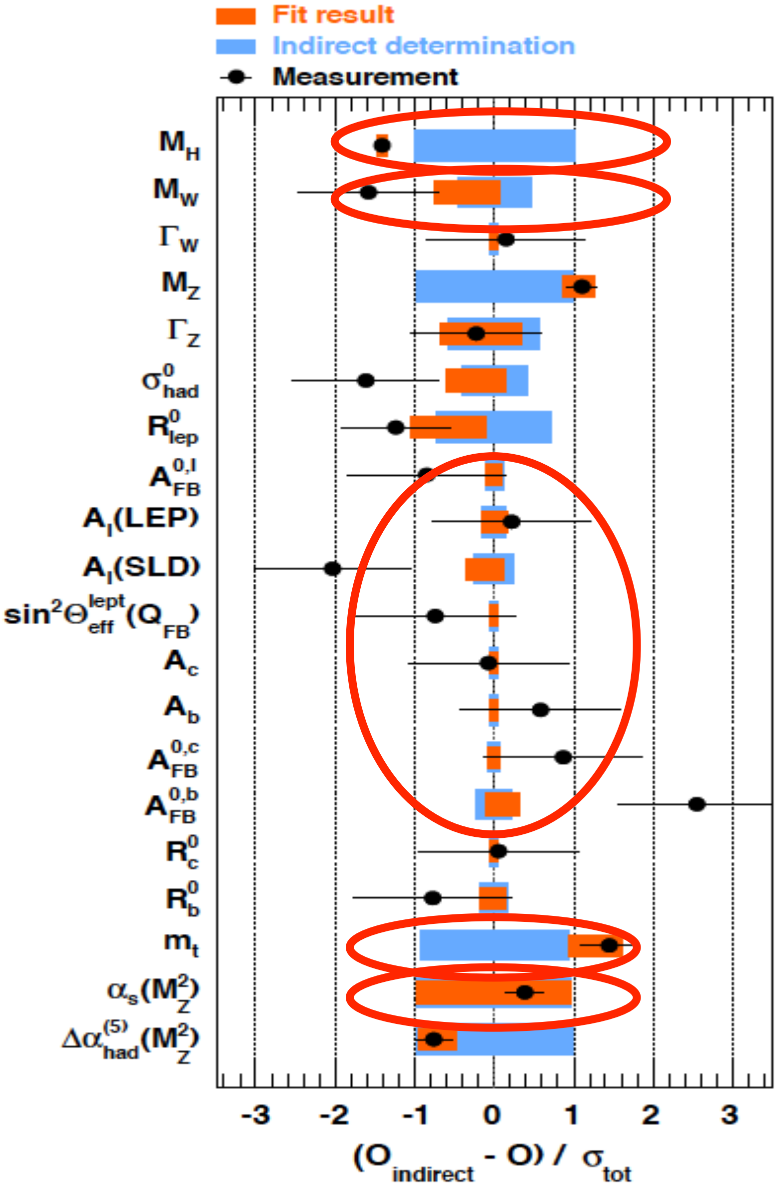


Electroweak Fit – SM Fit Results

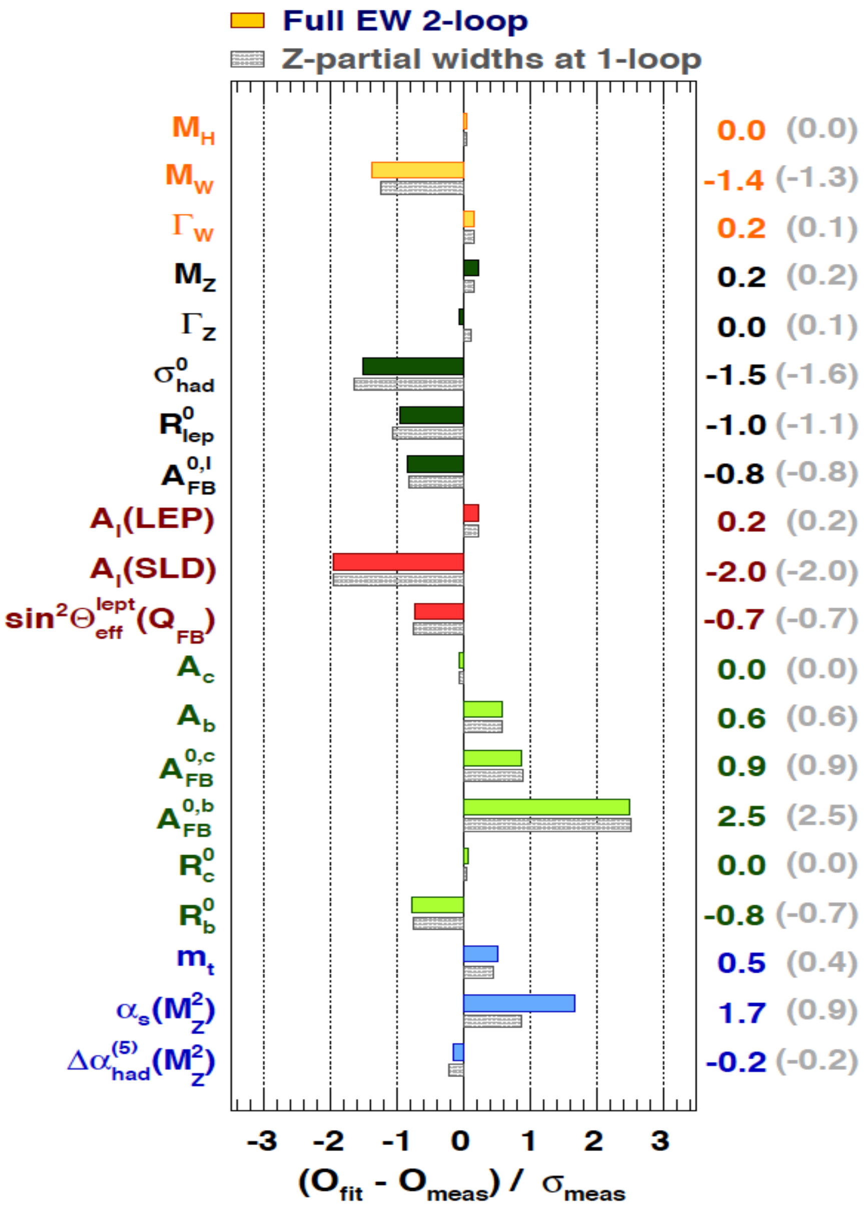
- Results drawn as *pull values:* → deviations to the *indirect* determinations, divided by *total error*.
- Total error: *error of direct measurement plus error from indirect determination.*

- Black: direct measurement (data)
- Orange: full fit
- Light-blue: fit excluding input from the row

- *The prediction (light blue) is often more precise than the measurement!*



Electroweak Fit – SM Fit Results



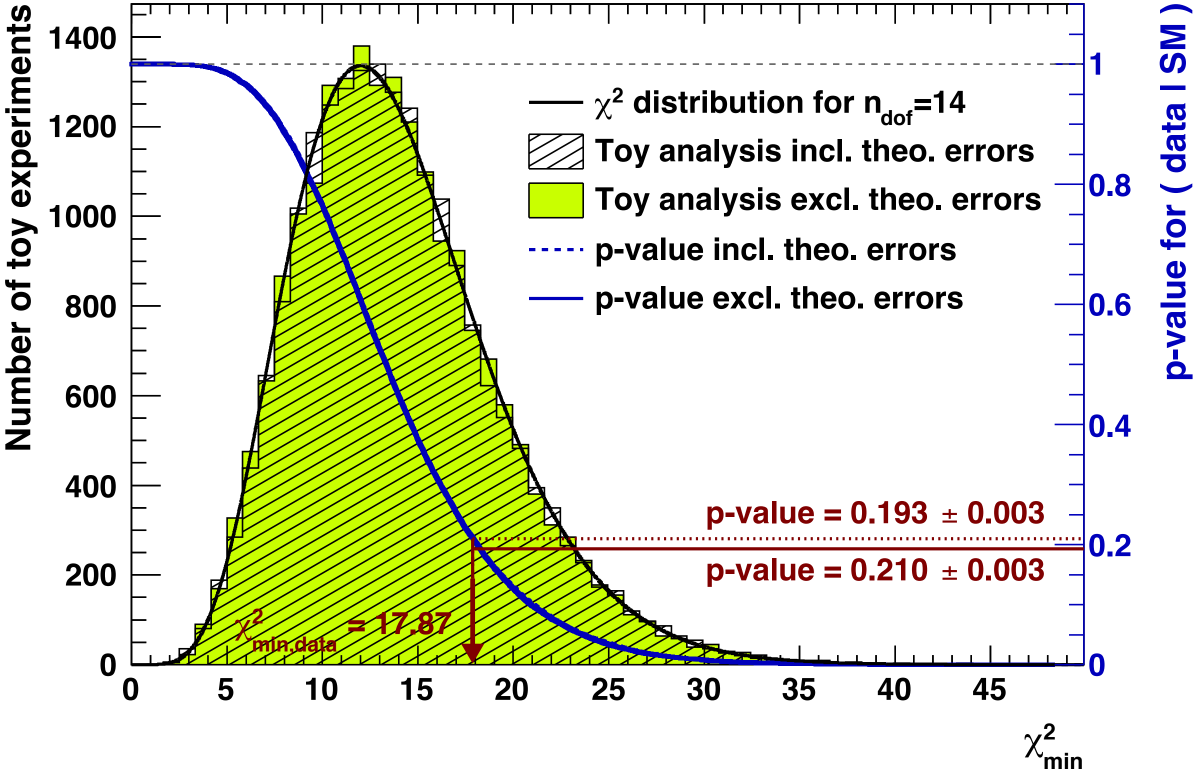
- No individual value exceeds 3σ

- Largest deviations in b-sector: $A^{0,b}_{FB}$ with 2.5σ
  - → largest contribution to χ^2

- Small pulls for M_H, M_Z, $\Delta\alpha_{had}^{(5)}(M_Z^2)$, $\overline{m}_c$, $\overline{m}_b$ indicate that input accuracies exceed fit requirements

- Small changes from switching between 1 and 2-loop calc. for partial Z widths and small M_W correction.
  - **χ^2_{min}(complete setup) = 17.8**
  - χ^2_{min}(1-loop Z width) = 18.0
  - χ^2_{min}(no M_W correction) = 17.4
  - χ^2_{min}(no extra theory errors) = 18.2

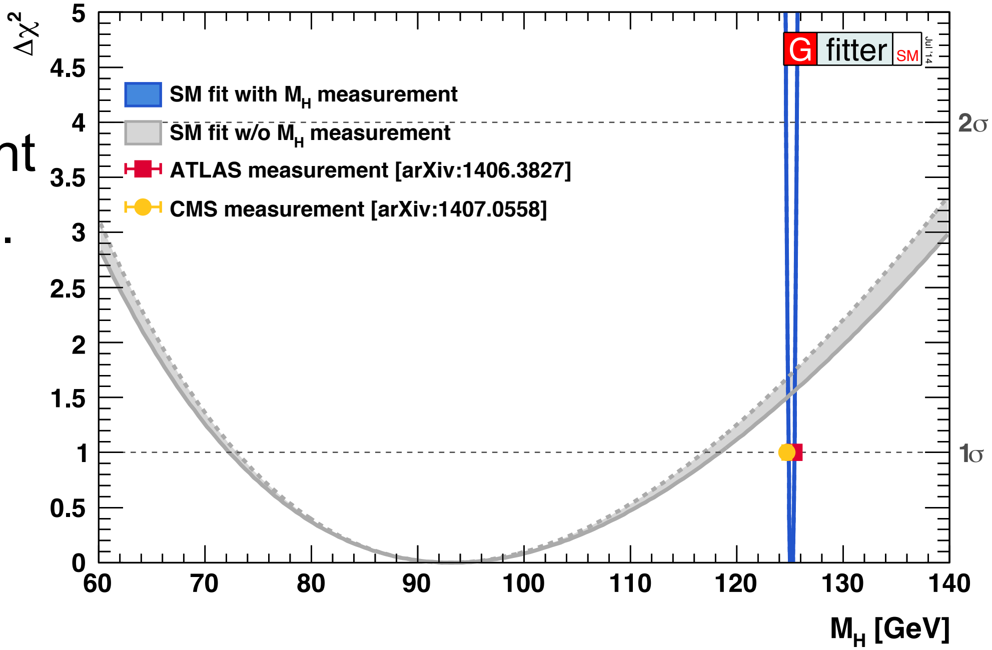
Goodness of Fit



- Toy analysis: p-value for wrongly rejecting the SM = 21 ± 2 (theo) %
  - p-value is equivalent to 0.8σ
  - Evaluated with 20k pseudo experiments – follows χ^2 with 14 d.o.f.
  - For comparison: χ^2_{min}= 17.8 → Prob(χ^2_{min}, 14) = 21 %
- Large value of χ^2_{min} *not* due to inclusion of M_H measurement.
  - Without M_H measurement: χ^2_{min}= 16.3 → Prob(χ^2_{min}, 13) = 23%

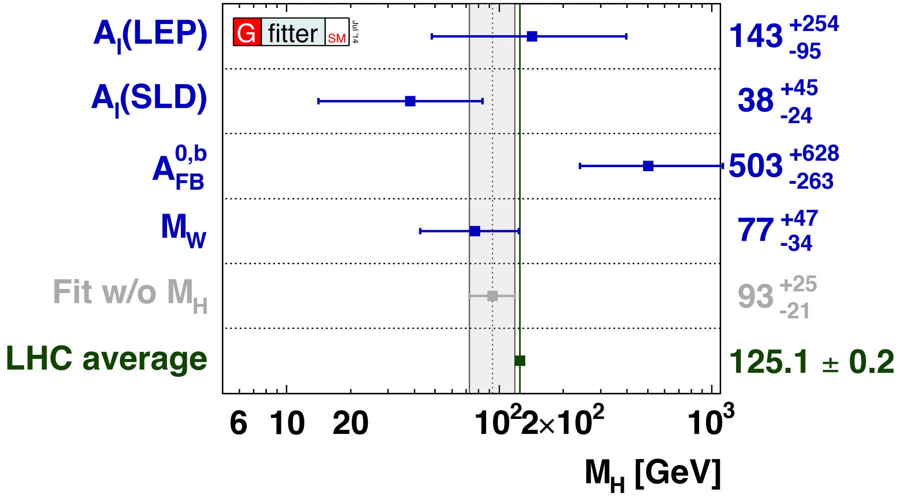
Higgs results of the EW fit

- Scan of $\Delta\chi^2$ profile versus M_H
  - Grey band: fit w/o M_H measurement
  - Blue line: full SM fit, with M_H meas.
  - Fit w/o M_H measurement gives: $M_H = 93^{+25}_{-21}$ GeV
  - Consistent at 1.3σ with LHC measurements.

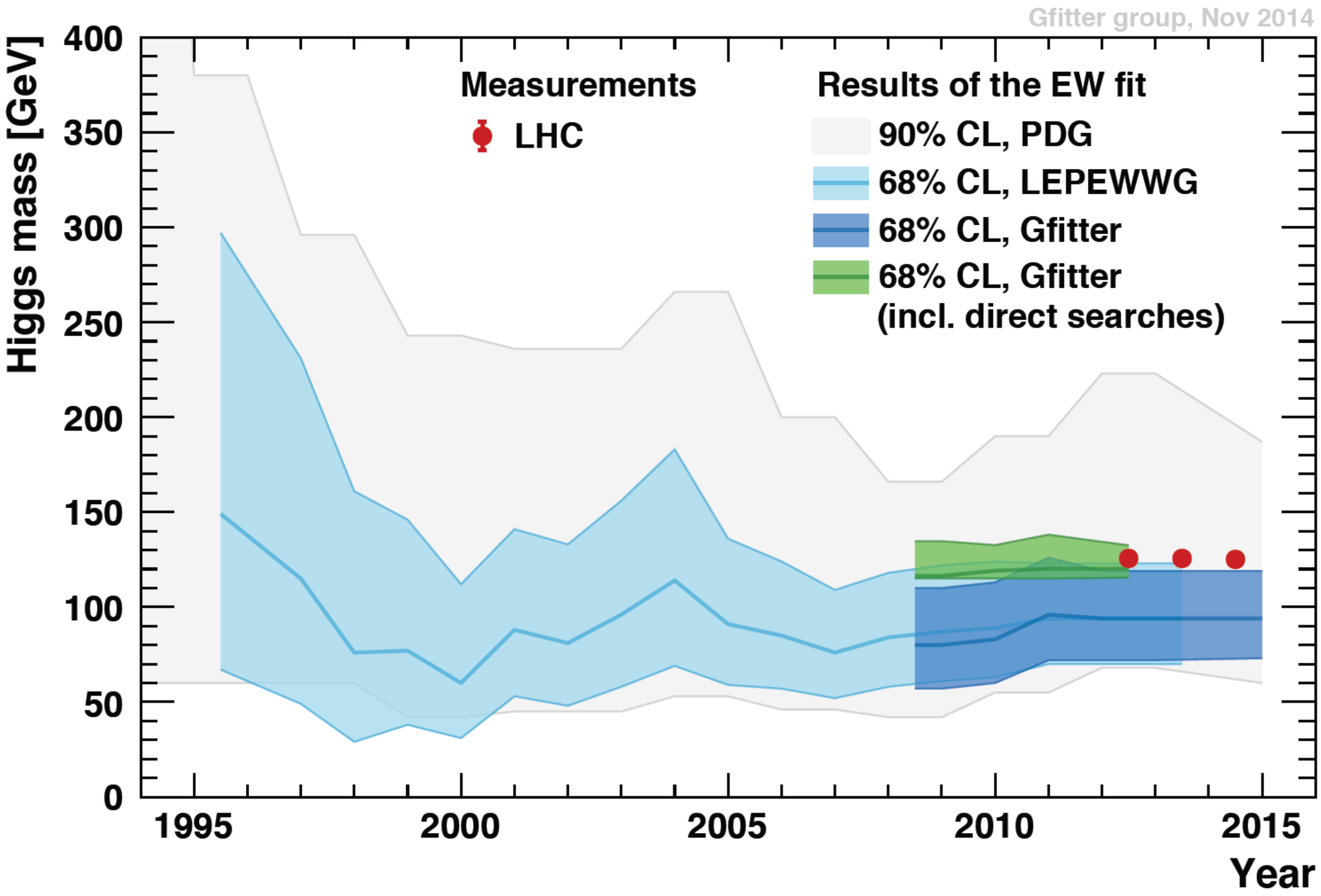


- Bottom plot: impact of other most sensitive Higgs observables
  - Determination of M_H removing all sensitive observables except the given one.
  - Known tension (2.5σ) between A_l(SLD), $A^{0,b}_{FB}$, and M_W clearly visible.

fit below only includes the given observable

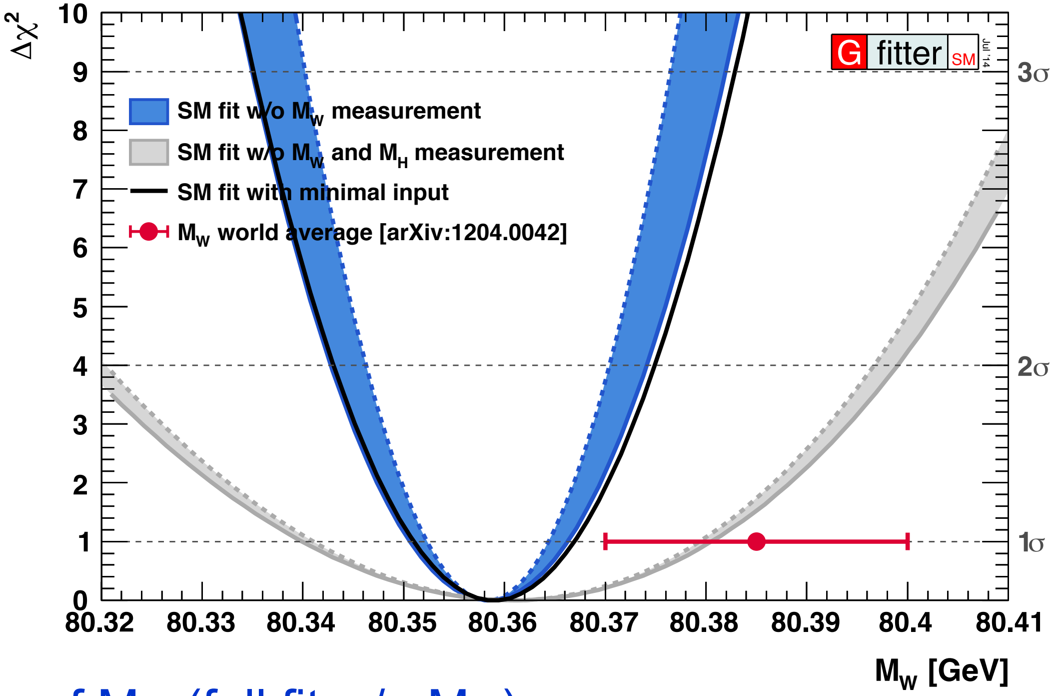


History of Higgs mass predictions



- The EW fits have always been able to predict the Higgs mass correctly!

Prediction of W mass



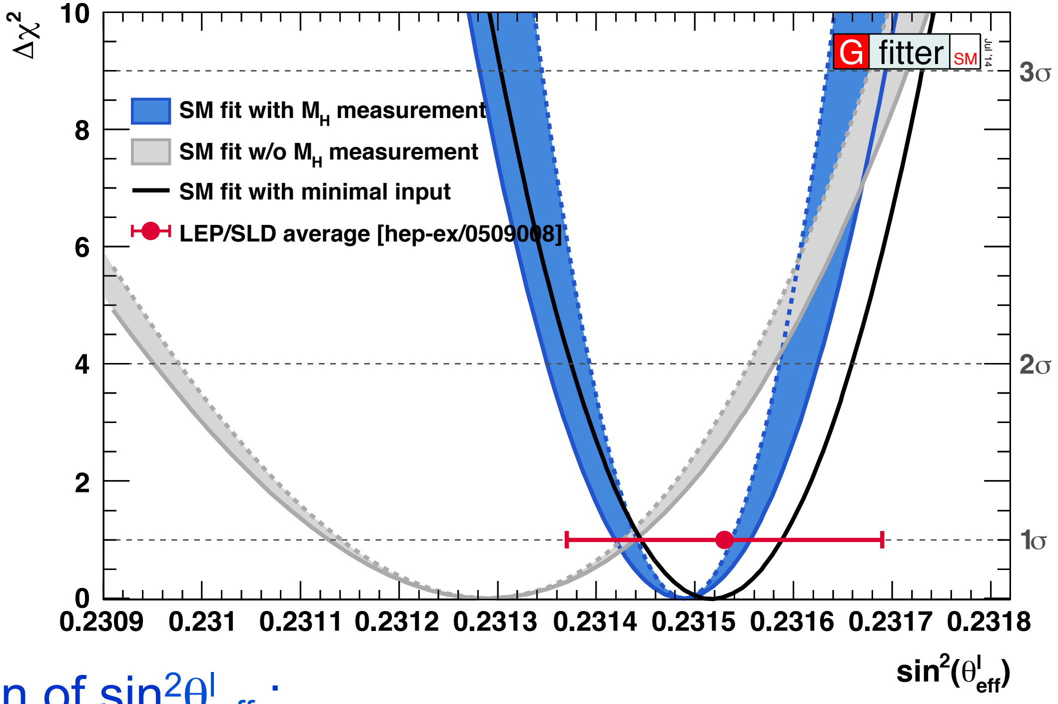
- Scan of $\Delta\chi^2$ profile versus M_W
  - Also shown: SM fit with minimal inputs: M_Z, G_F, $\Delta\alpha_{had}^{(5)}(M_Z)$, $\alpha_s(M_Z)$, M_H, and fermion masses
  - Good consistency between total fit and SM w/ minimal inputs
- M_H measurement allows for precise constraint on M_W
  - Agreement at 1.4σ
- Fit result for indirect determination of M_W (full fit w/o M_W):

$$\begin{aligned} M_W &= 80.3584 \pm \boxed{0.0046_{m_t}} \pm \boxed{0.0030_{\delta_{\rm theo} m_t}} \pm 0.0026_{M_Z} \pm 0.0018_{\Delta\alpha_{\rm had}} \\ &\qquad \pm 0.0020_{\alpha_S} \pm 0.0001_{M_H} \pm \boxed{0.0040_{\delta_{\rm theo} M_W}}\ {\rm GeV}\,, \\ &= 80.358 \pm 0.008_{\rm tot}\ {\rm GeV}\,. \end{aligned}$$

Obtained with simple error propagation

- More precise estimate of M_W than the direct measurements!
  - Uncertainty on world average measurement: 15 MeV

Prediction of effective weak mixing angle



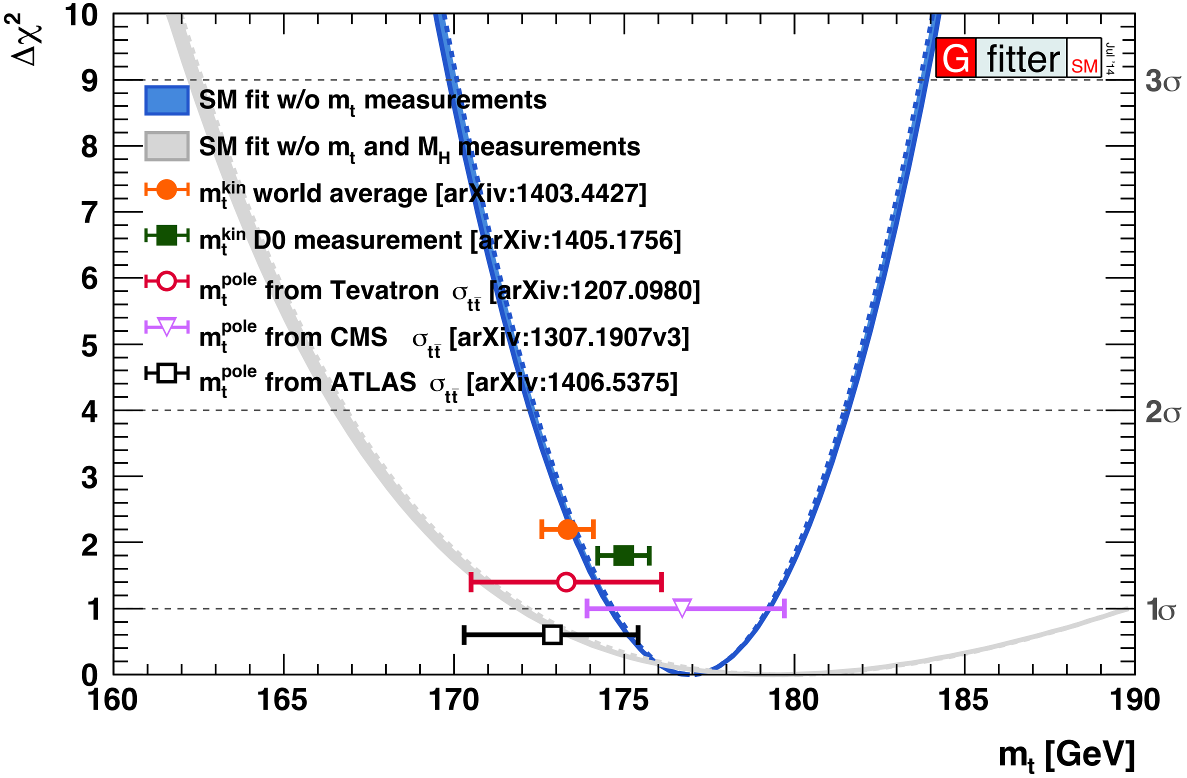
- Right: scan of $\Delta\chi^2$ profile versus $\sin^2\theta^l_{\rm eff}$
  - All sensitive measurements removed from the SM fit.
  - Also shown: SM fit with minimal inputs
- M_H measurement allows for very precise constraint on $\sin^2\theta^l_{\rm eff}$
- Fit result for indirect determination of $\sin^2\theta^l_{\rm eff}$:

$$\begin{aligned}\sin^2\theta^\ell_{\rm eff} &= 0.231488 \pm \boxed{0.000024_{m_t}} \pm 0.000016_{\delta_{\rm theo} m_t} \pm 0.000015_{M_Z} \pm \boxed{0.000035_{\Delta\alpha_{\rm had}}} \\ &\qquad \pm 0.000010_{\alpha_S} \pm 0.000001_{M_H} \pm \boxed{0.000047_{\delta_{\rm theo}\sin^2\theta^f_{\rm eff}}} , \\ &= 0.23149 \pm 0.00007_{\rm tot} ,\end{aligned}$$

Obtained with simple error propagation

- More precise than direct determination (from LEP/SLD) !
  - Uncertainty on LEP/SLD average: 1.6×10^{-4}

Prediction of top mass



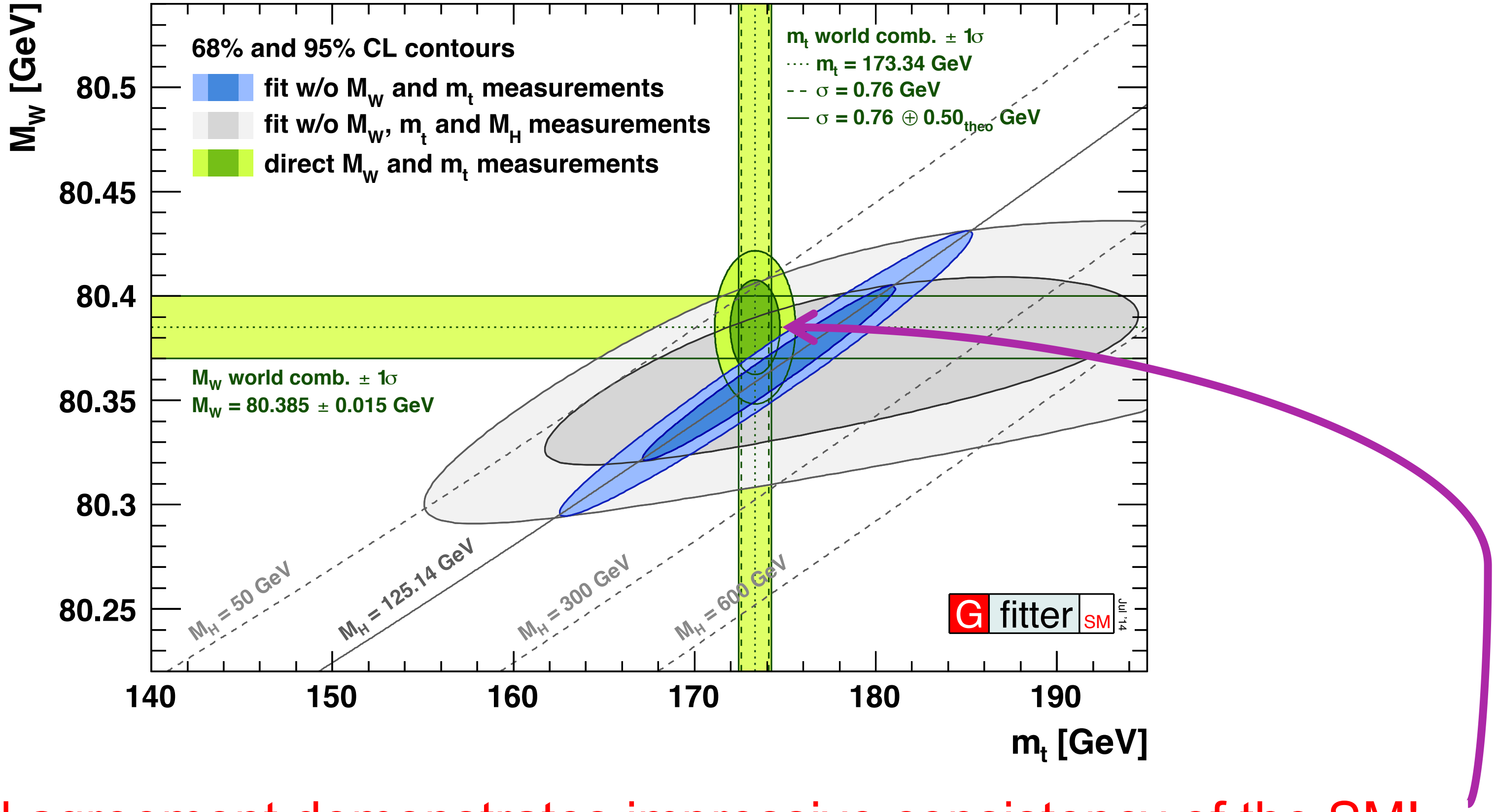
- Shown: scan of $\Delta\chi^2$ profile versus m_t (without m_t measurement)
  - M_H measurement allows for significant better constraint of m_t
  - Indirect determination consistent with direct measurements
    - Remember: fully obtained from radiative corrections!

- Indirect result: $m_t = 177.0^{+2.3}_{-2.4}$ GeV

Tevatron+LHC: 173.34 ± 0.76 GeV
new Tevatron-only: 174.34 ± 0.64 GeV

State of the SM: W versus top mass

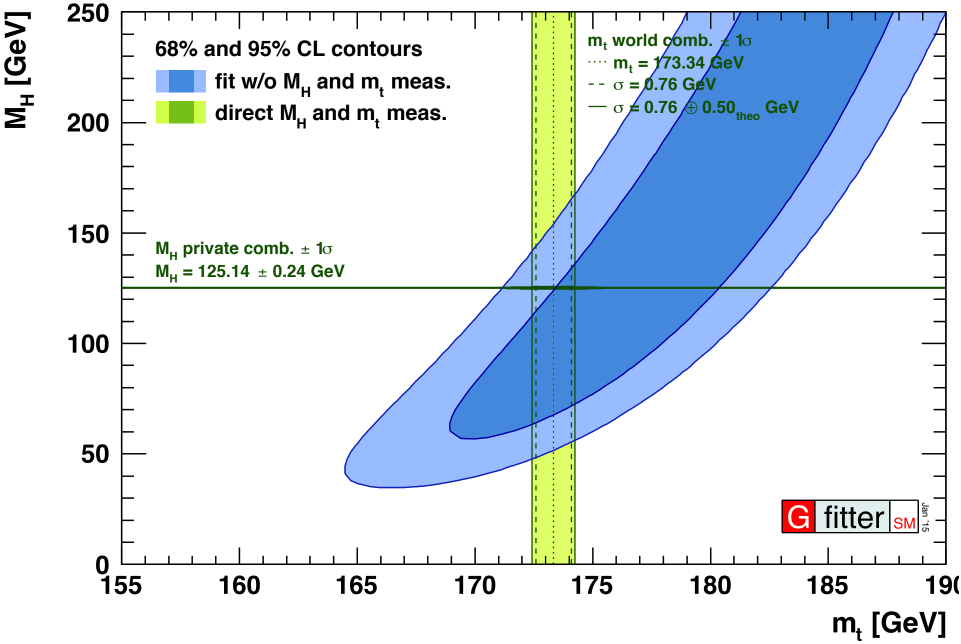
- Scan of M_W vs m_t, with the direct measurements excluded from the fit.
- Results from Higgs measurement significantly reduces allowed indirect parameter space → corners the SM!



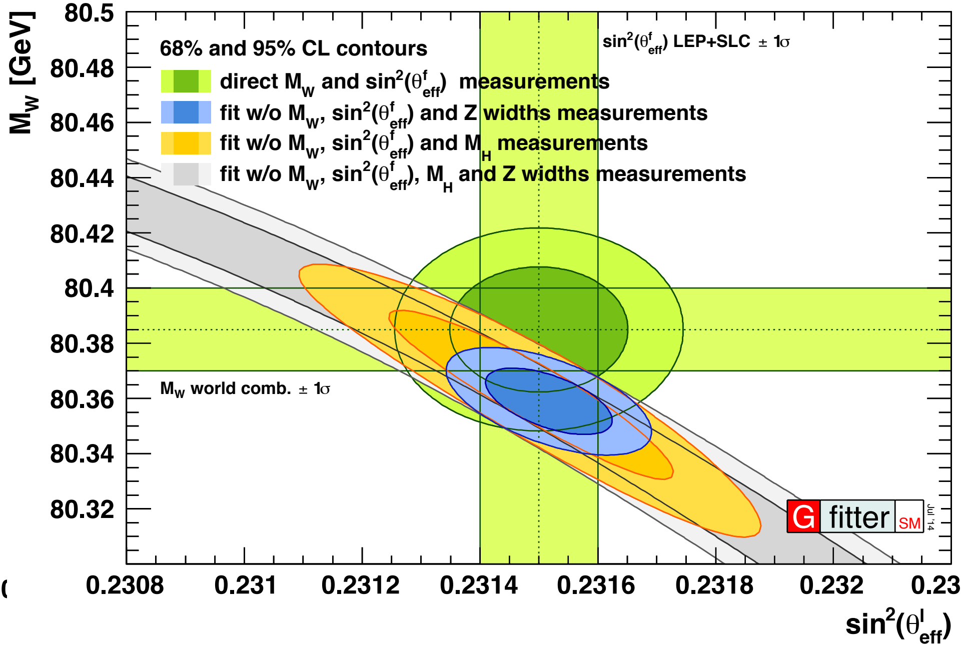
- Observed agreement demonstrates impressive consistency of the SM!

State of the SM: loop vs tree-level observables

- Scan of M_H vs m_{top} (left) and M_W vs $\sin^2\theta^l_{eff}$ (right), with direct measurements excluded from the fit.
- Again, significant reduction allowed indirect parameter space from Higgs mass measurement.



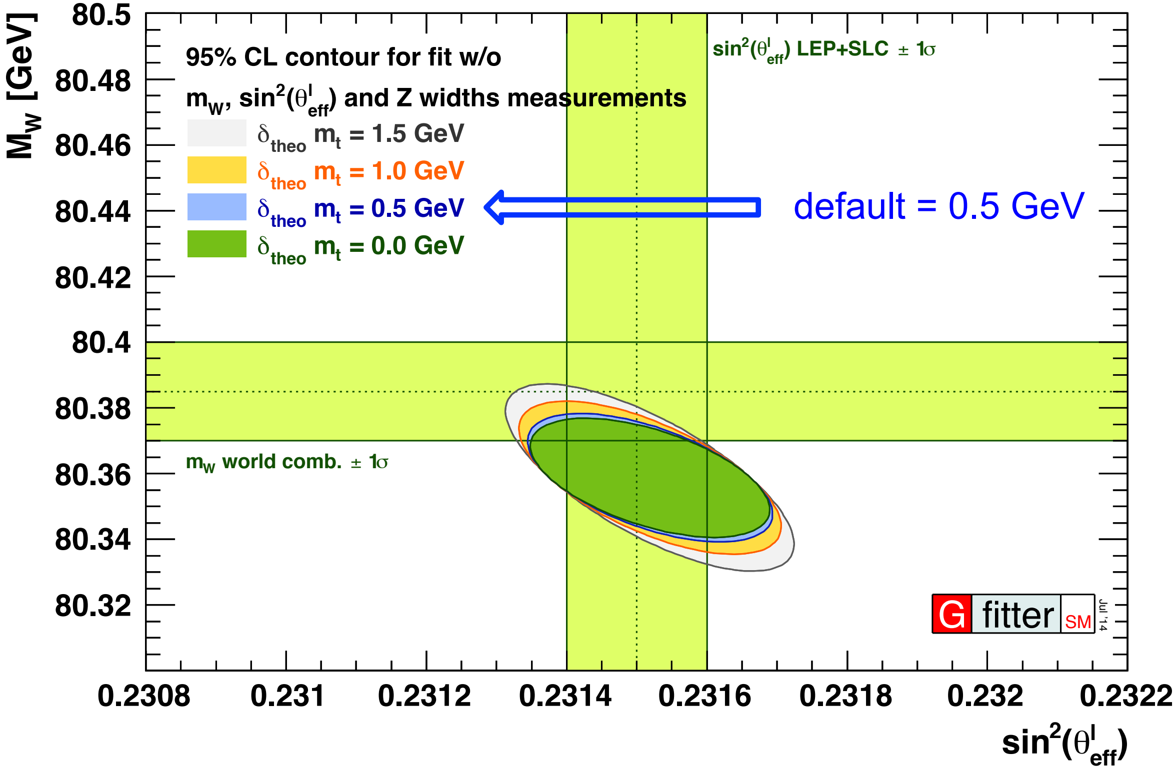
Observables from radiative corrections



“Tree-level” observables

- M_W and $\sin^2\theta^l_{eff}$ have become *the* sensitive probes of new physics!
  - Reason: both are ‘tree-level’ SM predictions.

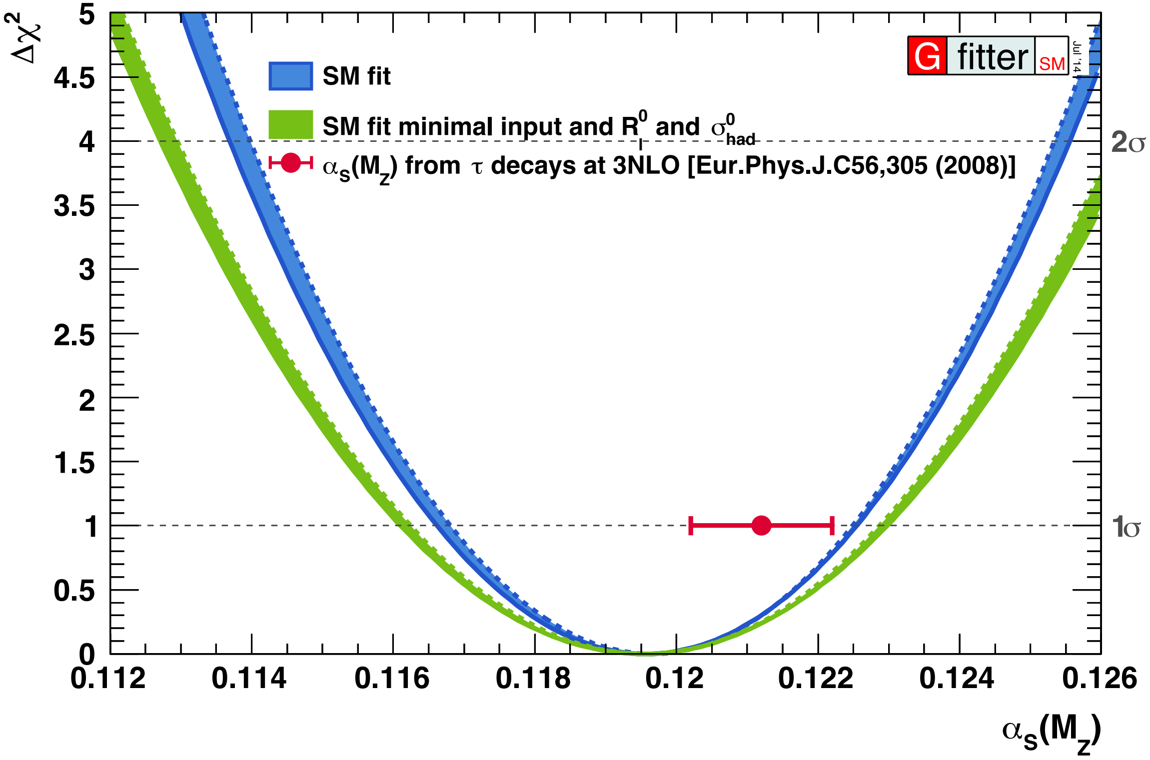
Theoretical uncertainty on m_{top}



- *δ_{theo} m_t : unc. on conversion of measured top mass to MS-bar mass*
  - Sources: ambiguity top mass definition, fragmentation process, pole→MS conv.
  - Predictions for δ_{theo} m_t : *between 0.25 – 0.9 GeV or greater*.
    [Moch etal, aX:1405.4781, Mangano: TOP'12, Buckley etal, aX:1101.2599, Juste etal: aX:1310.0799]
  - δ_{theo} m_t varied here between 0 and 1.5 GeV, in steps of 0.5 GeV.
- *Better assessment of δ_{theo} m_t of relevance for the EW fit.* *(see also backup)*

Prediction for $\alpha_s(M_Z)$ from Z→hadrons

- Scan of $\Delta\chi^2$ versus α_s
  - Also shown: SM fit with minimal inputs: M_Z, G_F, $\Delta\alpha_{had}^{(5)}(M_Z)$, $\alpha_s(M_Z)$ M_H, and fermion masses
- Determination of α_s at full N^{2}LO and partial N^{3}LO.
  - Most sensitive through total hadronic cross-section σ^0_{had} and partial leptonic width R^0_l



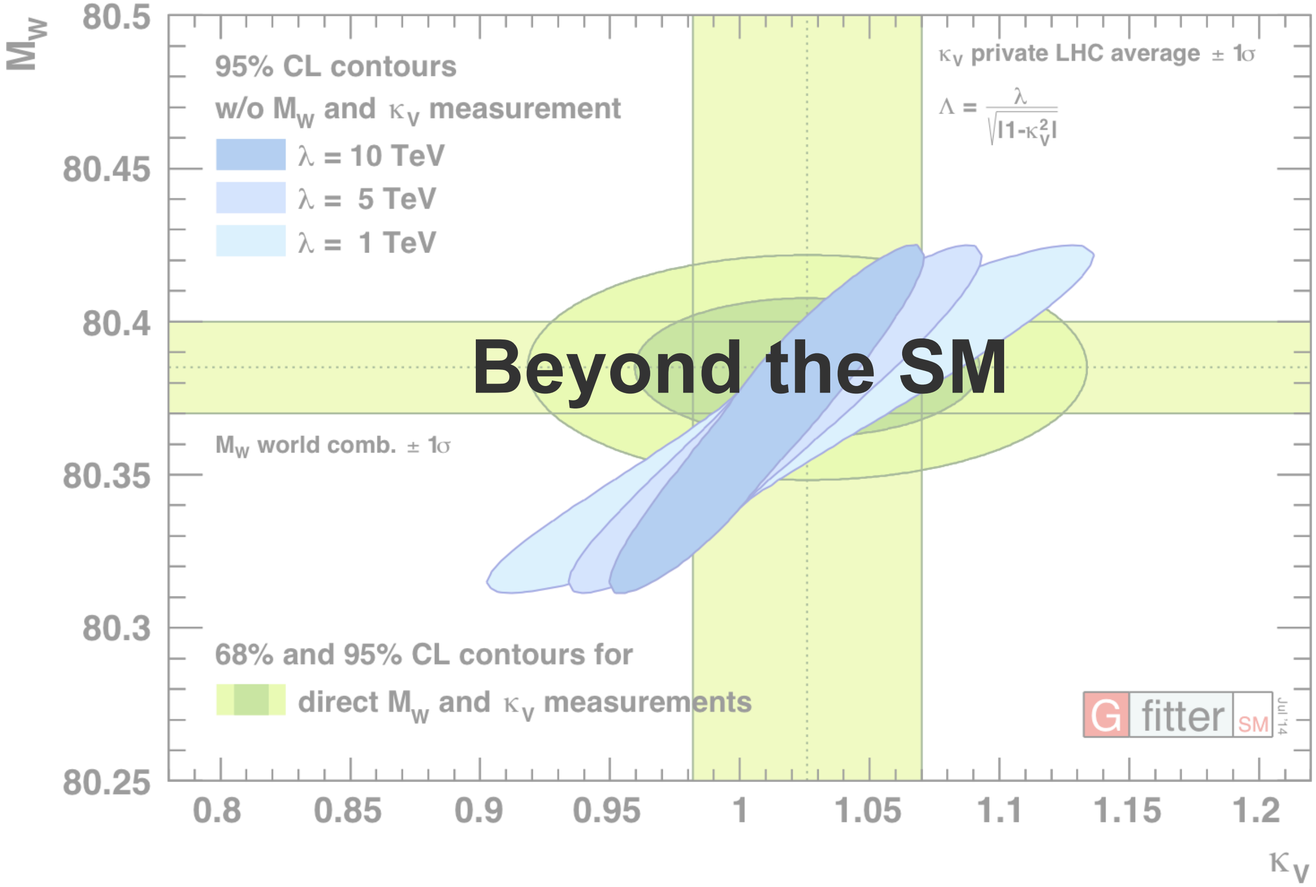
$$
\begin{aligned}
\alpha_s(M_Z^2) &= 0.1196 \pm 0.0028_{\rm exp} \pm \boxed{0.0006_{\delta_{\rm theo}\mathcal{R}_{V,A}} \pm 0.0006_{\delta_{\rm theo}\Gamma_i} \pm 0.0002_{\delta_{\rm theo}\sigma^0_{\rm had}}} \\
&= 0.1196 \pm 0.0030_{\rm tot}\ ,
\end{aligned}
$$

Most affected by new theory uncertainties

Before: δ_{theo} = 0.0001

- *In good agreement with value from τ decays, at N^{3}LO, and with WA.*
  - (Improvements in precision only expected with ILC/GigaZ. See later.)

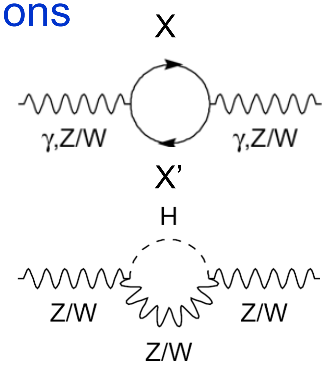
Beyond the SM



Constraints on BSM models

Gfitter BSM

- If energy scale of NP is high, BSM physics could appear dominantly through vacuum polarization corrections
  - Aka, “oblique corrections”
- Oblique corrections reabsorbed into electroweak form factors
  - $\Delta\rho$, $\Delta\kappa$, Δr parameters, appearing in: M_W^2, $\sin^2\theta_{eff}$, G_F, α, etc.
- Electroweak fit sensitive to BSM physics through oblique corrections
  - Similar to sensitivity to top and Higgs loop corrections.



- Oblique corrections from New Physics described through STU parametrization

[Peskin and Takeuchi, Phys. Rev. D46, 1 (1991)]

$$\boxed{O_{meas} = O_{SM,REF}(m_H,m_t) + c_S S + c_T T + c_U U}$$

- S : New Physics contributions to neutral currents
- T : Difference between neutral and charged current processes – sensitive to weak isospin violation
- U : (+S) New Physics contributions to charged currents. U only sensitive to W mass and width, usually very small in NP models (often: U=0)
- Also implemented: extended parameters (VWX), correction to Z→bb couplings.

[Burgess et al., Phys. Lett. B326, 276 (1994)]
[Burgess et al., Phys. Rev. D49, 6115 (1994)]

Fit results for S, T, U

- S,T,U parameters obtained directly from fit to the EW observables.

- SM: M_H = 125 GeV, m_t = 173 GeV
  - This defines (S,T,U) = (0,0,0)
- S, T depend logarithmically on M_H

- Fit result (with U floating):

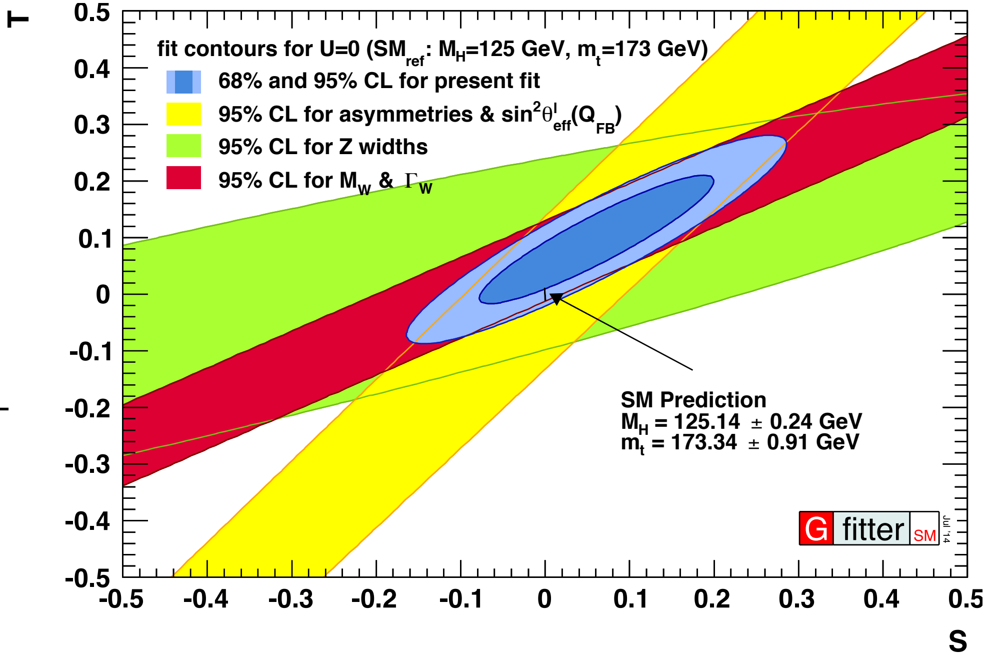
$S = 0.05 \pm 0.11$

$T = 0.09 \pm 0.13$

$U = 0.01 \pm 0.11$

	S	T	U
S	1	+0.90	-0.59
T		1	-0.83
U			1

- Also results for $Z \rightarrow b\bar{b}$ correction (see backup)



- Stronger constraints with U=0.

- *No indication for new physics.*
- Use this to constrain 4th gen, Ex-Dim, T-C, *Higgs couplings* (in backup)

Modified Higgs couplings

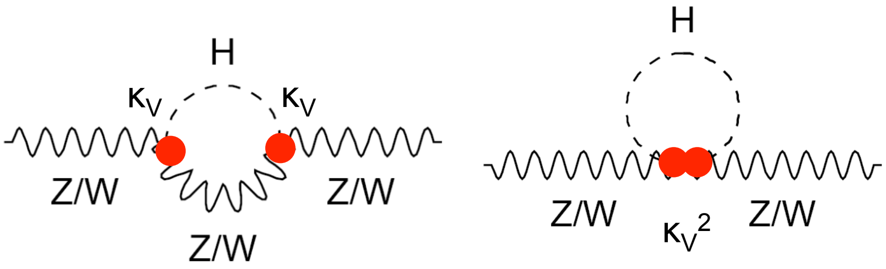
- Study of potential deviations of Higgs couplings from SM.
- BSM modeled as extension of SM through effective Lagrangian.
  - Consider leading corrections only.

- Popular benchmark model:
  - Scaling of Higgs-vector boson (κ_V) and Higgs-fermion couplings (κ_F) with no invisible/undetectable width
  - (Custodial symmetry is assumed.)
  - *"Kappa parametrization"*

$$L_V = \frac{h}{v}\left(2\kappa_V m_W^2 \, W_\mu W^\mu + \kappa_V m_Z^2 \, Z_\mu Z^\mu\right)$$

$$L_F = -\frac{h}{v}\left(\kappa_F m_t \, \bar{t}t + \kappa_F m_b \, \bar{b}b + \kappa_F m_\tau \, \bar{\tau}\tau\right)$$

- Main effect on EWPO due to modified Higgs coupling to gauge bosons (κ_V)
  - Involving the longitudinal d.o.f.



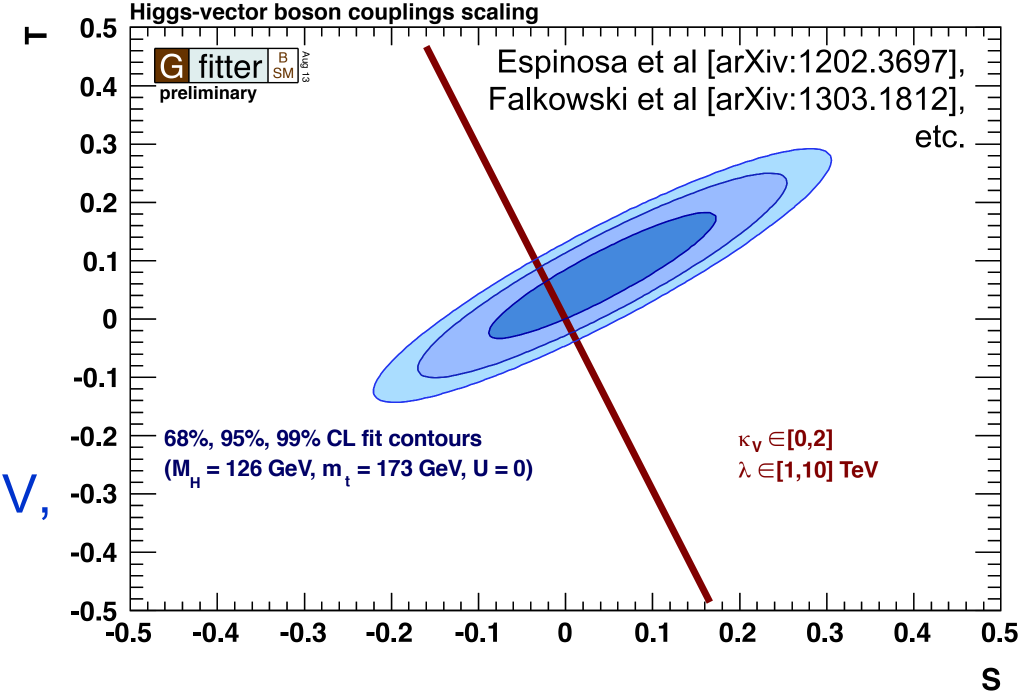
- Most BSM models: $\kappa_V < 1$
  - Additional Higgses typically give *positive* contribution to M_W.

Modified Higgs couplings

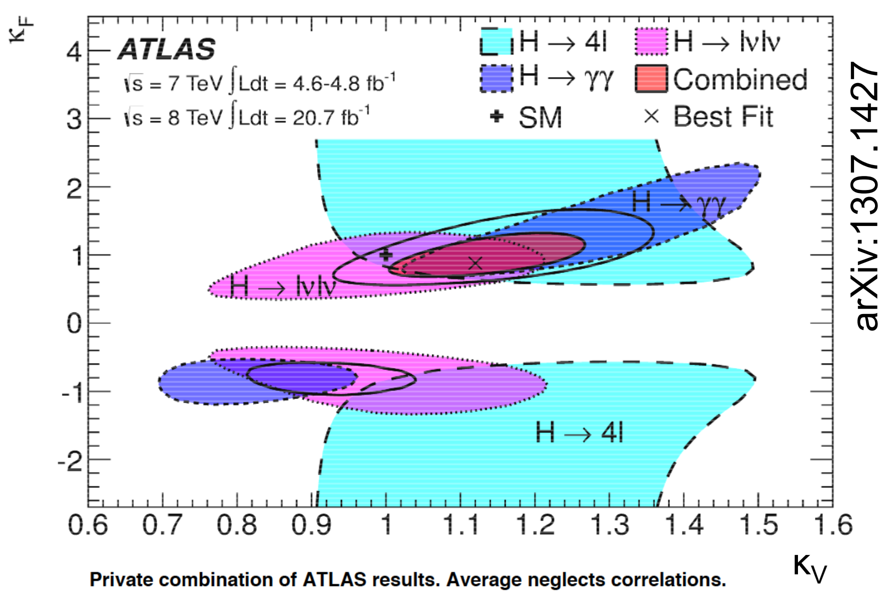
- Main effect on EWPO due to Higgs coupling to gauge bosons (κ_V).
  - $$S = \frac{1}{12\pi}(1 - \kappa_V^2) \log\left(\frac{\Lambda^2}{M_H^2}\right), \quad T = -\frac{3}{16\pi c_W^2}(1 - \kappa_V^2) \log\left(\frac{\Lambda^2}{M_H^2}\right), \quad \Lambda = \frac{\lambda}{\sqrt{|1 - \kappa_V^2|}}$$
  - Formulas from: Espinosa et al [arXiv:1202.3697]

- Cut-off scale Λ represents mass scale of new states that unitarize longitudinal gauge-boson scattering.
  - (As required in this model.)

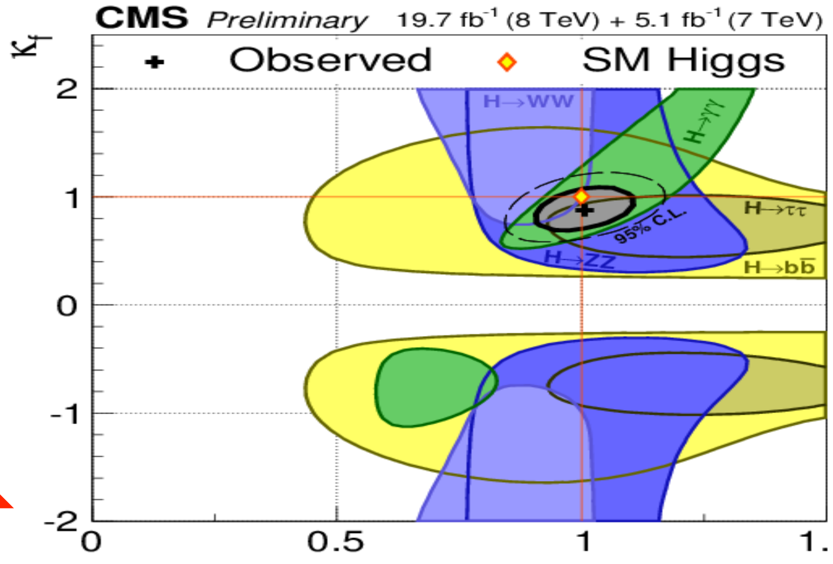
- λ is varied between 1 and 10 TeV, nominally fixed to 3 TeV ($4\pi v$).



Reproduction of ATLAS and CMS results

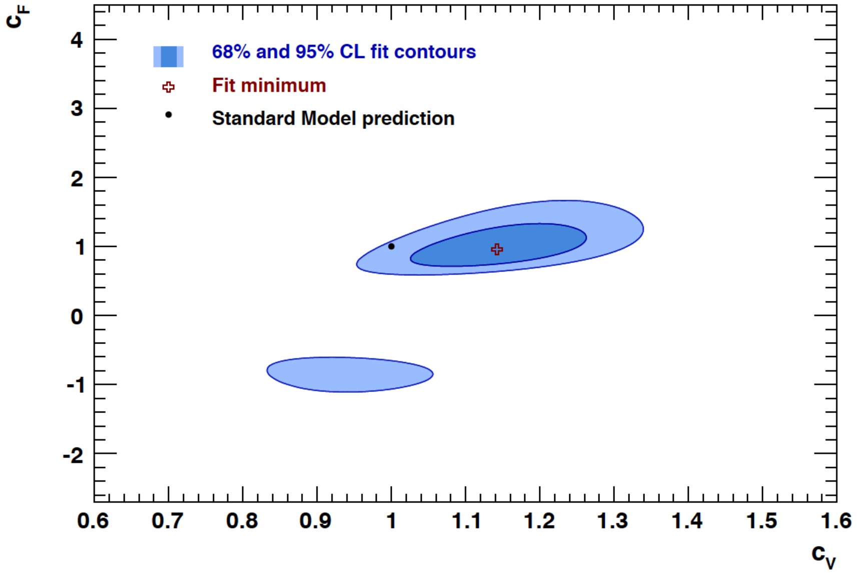


arXiv:1307.1427

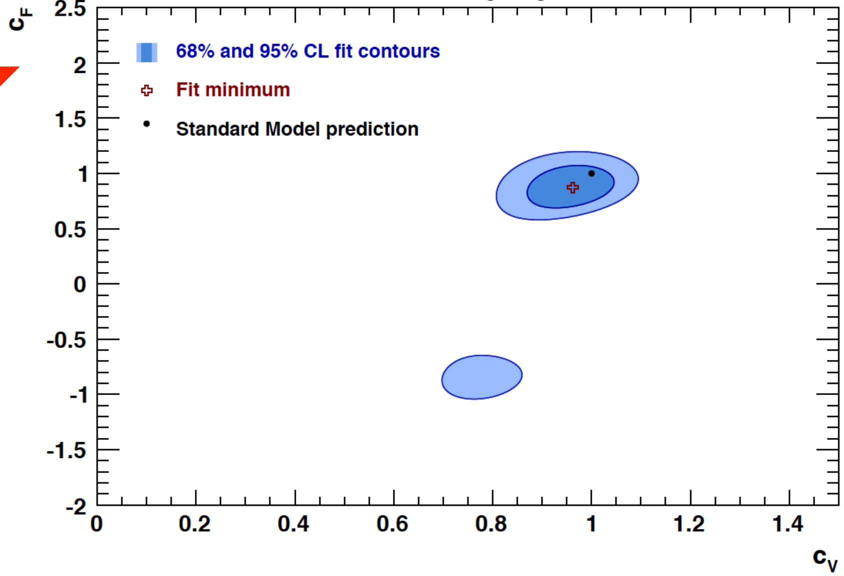


CMS-PAS-HIG-14-009

Private combination of ATLAS results. Average neglects correlations.



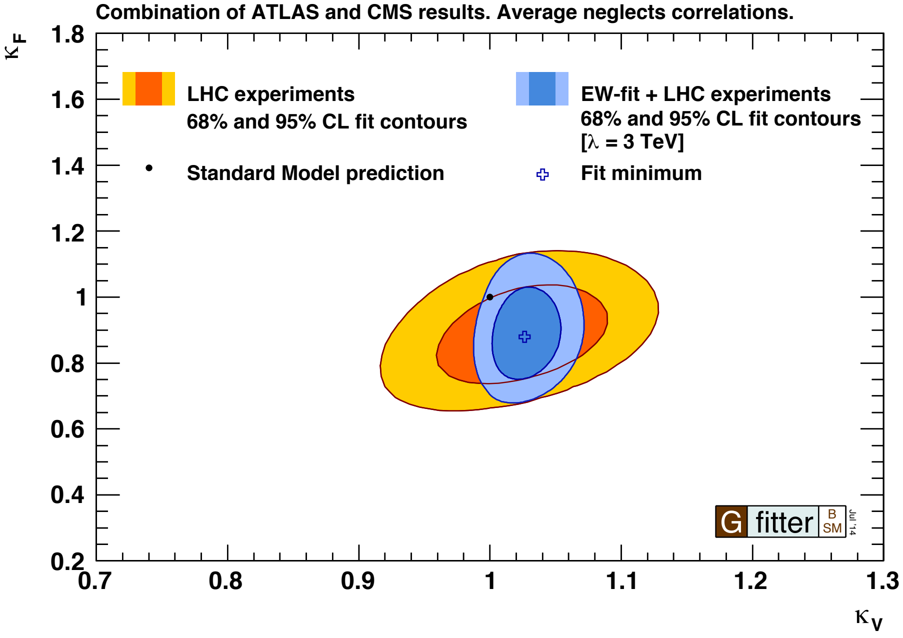
Private combination of CMS results. Average neglects correlations.

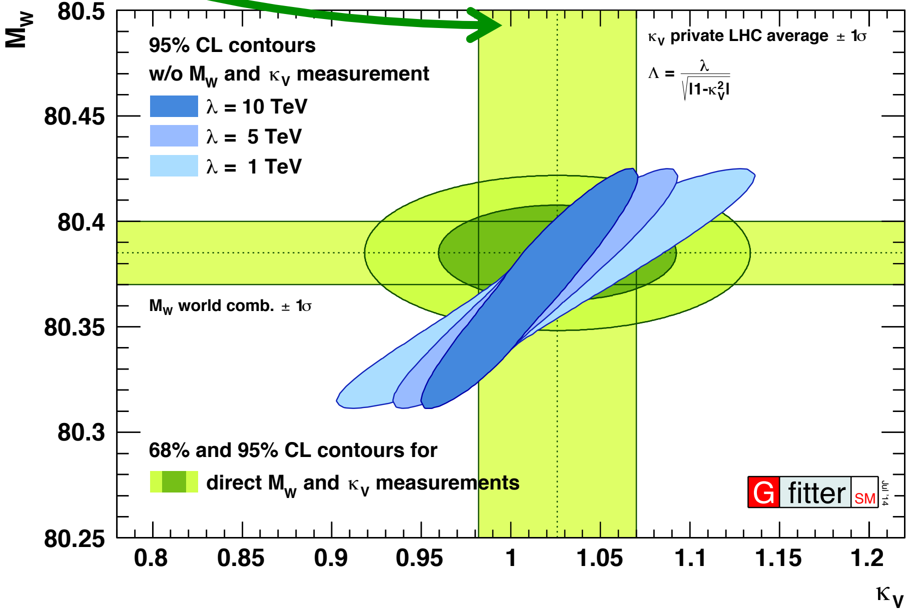


- Approximate reproduction of ATLAS/CMS results within limited public-info available.

Higgs coupling results

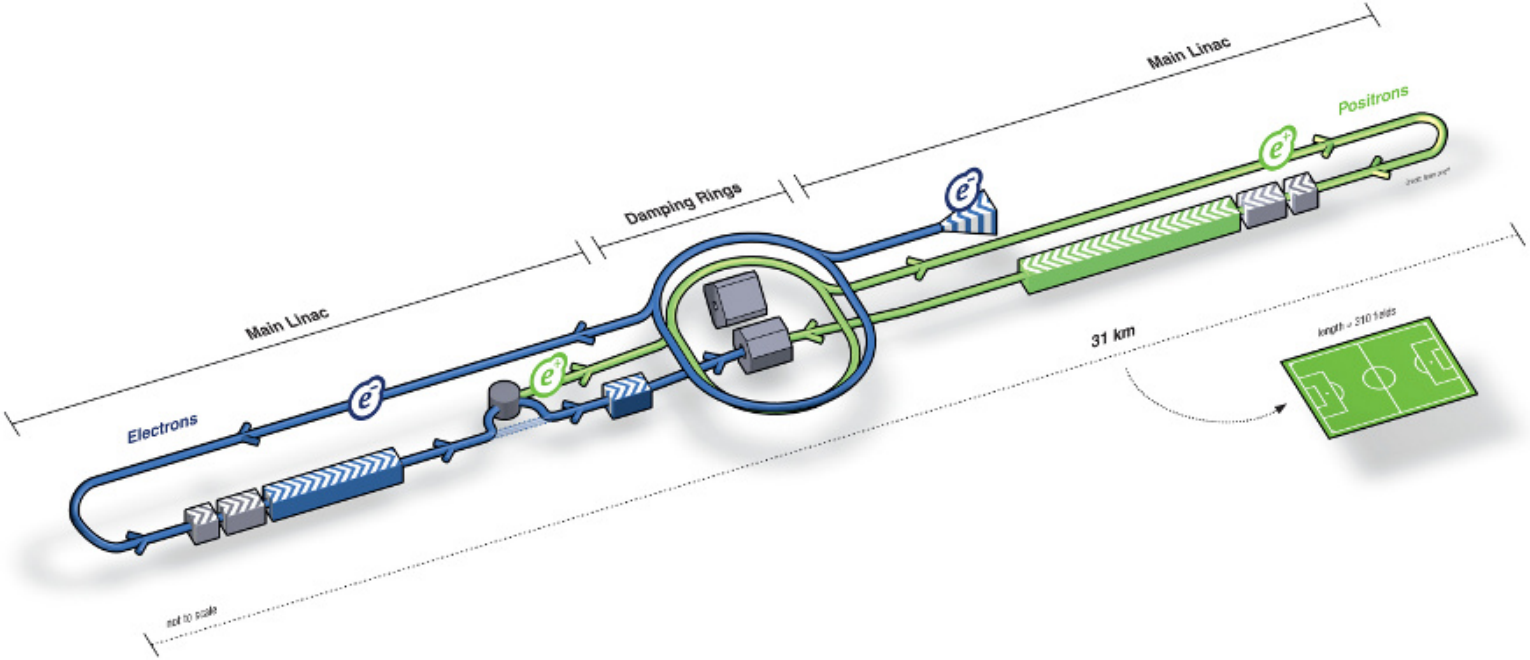
- Private LHC combination:
  - $\kappa_V = 1.026^{+0.043}_{-0.043}$
  - $\kappa_F = 0.88^{+0.10}_{-0.09}$
- Result from stand-alone EW fit:
  - $\kappa_V = 1.03 \pm 0.02$ (using λ=3 TeV)
  - *Implies NP-scale of* $\Lambda \gtrsim 13$ *TeV.*





- Some dependency for κ_V in central value [1.02-1.04] and error [0.02-0.03] on cut-off scale λ [1-10 TeV].

1. EW fit sofar more precise result for κ_V than current LHC experiments.
2. EW fit has positive deviation of κ_V from 1.0.
   - (Many BSM models: $\kappa_V < 1$)



Prospects for the Standard Model fit

Two prospects scenarios: LHC, ILC/GigaZ

Prospects of EW fit tested for two (three) scenarios:

1. LHC Phase-1 = *before HL upgrade*
2. ILC **with GigaZ** (*)
3. (FCC-ee in backup)

(*) GigaZ:

- Operation of ILC at lower energies like Z-pole or WW threshold.
  - Allows to perform precision measurements of EW sector of the SM.
- At Z-pole, several billion Z's can be studied within ~1-2 months.
  - Physics of LEP1 and SLC can be revisited with few days of data.

In following studies:
central values of input measurements adjusted to M_H = 125 GeV.

- *(Except where indicated.)*

Prospects of EW fit for: ILC with Giga Z

Future Linear Collider can improve precision of EWPO's tremendously.

- *WW threshold scan + kinematic reconstruction, to obtain M_W*
  - From threshold scan: δM_W : 15 → 5 MeV
- *ttbar threshold scan, to obtain m_t*
  - Obtain m_t indirectly from production cross section: δm_t : 0.8 → 0.1 GeV
    - Dominated by conversion from threshold to MSbar mass.
- *Z pole measurements*
  - High statistics: 10^9 Z decays: $\delta R^0_{lep} : 2.5 \cdot 10^{-2} \rightarrow 4 \cdot 10^{-3}$
  - With polarized beams, uncertainty on $\delta A^{0,f}_{LR}$: $10^{-3} \rightarrow 10^{-4}$, which translates to $\delta \sin^2\theta^l_{eff} : 1.6 \cdot 10^{-4} \rightarrow 1.3 \cdot 10^{-5}$
- *H→ZZ and H→WW couplings*: measured at 1% precision.

ILC prospects: from ILC TDR (Vol-2).

Prospects of EW fit for: LHC Phase-1

LHC Phase-1 (300/fb)

- *W mass measurement* : δM_W : 15 → 8 MeV
- *Final top mass measurement m_t* : δm_t : 0.8 → 0.6 GeV
- *H→ZZ and H→WW couplings*: measured at 3% precision.

LHC prospects: possibly optimistic scenario, but not impossible.

Prospects of EW fit

LHC Phase-1 (300/fb)

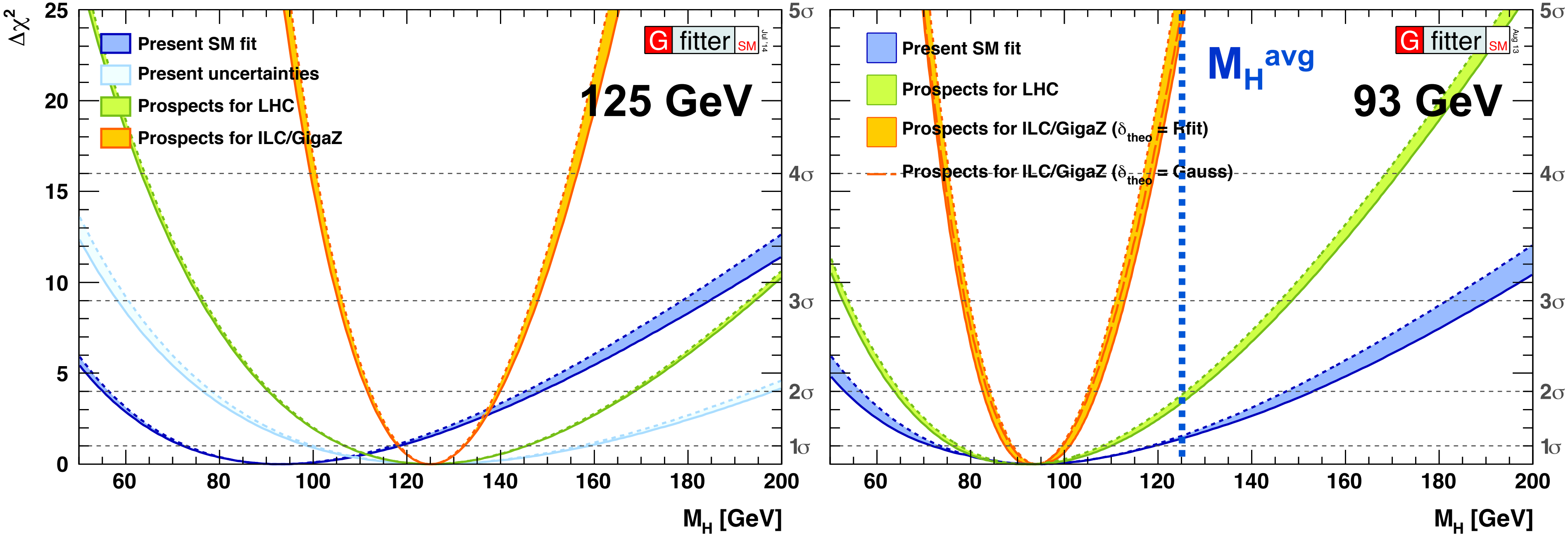
- *W mass measurement* : δM_W : 15 → 8 MeV
- *Final top mass measurement m_t* : δm_t : 0.8 → 0.6 GeV
- *H→ZZ and H→WW couplings*: measured at 3% precision.

For both LHC and ILC:

- Low-energy data results to improve $\Delta\alpha_{had}$:
  - ISR-based (BABAR), KLOE-II, VEPP-2000 (at energy below cc resonance), and BESIII e^+e^- cross-section measurements (around cc resonance).
  - Plus: improved α_s (from reliable Lattice predictions): $\Delta\alpha_{had}$: $10^{-4} \rightarrow 5 \cdot 10^{-5}$

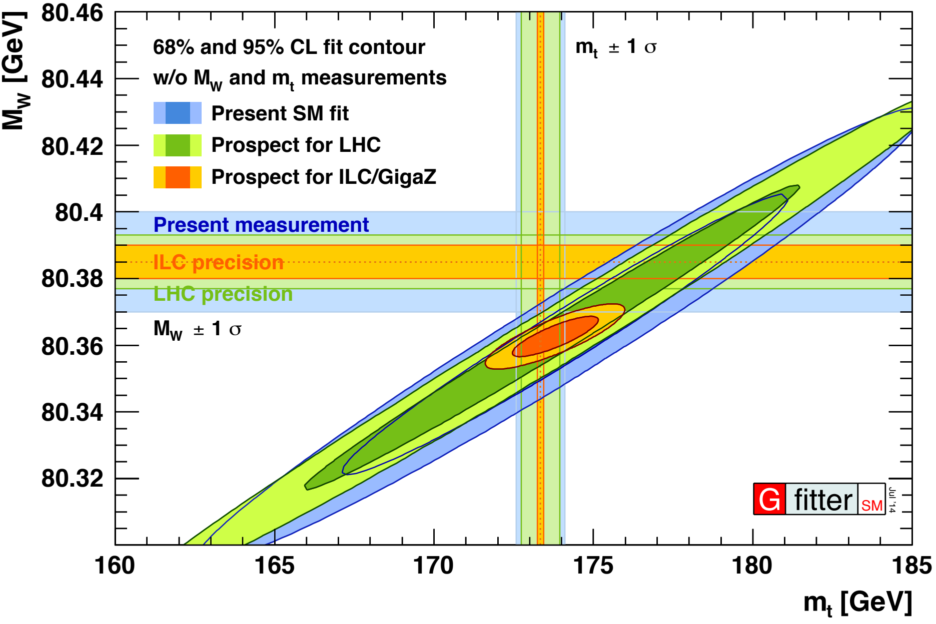
- Assuming ~25% of today's theoretical uncertainties on M_W and $\sin^2\theta^l_{eff}$
  - *Implies ambitions three-loop electroweak calculations!*
    - δM_W (4→1 MeV), $\delta\sin^2\theta^l_{eff}$ ($4.7\times10^{-5} \rightarrow 1\times10^{-5}$) (from Snowmass report)
  - Partial Z decay widths at 3-loop level: factor 4 improvement
  - LHC: top quark mass theo uncertainty: 0.50 → 0.25 GeV

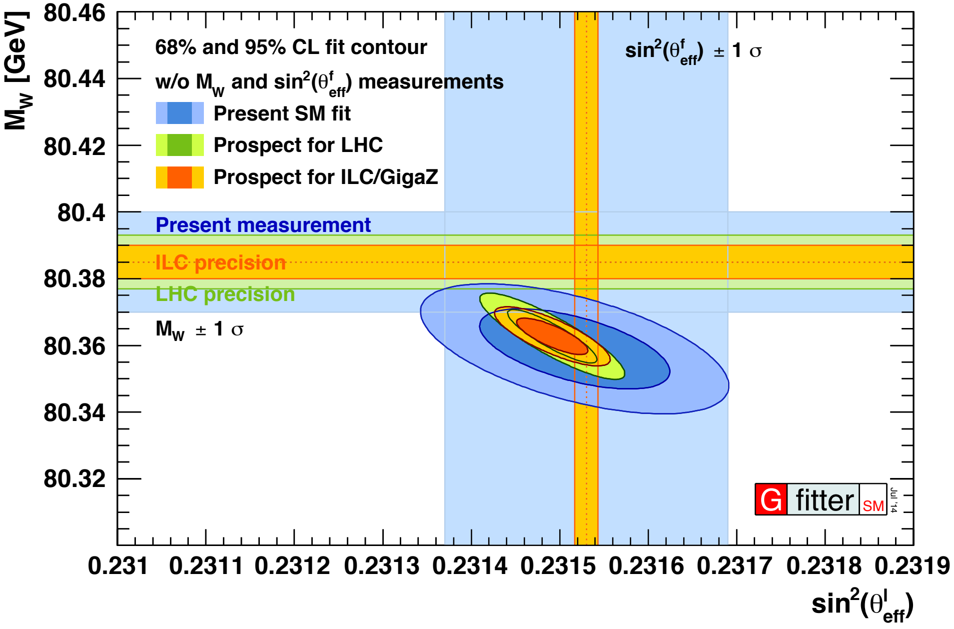
Prospects of EW fit



- Indirect prediction M_H dominated by experimental uncertainties.
  - Present: $\sigma(M_H) = {}^{+31}_{-26}$ (exp) ${}^{+10}_{-8}$ (theo) GeV
  - LHC: $\sigma(M_H) = {}^{+20}_{-18}$ (exp) ${}^{+3.9}_{-3.8}$ (theo) GeV
  - ILC: $\sigma(M_H) = {}^{+6.9}_{-6.6}$ (exp) ${}^{+2.5}_{-2.3}$ (theo) GeV
- Logarithmic dependency on M_H → *cannot compete with direct M_H meas.*
- If EWP-data central values unchanged, i.e. keep favoring low value of Higgs mass (93 GeV), ~5σ discrepancy with measured Higgs mass.

Prospects of EW fit

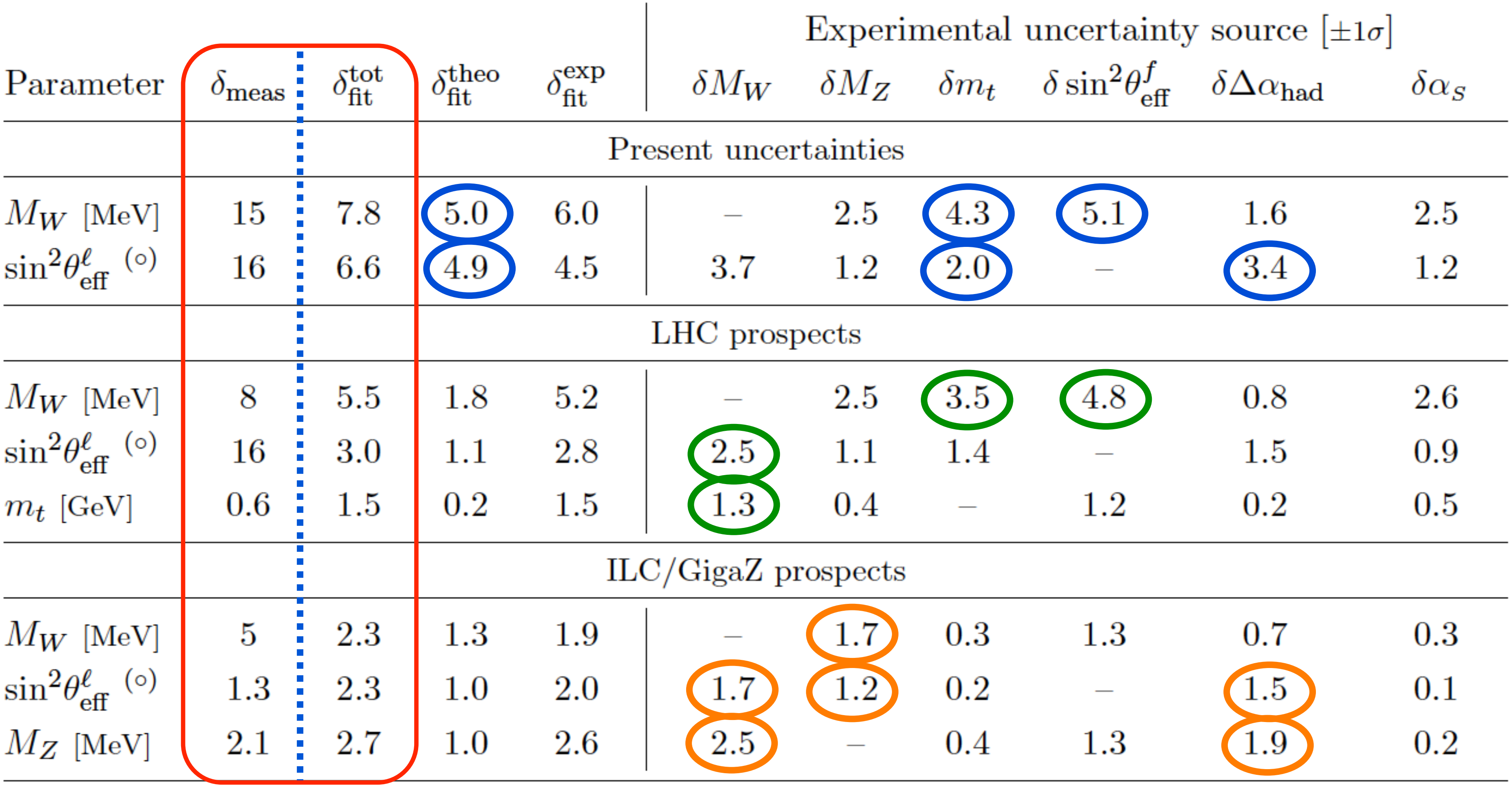




- Huge reduction of uncertainty on indirect determinations of m_t, m_W, and $\sin^2\theta^l_{eff}$, ***by a factor of 3 or more.***
- Assuming central values of m_t and M_W do not change, (at ILC) a deviation between the SM prediction and the direct measurements would be prominently visible.

Impact of individual uncertainties

- Breakdown of individual contributions to errors of M_W and $\sin^2\theta^l_{eff}$

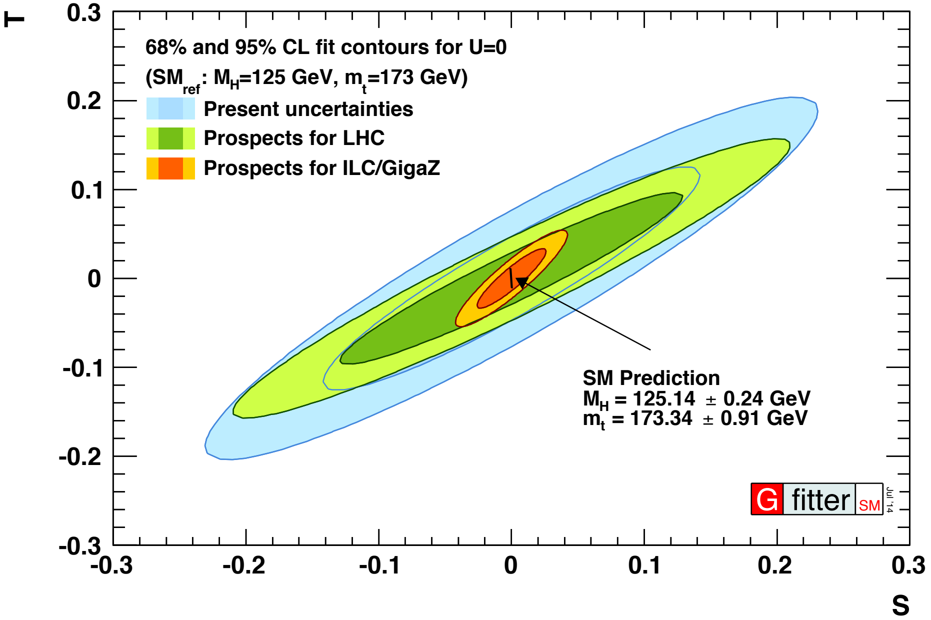


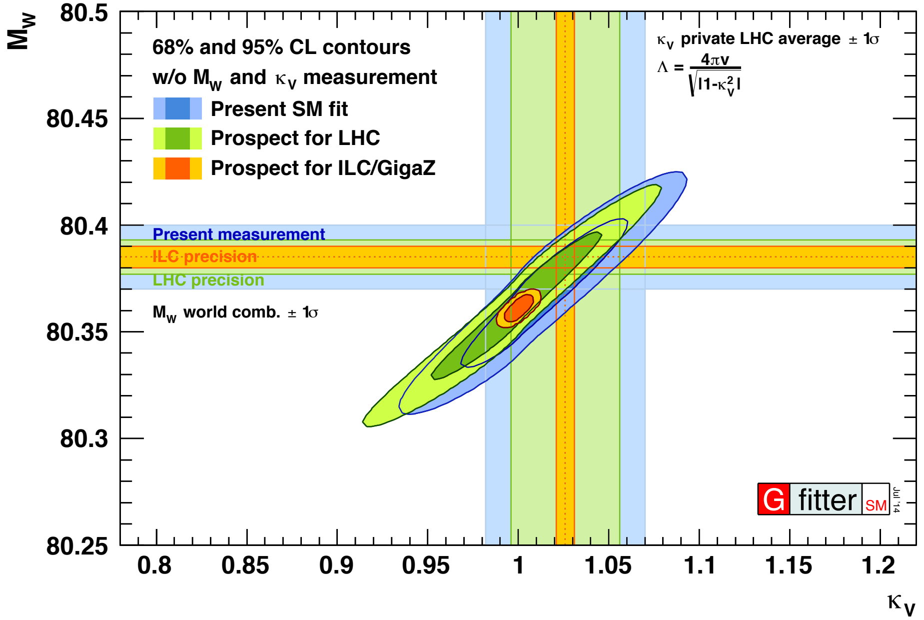
Parameter	δ_{meas}	$\delta_{\text{fit}}^{\text{tot}}$	$\delta_{\text{fit}}^{\text{theo}}$	$\delta_{\text{fit}}^{\text{exp}}$	Experimental uncertainty source [$\pm 1\sigma$]					
					δM_W	δM_Z	δm_t	$\delta \sin^2\theta^f_{\text{eff}}$	$\delta\Delta\alpha_{\text{had}}$	$\delta\alpha_S$
Present uncertainties										
M_W [MeV]	15	7.8	5.0	6.0	–	2.5	4.3	5.1	1.6	2.5
$\sin^2\theta^\ell_{\text{eff}}$ $^{(\circ)}$	16	6.6	4.9	4.5	3.7	1.2	2.0	–	3.4	1.2
LHC prospects										
M_W [MeV]	8	5.5	1.8	5.2	–	2.5	3.5	4.8	0.8	2.6
$\sin^2\theta^\ell_{\text{eff}}$ $^{(\circ)}$	16	3.0	1.1	2.8	2.5	1.1	1.4	–	1.5	0.9
m_t [GeV]	0.6	1.5	0.2	1.5	1.3	0.4	–	1.2	0.2	0.5
ILC/GigaZ prospects										
M_W [MeV]	5	2.3	1.3	1.9	–	1.7	0.3	1.3	0.7	0.3
$\sin^2\theta^\ell_{\text{eff}}$ $^{(\circ)}$	1.3	2.3	1.0	2.0	1.7	1.2	0.2	–	1.5	0.1
M_Z [MeV]	2.1	2.7	1.0	2.6	2.5	–	0.4	1.3	1.9	0.2

$^{(\circ)}$In units of 10^{-5}.

- M_W and $\sin^2\theta^l_{eff}$ are sensitive probes of new physics! For all scenarios.
- At ILC/GigaZ, precision of M_Z will become important again.

BSM prospects of EW fit





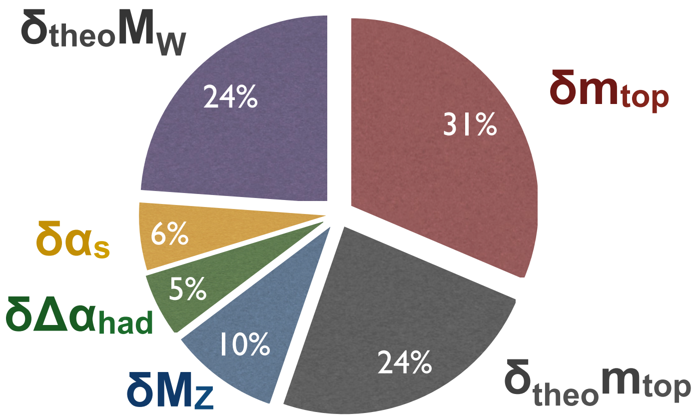
- For STU parameters, ***improvement of factor of >3*** is possible at ILC.
- Again, at ILC a deviation between the SM predictions and direct measurements would be prominently visible.
- Competitive results between EW fit and Higgs coupling measurements!
  - (At level of 1%.)

Conclusion and Today's prospects

- Including M_H measurement, for first time SM is fully over-constrained!
  - M_H consistent at 1.3σ with indirect prediction from EW fit.
  - p-Value of global electroweak fit of SM: 21% (pseudo-experiments)
- New: N^{2}LO calcs and theo. uncertainties for all relevant observables.
  - δ_{theo} m_t starting to become relevant.

- Knowledge of M_H dramatically improves SM prediction of key observables
  - M_W (28→8 MeV), $\sin^2\theta^l_{eff}$ (2.3x10^{-5}→0.7x10^{-5}), m_t (6.2→2.5 GeV)
- Improved accuracies set benchmark for new direct measurements!

- δM_W (indirect) = 8 MeV
  - Large contributions to δM_W from top and unknown higher-order EW corrections
- δM_W (direct) = 15 MeV



- Including new data electroweak fits remain very interesting in the next years!
- Latest results always available at: http://cern.ch/Gfitter

Thanks!

Backup

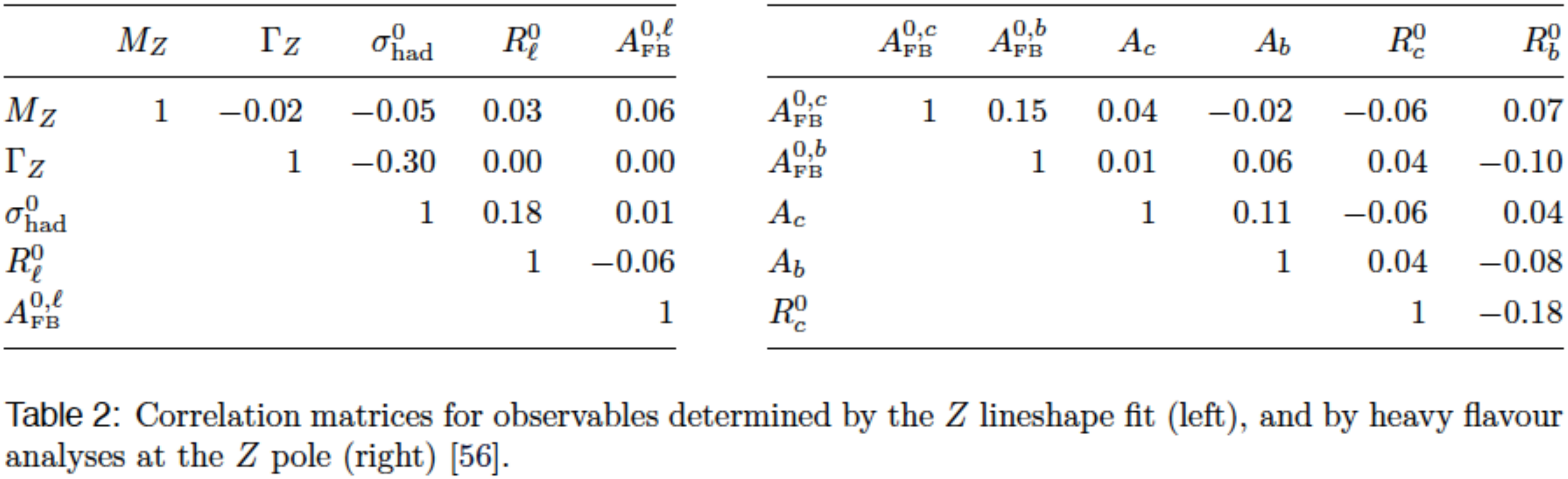
G fitter

A Generic Fitter Project for HEP Model Testing

Backup

Input correlations of the EW fit

- Input correlation coefficients between Z pole measurements



	M_Z	Γ_Z	σ^0_{had}	R^0_ℓ	$A^{0,\ell}_{\text{FB}}$
M_Z	1	-0.02	-0.05	0.03	0.06
Γ_Z		1	-0.30	0.00	0.00
σ^0_{had}			1	0.18	0.01
R^0_ℓ				1	-0.06
$A^{0,\ell}_{\text{FB}}$					1

	$A^{0,c}_{\text{FB}}$	$A^{0,b}_{\text{FB}}$	A_c	A_b	R^0_c	R^0_b
$A^{0,c}_{\text{FB}}$	1	0.15	0.04	-0.02	-0.06	0.07
$A^{0,b}_{\text{FB}}$		1	0.01	0.06	0.04	-0.10
A_c			1	0.11	-0.06	0.04
A_b				1	0.04	-0.08
R^0_c					1	-0.18

Table 2: Correlation matrices for observables determined by the Z lineshape fit (left), and by heavy flavour analyses at the Z pole (right) [56].

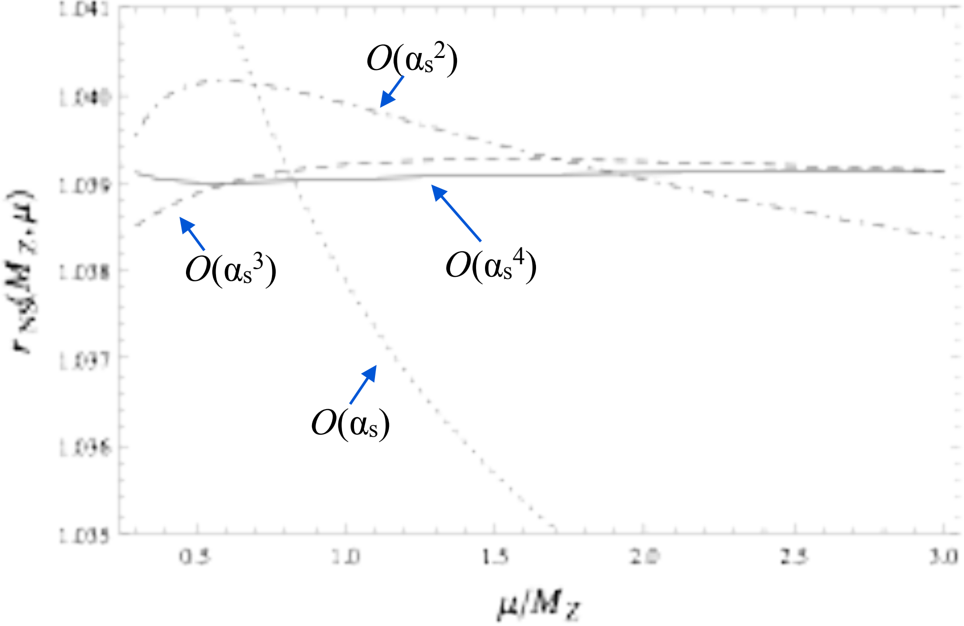
Radiator Functions

- Partial widths are defined inclusively:
  contain both QCD and QED contributions.
- Corrections expressed as so-called radiator functions $R_{A,f}$ and $R_{V,f}$

$$\Gamma_{f\bar{f}} = N_c^f \frac{G_F M_Z^3}{6\sqrt{2}\pi} \left(|g_{A,f}|^2 R_{A,f} + |g_{V,f}|^2 R_{V,f} \right)^2$$

- High sensitivity to the strong coupling α_s
- Recently, full four-loop calculation of QCD Adler function became available (N^{3}LO)
- Much-reduced scale dependence!
- *Theoretical uncertainty of ~0.15 MeV, compared with experimental uncertainty of 2.0 MeV.*

[D. Bardin, G. Passarino, “The Standard Model in the Making”, Clarendon Press (1999)]

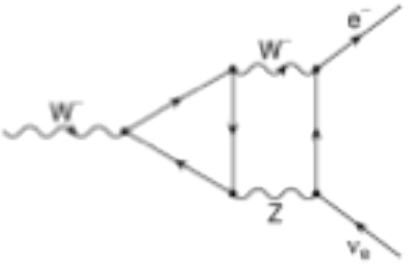


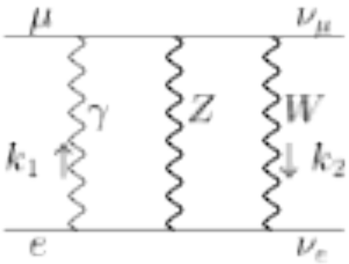
[P. Baikov et al., Phys. Rev. Lett. 108, 222003 (2012)]
[P. Baikov et al Phys. Rev. Lett. 104, 132004 (2010)]

Calculation of M_W

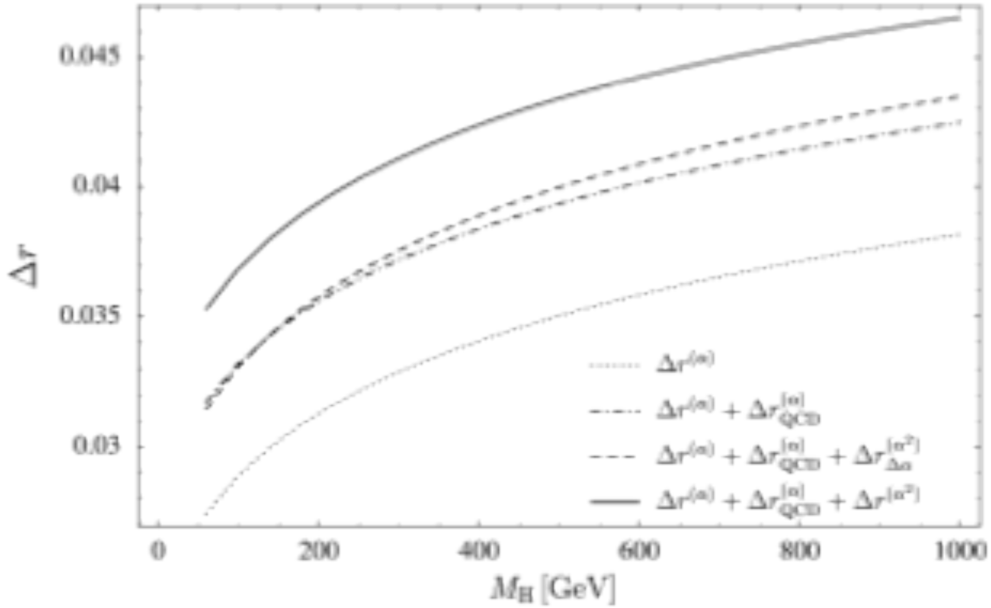
- Full EW one- and two-loop calculation of fermionic and bosonic contributions.
- One- and two-loop QCD corrections and leading terms of higher order corrections.
- Results for Δr include terms of order $O(\alpha)$, $O(\alpha\alpha_s)$, $O(\alpha\alpha_s^2)$, $O(\alpha^2_{ferm})$, $O(\alpha^2_{bos})$, $O(\alpha^2\alpha_s m_t^4)$, $O(\alpha^3 m_t^6)$
- Uncertainty estimate:
  - Missing terms of order $\mathbf{O(\alpha^2\alpha_s)}$: about 3 MeV (from $O(\alpha^2\alpha_s m_t^4)$)
  - Electroweak three-loop correction $\mathbf{O(\alpha^3)}$: < 2 MeV
  - Three-loop QCD corrections $\mathbf{O(\alpha_s^3)}$: < 2 MeV
- Total: $\delta M_W \approx 4$ MeV

[M Awramik et al., Phys. Rev. D69, 053006 (2004)]
[M Awramik et al., Phys. Rev. Lett. 89, 241801 (2002)]





[A Freitas et al., Phys. Lett. B495, 338 (2000)]



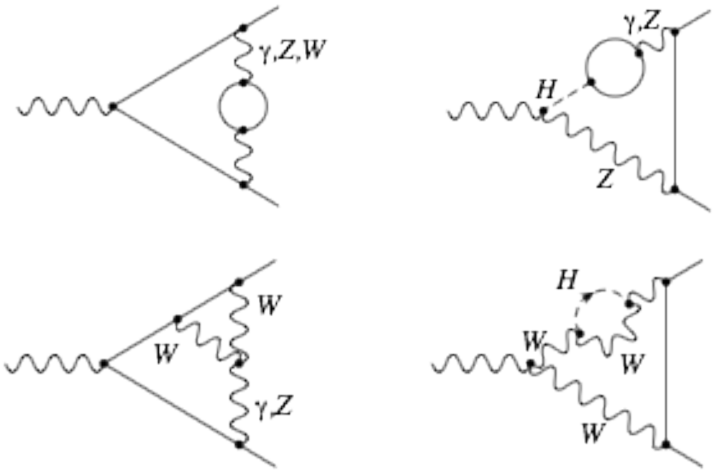
Calculation of $\sin^2(\theta^l_{\text{eff}})$

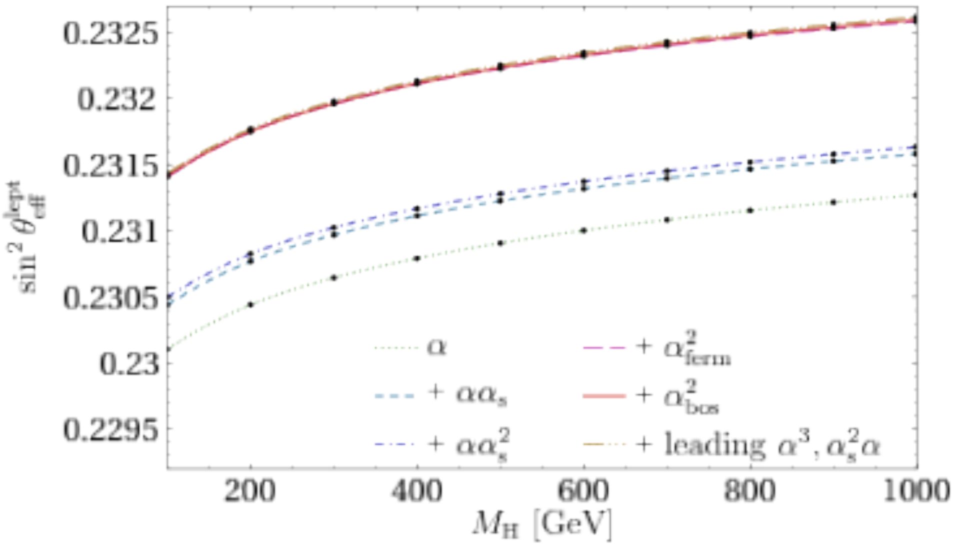
[M Awramik et al, Phys. Rev. Lett. 93, 201805 (2004)]
[M Awramik et al., JHEP 11, 048 (2006)]

- Effective mixing angle:

$$\sin^2\theta^{\text{lept}}_{\text{eff}} = \left(1 - M_W^2/M_Z^2\right)(1 + \Delta\kappa)$$

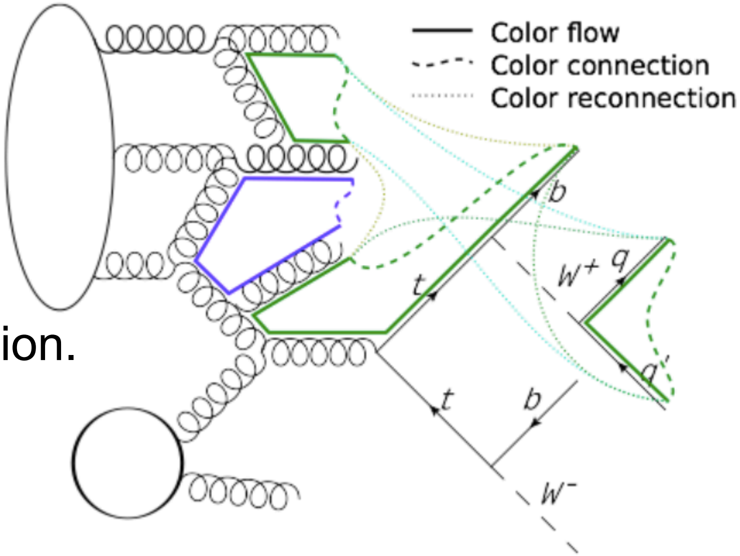
- Two-loop EW and QCD correction to Δκ known, leading terms of higher order QCD corrections.
- Fermionic two-loop correction about 10^{-3}, whereas bosonic one 10^{-5}.
- Uncertainty estimate obtained with different methods, geometric progression, leading to total of: $\delta\sin^2(\theta^l_{\text{eff}}) = 4.7\text{x}10^{-5}$





Uncertainty in Top mass definition

- Difficult to define a pole mass for heavy, unstable and colored particle.
    - Single top decays before hadronizing. To have colorless final states, additional quarks needed.
    - *Non-perturb.* color-reconnection effects in fragmentation → biases in simulation.
    - ‘Renormalon’ ambiguity in top mass definition.
        - For pole mass, not for MS-bar scheme.
    - Impact of finite top width effects.

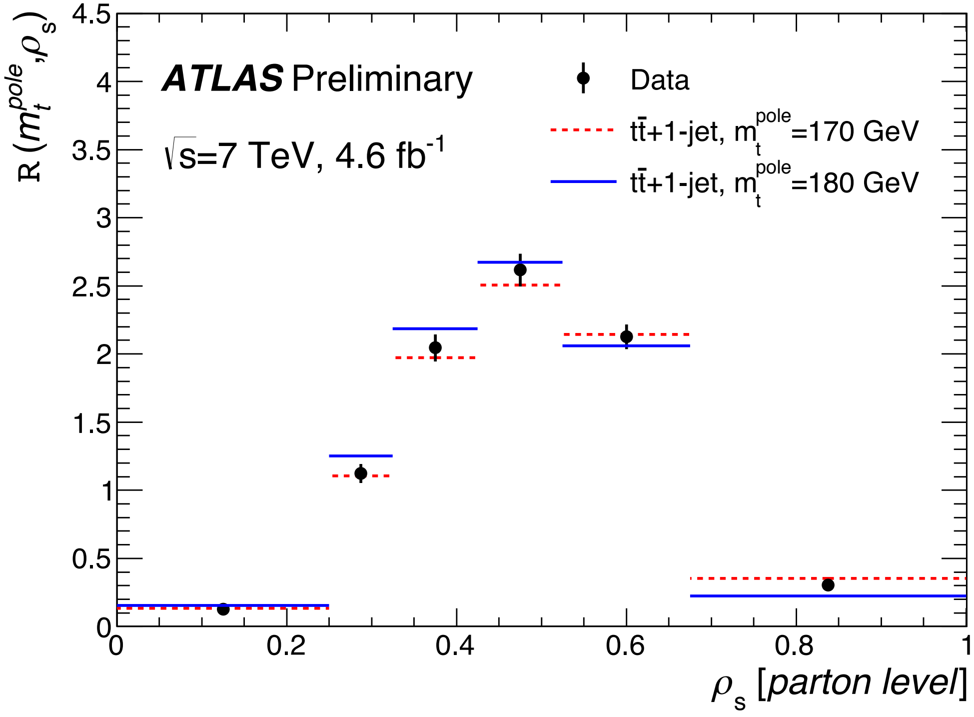


- Result: $m_t^{exp} \not\equiv m_t^{pole}$, and event-dependent.
- The top mass extracted in hadron collisions is not well defined below a precision of $O(\Gamma_t)$ ~ 1 GeV

- Hard to estimate additional theo. uncertainties. With 0.5 GeV on m_t:
    - $M_H = 90^{+34}_{-21}$ GeV, $M_W = 80.359 \pm 0.013$ GeV, $\sin^2\theta^l_{eff} = 0.23148 \pm 0.00010$.
    - → Sofar only small deterioration in precision.

Interesting Top pole mass measurement

- From: ATLAS-CONF-2014-053:
  "top-quark pole mass measurement from ttbar+1jet events"
  - Through study of inverse of invariant mass of ttbar+1jet system (quantity: ρ_S).
  - Free of MC→pole mass conversion uncertainty.



- $\Rightarrow m_t^{pole} = 173.7 \pm 1.5\ (\text{stat}) \pm 1.4\ (\text{syst})\ ^{+1.0}_{-0.5}\ (\text{theo})$ GeV
- Great to see these efforts ongoing!
  - Similar measurements / tests ongoing at CMS.

Moriond 2011: Prediction for Higgs mass

- LEP + Tevatron (Fall 2010) :
  - CL^{2s}_{s+b} central value ±1σ: $M_H = 120.2^{+17.9}_{-5.2}$ GeV
  - 2σ interval:

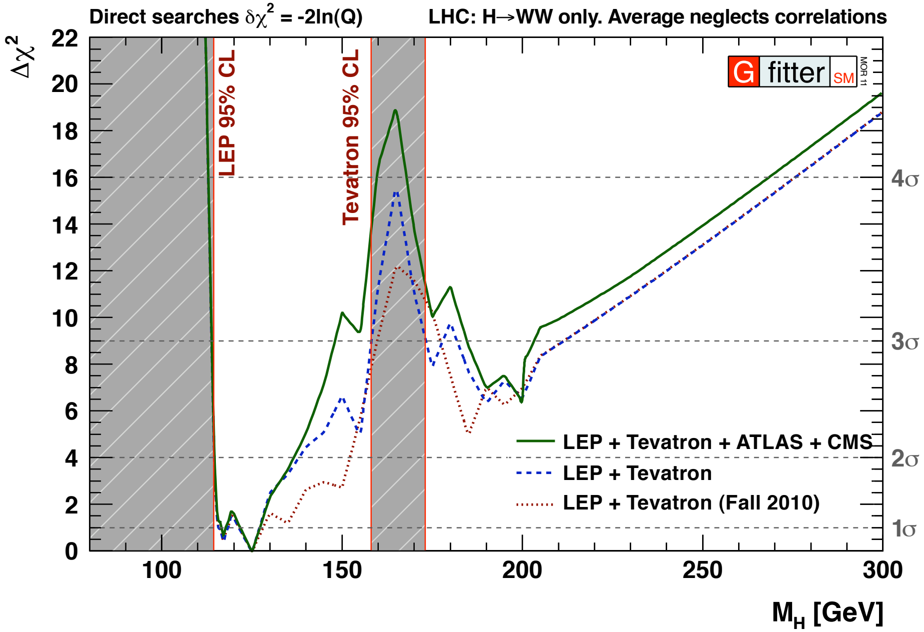
$$-2\ln Q: \quad [115,152]\ \mathrm{GeV}$$
$$CL_{s+b}^{2-sided}: \quad [114,155]\ \mathrm{GeV}$$

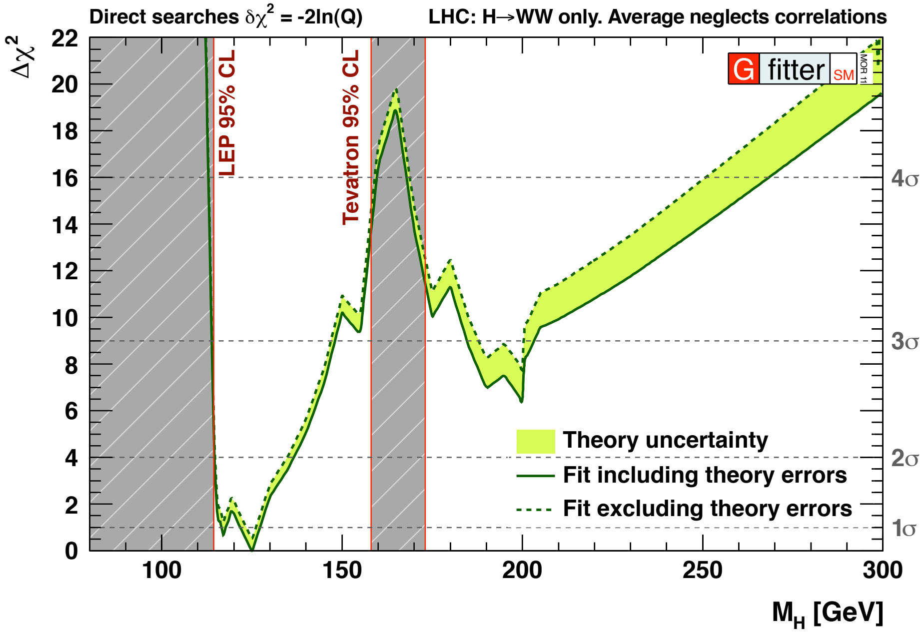
- LEP + Tevatron (Moriond 2011) :
  - CL^{2s}_{s+b} central value ±1σ: $M_H = 120.2^{+12.3}_{-4.7}$ GeV
  - 2σ interval:

$$-2\ln Q: \quad [115,138]\ \mathrm{GeV}$$
$$CL_{s+b}^{2-sided}: \quad [114,149] \cup [152,155]\ \mathrm{GeV}$$

- Fit with LEP + Tevatron + LHC (H➔WW) searches (Moriond 2011) :
  - Central value unchanged
  - 2σ interval:

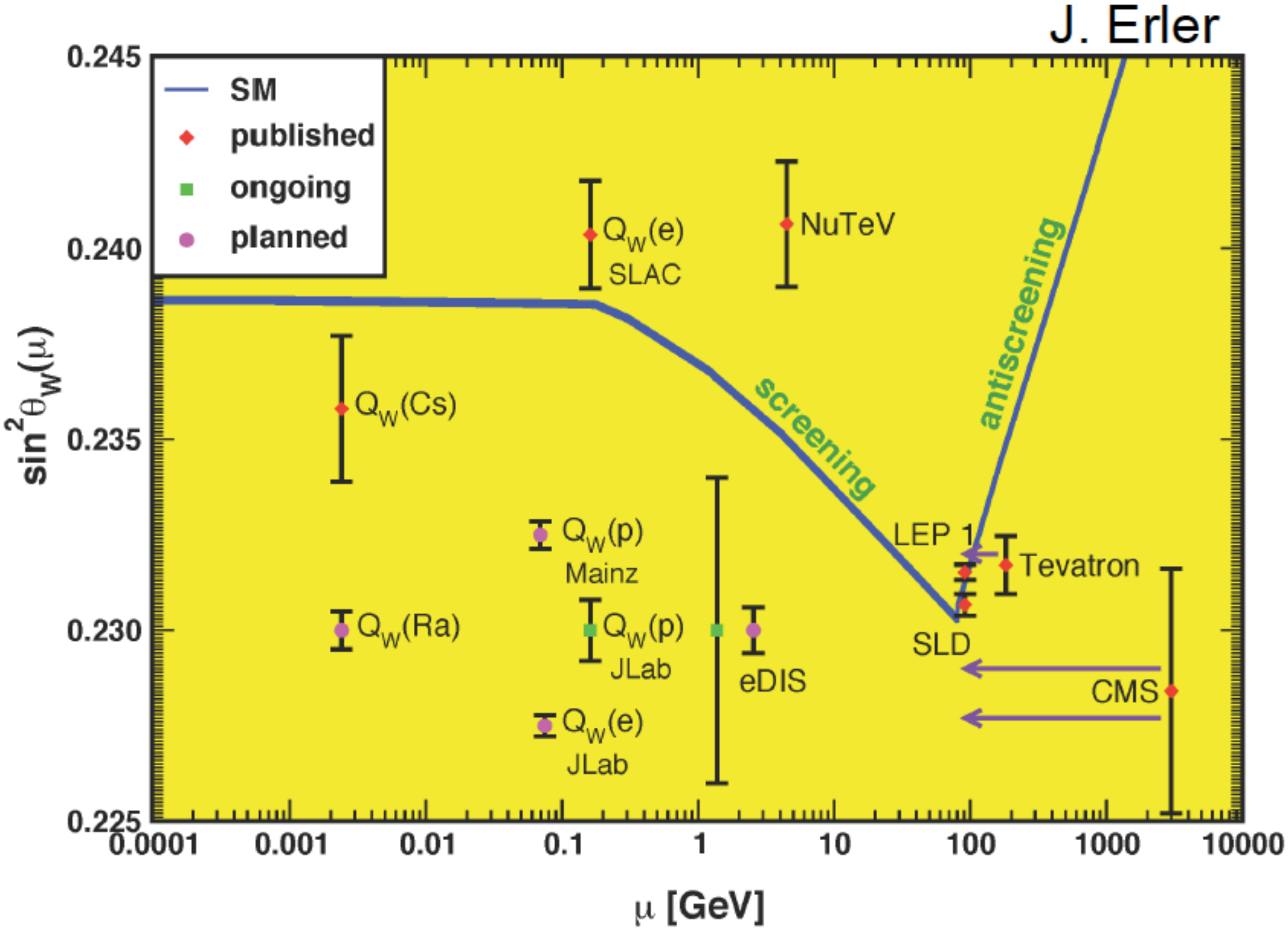
$$-2\ln Q: \quad [115,137]\ \mathrm{GeV}$$
$$CL_{s+b}^{2-sided}: \quad [114,14?]\ \mathrm{GeV}$$





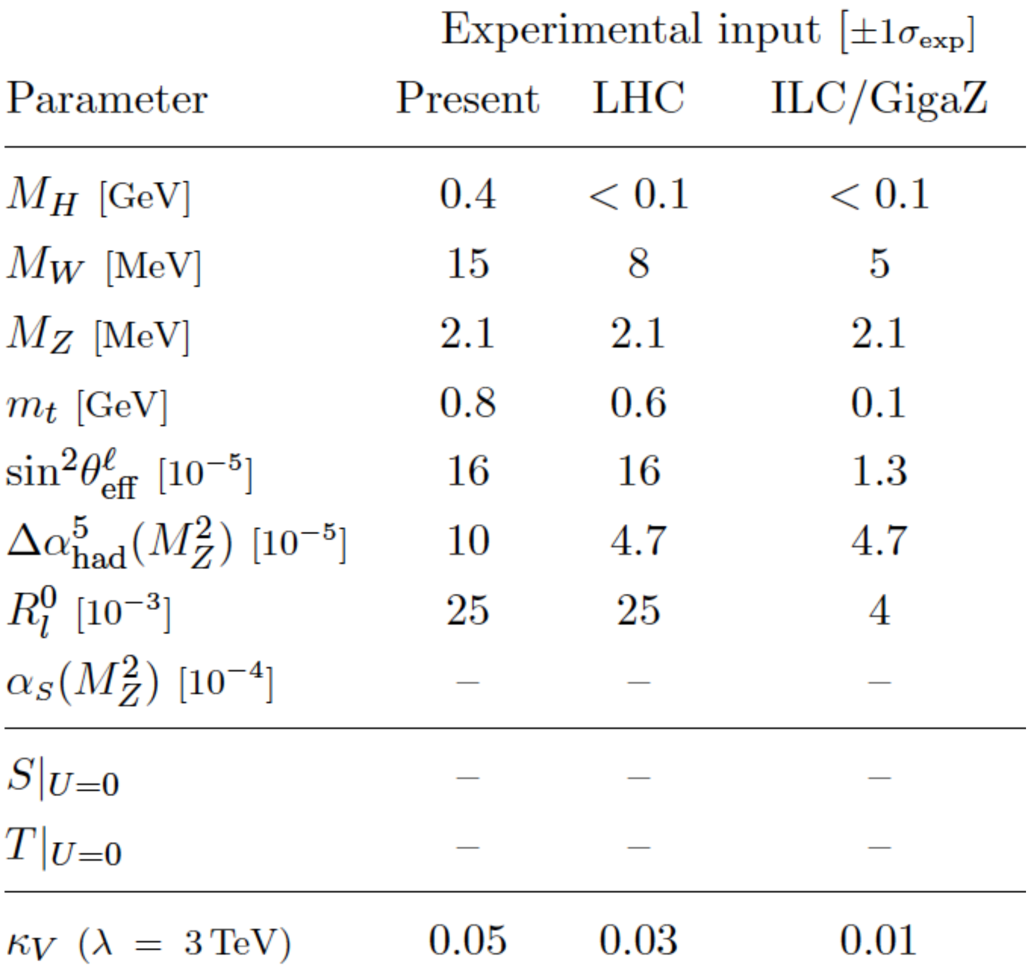
Low energy observables

- Low energy observables with interesting precision will soon become available.



Two prospects scenarios: LHC, ILC/GigaZ

- Uncertainty estimates used:



Parameter	Experimental input $[\pm 1\sigma_{\rm exp}]$ Present	 LHC	 ILC/GigaZ
M_H [GeV]	0.4	< 0.1	< 0.1
M_W [MeV]	15	8	5
M_Z [MeV]	2.1	2.1	2.1
m_t [GeV]	0.8	0.6	0.1
$\sin^2\theta^\ell_{\rm eff}$ $[10^{-5}]$	16	16	1.3
$\Delta\alpha^5_{\rm had}(M_Z^2)$ $[10^{-5}]$	10	4.7	4.7
R^0_l $[10^{-3}]$	25	25	4
$\alpha_s(M_Z^2)$ $[10^{-4}]$	–	–	–
$S\|_{U=0}$	–	–	–
$T\|_{U=0}$	–	–	–
κ_V $(\lambda = 3\,\text{TeV})$	0.05	0.03	0.01

  - ILC prospects from: ILC TDR (Vol-2).
  - Theoretical uncertainty estimates from recent Snowmass report
- Central values of input measurements adjusted to M_H = 126 GeV.

FCC-ee prospects

- From TLEP prospects: arXiv:1308.6176

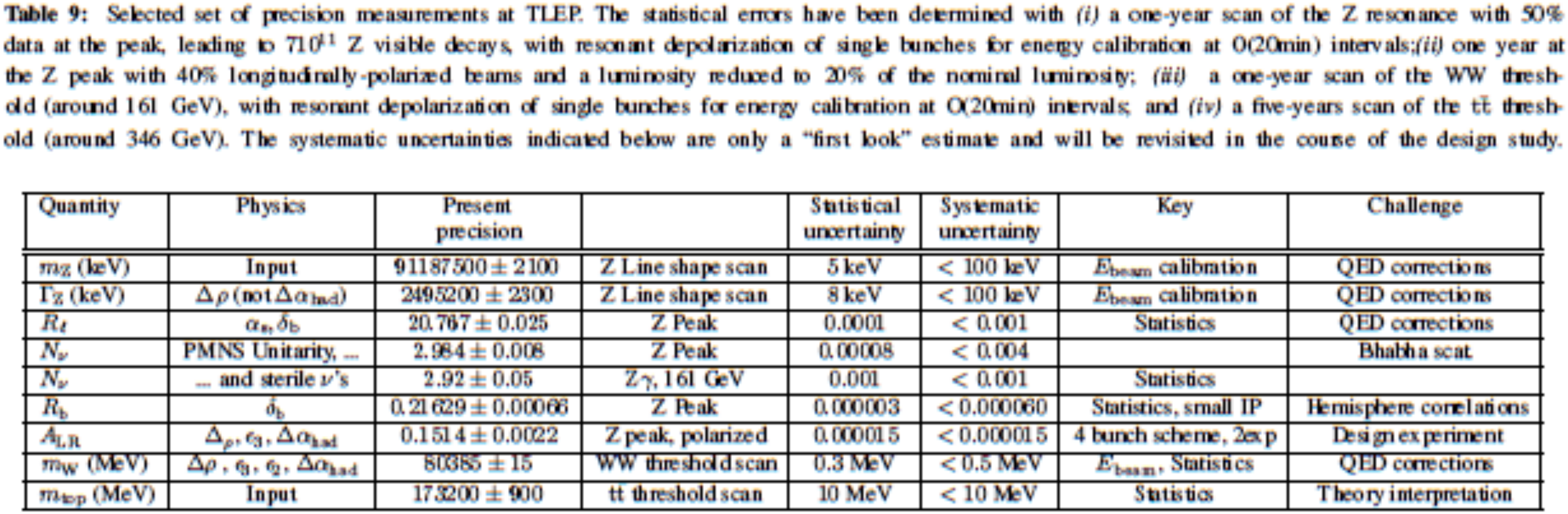


Table 9: Selected set of precision measurements at TLEP. The statistical errors have been determined with *(i)* a one-year scan of the Z resonance with 50% data at the peak, leading to $7\,10^{11}$ Z visible decays, with resonant depolarization of single bunches for energy calibration at O(20min) intervals;*(ii)* one year at the Z peak with 40% longitudinally-polarized beams and a luminosity reduced to 20% of the nominal luminosity; *(iii)* a one-year scan of the WW threshold (around 161 GeV), with resonant depolarization of single bunches for energy calibration at O(20min) intervals; and *(iv)* a five-years scan of the $t\bar{t}$ threshold (around 346 GeV). The systematic uncertainties indicated below are only a "first look" estimate and will be revisited in the course of the design study.

Quantity	Physics	Present precision		Statistical uncertainty	Systematic uncertainty	Key	Challenge
m_Z (keV)	Input	91187500 ± 2100	Z Line shape scan	5 keV	< 100 keV	$E_{\rm beam}$ calibration	QED corrections
Γ_Z (keV)	$\Delta\rho$ (not $\Delta\alpha_{\rm had}$)	2495200 ± 2300	Z Line shape scan	8 keV	< 100 keV	$E_{\rm beam}$ calibration	QED corrections
R_ℓ	α_s, δ_b	20.767 ± 0.025	Z Peak	0.0001	< 0.001	Statistics	QED corrections
N_ν	PMNS Unitarity, ...	2.984 ± 0.008	Z Peak	0.00008	< 0.004		Bhabha scat.
N_ν	... and sterile ν's	2.92 ± 0.05	Zγ, 161 GeV	0.001	< 0.001	Statistics	
R_b	δ_b	0.21629 ± 0.00066	Z Peak	0.000003	< 0.000060	Statistics, small IP	Hemisphere correlations
A_{LR}	$\Delta_\rho, \epsilon_3, \Delta\alpha_{\rm had}$	0.1514 ± 0.0022	Z peak, polarized	0.000015	< 0.000015	4 bunch scheme, 2exp	Design experiment
m_W (MeV)	$\Delta\rho$, ϵ_3, ϵ_2, $\Delta\alpha_{\rm had}$	80385 ± 15	WW threshold scan	0.3 MeV	< 0.5 MeV	$E_{\rm beam}$, Statistics	QED corrections
$m_{\rm top}$ (MeV)	Input	173200 ± 900	tt threshold scan	10 MeV	< 10 MeV	Statistics	Theory interpretation

Experimental inputs – Predicted uncertainties

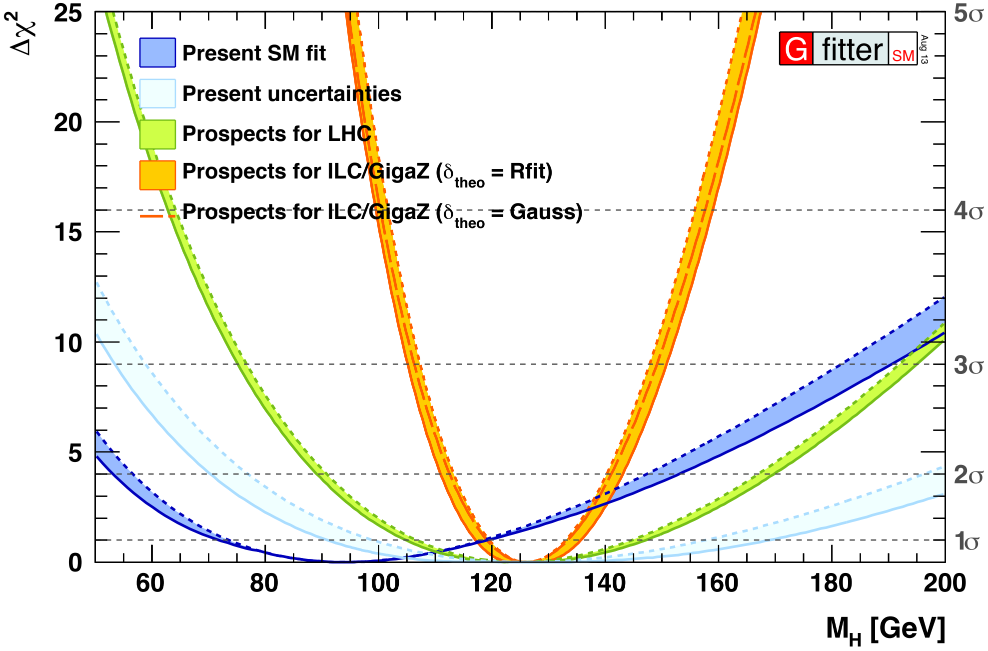
Parameter	Experimental input [$\pm 1\sigma$]			
	Present	LHC	ILC/GigaZ	TLEP
M_H [GeV]	0.4	⇨ < 0.1	< 0.1	< 0.1
M_W [MeV]	15	⇨ 8	⇨ 5	⇨ 1.3
M_Z [MeV]	2.1	2.1	2.1	⇨ 0.1
m_t [GeV]	0.9	⇨ 0.6	0.1	0.08
Γ_Z [MeV]	2.3	2.3	⇨ 0.8	⇨ 0.1
$\sin^2\theta^\ell_{\text{eff}}$ [$\cdot 10^{-5}$]	16	16	⇨ 1.3	⇨ 0.3
R^0_l [$\cdot 10^{-3}$]	25	25	⇨ 4	⇨ 1.3
$\Delta\alpha^5_{\text{had}}(M^2_Z)$ [$\cdot 10^{-5}$]	10	⇨ 4.7	4.7	4.7
$\alpha_S(M^2_Z)$ [$\cdot 10^{-4}$]	–	–	–	–
$\delta_{\text{th}} M_W$ [MeV]	4	⇨ 1	1	1
$\delta_{\text{th}} \sin^2\theta^\ell_{\text{eff}}$ [$\cdot 10^{-5}$]	4.7	⇨ 1	1	1

FCC-ee scenario:

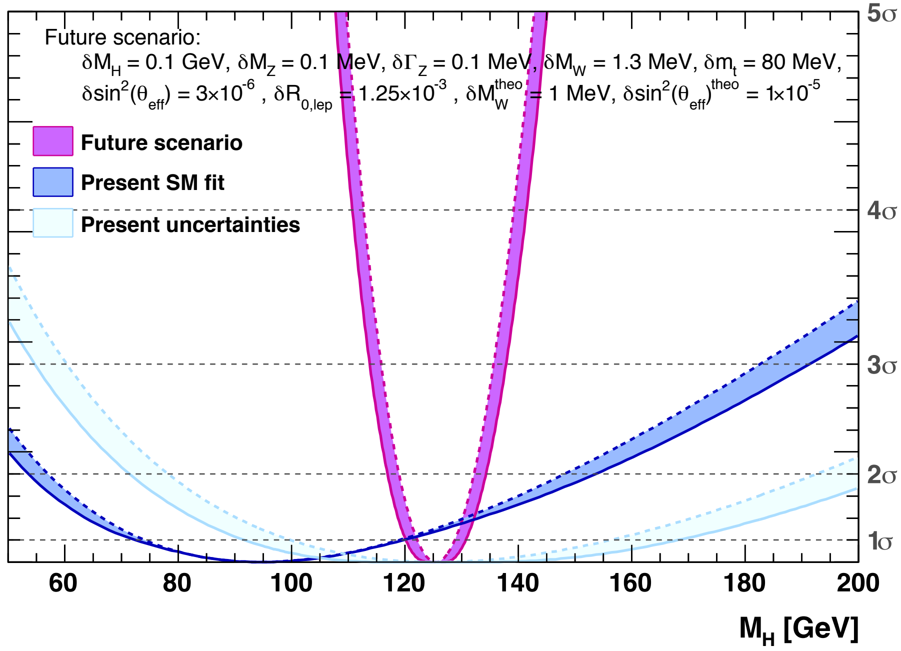
- *Preliminary estimates*
- Clearly not the same level of understanding as LHC or ILC.
- Uncertainties may turn out completely different.
  - From arXiv:1308.6176,
  - and Snowmass report.
    - Of these two, we take most conservative estimate.
- Note: top mass dominated by theoretical uncertainty.
- Higher statistics
- From beam energy precision: improved M_Z and Γ_Z

Prospects of the EW fit: Higgs mass (126 GeV)

Present / LHC / ILC

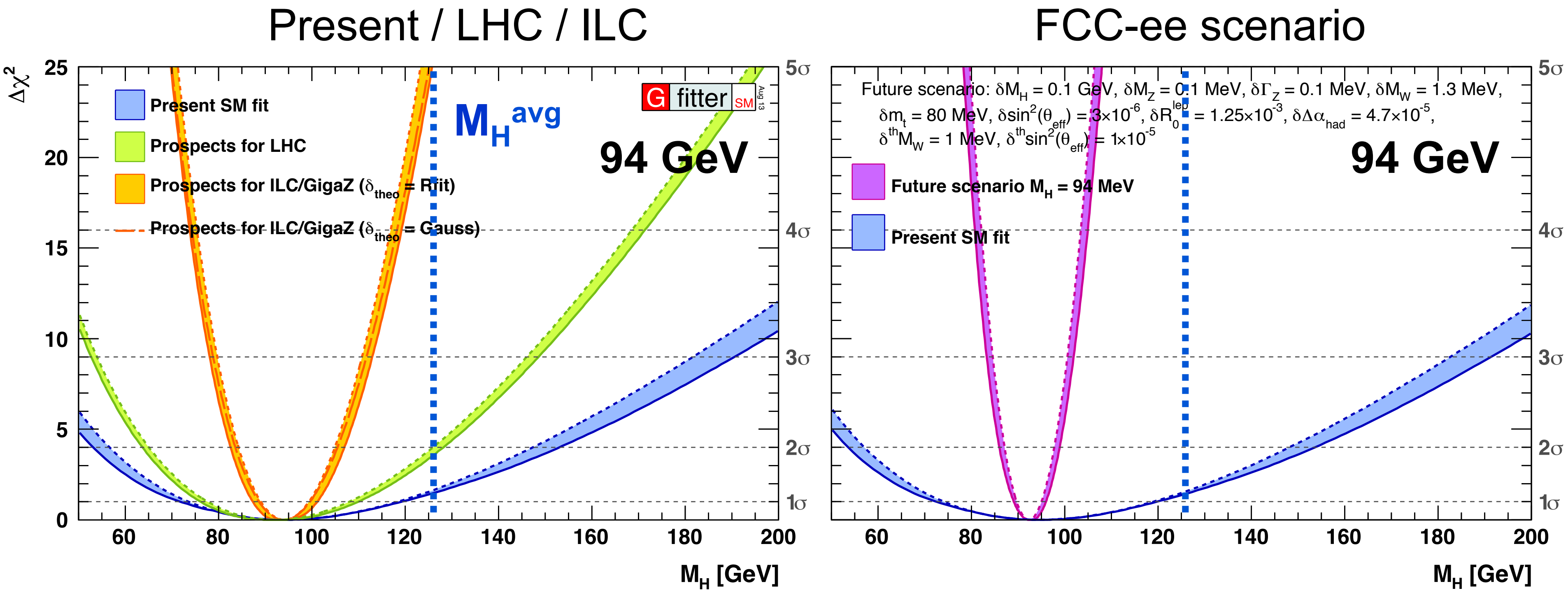


FCC-ee scenario



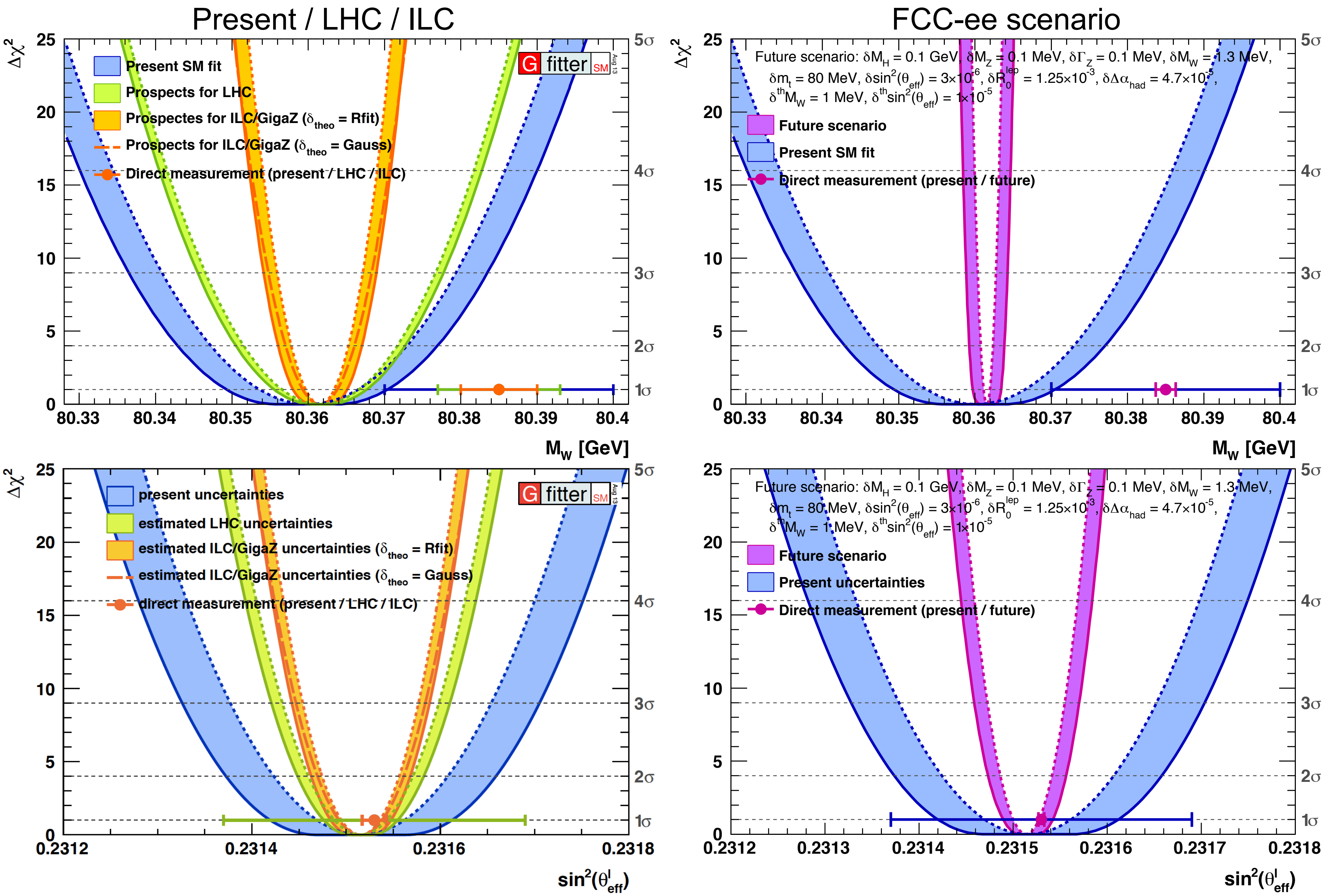
- Logarithmic dependency on M_H → cannot compete with direct M_H meas.
- Indirect prediction M_H dominated by theory uncertainties.
  - ILC with (without) theory errors: $M_H = 126^{+10}_{-9}$ (±7) GeV
  - ILC with present-day theory uncertainties: $M_H = 126^{+20}_{-17}$ GeV
  - FCC-ee with (without) theory errors: $M_H = 126 \pm 5$ (±3) GeV

Prospects of the EW fit: Higgs mass (94 GeV)



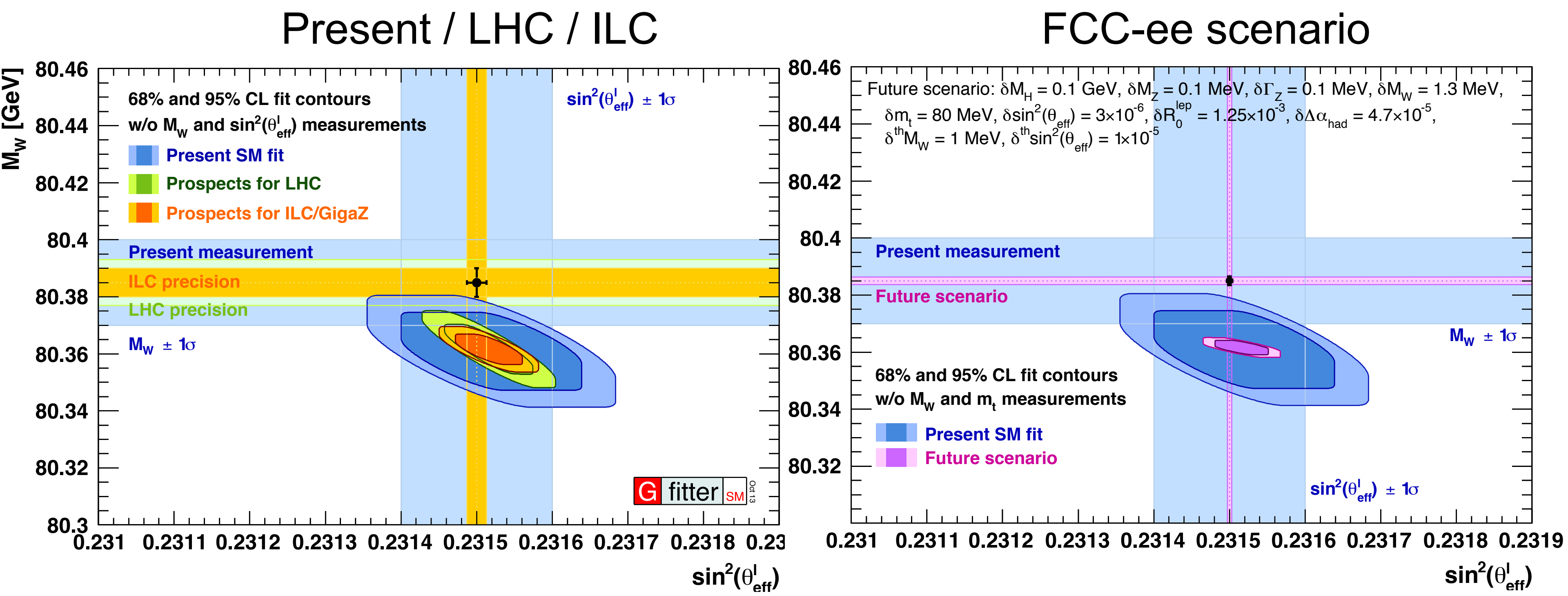
- If EWP-data central values are unchanged, i.e. they keep favoring low value of Higgs mass (94 GeV), >5σ discrepancy with measured Higgs mass.
  - In both ILC and FCC-ee scenarios.

Prospects of the EW fit: W mass and $\sin^2\theta^l_{eff}$


Present / LHC / ILC

FCC-ee scenario

Prospects of the EW fit: W mass versus $\sin^2\theta^l_{eff}$



Present / LHC / ILC

FCC-ee scenario

- Huge reduction of uncertainty on indirect determinations of m_W, and $\sin^2\theta^l_{eff}$, by a factor of ≳3 (≳4-5) at ILC (FCC-ee).

- Assuming central values of M_W and $\sin^2\theta^l_{eff}$ do not change, a deviation between the SM prediction and the direct measurements would be prominently visible, at both ILC and FCC-ee.
  - But also in LHC-300 scenario, from improved theory uncertainties.

Confrontation of measurement and prediction

- Breakdown of individual contributions to errors of M_W and $\sin^2\theta^l_{eff}$
- Parametric uncertainties (not the full fit).

error due to uncertainty ($\pm 1\sigma$)

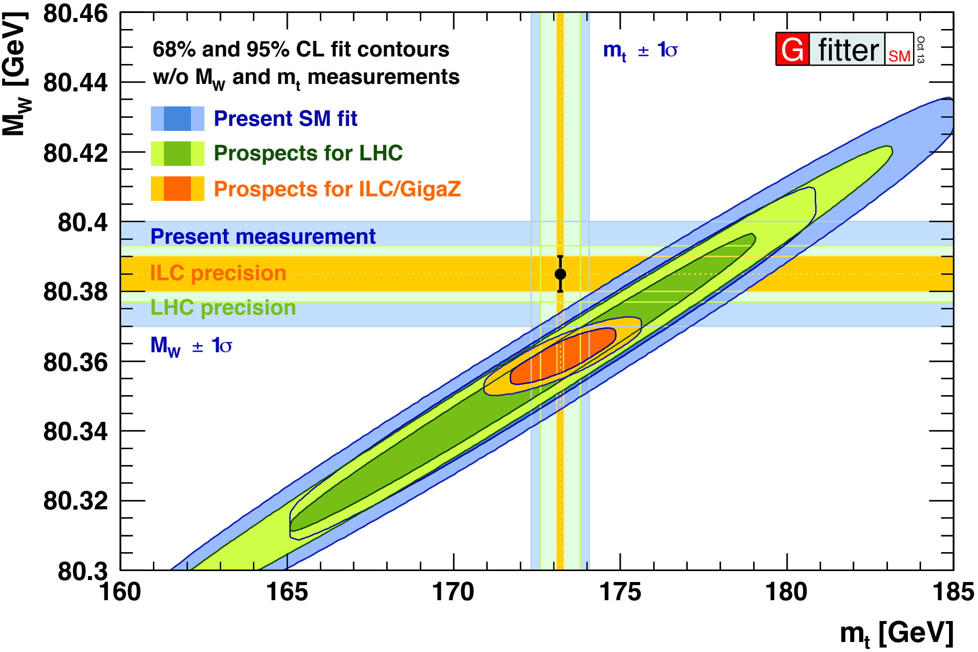
Parameter	Scenario	δ_{meas}	δ_{pred}	δ_{exp}	δM_Z	δm_t	$\delta\Delta\alpha_{\text{had}}$	$\delta\alpha_S$	δ_{theo}
M_W [MeV]	Present	15	10.4	6.4	2.6	5.2	1.8	1.7	4.0
	LHC	8	5.8	4.8	2.6	3.6	0.9	1.7	1.0
	ILC	5	3.8	2.8	2.6	0.6	0.9	0.4	1.0
	Future	1.3	2.0	1.0	0.1	0.5	0.9	0.3	1.0
$\sin^2\theta^\ell_{\text{eff}}$ $^{(\circ)}$	Present	16	9.5	4.8	1.5	2.8	3.5	1.0	4.7
	LHC	16	4.1	3.1	1.5	1.9	1.6	1.0	1.0
	ILC	1.3	3.2	2.2	1.5	0.3	1.6	0.2	1.0
	Future	0.3	2.7	1.7	0.1	0.3	1.6	0.2	1.0

$^{(\circ)}$In units of 10^{-5}.

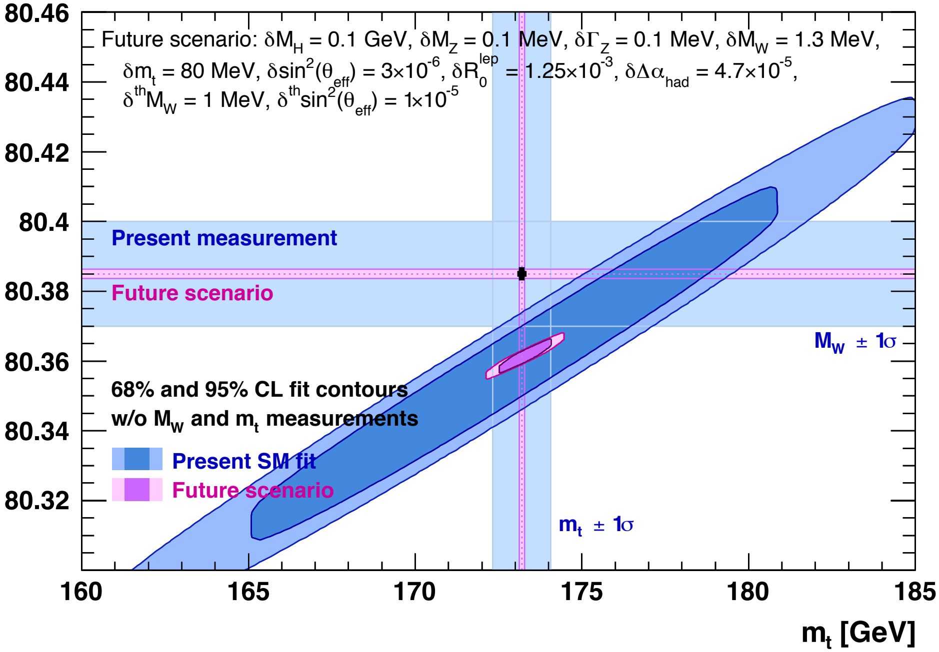
- M_W and $\sin^2\theta^l_{eff}$ are sensitive probes of new physics! In all scenarios.
- At ILC/GigaZ, precision of M_Z will become important again.
- At FCC-ee ('Future'), limited by external inputs: theory errors and $\Delta\alpha_{had}$

Prospects of the EW fit: W versus top mass

Present / LHC / ILC



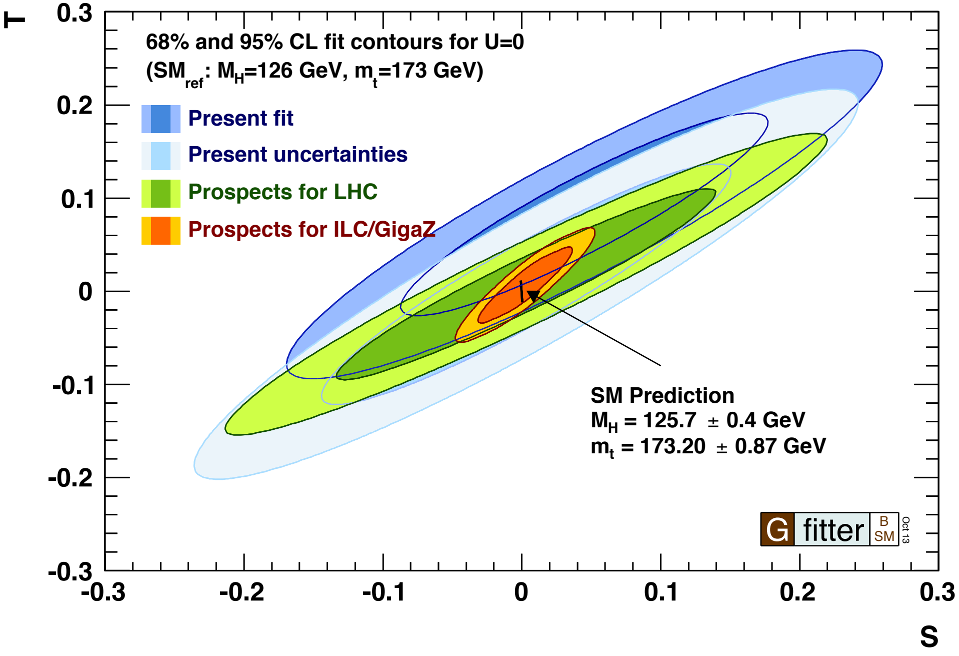
FCC-ee scenario

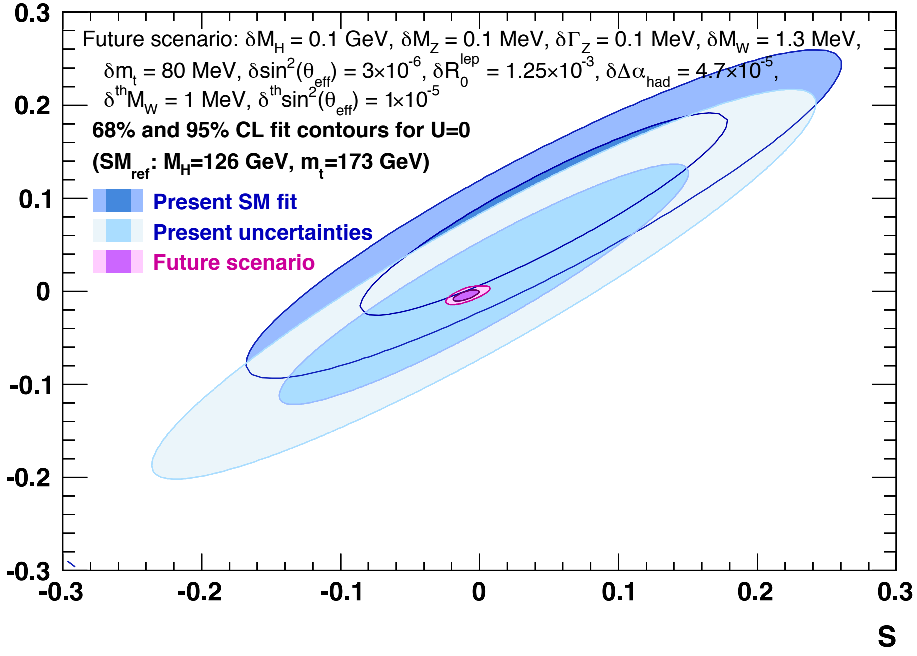


- Huge reduction of uncertainty on indirect determinations of m_t and m_W by a factor of ≳3 (≳5) at ILC (FCC-ee).

- Assuming central values of m_t and M_W do not change, a deviation between the SM prediction and the direct measurements would be prominently visible.

Prospects of EW fit: S versus T


Present / LHC / ILC


FCC-ee scenario

- For STU parameters, improvement of factor of ≳4 (≳10) is possible at ILC (FCC-ee).
- Again, at both ILC and FCC-ee a deviation between the SM predictions and direct measurements would be prominently visible.

Predicted uncertainties from EW fit

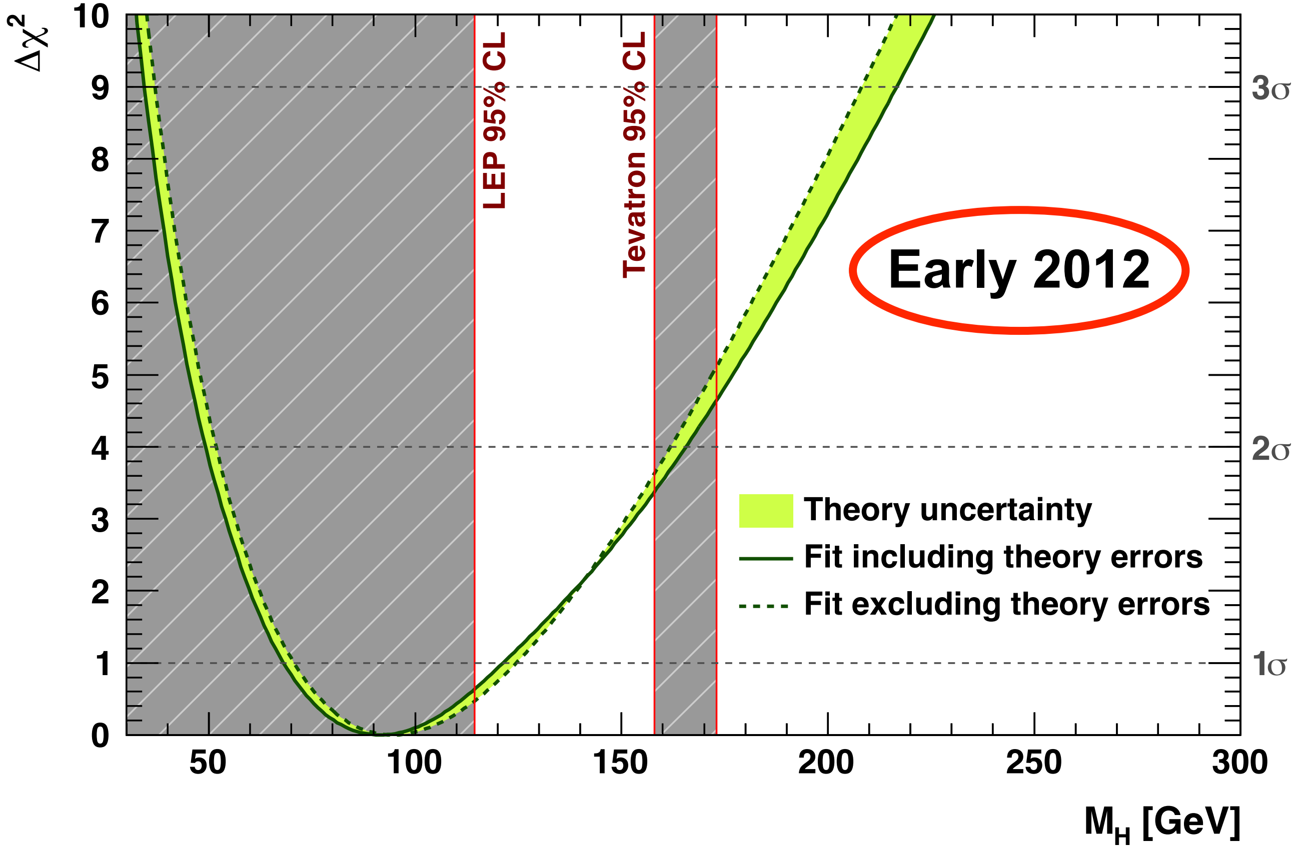
Parameter	δ_{meas}	$\delta_{\text{fit}}^{\text{tot}}$	$\delta_{\text{fit}}^{\text{exp}}$	$\delta_{\text{fit}}^{\text{theo}}$	error due to uncertainty $(\pm 1\sigma)$					
					δM_W	δM_Z	δm_t	$\delta \sin^2\theta^{\ell}_{\text{eff}}{}^{(\circ)}$	$\delta\Delta\alpha_{\text{had}}{}^{(\circ)}$	$\delta\alpha_S{}^{(\triangle)}$
					ILC prospects					
M_H [GeV]	< 0.1	$^{+9.6}_{-9.0}$	$^{+6.9}_{-6.6}$	$^{+2.7}_{-2.4}$	$^{+4.2}_{-0.8}$	$^{+4.4}_{-4.0}$	$^{+0.9}_{-0.8}$	$^{+3.1}_{-3.3}$	$^{+4.2}_{-4.1}$	$^{+0.6}_{-0.6}$
M_W [MeV]	5	3.6	1.9	1.7	–	1.7	0.3	1.2	0.7	0.2
M_Z [MeV]	2.1	3.7	2.6	1.1	2.4	–	0.5	1.3	1.9	0.3
m_t [GeV]	0.1	1.0	0.7	$^{+0.3}_{-0.2}$	$^{+0.5}_{-0.6}$	0.5	–	$^{+0.3}_{-0.2}$	0.4	–
$\sin^2\theta^{\ell}_{\text{eff}}$ $^{(\circ)}$	1.3	3.2	2.0	1.2	1.7	1.2	0.2	–	1.5	0.1
$\Delta\alpha_{\text{had}}$ $^{(\circ)}$	4.7	8.6	5.7	2.9	2.5	4.2	0.8	3.9	–	0.5
					Future prospects					
M_H [GeV]	< 0.1	5.3	3.3	2.0	3.0	0.3	1.0	$^{+0.0}_{-1.2}$	3.2	0.6
M_W [MeV]	1.3	1.9	0.4	1.5	–	0.1	0.3	0.2	0.1	0.1
M_Z [MeV]	0.1	1.5	1.0	0.5	1.0	–	0.3		0.9	0.4
m_t [GeV]	0.08	0.38	0.24	0.14	0.24	0.03	–	0.01	0.22	0.02
$\sin^2\theta^{\ell}_{\text{eff}}$ $^{(\circ)}$	0.3	$^{+2.8}_{-2.4}$	1.4	$^{+1.5}_{-1.1}$	1.2		0.1	–	1.3	0.5
$\Delta\alpha_{\text{had}}$ $^{(\circ)}$	4.7	0.4	0.1	0.3			0.1	0.1	–	

$^{(\circ)}$In units of 10^{-5}. $^{(\triangle)}$In units of 10^{-4}

- Breakdown of uncertainties derived from EW fit. (Note: *correlated* errors.)
- Compared to parametric breakdown: reduced experimental, but increased theory errors. Slightly smaller total errors.

Hunt for the Higgs

Gfitter group, EPJC 72, 2003 (2012)



- M_H was last missing input parameter of the electroweak fit
- Indirect determination from EW fit (2012): $M_H = 96^{+31}_{-24}$ GeV
  - With direct limits incorporated in the EW fit: $M_H = 120^{+12}_{-5}$ GeV