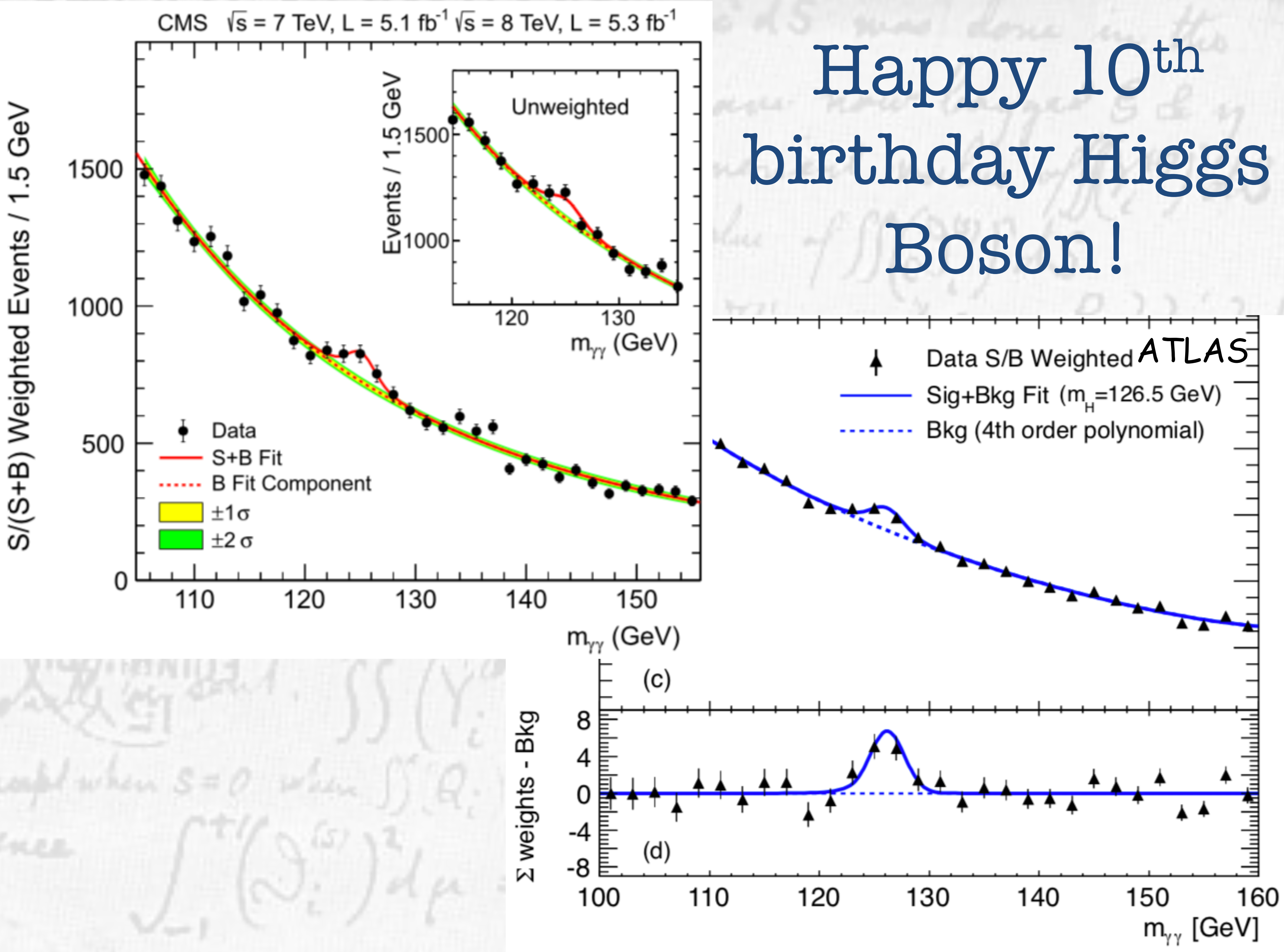
The Higgs Boson Under a Microscope

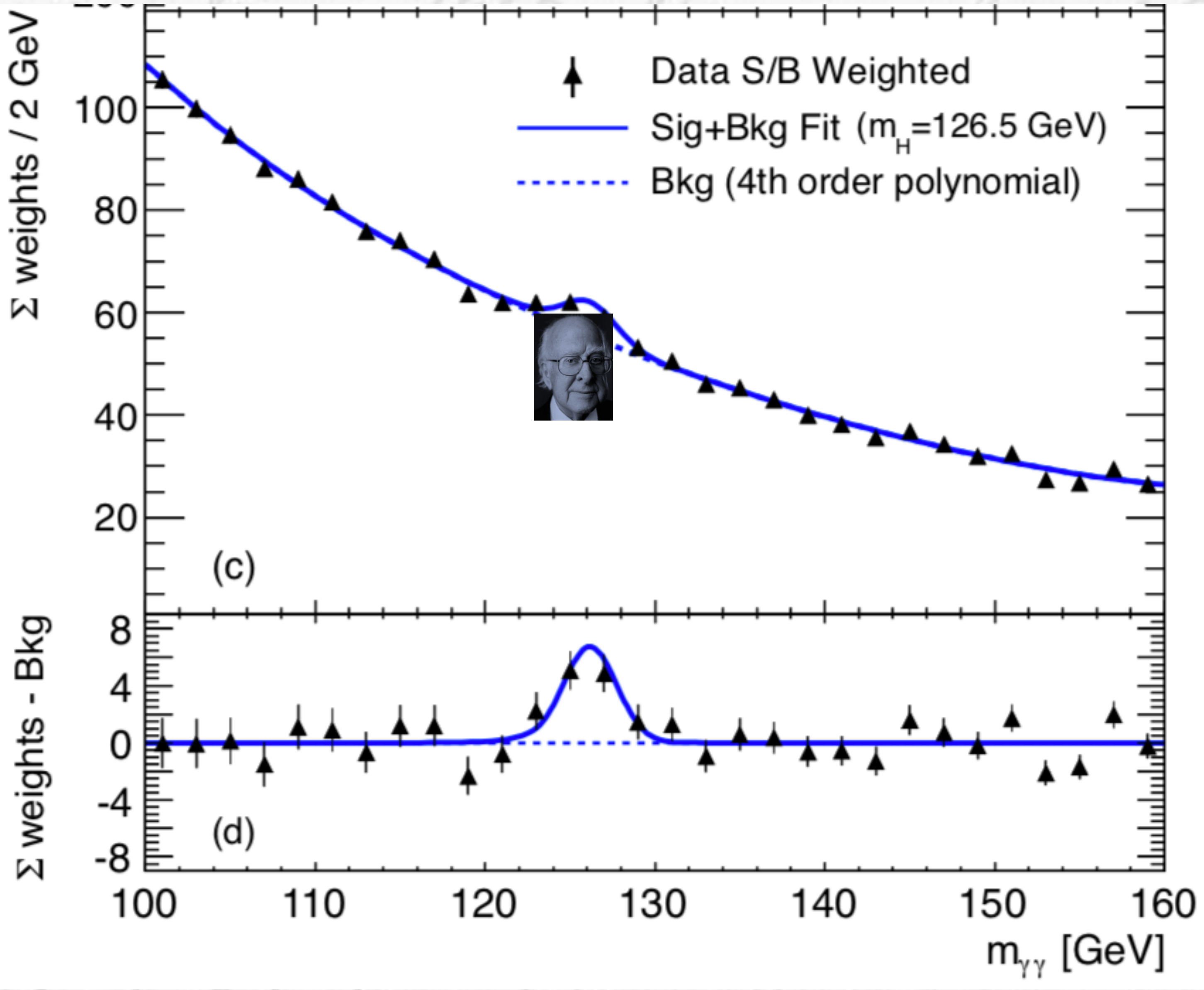
Birmingham, Nov 30th 2022

Matthew McCullough
CERN

Happy 10th birthday Higgs Boson!

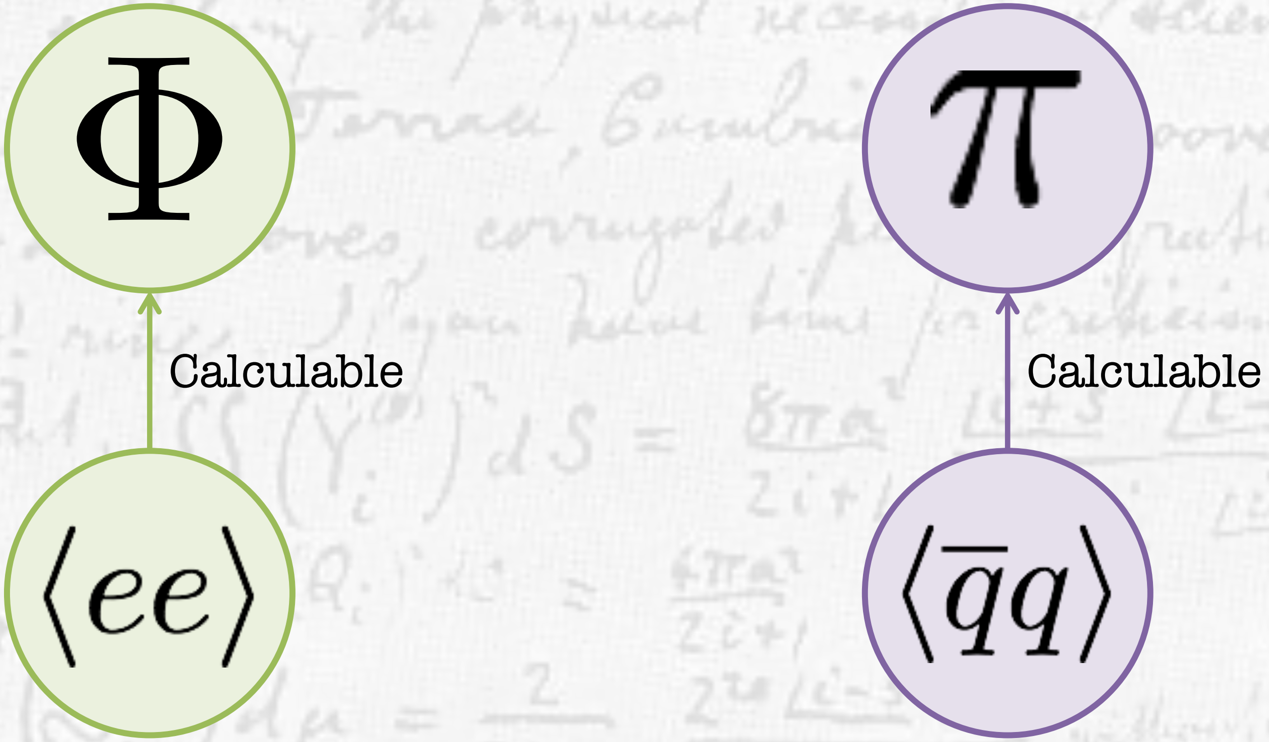


How well do we know the Higgs?



How well do we know the Higgs?

Every scalar we encountered until now has properties (mass, vev, etc) that are calculable within some more fundamental theory:



Φ

Calculable

$\langle ee \rangle$

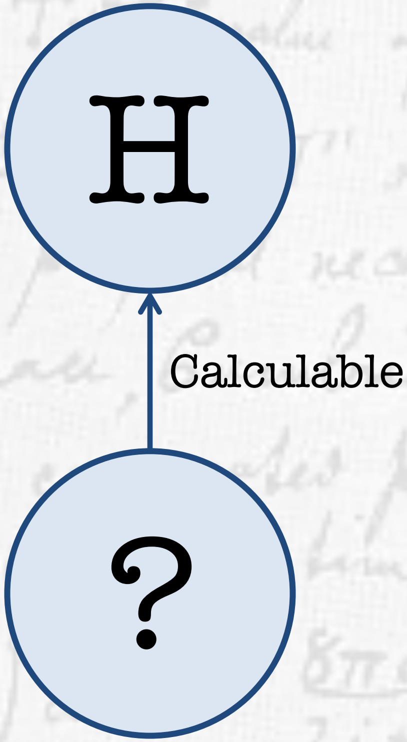
π

Calculable

$\langle \bar{q}q \rangle$

How well do we know the Higgs?

What about the Higgs?



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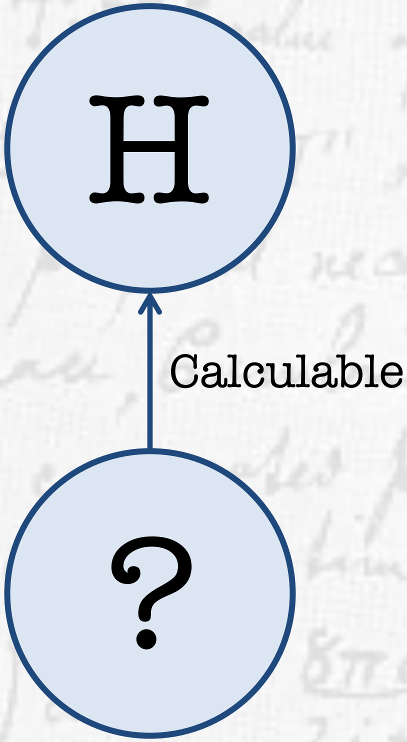

H

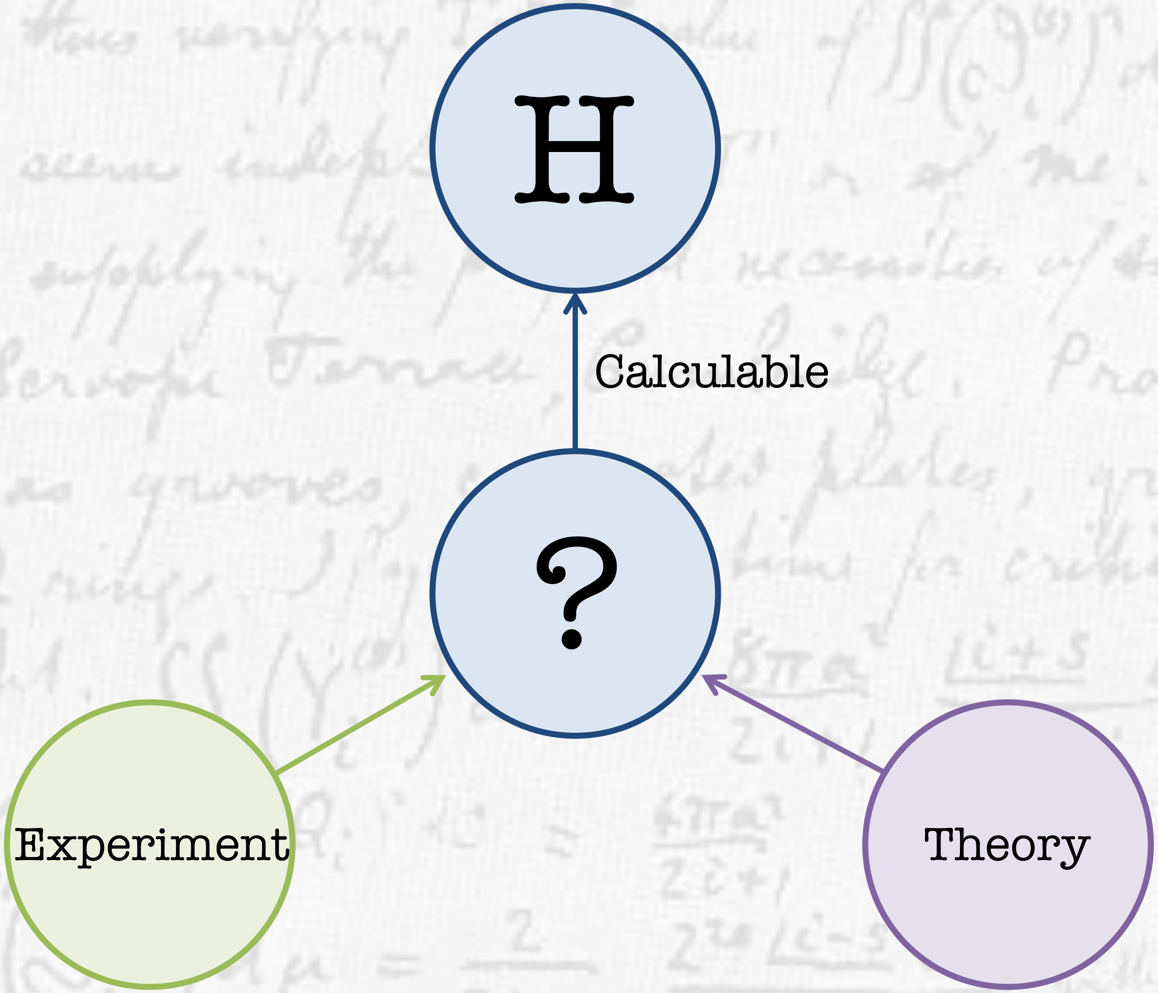
Calculable

?

The Standard Model breaks down: It is an effective field theory, to be replaced by something more fundamental at shorter distance scales.

How well do we know the Higgs?

What about the Higgs?



H

Calculable

?

Experiment

Theory

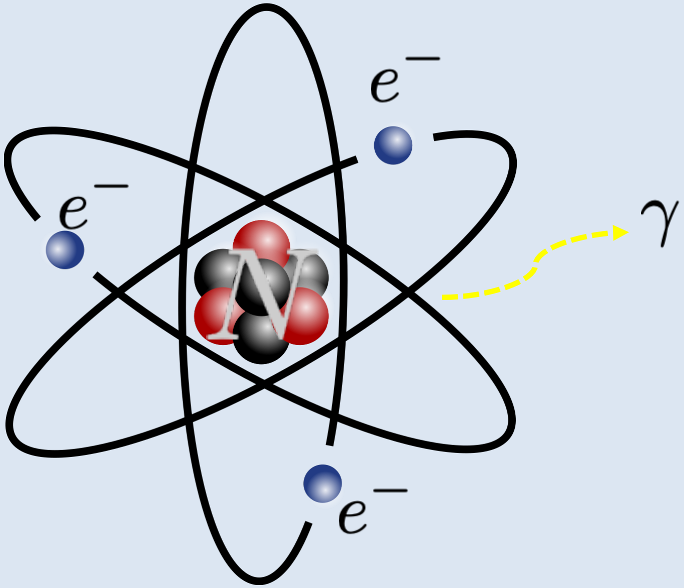
Zooming in on the Higgs

Zooming in on the Higgs

Is there some substructure yet to be revealed?

Effective Field Theory Basics

Consider exploring a neutral atom at eV energies:



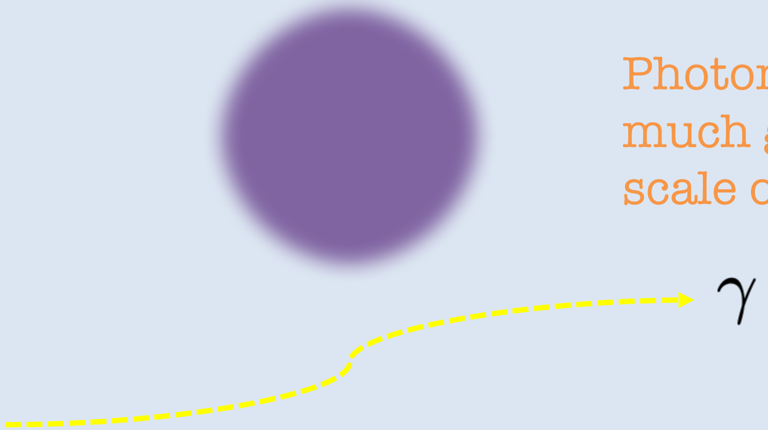
Photon wavelength on scale of orbitals.

The appropriate theory at this length scale contains the photon, electrons and nucleus:

$$\mathcal{L} = \mathcal{L}(\gamma, e^-, N)$$

Effective Field Theory Basics

Consider exploring a neutral atom at much lower energies:



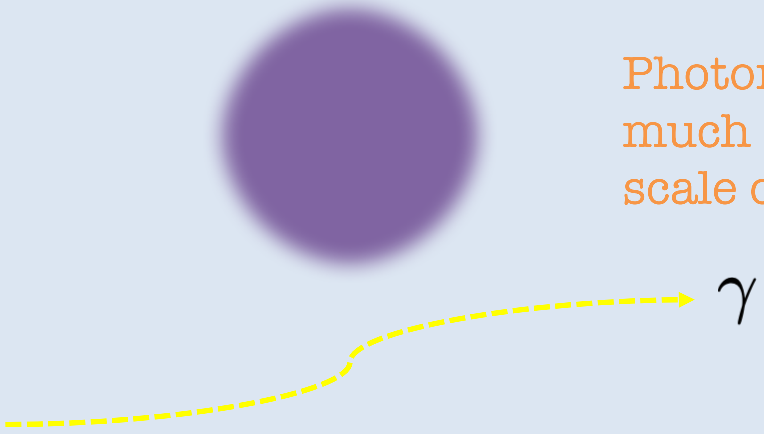
Photon wavelength much greater than scale of orbitals.

The appropriate theory at this length scale contains the photon and neutral atom...

$$\mathcal{L} = \mathcal{L}(\gamma, \chi)$$

Effective Field Theory Basics

Consider exploring a neutral atom at much lower energies:



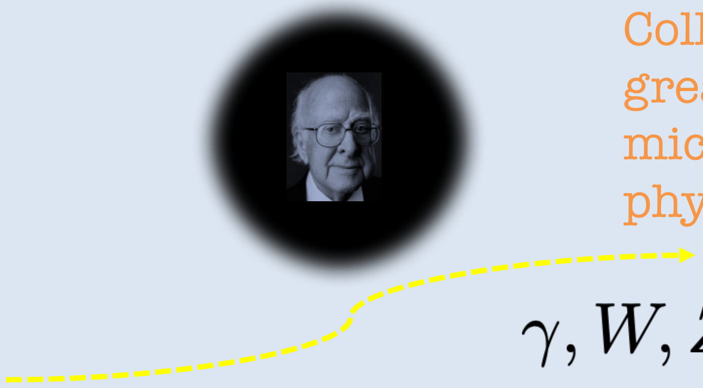
Photon wavelength much greater than scale of orbitals.

Crucially, the substructure is encoded in “higher dimension operators”, like dipoles or Rayleigh...

$$\mathcal{L} = ... + \frac{\chi^2}{\Lambda^2} F^{\mu\nu} F_{\mu\nu} + ...$$

Effective Field Theory Basics

The same is true for the Higgs boson!

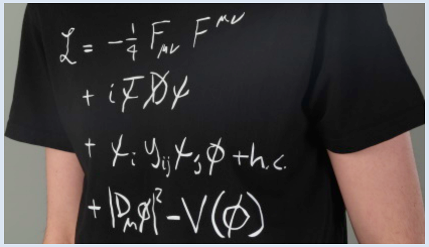


Collider wavelength greater than scale of microscopic new physics...

$\gamma, W, Z, g, \ldots$

The Standard Model is an “Effective Field Theory”. Unknown smaller distance physics in extra “operators”:

$$\mathcal{L} =$$



$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + i \bar{\psi} \not{D} \psi + \psi_i y_{ij} \psi_j \phi + h.c. + |D_\mu \phi|^2 - V(\phi)$$

$$+\sum_{jk} \frac{c_j}{\Lambda^k} \mathcal{O}_{jk}$$

Organizing the Unknown

To understand the origin and nature of the Higgs boson, we need to study how it behaves.

$$c_T \quad (H^\dagger \overleftrightarrow{D}^\mu H)^2$$

$$\mathcal{O}_W = \frac{ig\, c_W}{2M^2}(H^\dagger \sigma^a \overleftrightarrow{D}^\mu H) D^\nu W^a_{\mu\nu}$$

$$\mathcal{O}_{2B} = -\frac{c_{2B}}{4M^2}(\partial_\rho B_{\mu\nu})^2$$

$$c_\Box \quad |\Box H|^2$$

$$\mathcal{O}_{WW} = \frac{g^2\, c_{WW}}{M^2}|H|^2 W^{a\,\mu\nu} W^a_{\mu\nu}$$

$$g^2\, c_{GG} \; |H|^2 G^{a,\mu\nu} G^a_{\mu\nu}$$

$$\mathcal{O}_B = \frac{ig'\, c_B}{2M^2}(H^\dagger D \quad H)$$

$$\mathcal{O}_H = \frac{c_H}{2M^2}\left(\partial^\mu |H|^2\right)$$

$$\mathcal{O}_R = \quad \frac{}{M^2}$$

$$\mathcal{O}_{BB} = \frac{g'^2\, c_{BB}}{M^2}|H|^2 B^{\mu\nu} B_{\mu\nu}$$

$$\mathcal{O}_{2W} = -\frac{c_{2W}}{4M^2}(D_\rho W^a_{\mu\nu})^2$$

$$\mathcal{O}_{WB} = \frac{gg'\, c_{WB}}{M^2} H^\dagger \sigma^a H B^{\mu\nu} W^a_{\mu\nu}$$

Let us suppose that any new physics is heavy. Not necessary, just for our purposes.

Operators like those above capture leading effects of heavy physics beyond the standard model. Probing them could reveal origins.

Organizing the Unknown

To understand the origin and nature of the Higgs boson, we need to study how it behaves.

$$\mathcal{O}_T = \frac{c_T}{2M^2}(H^\dagger \overleftrightarrow{D}^\mu H)^2 \qquad \mathcal{O}_W = \frac{ig\, c_W}{2M^2}(H^\dagger \sigma^a \overleftrightarrow{D}^\mu H) D^\nu W^a_{\mu\nu} \qquad \mathcal{O}_{2B} = -\frac{c_{2B}}{4M^2}(\partial_\rho B_{\mu\nu})^2$$

$$\mathcal{O}_{2G} = -\frac{c_{2G}}{4M^2}(D_\rho G^a_{\mu\nu})^2 \quad \mathcal{O}_\Box = \frac{c_\Box}{M^2}|\Box H|^2 \quad \mathcal{O}_{WW} = \frac{g^2\, c_{WW}}{M^2}|H|^2 W^{a\,\mu\nu} W^a_{\mu\nu}$$

$$\mathcal{O}_B = \frac{ig'\, c_B}{2M^2}(H^\dagger \overleftrightarrow{D}^\mu H)\partial^\nu B_{\mu\nu} \qquad \mathcal{O}_6 = \frac{c_6}{M^2}|H|^6 \qquad \mathcal{O}_{GG} = \frac{g_s^2\, c_{GG}}{M^2}|H|^2 G^{a,\mu\nu} G^a_{\mu\nu}$$

$$\mathcal{O}_H = \frac{c_H}{2M^2}\left(\partial^\mu |H|^2\right)^2 \qquad \mathcal{O}_R = \frac{c_R}{M^2}|H|^2 |D^\mu H|^2$$

$$\mathcal{O}_{BB} = \frac{g'^2\, c_{BB}}{M^2}|H|^2 B^{\mu\nu} B_{\mu\nu}$$

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Operators like those above capture leading effects of heavy physics beyond the standard model. Probing them could reveal origins.

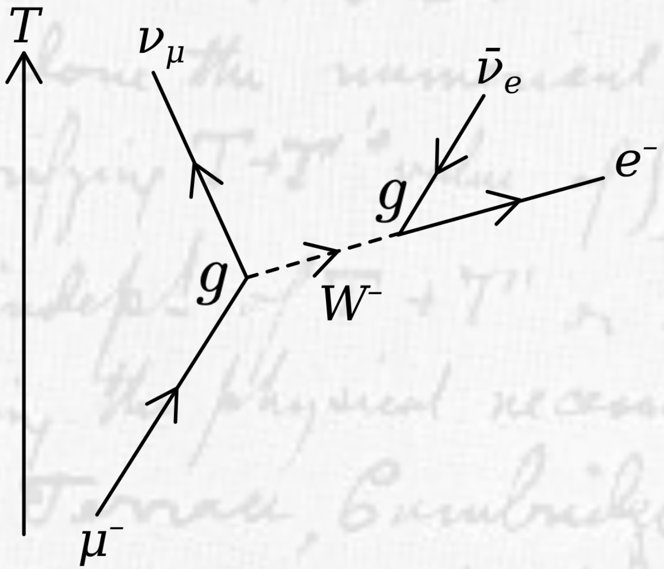
Organizing the Unknown

Naïve dimensional analysis:

$$[H] = [A_\mu] = \frac{1}{LC} \quad , \quad [\psi] = \frac{1}{L^{3/2}C}$$

Fields carry not only dimension of inverse length, but also inverse coupling.

Example: Muon Decay



Fermi Scale

Interaction: $\mathcal{L} \sim \dfrac{\psi^4}{\Lambda^2}$

UV-completion

Dimension: $[\Lambda] = [G_F^{-1/2}] = \dfrac{[M_W]}{[g]}$

Coupling

Organizing the Unknown

Higgs Only

$[\boldsymbol{g_*^0}]$

$$\mathcal{O}_\Box = \frac{c_\Box}{M^2}|\Box H|^2$$

$[\boldsymbol{g_*^2}]$

$$\mathcal{O}_H = \frac{c_H}{2M^2}\left(\partial^\mu |H|^2\right)^2$$

$$\mathcal{O}_T = \frac{c_T}{2M^2}(H^\dagger \overleftrightarrow{D}^\mu H)^2$$

$$\frac{c_R}{}|H|^2|D^\mu H|^2$$

$[\boldsymbol{g_*^4}]$

$$\mathcal{O}_6 = \frac{c_6}{M^2}|H|^6$$

Any new physics concerning primarily with Higgs and gauge sectors matches, at leading order, to these operators.

$$\mathcal{O}_{2G} = -\frac{c_{2G}}{4M^2}(D_\rho G^a_{\mu\nu})^2$$

Mixed

$$\mathcal{O}_B = \frac{ig'\,c_B}{2M^2}(H^\dagger \overleftrightarrow{D}^\mu H)\partial^\nu B_{\mu\nu}$$

$$\mathcal{O}_W = \frac{ig\,c_W}{2M^2}(H^\dagger \sigma^a \overleftrightarrow{D}^\mu H)D^\nu W^a_{\mu\nu}$$

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Organizing the Unknown

Higgs Only

$[g_*^0]$

$$\mathcal{O}_\Box = \frac{c_\Box}{M^2}|\Box H|^2$$

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$$\mathcal{O}_R = \frac{c_R}{M^2}|H|^2|D^\mu H|^2$$

$[g_*^4]$

$$\mathcal{O}_6 = \frac{c_6}{M^2}|H|^6$$

Gauge Only

$$\mathcal{O}_{2G} = -\frac{c_{2G}}{4M^2}(D_\rho G^a_{\mu\nu})^2$$

$$\mathcal{O}_{2W} = -\frac{c_{2W}}{4M^2}(D_\rho W^a_{\mu\nu})^2$$

$$\mathcal{O}_{2B} = -\frac{c_{2B}}{4M^2}(\partial_\rho B_{\mu\nu})^2$$

Mixed

$$\mathcal{O}_B = \frac{ig'\,c_B}{2M^2}(H^\dagger \overleftrightarrow{D}^\mu H)\partial^\nu B_{\mu\nu}$$

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Let's not overlook the outlier operators...

Higgs Only

$[\boldsymbol{g_*^0}]$

$$\mathcal{O}_\Box = \frac{c_\Box}{M^2}|\Box H|^2$$

$[\boldsymbol{g_*^2}]$

$$\mathcal{O}_H = \frac{c_H}{2M^2}\left(\partial^\mu |H|^2\right)^2$$

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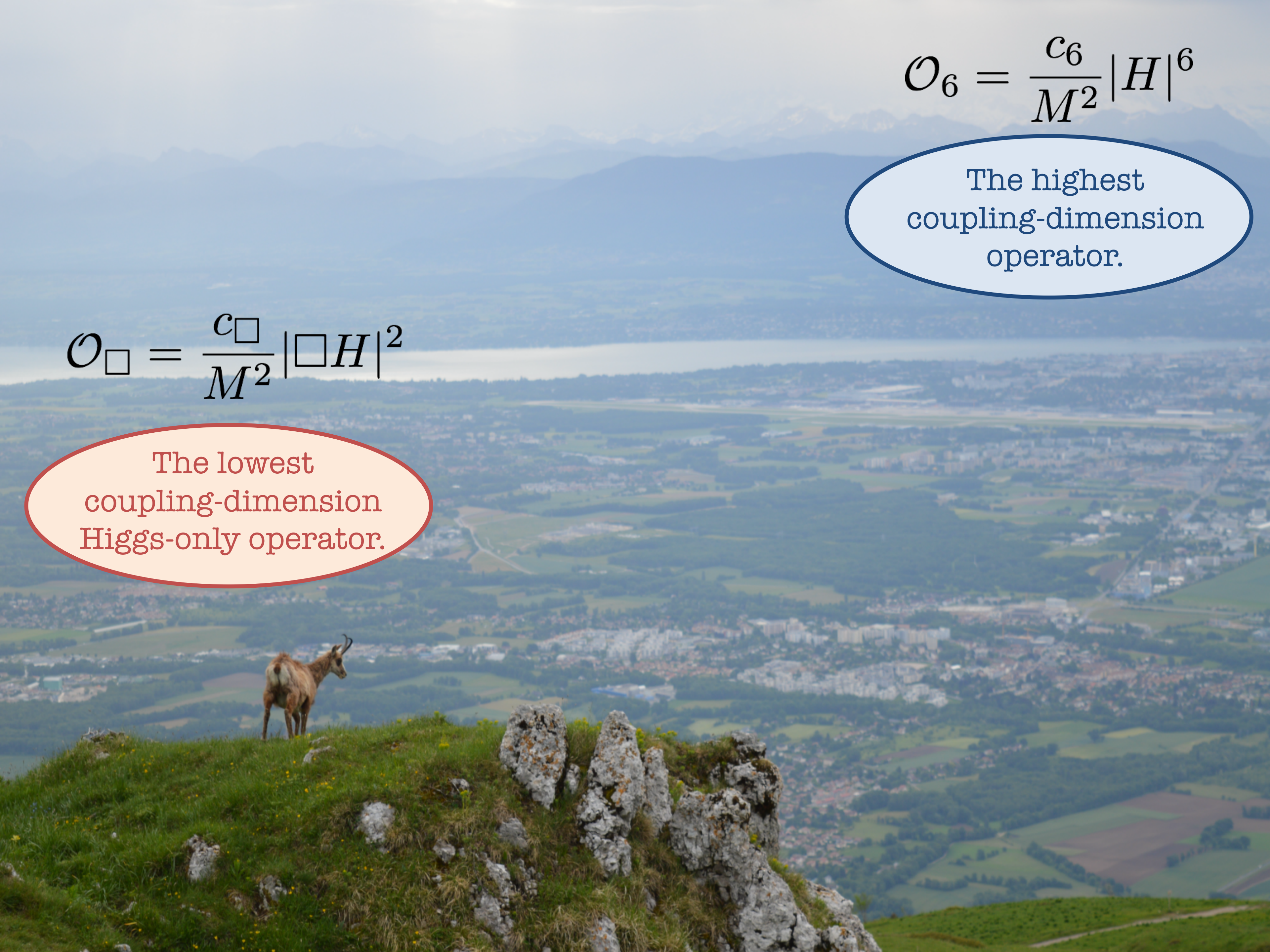
$$\mathcal{O}_R = \frac{c_R}{M^2}|H|^2|D^\mu H|^2$$

$[\boldsymbol{g_*^4}]$

$$\mathcal{O}_6 = \frac{c_6}{M^2}|H|^6$$

which determine the dynamics of the Higgs, from how it moves to the shape of the Higgs potential.

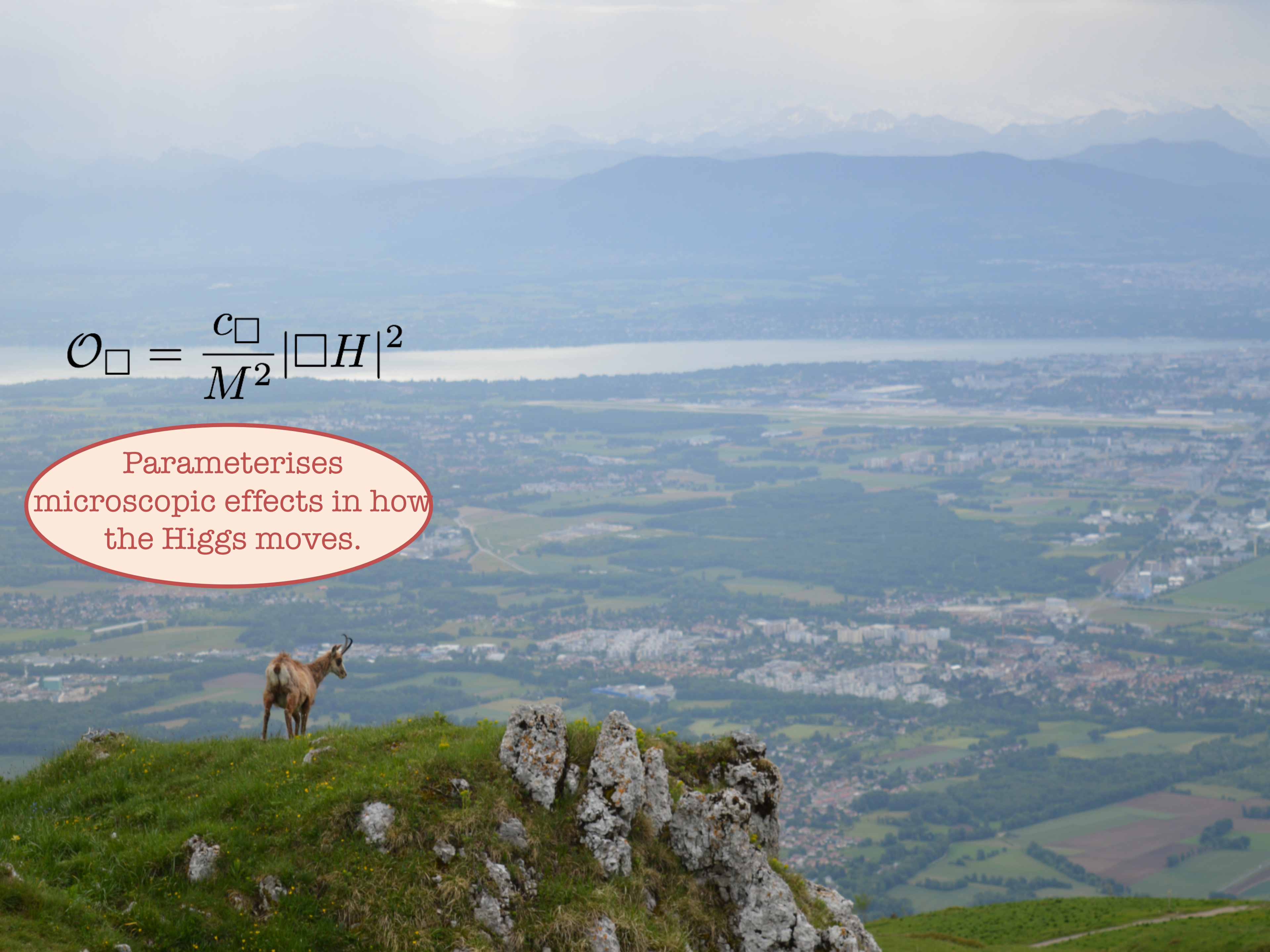


$$\mathcal{O}_6 = \frac{c_6}{M^2}|H|^6$$

The highest coupling-dimension operator.

$$\mathcal{O}_\Box = \frac{c_\Box}{M^2}|\Box H|^2$$

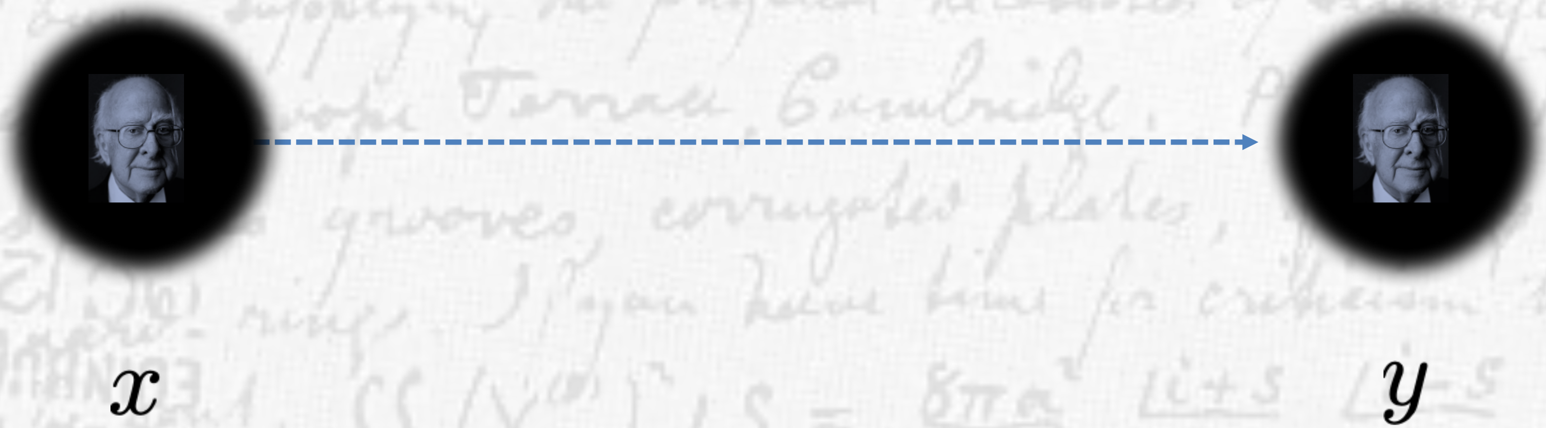
The lowest coupling-dimension Higgs-only operator.



$$\mathcal{O}_{\Box} = \frac{c_{\Box}}{M^2}|\Box H|^2$$

Parameterises microscopic effects in how the Higgs moves.

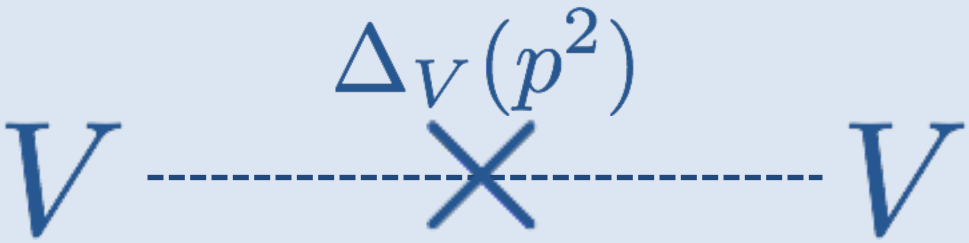
How does the Higgs move?



“Oblique” Corrections

Oblique corrections have formerly been a formidable toolkit in the effort to explore propagation in the electroweak sector.

- S-parameter
- T-parameter
- W-parameter
- Y-parameter



The latter two contribute to processes in an “energy-growing” manner:

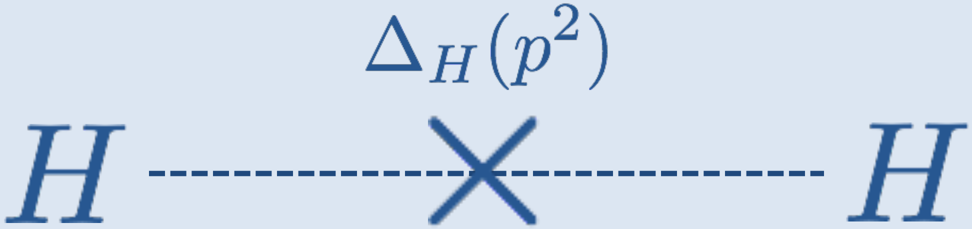
$$\Delta_W(p^2) \approx \frac{1}{p^2 - M_W^2} - \frac{\hat{W}}{M_W^2}$$

Making these oblique parameters an excellent target for high energy colliders...

“Oblique” Corrections

Makes sense to extend to the Higgs sector. Especially since the Higgs can easily interact with new states…

- H-parameter:



1903.07725

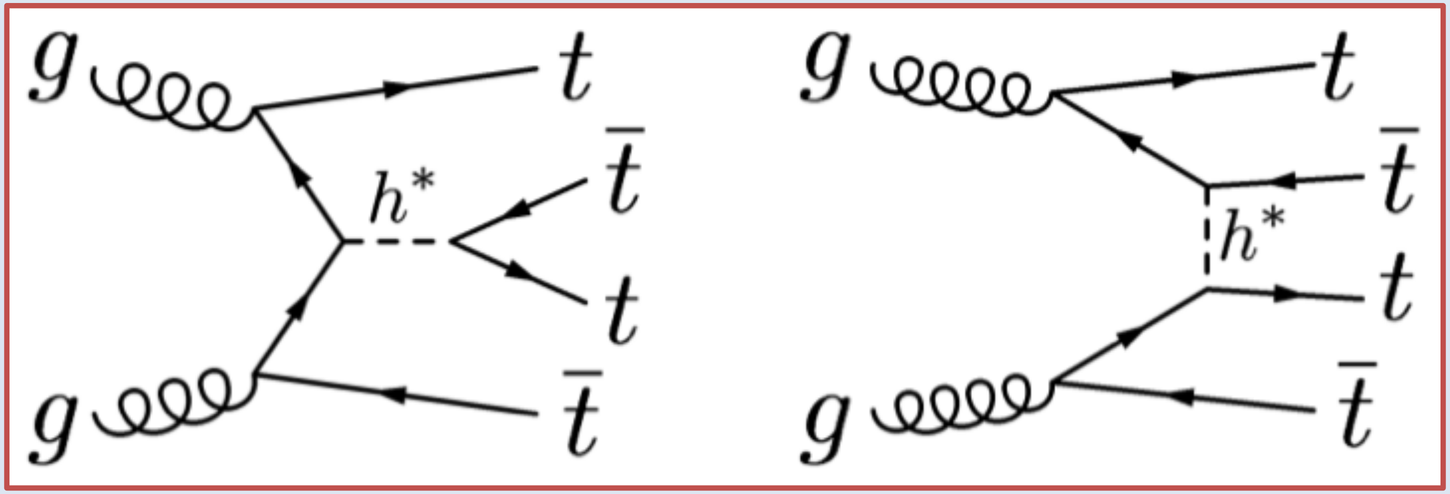
This also contributes to processes in an “energy-growing” manner:

$$\Delta_H(p^2) \approx \frac{1}{p^2 - m_h^2} - \frac{\hat{H}}{m_h^2} + \ldots$$

However, one needs to take the Higgs momentum far from mass-scale, which isn't easy…

Oblique Corrections

Most promising avenue to take this Higgs momentum high is through four-top production:

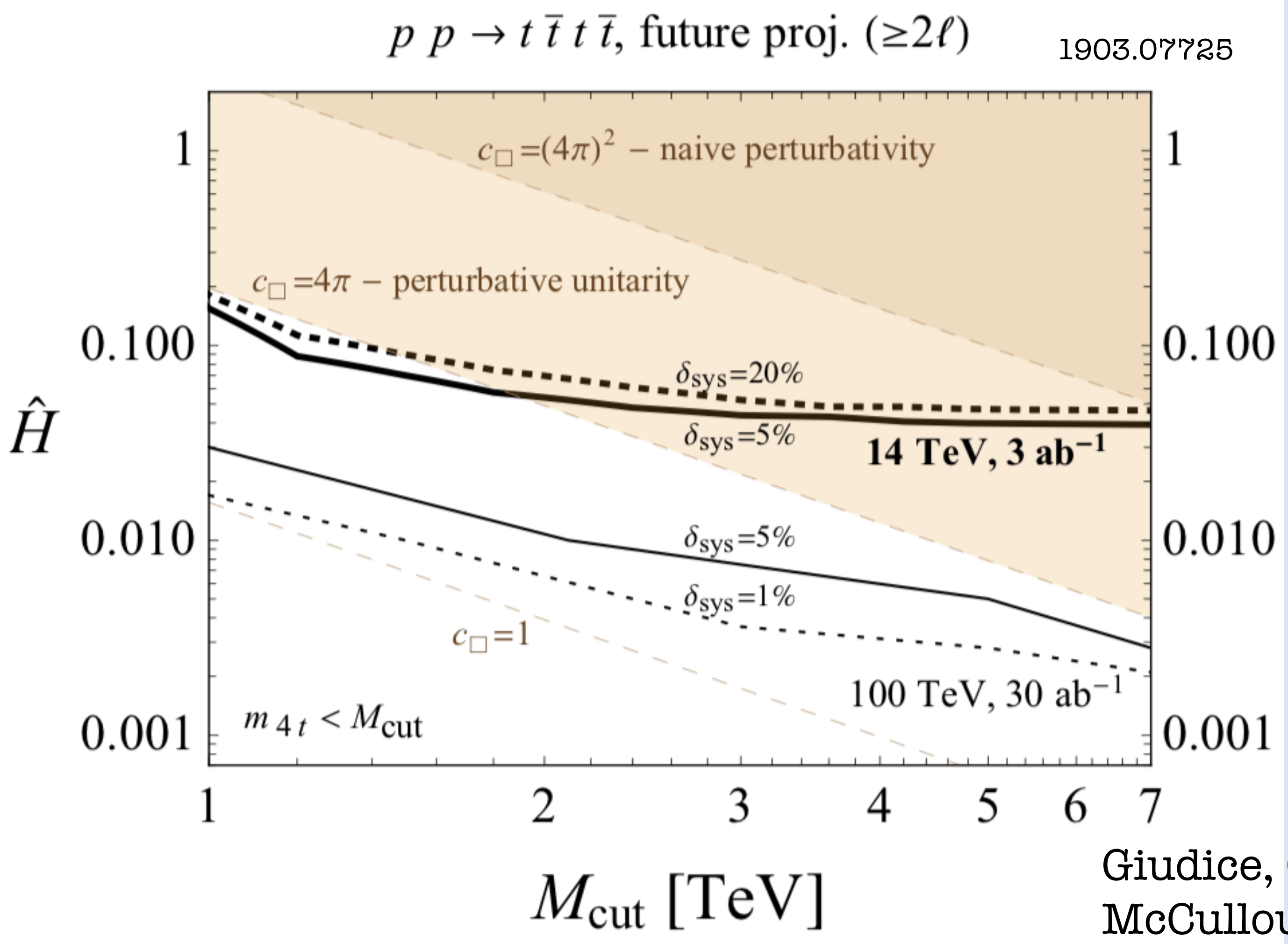


We may relate the effective field theory coefficient to the scale of new physics as:

$$\frac{\hat{H}}{m_h^2} = \frac{c_\Box}{M^2}$$

Oblique Corrections

Our estimate suggests the practical way to probe this special operator is with future colliders:



Giudice, Greljo, McCullough, 2019.

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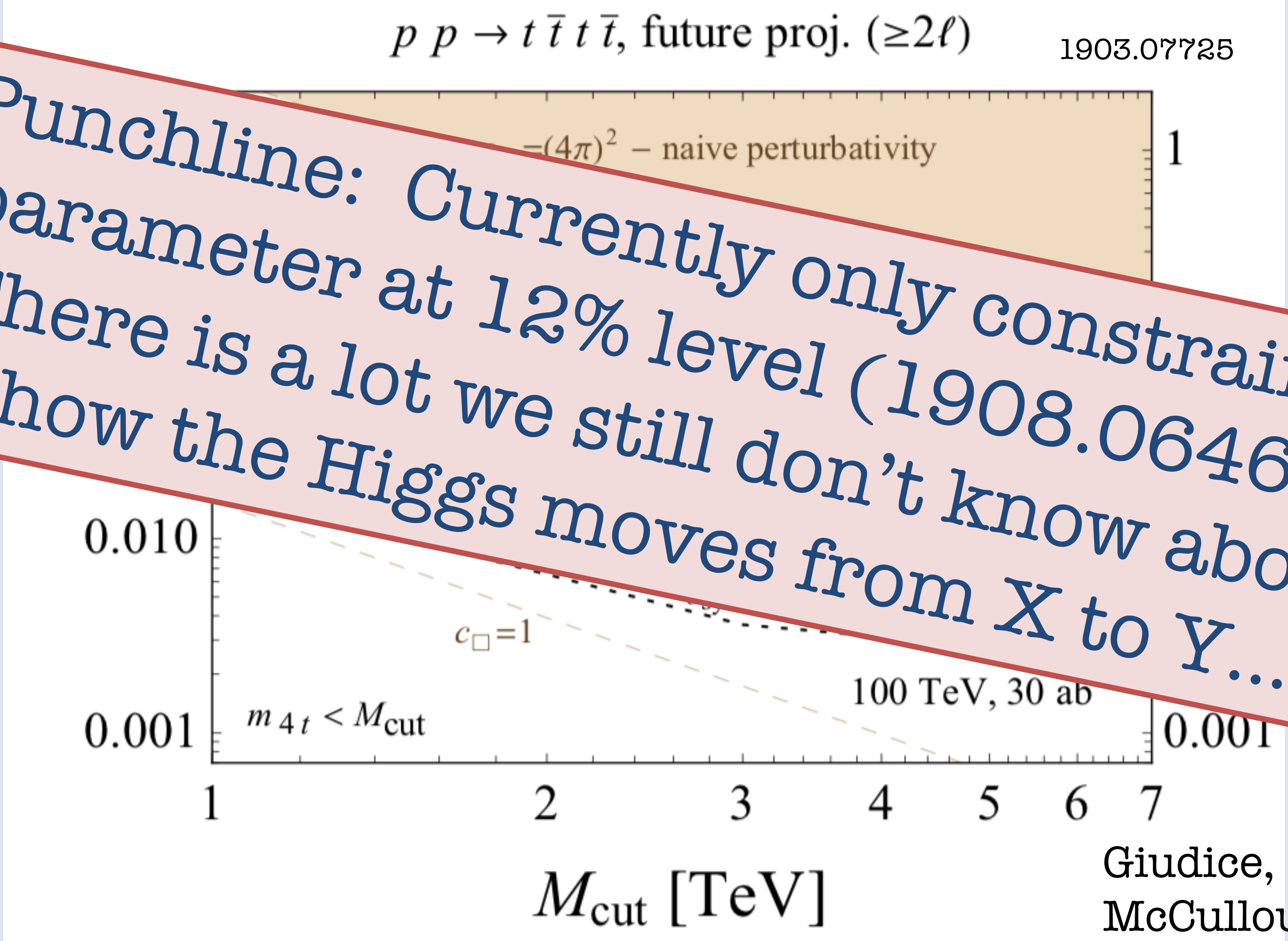

1903.07725

Punchline: Currently only constrain H-parameter at 12% level (1908.06463). There is a lot we still don't know about how the Higgs moves from X to Y...

Giudice, Greljo, McCullough, 2019.

Oblique Corrections

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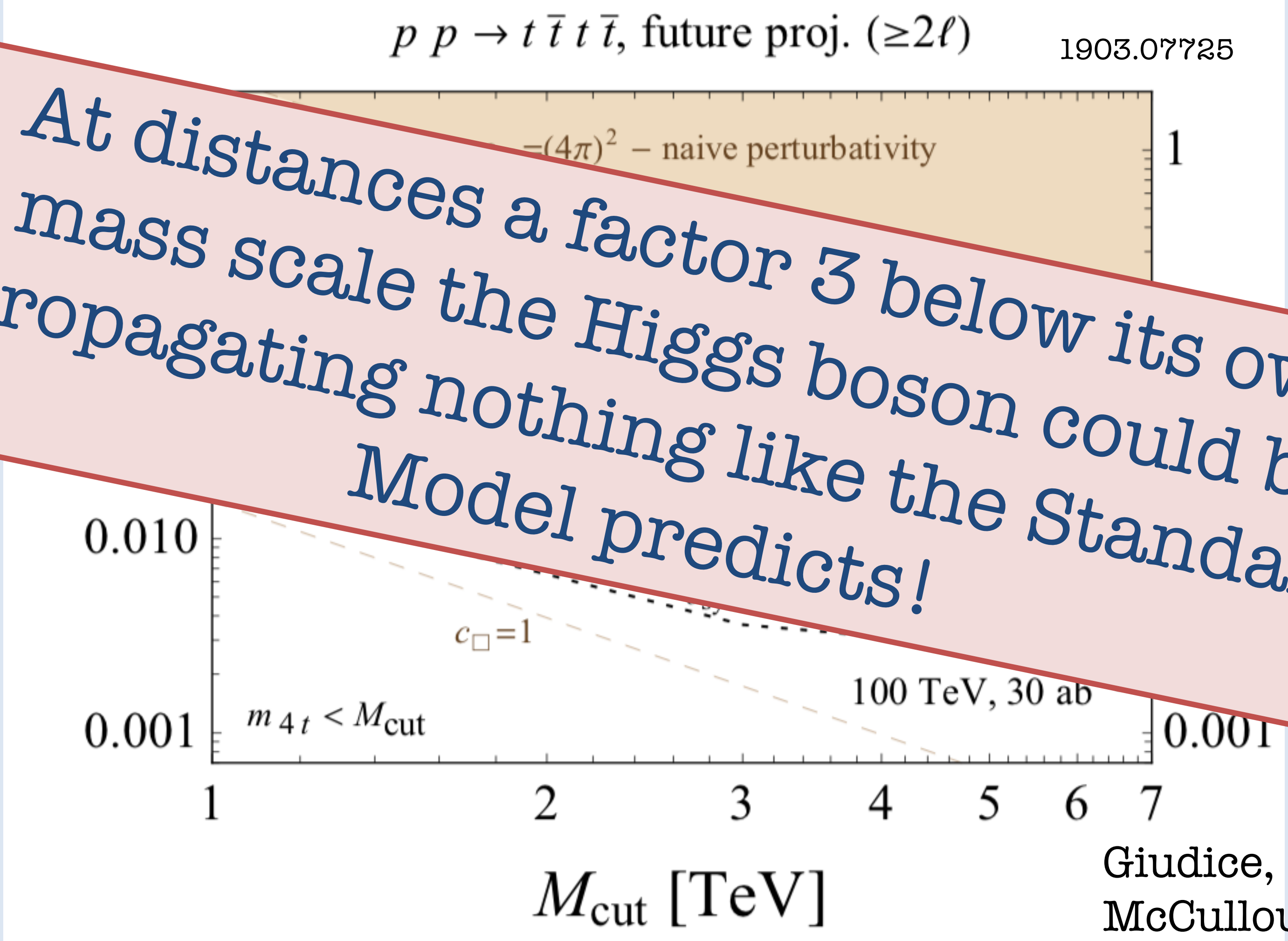


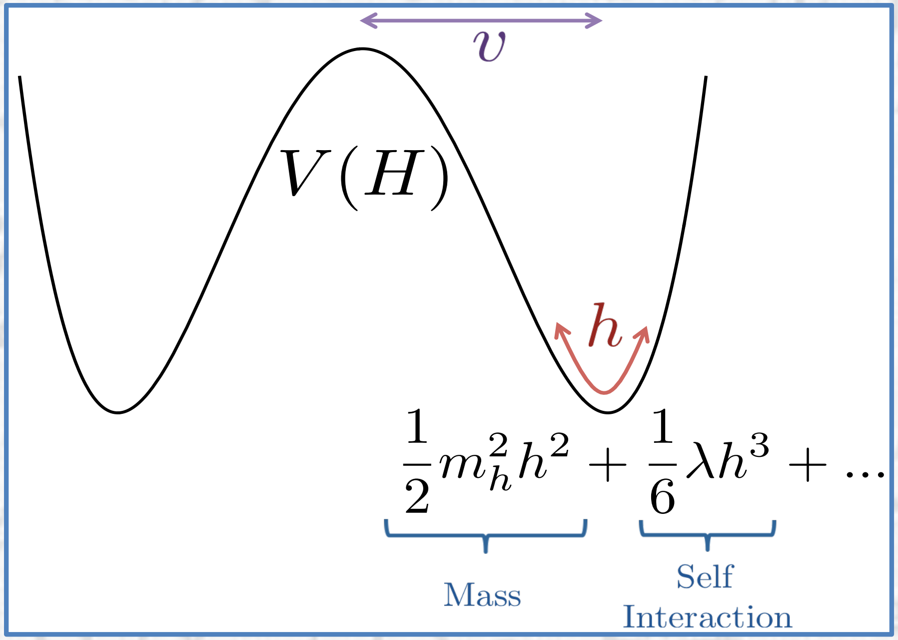
At distances a factor 3 below its own mass scale the Higgs boson could be propagating nothing like the Standard Model predicts!

Giudice, Greljo, McCullough, 2019.

$$\mathcal{O}_6 = \frac{c_6}{M^2}|H|^6$$

Parameterises
BSM deviations in sole
self-interaction of SM.

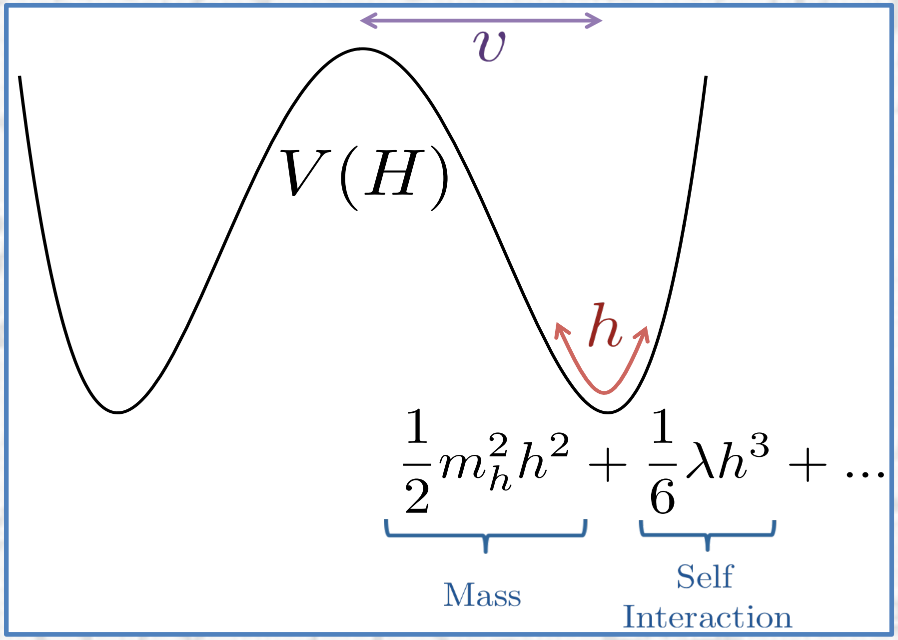
Why do we need to know about the Higgs Field Potential?



$$\underbrace{\frac{1}{2}m_h^2h^2}_{\text{Mass}} + \underbrace{\frac{1}{6}\lambda h^3}_{\text{Self Interaction}} + \ldots$$

Because it determines how the Universe froze in the EW sector, giving mass to gauge bosons, fermions, the Higgs…

Why do we need to know about the Higgs Field Potential?

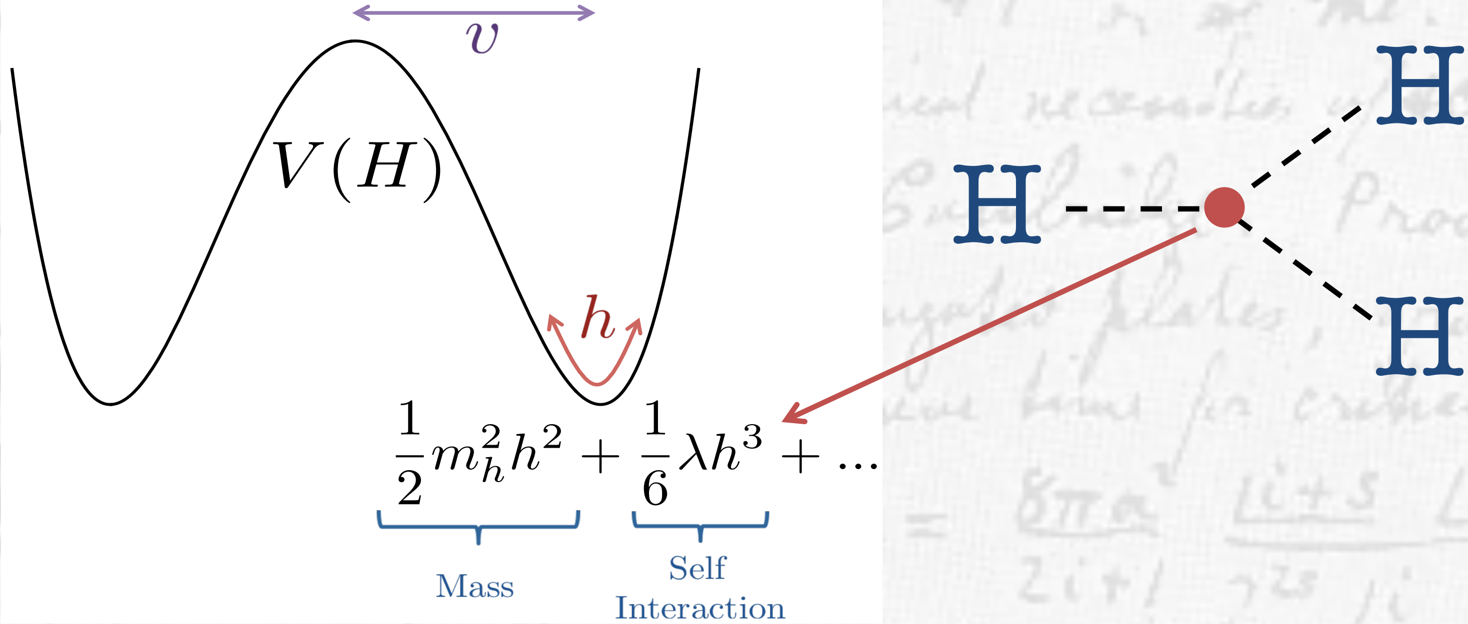


$$\underbrace{\frac{1}{2}m_h^2 h^2}_{\text{Mass}} + \underbrace{\frac{1}{6}\lambda h^3}_{\text{Self Interaction}} + \ldots$$

...because it determines how the Universe will end...

Naïve Dimensional Analysis

It's known that O_6 contributes to Higgs self-interaction, how it gives mass to itself, etc.



$$\underbrace{\frac{1}{2}m_h^2 h^2}_{\text{Mass}} + \underbrace{\frac{1}{6}\lambda h^3}_{\text{Self Interaction}} + \ldots$$

But less-well appreciated are the NDA aspects underlying it…

Naïve Dimensional Analysis

The fact that

$$[c_6] = [g^4]$$

and all other operator coefficients have

$$[c_j] \leq [g^2]$$

makes the self-coupling special, with one important implication I’ll highlight today.

Self-Coupling Dominance

Suppose in fundamental theory leading interaction with microscopic physics is through parameter of coupling dimension

$$[y] = [g^2]$$

arising from a lower-dimension coupling with rule:

$$\kappa \propto y^2 \quad , \quad y \to -y$$

Then the only operator at $\hbar^0$ you can have is

$$\frac{\kappa |H|^6}{M^2}$$

all other dim-6 at least quantum-loop suppressed!

Self-Coupling Dominance

In other words, no obstruction to having Higgs self-coupling modifications a “loop factor” greater than **all** other couplings. Could have

$$\left|\frac{\delta_{h^3}}{\delta_{VV}}\right| \lesssim \min\left[\left(\frac{4\pi v}{m_h}\right)^2, \left(\frac{M}{m_h}\right)^2\right]$$

without fine-tuning any parameters, as big as,

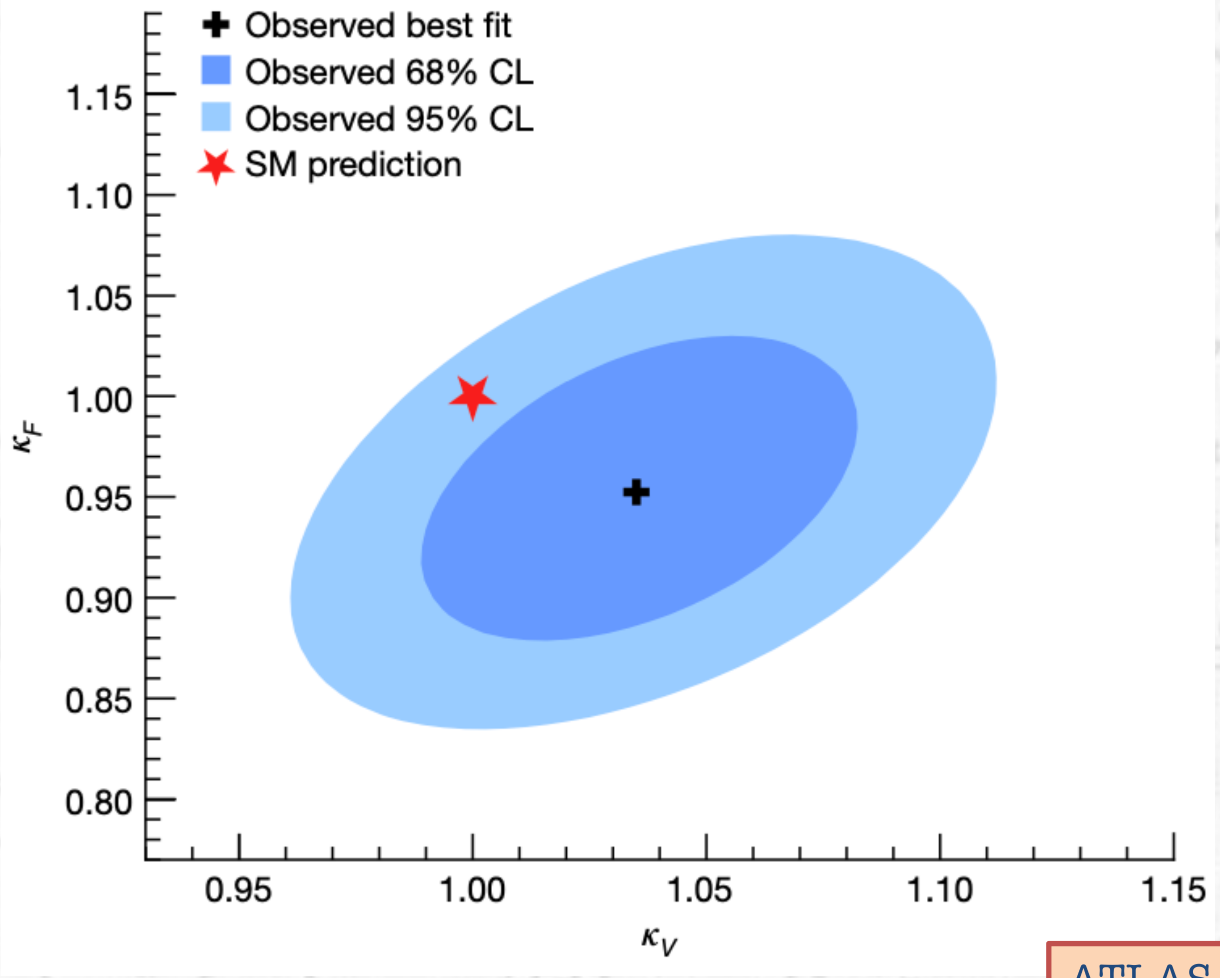
$$(4\pi v/m_h)^2 \approx 600$$

which is significant!

Durieux, MM, Salvioni. 2022

Status of Higgs Couplings

What are experimental limits on modifications of couplings relative to Standard Model prediction?



ATLAS, Nature, 2022

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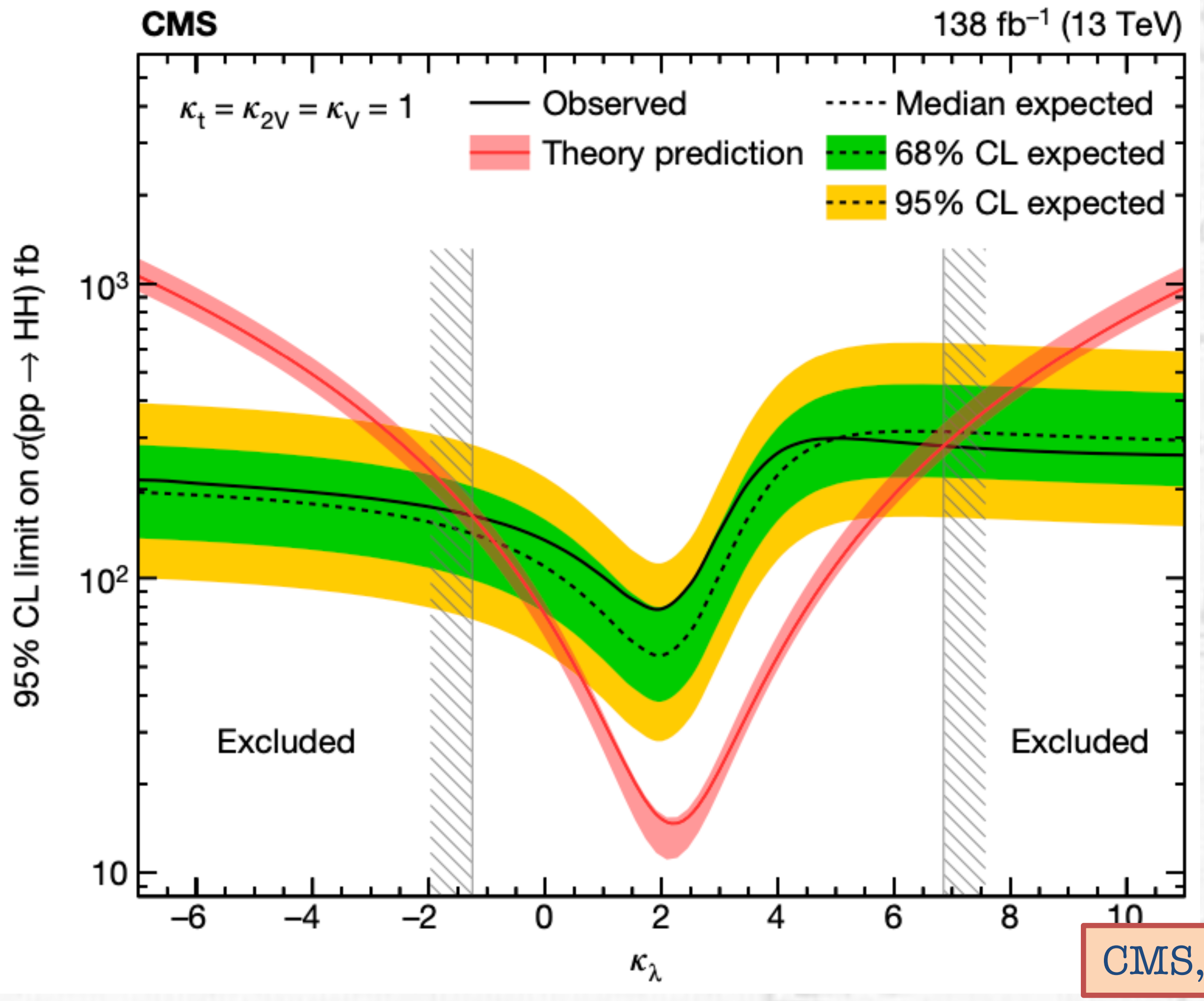

CMS, Nature, 2022

Status of Higgs Couplings

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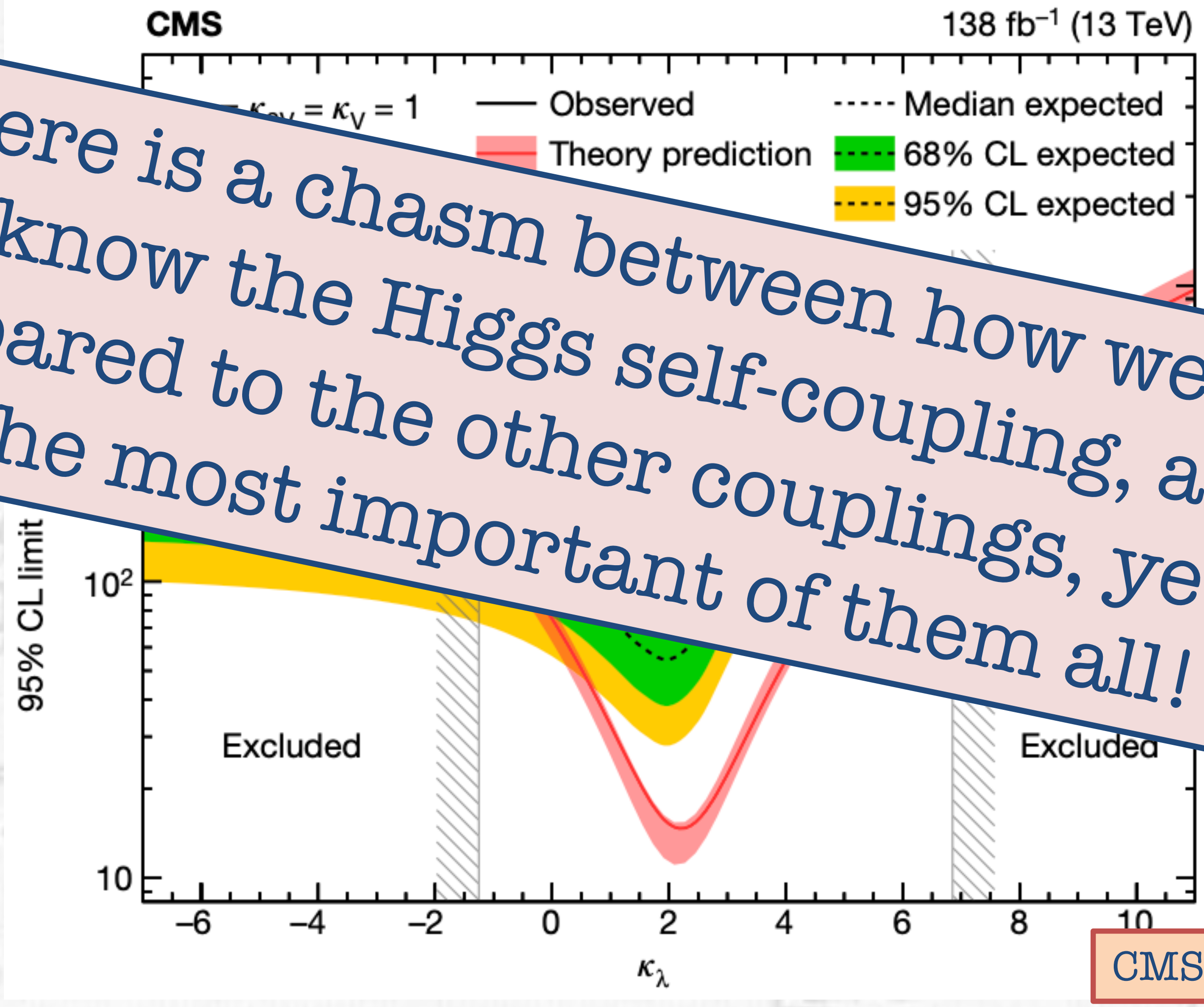


There is a chasm between how well we know the Higgs self-coupling, as compared to the other couplings, yet it is the most important of them all!

CMS, Nature, 2022

Self-Coupling Dominance

In other words, no obstruction from to having Higgs self-coupling modifications a loop factor greater than **all** other couplings. Could have

$$\left|\frac{}{\delta_{VV}}\right| \quad \left[\left(\frac{4\pi v}{}\right)^2, \left(\frac{M}{m_h}\right)^2\right]$$

But can such a theory exist in practise?

without fine-tuning any parameters, as

$$(4\pi v/m_h)^2 \approx 600$$

which is significant!

Durieux, MM, Salvioni. 2022

Custodial Quadruplet

This is all well and good, but does such a theory exist? Yes: The custodial quadruplet scalar. Projecting the $(\mathbf{4},\mathbf{4})$ of $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ onto EW group we have

$$(\mathbf{4},\mathbf{4}) \quad\rightarrow\quad \mathbf{4}_{1/2} + \mathbf{4}_{3/2}$$

and including all couplings to the Higgs we have for scalar quadruplet

$$\mathcal{L}_{\mathrm{SO}(4)} = -\lambda\Big(H^*H^*(\epsilon H)\Phi + \frac{1}{\sqrt{3}}H^*H^*H^*\widetilde{\Phi}\Big) + \mathrm{h.c.}$$

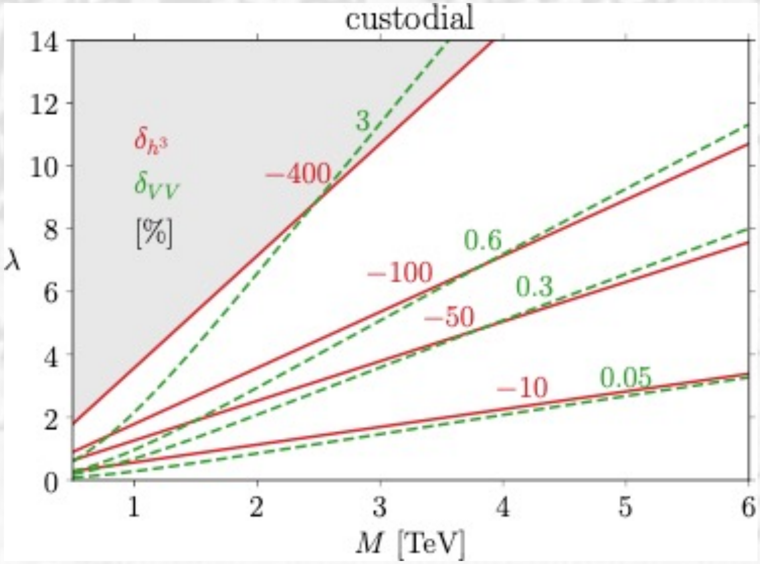
which has exactly the pattern described.

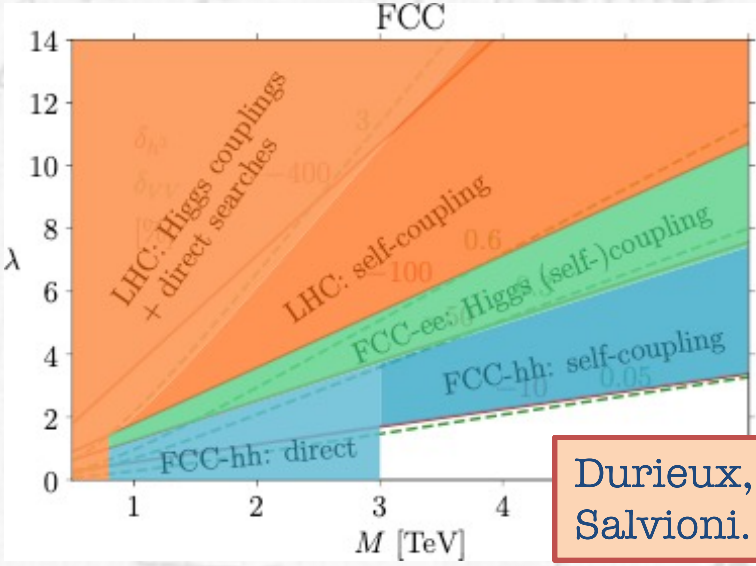
Custodial Quadruplet

Higgs self-coupling is modified at dim-6 at tree-level, all other couplings modified at dim-6 one-loop, or dim-8. All calculable, giving

$$-\frac{\delta_{VV}}{\delta_{h^3}} = 3\left(\frac{m_h}{4\pi v}\right)^2 + \left(\frac{m_h}{M}\right)^2 \approx \frac{1}{200} + \frac{1}{580}\left(\frac{3\text{ TeV}}{M}\right)^2$$

Remarkably close to NDA estimate!





Durieux, MM, Salvioni. 2022

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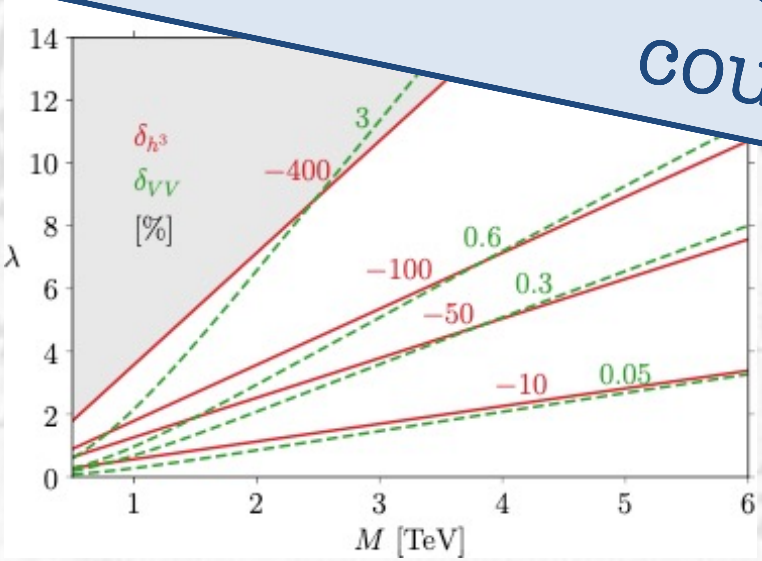
Higgs self-coupling is modified at dim-6 at tree-

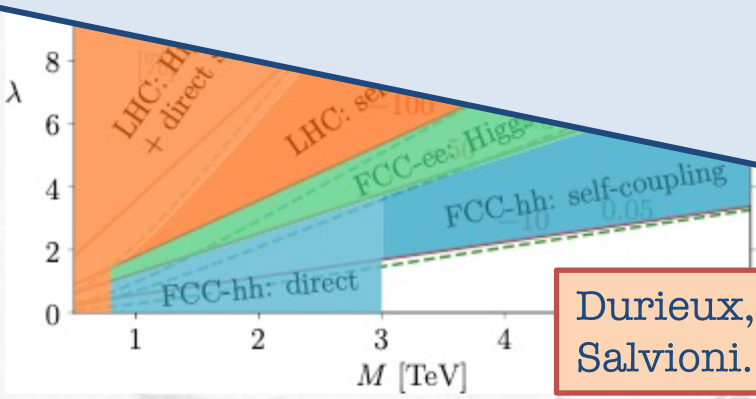
lev ll other couplings modified at dim-6 one-

loo lculable, giving

$$1 \qquad \left(3\ \text{TeV}\right)^2$$

Punchline: Currently only know the self-interaction at the level of 100's %. There is plenty of room for enormous new physics effects to show up in the self-coupling!

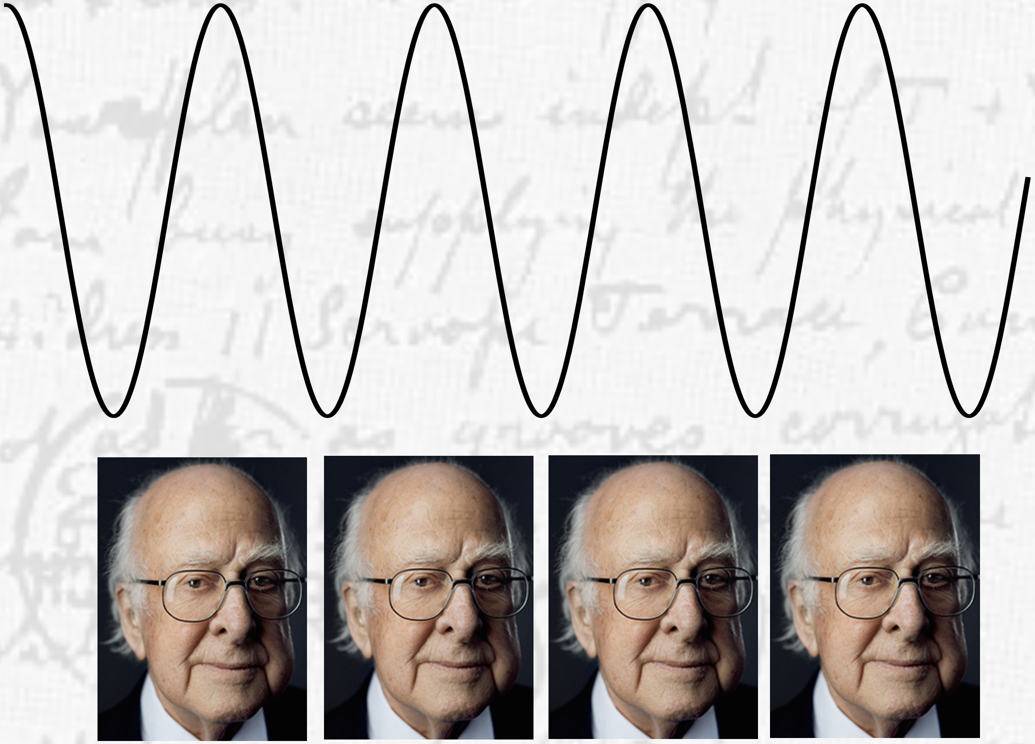




Durieux, MM, Salvioni. 2022

Is the Higgs Fundamental?

The Higgs boson has a size/wavelength. What's inside?



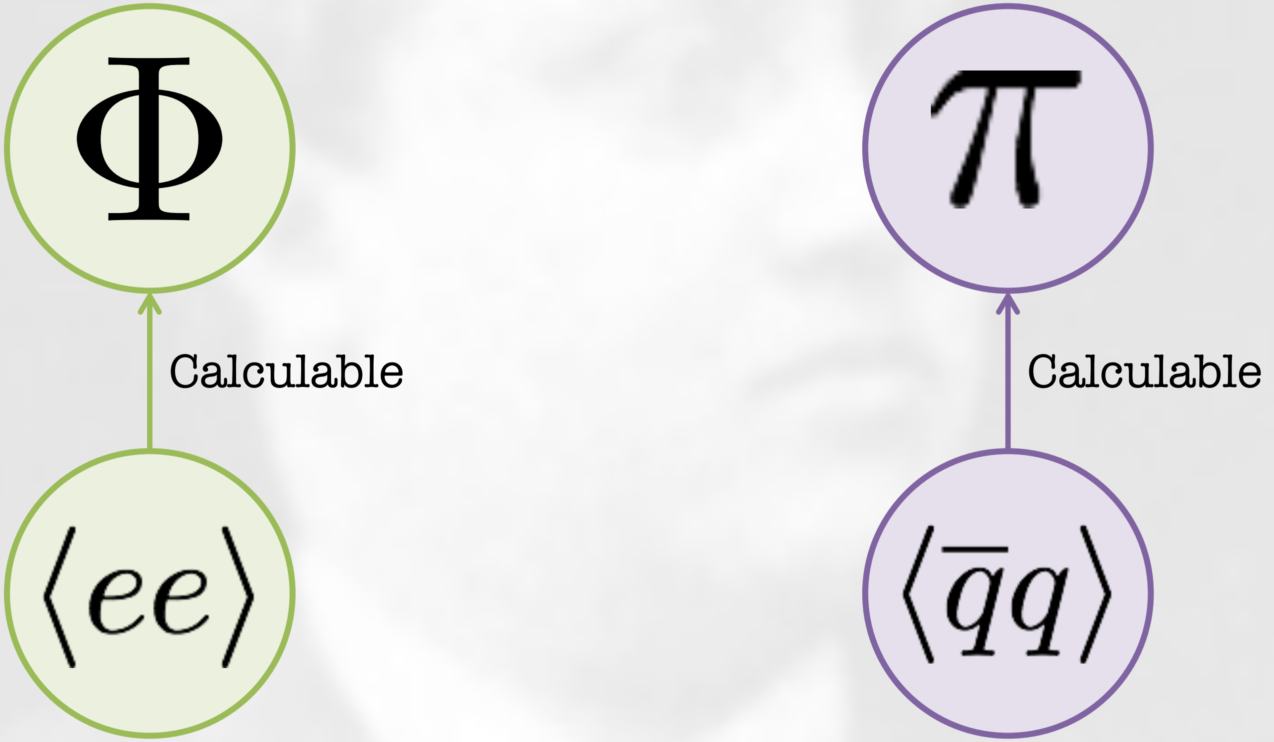
Precision measurements are different ways of probing the "compositeness of the Higgs".

$$\lambda_h \approx 10^{-17} \text{ m}$$

$$\lambda_{10 \text{ TeV}} \approx 10^{-19} \text{ m}$$

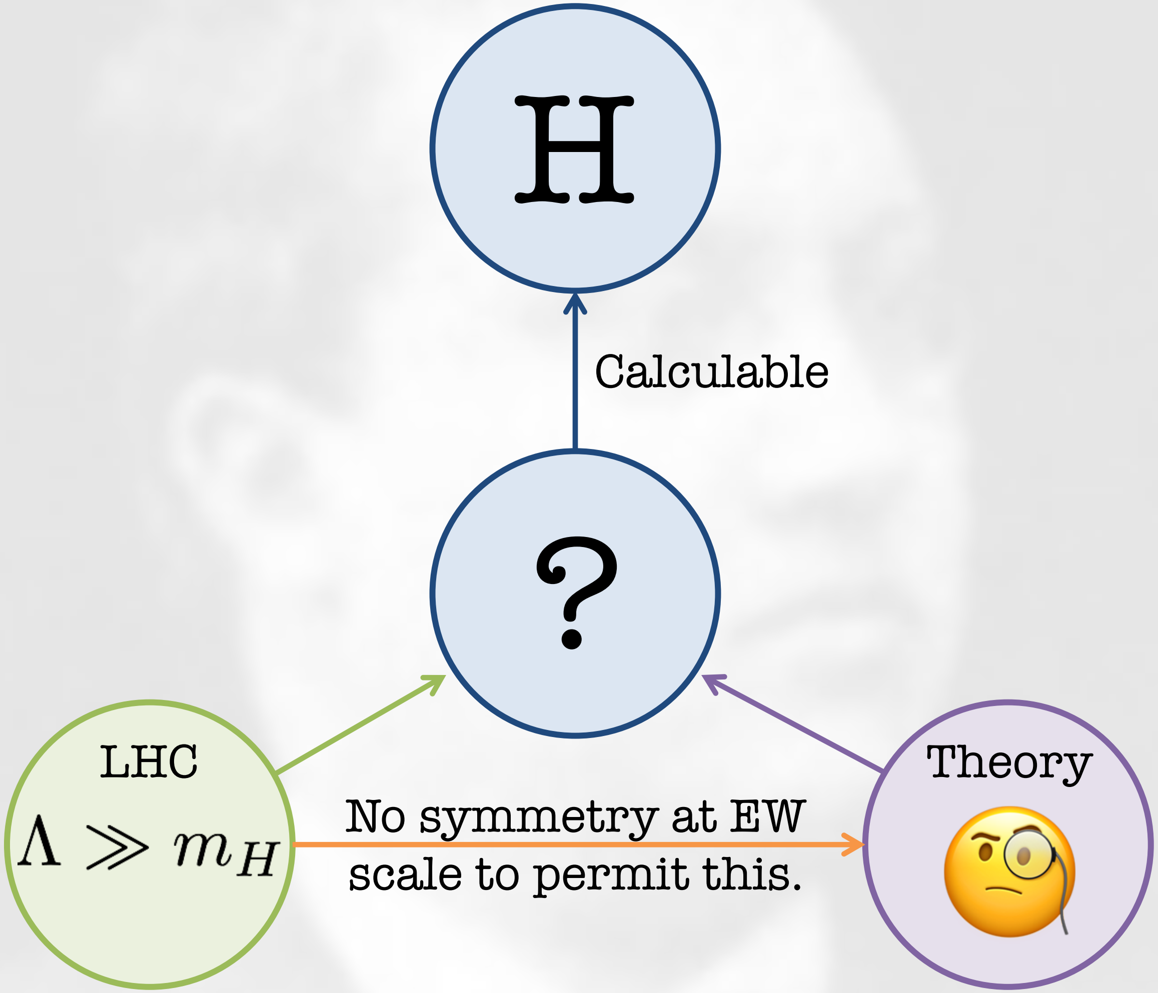
Backdrop

Every scalar we encountered until now has properties (mass, vev, etc) that are calculable within some more fundamental theory:



Backdrop

What about the Higgs?



H

Calculable

?

LHC

$\Lambda \gg m_H$

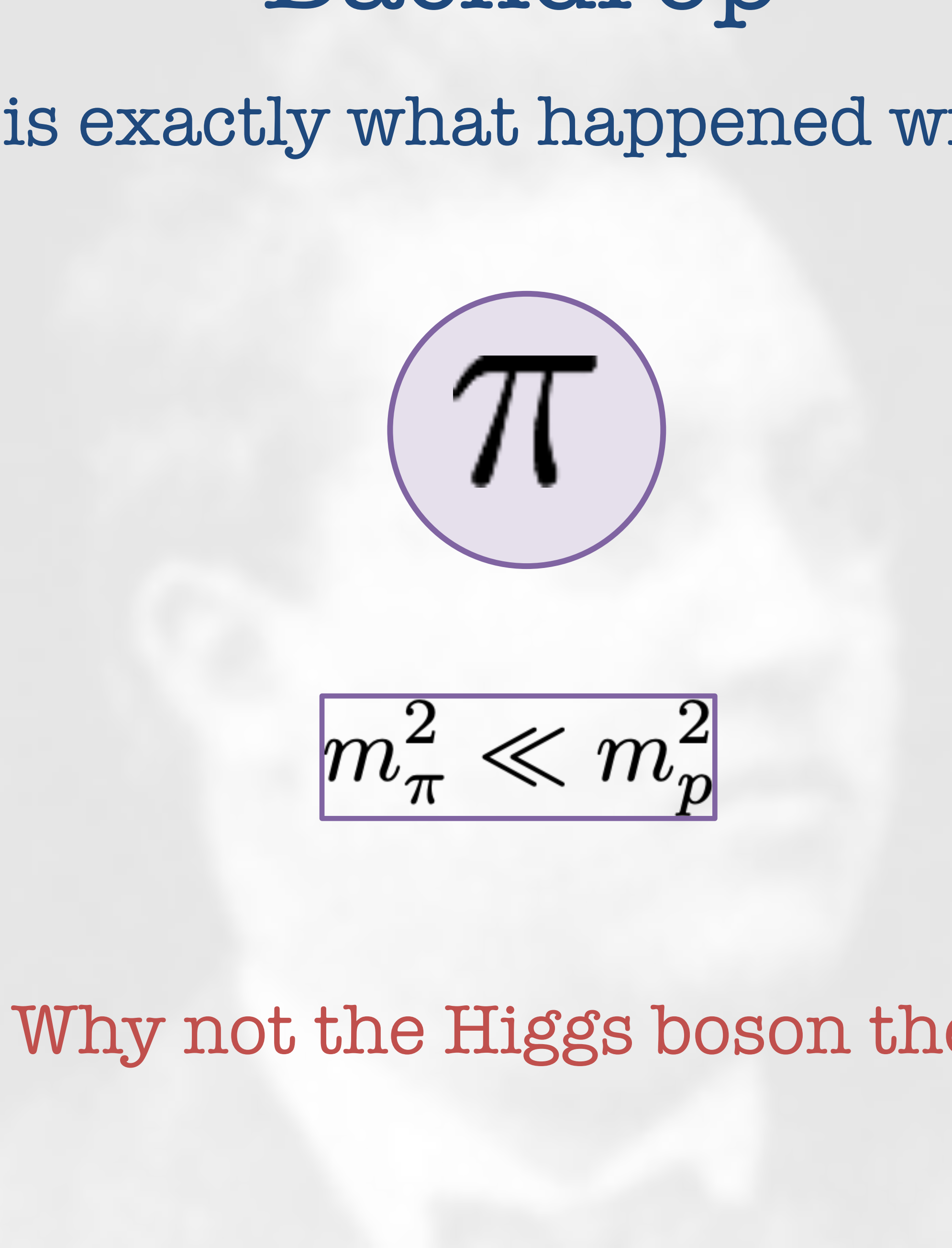
No symmetry at EW scale to permit this.

Theory

🧐

Backdrop

But this is exactly what happened with the pions...



π

$$\boxed{m_\pi^2 \ll m_p^2}$$

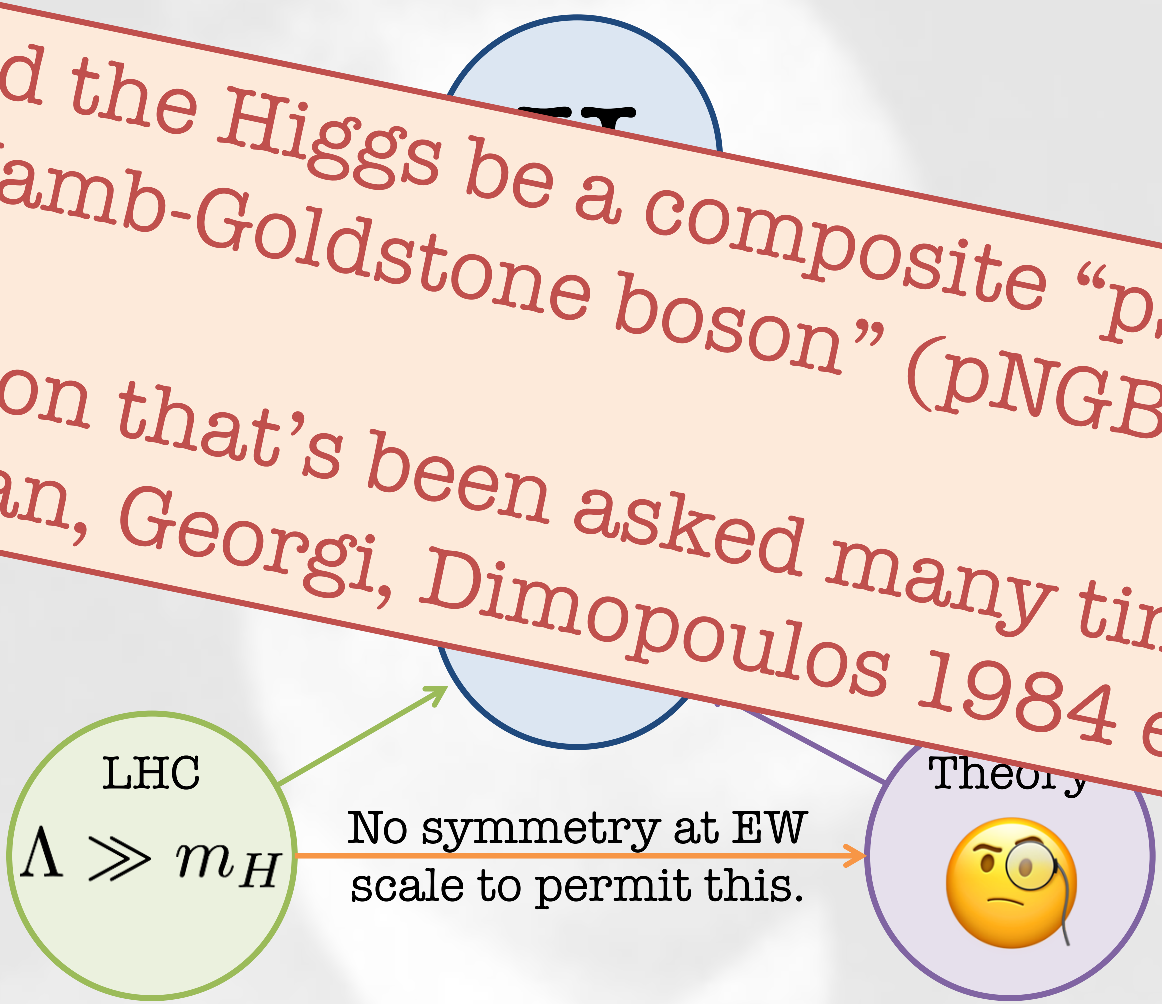
Why not the Higgs boson then?

Backdrop

What about the Higgs?

Could the Higgs be a composite "pseudo-Namb-Goldstone boson" (pNGB)?

Question that's been asked many times… Kaplan, Georgi, Dimopoulos 1984 etc.



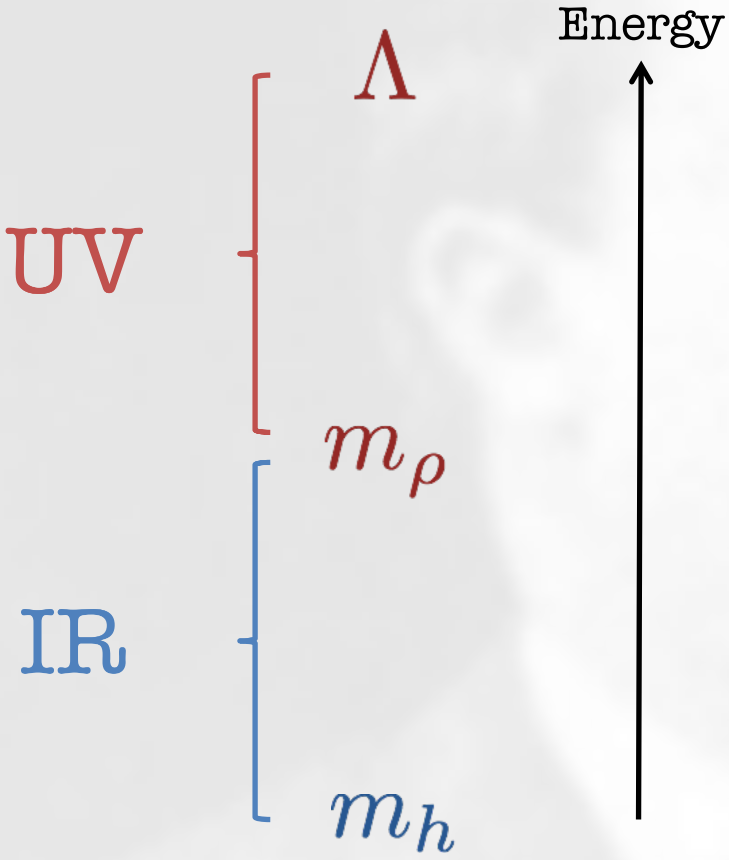
LHC

$\Lambda \gg m_H$

No symmetry at EW scale to permit this.

Naturalness

If the Higgs is a pNGB and the microscopic theory isn't fine-tuned, then properties such as field and mass are quantum-stable at all scales.



The Higgs potential receives contributions from physics at all scales.

$$V_{\mathrm{IR}}(h)$$

Its properties, including the position of the minimum and mass

$$V'_{\mathrm{IR}}(v) = 0$$

$$V''_{\mathrm{IR}}(v) = m_h^2$$

should not change significantly across scales. Otherwise fine-tuning between physics at different scales.

Naturalness – Composite Higgs

Vanilla composite Higgs scenarios have a potential which looks like

“Compositeness” Scale

$$V(h) = \epsilon f^2 \Lambda^2 F(h/f)$$

Where F is a generic function. Not so difficult to have a light Higgs

$$m_h^2 \sim \epsilon \Lambda^2$$

If one has $\epsilon \ll 1$. This is not fully possible in concrete models, since this is controlled by a symmetry which is already broken in SM.

However…

Naturalness – Composite Higgs

Vanilla composite Higgs scenarios have a potential which looks like

“Compositeness” Scale

$$V(h) = \epsilon f^2 \Lambda^2 F\left(h/f\right)$$

Where F is a generic function. The position of the minimum of the potential doesn’t care about this parameter:

$$V'(h) = 0 \Leftrightarrow F'(h/f) = 0$$

So, if this is to occur at $h = v \ll f$ then one has to fine-tune the contributions to the potential from the composite physics.

Naturalness – Composite Higgs

Vanilla composite Higgs scenarios have a potential which looks like

$$V(h) = \epsilon f^2 \Lambda^2 F\left(h/f\right)$$

Compositeness Scale

Where F is a generic function. However, it is generic, like for pions, that the operator

$$\mathcal{O}_H \sim \frac{1}{f^2}\left(\partial^\mu |H|^2\right)^2$$

is generated. This modifies all Higgs couplings by an amount

$$\delta_\kappa \sim \frac{v^2}{f^2}$$

Naturalness – Composite Higgs

Vanilla composite Higgs scenarios have a p ial which looks like

Compositeness Scale

$F(h/f)$

So, in vanilla scenarios, Higgs coupling measurements suggest that if the Higgs is composite then there must be some fine-tuning of parameters at least at the 10% or so level!

is generated. This modifies all Higg

by an amount

$$\delta_\kappa \sim \frac{v^2}{f^2}$$

Naturalness – Composite Higgs

Let’s scrutinize the assumptions...

$$V(h) = \epsilon f^2 \Lambda^2 F(h/f)$$

How much symmetry breaking

How the symmetry is broken...

Assumption until now has been that the symmetry is broken in the most minimal ways.

Technically: Breaking “spurion” is in a low-index irrep of the global symmetry.

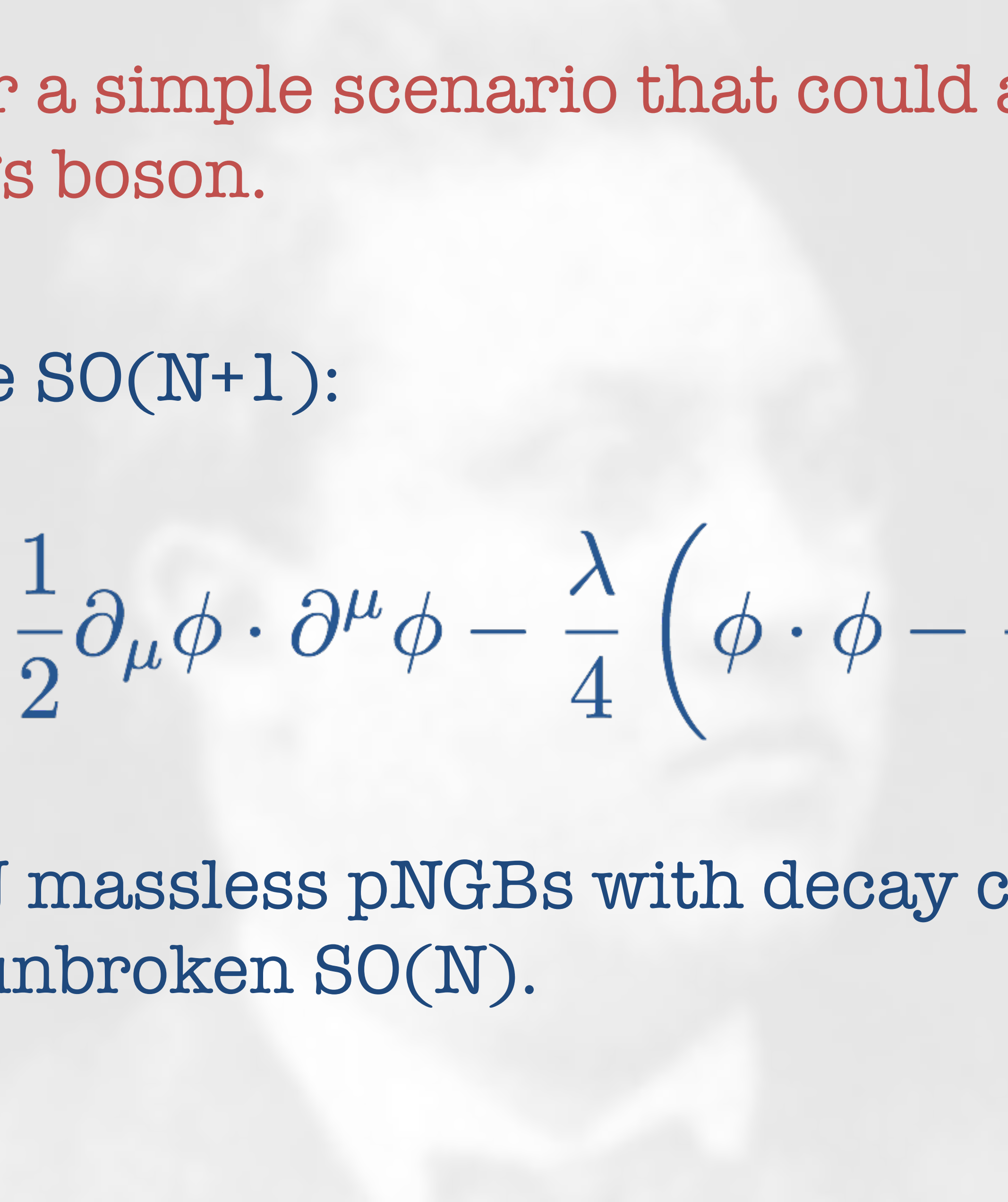
Beyond Minimality

Consider a simple scenario that could apply to the Higgs boson.

Example SO(N+1):

$$\mathcal{L} = \frac{1}{2}\partial_\mu \phi \cdot \partial^\mu \phi - \frac{\lambda}{4}\left(\phi \cdot \phi - \frac{f^2}{2}\right)^2$$

We get N massless pNGBs with decay constant “f” and unbroken SO(N).



Beyond Minimality

Consid

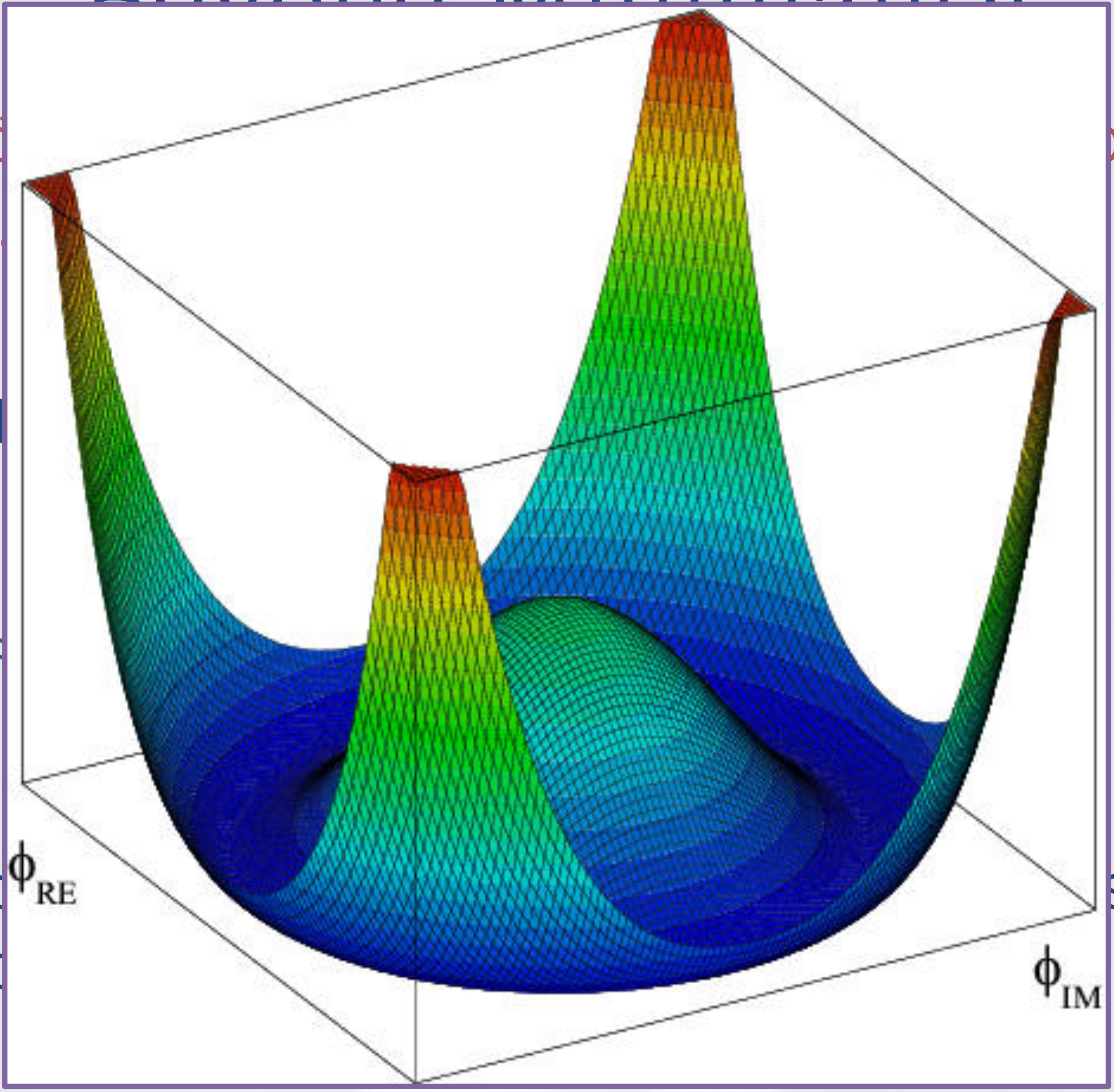
ly to

the Hi

Examp

$$\mathcal{L} =$$

$$\Big)^2$$



We get

stant

“f” anc

Beyond Minimality

Now assume some small explicit breaking "spurion" in a symmetric irrep with "n" indices:

$$V_\epsilon = \frac{\lambda}{f^{n-4}} \epsilon_{a_1,a_2,\ldots,a_n} \phi^{a_1} \phi^{a_2} \ldots \phi^{a_n}$$

How the symmetry is broken...

For the pNGB fields this generates a potential:

$$V = \epsilon m_\rho^2 f^2 \, G_n^{(N-1)/2}(\cos \Pi / f)$$

Gegenbauer function!

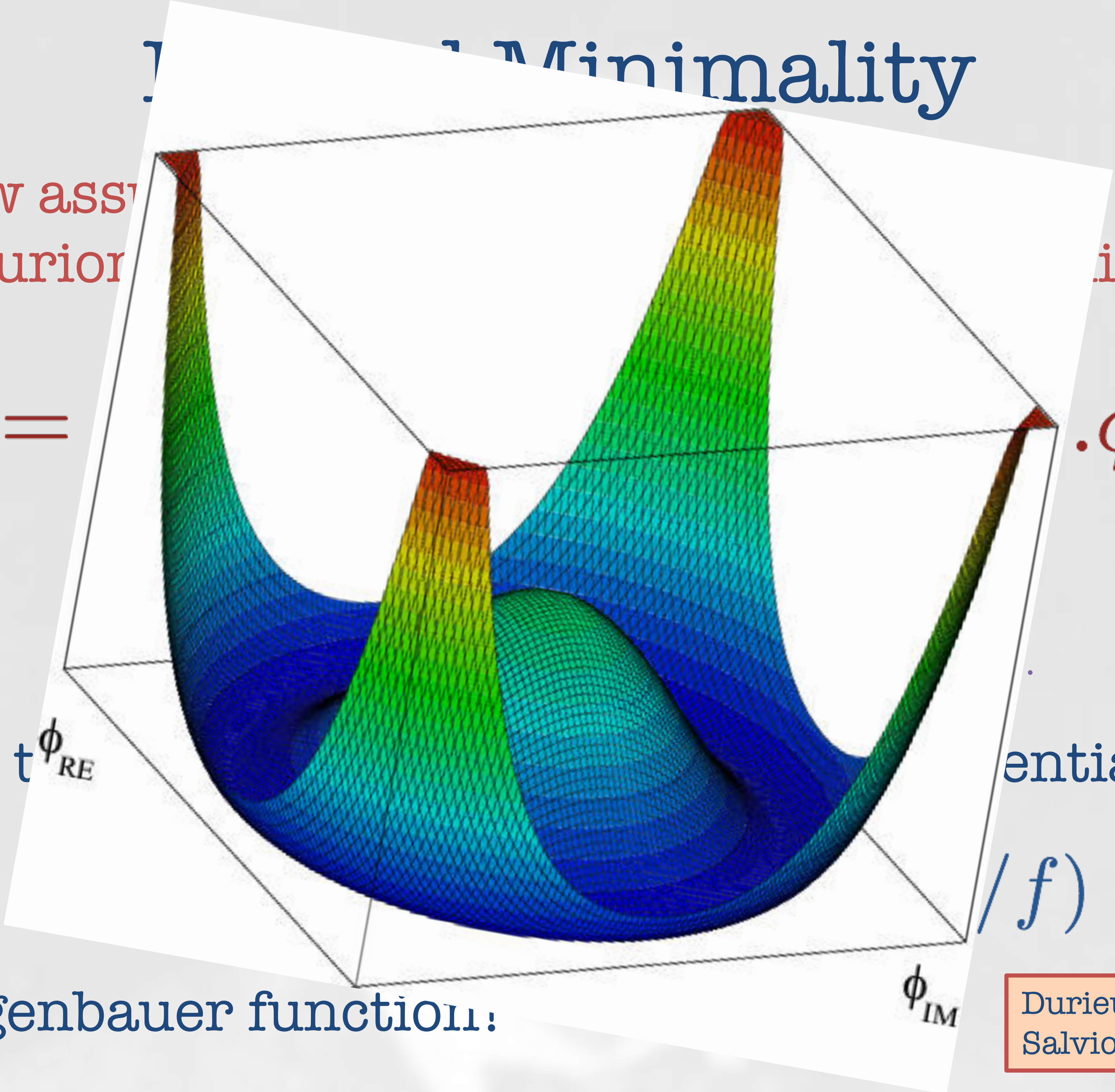
Durieux, MM, Salvioni. 2021

Minimality

Now ass

“spurior ices:

$$V_\epsilon = \quad .\phi^{a_n}$$



For t ential:

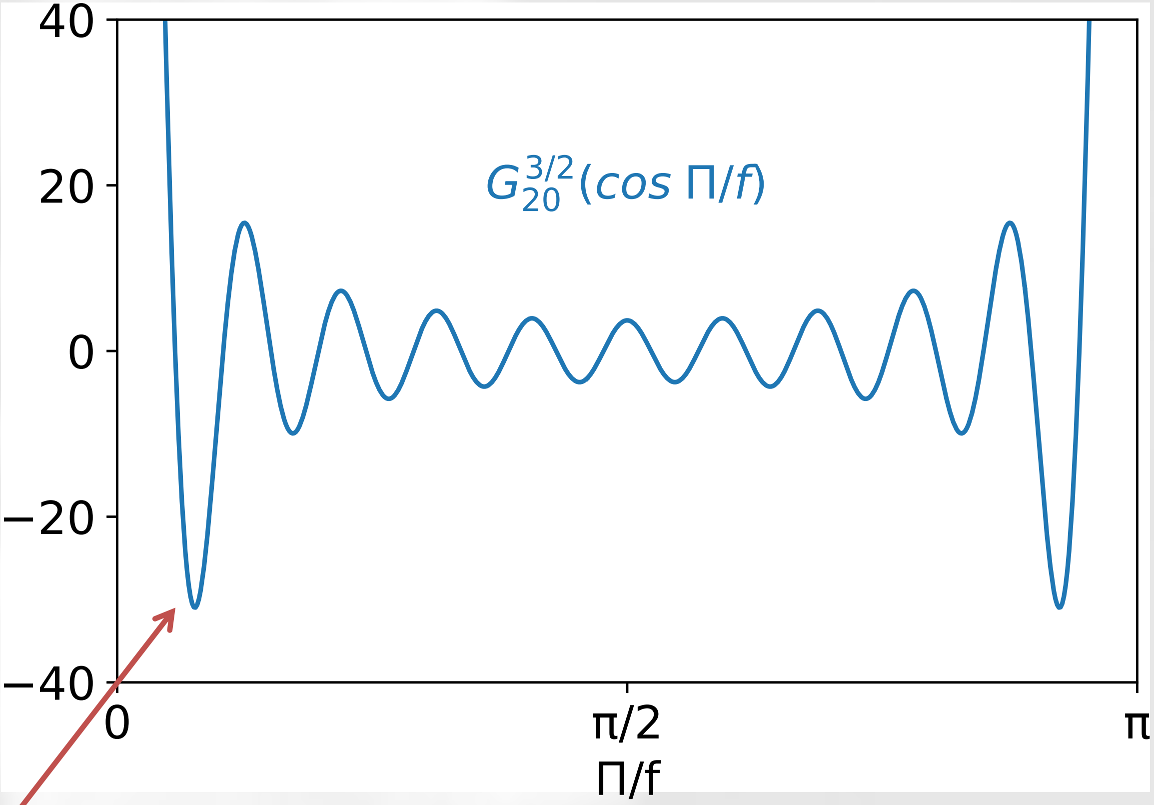
$$/ f)$$

Gegenbauer function:

Durieux, MM, Salvioni. 2021

Getting to know Gegenbauer

The Gegenbauer potential looks like:



Global minimum at naturally small field values:

$$\frac{\langle \Pi \rangle}{f} \approx \frac{j_{\lambda+1/2,1}}{n+\lambda} \approx \frac{5.1}{n}$$

Gegenbauer's Twin

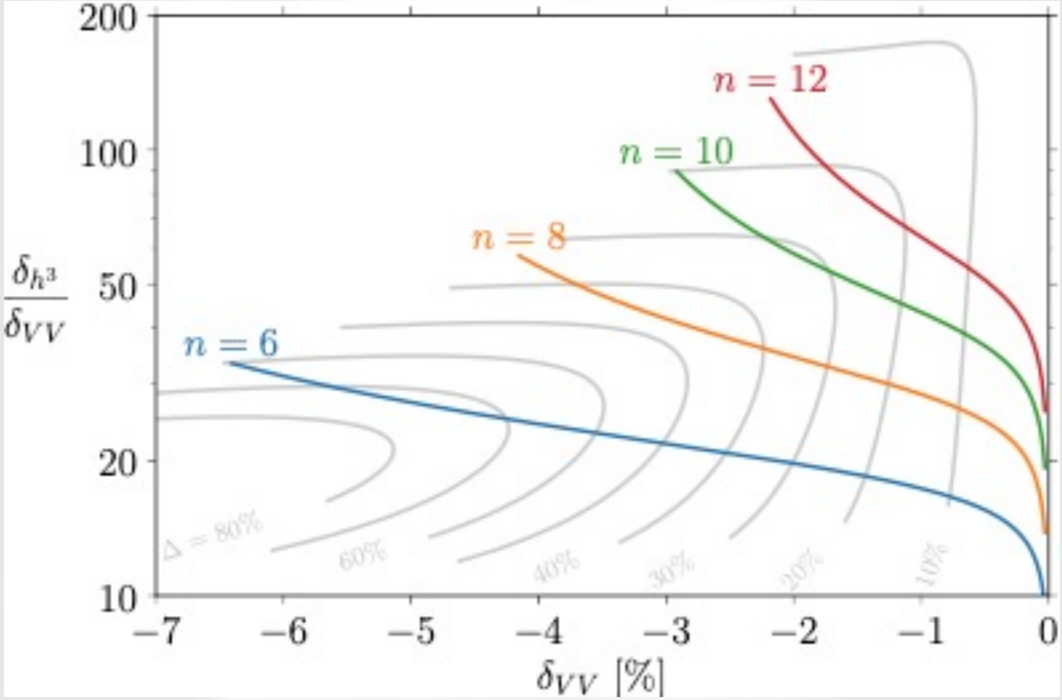
Durieux, MM, Salvioni. 2022

Gegenbauer contribution allows to naturally realise v<<f. On the other hand, for a standard composite Higgs model the top sector doesn't allow ϵ to be arbitrarily small...

Twin Higgs models, however, address that particular aspect. Could "Gegenbauer's Twin" allow both $\epsilon \ll 1$ and $v \ll f$?

Gegenbauer's Twin

Modifications to self-interaction relative to other couplings are huge:



Naturalness could show up in self-interaction!

Fine-tuning is small. The Higgs could still, naturally, be composite!

Conclusions

Higgs physics is still in its nascence. Pions were discovered in the early 1940's. Their fundamental origin, QCD, was developed theoretically in the early 1970's and only experimentally established in the late 1970's.

It has been ten years since the discovery of the Higgs boson.

We must be patient and determined to uncover its origins.

Conclusions

As it stands, we don't know how the Higgs behaves if we displace it by distances smaller than its Compton wavelength.

As it stands, we don't know how it interacts with itself; a property with far-reaching implications.

As it stands, we don't know if the Higgs boson is composite. However, some clues may already be pointing in a specific theory direction.

O.T.' R.U. ATOME? $\iint$ Sphere^2 dS was done in the most general form in 1867. I have now bagged ξ & η from T & T' and done the numerical value of $\iint (Y_i^{(s)})^2 dS$ in 4 lines, thus verifying T+T''s value of $\iint (\vartheta_i^{(s)})^2 dS$

Your plan seems indep^t of T + T' or of me. Publish! I am busy supplying the physical necessities of scientific life at home 11 Scroope Terrace, Cambridge. Prooves have got as far as grooves, corrugated plates, gratings & rings. If you have time for criticism then answer.

and $$\iint (Y_i^{(s)})^2 dS = \frac{8\pi a^2}{2i+1} \frac{|\underline{i+s}\ |\underline{i-s}}{2^{2s}\ |\underline{i}\ |\underline{i}}$$

except when $s=0$ when $\iint (Q_i)^2 dS = \frac{4\pi a^2}{2i+1}$

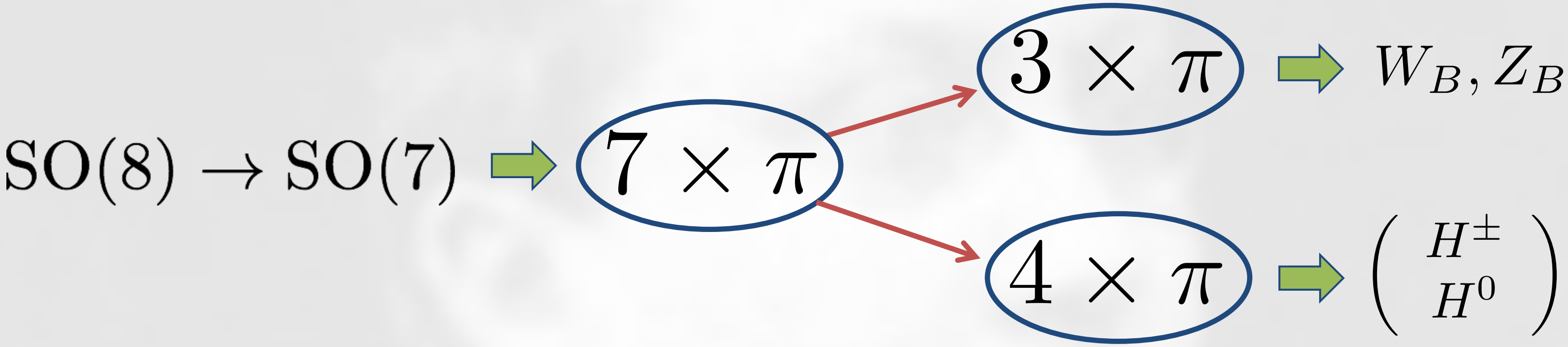
Hence $$\int_{-1}^{+1} (\vartheta_i^{(s)})^2 d\mu = \frac{2}{2i+1} \frac{2^{2s}\ |\underline{i-s}\ |\underline{s}\ |\underline{s}}{|\underline{i+s}}$$ without exception

yrs $\frac{dp}{dt}$

Composite Twin Higgs

Total symmetry-breaking pattern is: $\mathrm{SO}(8) \to \mathrm{SO}(7)$

Thus 7 pseudo-Goldstone bosons:



$$\begin{tikzcd}
 & & 3 \times \pi \arrow[r, Rightarrow] & W_B, Z_B \\
\mathrm{SO}(8) \to \mathrm{SO}(7) \arrow[r, Rightarrow] & 7 \times \pi \arrow[ur] \arrow[dr] & & \\
 & & 4 \times \pi \arrow[r, Rightarrow] & \begin{pmatrix} H^{\pm} \\ H^{0} \end{pmatrix}
\end{tikzcd}$$

The SM Higgs light because of the symmetry-breaking pattern!

Hierarchy problem solved all the way up to the scale: Λ

Chacko, Goh, Harnik 2005,...

Non-Abelian Goldstone Bosons

Writing usual CCWZ:

$$\phi = \frac{1}{\Pi} \sin\frac{\Pi}{f} \begin{pmatrix} \Pi_1 \\ \Pi_2 \\ \vdots \\ \Pi_N \\ \Pi \cot\frac{\Pi}{f} \end{pmatrix} , \qquad \text{with} \qquad \Pi = \sqrt{\mathbf{\Pi} \cdot \mathbf{\Pi}}$$

We find:

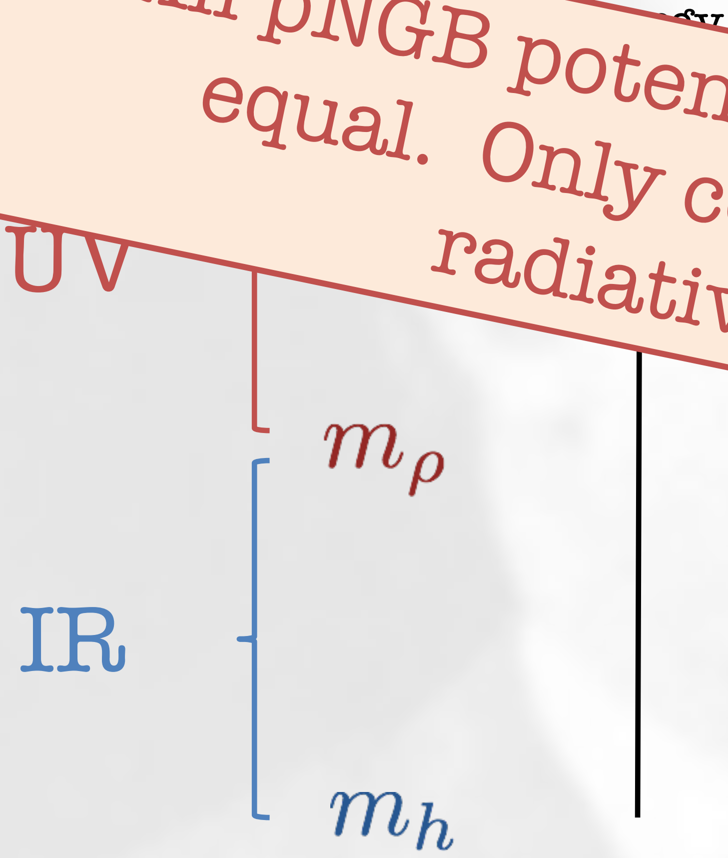
$$V = \epsilon m_\rho^2 f^2 \; G_n^{(N-1)/2}(\cos\Pi/f)$$

A Gegenbauer polynomial!

Durieux, MM,
Salvioni. 2021

Naturalness

If the Higgs is a pNGB and the UV-completion isn't fine-tuned then properties such as vev and are radiatively stable at all scales.



The IR Higgs potential receives from physics at all

of th

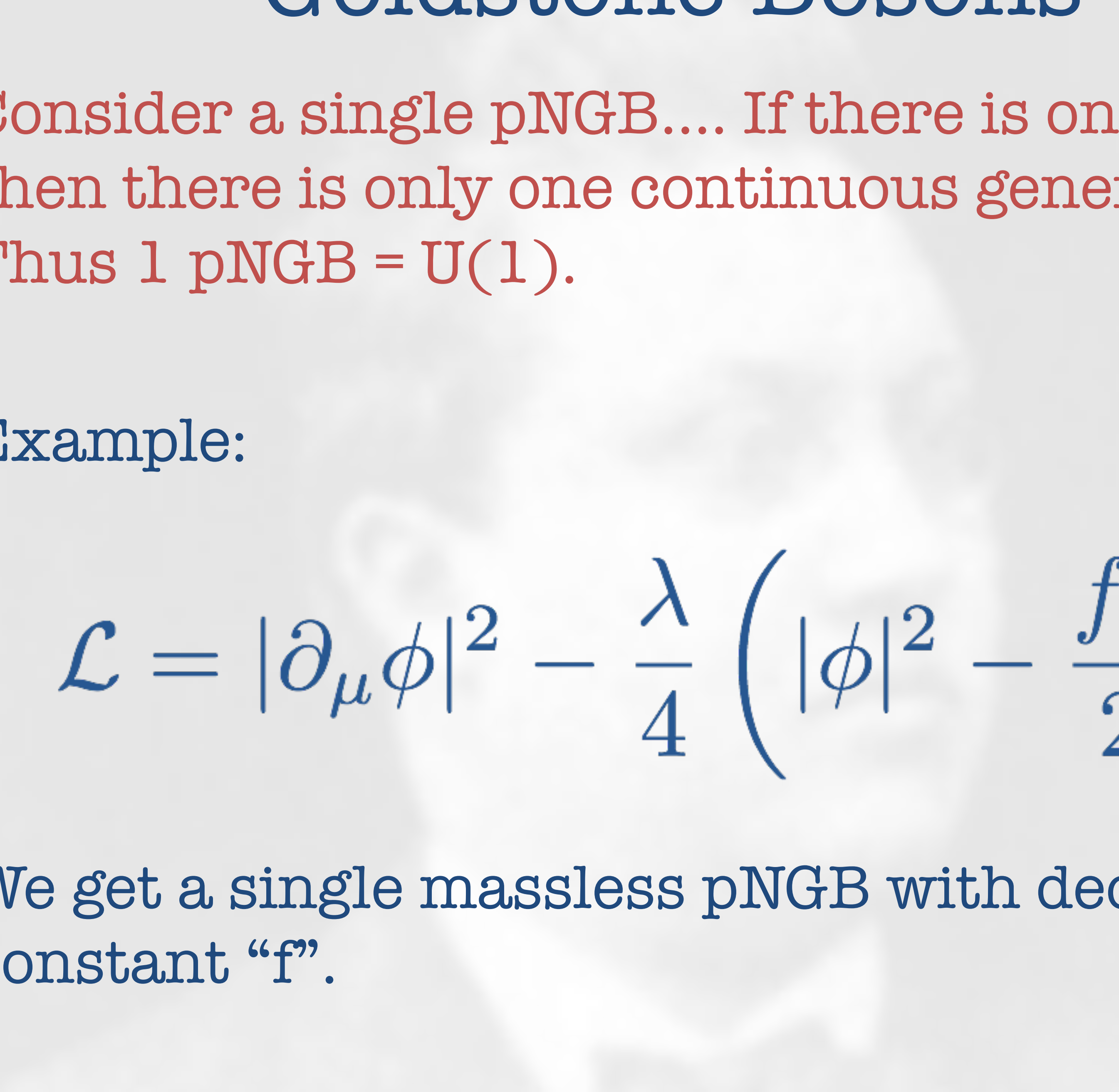
$$V'_{\text{IR}}(v) = 0$$

$$V''_{\text{IR}}(v) = m_h^2$$

should not change significantly across scales. Otherwise fine-tuning between physics at different scales.

All pNGB potentials are not created equal. Only certain classes are radiatively stable.

Goldstone Bosons



Consider a single pNGB.... If there is only one, then there is only one continuous generator. Thus 1 pNGB = U(1).

Example:

$$\mathcal{L} = |\partial_\mu \phi|^2 - \frac{\lambda}{4}\left(|\phi|^2 - \frac{f^2}{2}\right)^2$$

We get a single massless pNGB with decay constant “f”.

Goldstone Bosons

To generate a potential we assume some small explicit breaking “spurion” with charge “q”:

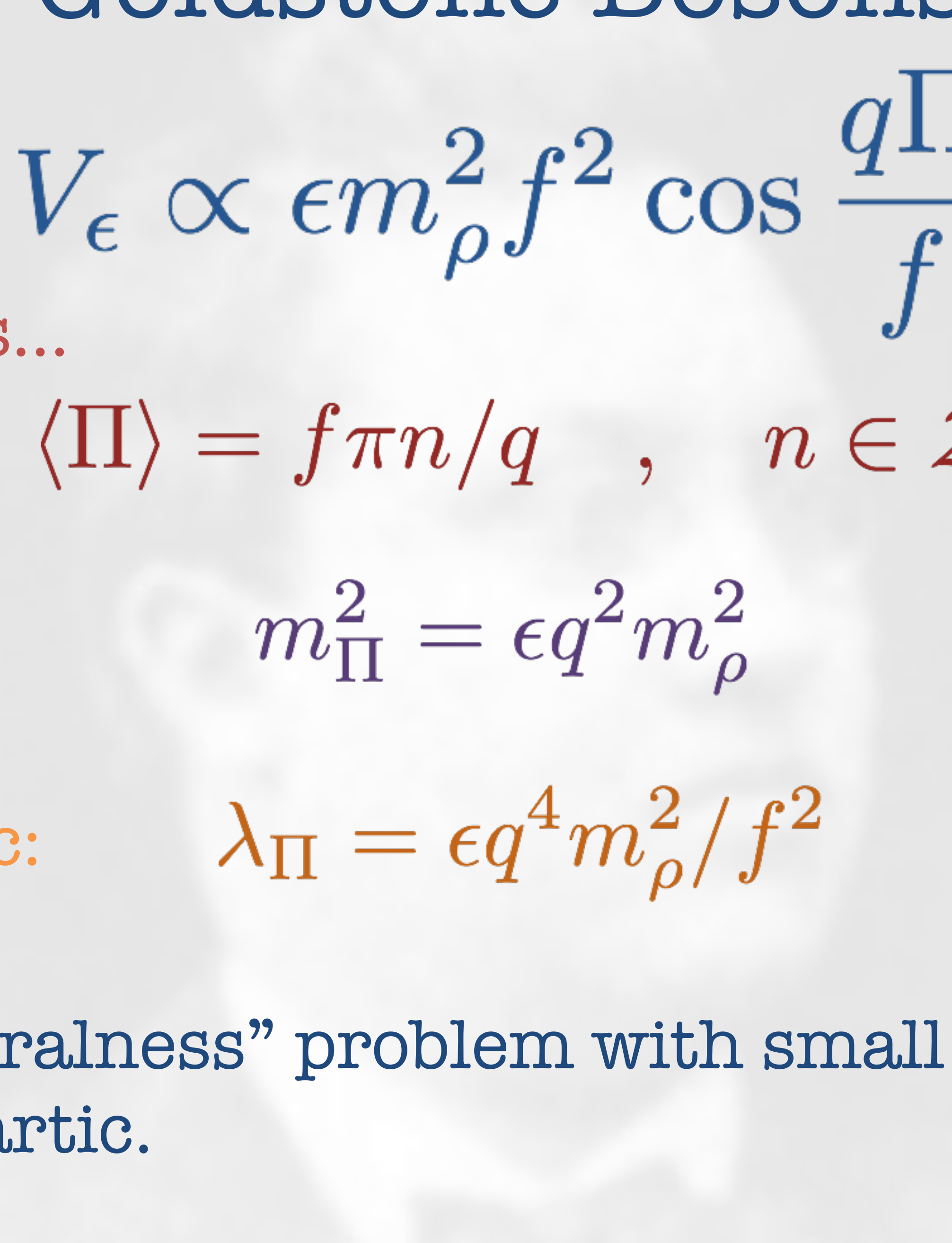
$$V_\epsilon = \epsilon \frac{\lambda}{f^{q-4}} \phi^q + h.c.$$

Which generates the potential:

$$V_\epsilon \propto \epsilon m_\rho^2 f^2 \cos \frac{q\Pi}{f}$$

Remarks...

Goldstone Bosons



$$V_\epsilon \propto \epsilon m_\rho^2 f^2 \cos \frac{q\Pi}{f}$$

Remarks...

- Vev: $\langle \Pi \rangle = f\pi n/q \quad , \quad n \in \mathcal{Z}$
- Mass: $m_\Pi^2 = \epsilon q^2 m_\rho^2$
- Quartic: $\lambda_\Pi = \epsilon q^4 m_\rho^2 / f^2$

No “naturalness” problem with small vev, mass, large quartic.

Goldstone Bosons

$$V_\epsilon \propto \epsilon m_\rho^2 f^2 \cos \frac{q\Pi}{f}$$

marks...

$$f\pi n/q \quad , \quad n \in \mathbb{Z}$$

This class of radiatively stable potentials we have all known for a long time...

- Mass: m_Π
- Quartic: $\lambda_\Pi = \epsilon q^4 m_\rho^2 / f^2$

No "naturalness" problem with small vev, mass, large quartic.

Radiative Stability (ii)

Consider the general potential:

$$V = \epsilon M^2 f^2 G(\cos \Pi / f)$$

One-loop Coleman-Weinberg potential is:

$$V_Q = \epsilon M^2 \left(f^2 G + \frac{\Lambda^2}{32\pi^2} \left(G'' + (N-1) \cot \frac{\Pi}{f} G' \right) \right)$$

But Gegenbauer polynomials are solutions to:

$$G_n^{\lambda \, \prime\prime} + 2\lambda \cot \frac{\Pi}{f} G_n^{\lambda \, \prime} + n(n+2\lambda) G_n^{\lambda} = 0$$

So iff “G” is a Gegenbauer polynomial with index

$$\lambda = (N-1)/2$$

then multiplicatively renormalised!

Radiative Stability (ii)

Consider the general potential:

$$V = \epsilon M^2 f^2 G(\cos \Pi / f)$$

Coleman-Weinberg potential is:

$$1) \cot \frac{\Pi}{f} G' \Big) \Big)$$

Radiative corrections are present but the functional form of the potential is unmodified.

But Gegenba

$$G_n^{\lambda\,\prime\prime} + 2\lambda \cot \frac{\Pi}{f} G_n^{\lambda\,\prime} + n(n + 2\lambda) G_n$$

So iff “G” is a Gegenbauer polynomial with index

$$\lambda = (N-1)/2$$

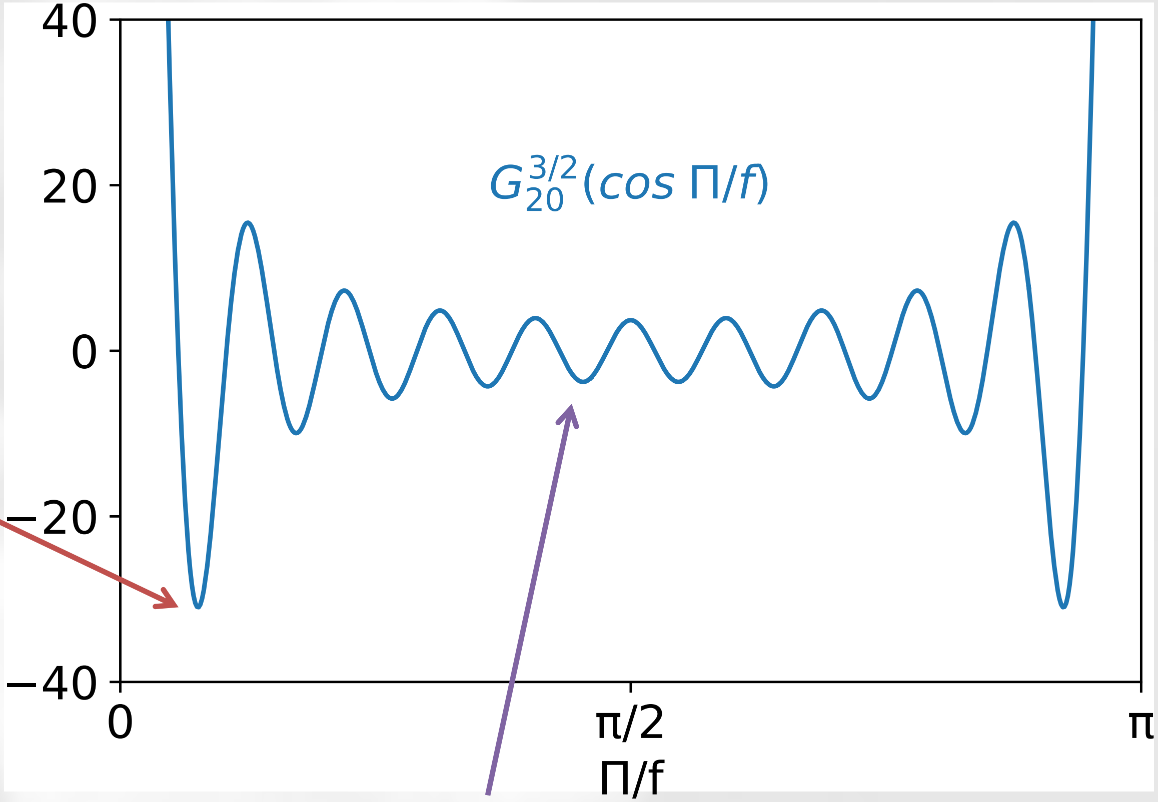
then multiplicatively renormalised!

Getting to know Gegenbauer

The Gegenbauer potential looks like:

Global minimum at naturally small field values:

$$\frac{\langle \Pi \rangle}{f} \approx \frac{j_{\lambda+1/2,1}}{n+\lambda} \approx \frac{5.1}{n}$$



Approximately periodic:

$$G_n^\lambda\left(\cos\frac{\Pi}{f}\right) \xrightarrow{n\gg 1} \frac{J_{\lambda-1/2}\left((n+\lambda)\frac{\Pi}{f}\right)}{\Pi^{\lambda-1/2}} \xrightarrow{\frac{\Pi}{f}\gg\frac{1}{n}} \frac{\cos\left((n+\lambda)\frac{\Pi}{f}-\lambda\frac{\pi}{2}\right)}{\Pi^\lambda}$$

Mini-Summary

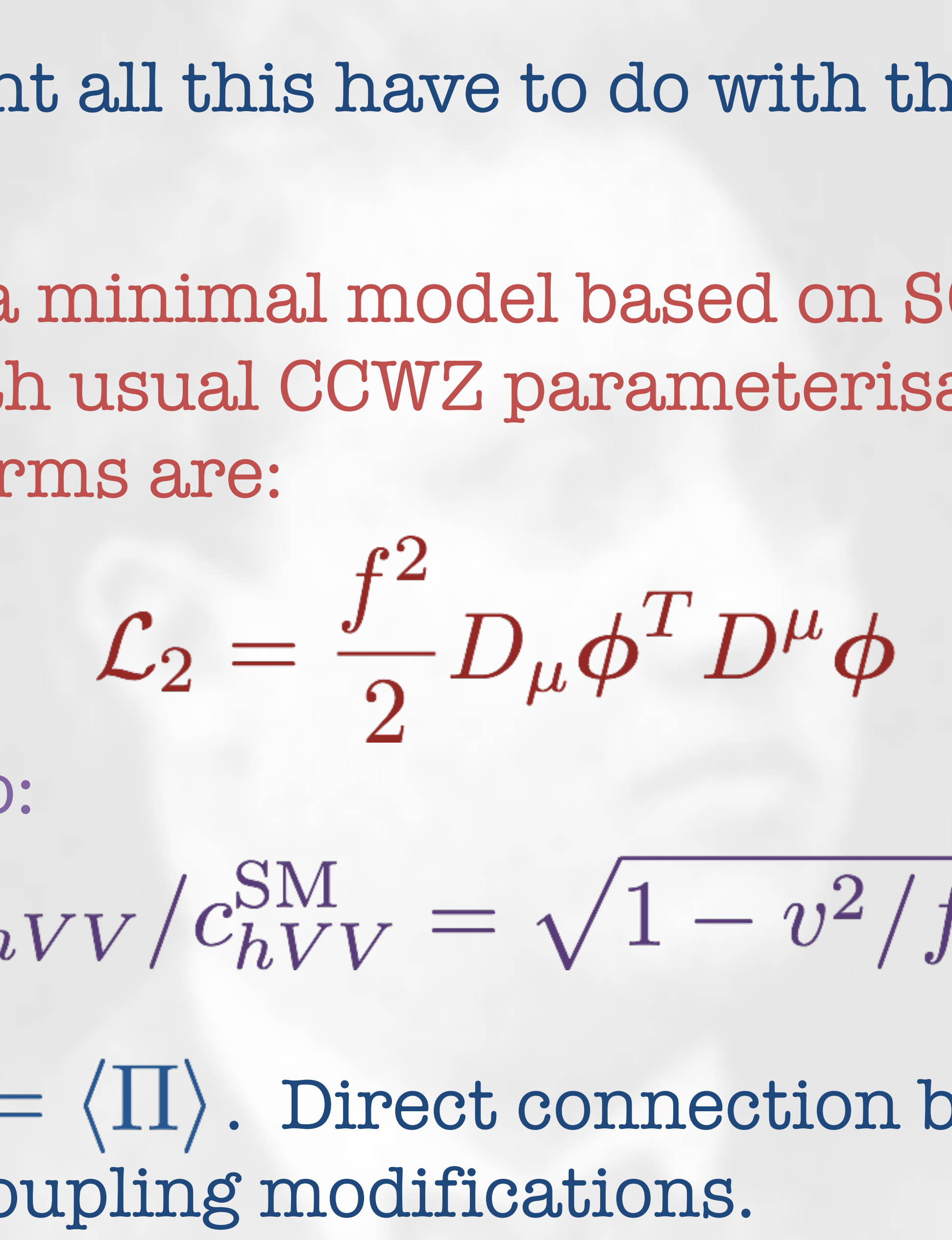
$$V = \epsilon m_\rho^2 f^2 \, G_n^{(N-1)/2}(\cos \Pi / f)$$

Remarks...

- Vev: $\langle \Pi \rangle \sim 5f/n \quad , \quad n \in \mathcal{Z}$
- Mass: $m_\Pi^2 \sim \epsilon n^2 m_\rho^2$
- Quartic: $\lambda_\Pi \sim \epsilon n^4 m_\rho^2 / f^2$

No “naturalness” problem with small vev, mass, large quartic also for non-Abelian pNGBs.

pNGB Higgs



What might all this have to do with the Higgs?

Consider a minimal model based on SO(5) to SO(4), with usual CCWZ parameterisation. Kinetic terms are:

$$\mathcal{L}_2 = \frac{f^2}{2} D_\mu \phi^T D^\mu \phi$$

Leading to:

$$c_{hVV}/c_{hVV}^{\mathrm{SM}} = \sqrt{1 - v^2/f^2}$$

Where $v = \langle \Pi \rangle$. Direct connection between vev and coupling modifications.

pNGB Higgs

But what determines the vev?

Explicit breaking in the top sector alone leads to a scalar potential that is typically of a form like:

$$V_t = \kappa \frac{N_c}{16\pi^2} y_t^2 f^2 M_T^2 \sin^2 \Pi/f$$

Up to model-dependent aspects, minimum typically at

$$v \sim, 0, \pi f, \ldots$$

Typically persists even when gauge loops are included. Higgs mass too big too...

Gegenbauer Higgs

For pions, sources of explicit symmetry breaking are very different: Quark masses, gauge couplings. Either could in principle have dominated.

$$V(\Pi) = \kappa \frac{N_c}{16\pi^2} y_t^2 f^2 M_T^2 \left[\sin^2 \Pi/f + \gamma\, G_n^{(N-1)/2}(\cos \Pi/f)\right]$$

Tunable parameter

Top contribution

Depends on magnitude of breaking

Gegenbauer

We propose that perhaps there is an additional source of breaking in the UV, not in a minimal irrep...

Fine-Tuning

Can consider some qualitative fine-tuning aspects like usual log-derivative. Two parameters are

κ and $$\Delta = \left(\frac{\partial \log f/v}{\partial \log \gamma} \right)^{-1} .$$

Quantitatively they scale as

$$\Delta \approx 30\% \left(\frac{f}{4v} \frac{5.1}{n} \right)^{-2.1} , \qquad \kappa \approx 30\% \left(\frac{f}{4v} \frac{5.1}{n} \frac{2\,\mathrm{TeV}}{M_T} \right)^{2}$$

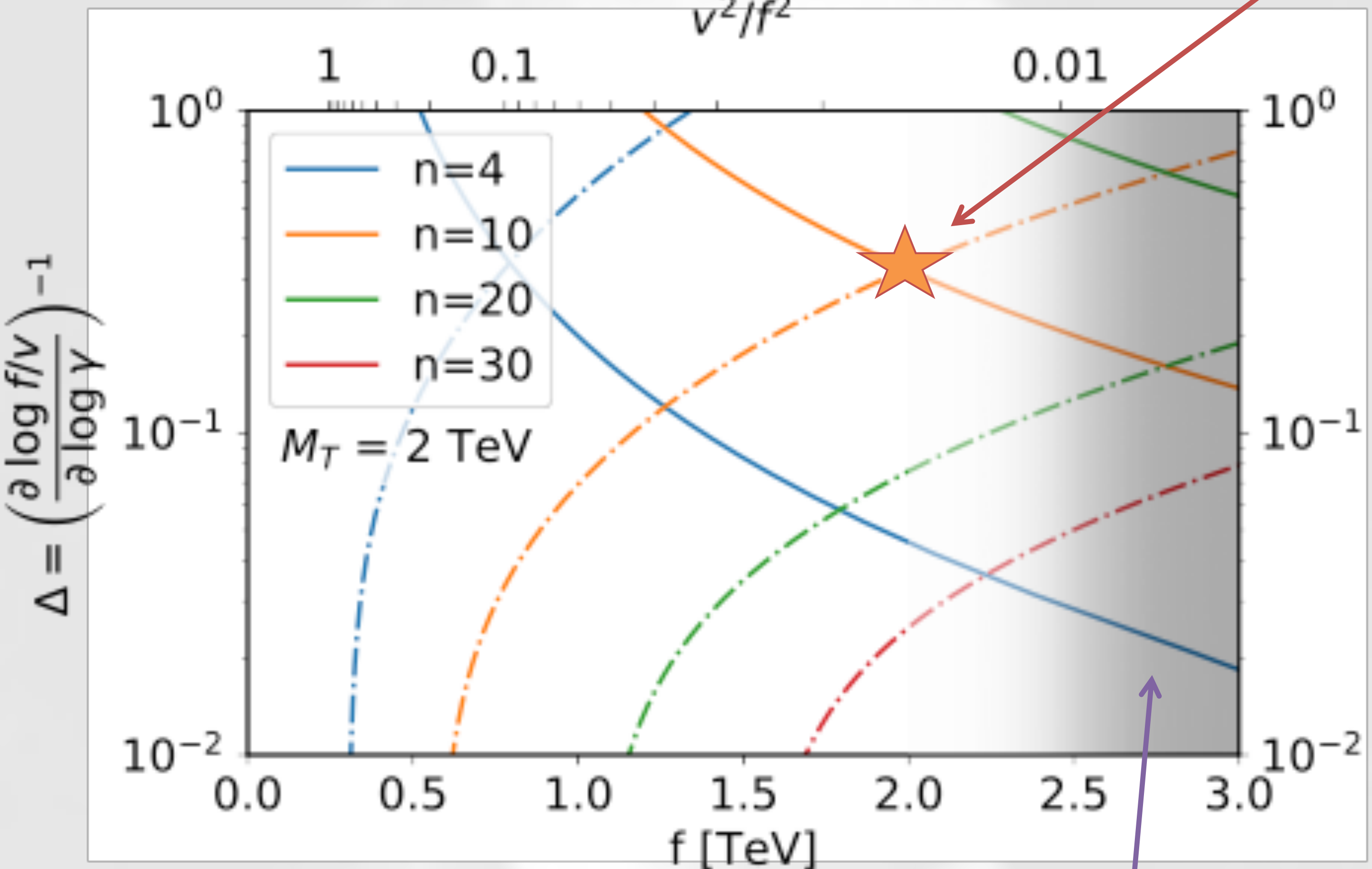
Where for a pure Gegenbauer. $5.1 f \sim nv$.

Fine-Tuning

Quantitatively:

Total tuning
less than 10%.

Solid



Dot-Dashed

Difficult to have f>>M_T.

Mini-Summary

The Higgs could be a pNGB with a naturally small vev and mass.

However top-sector corrections furnish the lowest order Gegenbauer potential, inevitably requiring some fine-tuning.

“Smoking gun” for Gegenbauer Higgs are very small Higgs single-coupling modifications, larger Higgs self-coupling modification.

Twin Tuning

While there is no tuning from the top sector the exact exchange symmetry predicts

$$v_A = 0 \quad , \quad v_B = f \qquad \text{or} \qquad v_A = v_B \, .$$

This would be fine, but SM-like Higgs couplings are corrected by a factor

$$\cos\theta = \sqrt{1 - \frac{v_A^2}{v_B^2}}$$

So we need $v_A \ll v_B$ hence exchange symmetry breaking, hence quadratic divergences, hence tuning...

Twin Tuning

While there is no tuning from the top sector the e[illegible] symmetry predicts

$$v_A = v_B \, .$$

In fact, in all realisations so far, however you calculate it, there has been an irreducible fine-tuning of

$$\Delta = 2\frac{v^2}{f^2}$$

...

So we need $v_A \ll v_B$ hence exchange symmetry breaking, hence quadratic divergences, hence tuning...

Gegenbauer’s Twin

Generalising the Gegenbauer story to the Twin setup for SO(8) → SO(7) and going to Unitary gauge the top sector contributions to the Higgs potential are

$$V_t \approx \frac{3y_t^4 f^4}{64\pi^2}\left[\sin^4\frac{h}{f}\log\frac{a}{\sin^2 h/f}+\cos^4\frac{h}{f}\log\frac{a}{\cos^2 h/f}\right]$$

Whereas the symmetric n-index irrep gives

$$V_G^{(n)} = \bar{\epsilon} f^4 G_n^{3/2}\left(\cos 2h/f\right)$$

Again, this is radiatively stable at all scales.

Gegenbauer's Twin

Generalising the Gegenbauer story to the Twin setup for SO(8) → SO(7) and going to Unitary gauge the top sector contributions to the Higgs potential are

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Whereas the symmetric n-index irrep gives

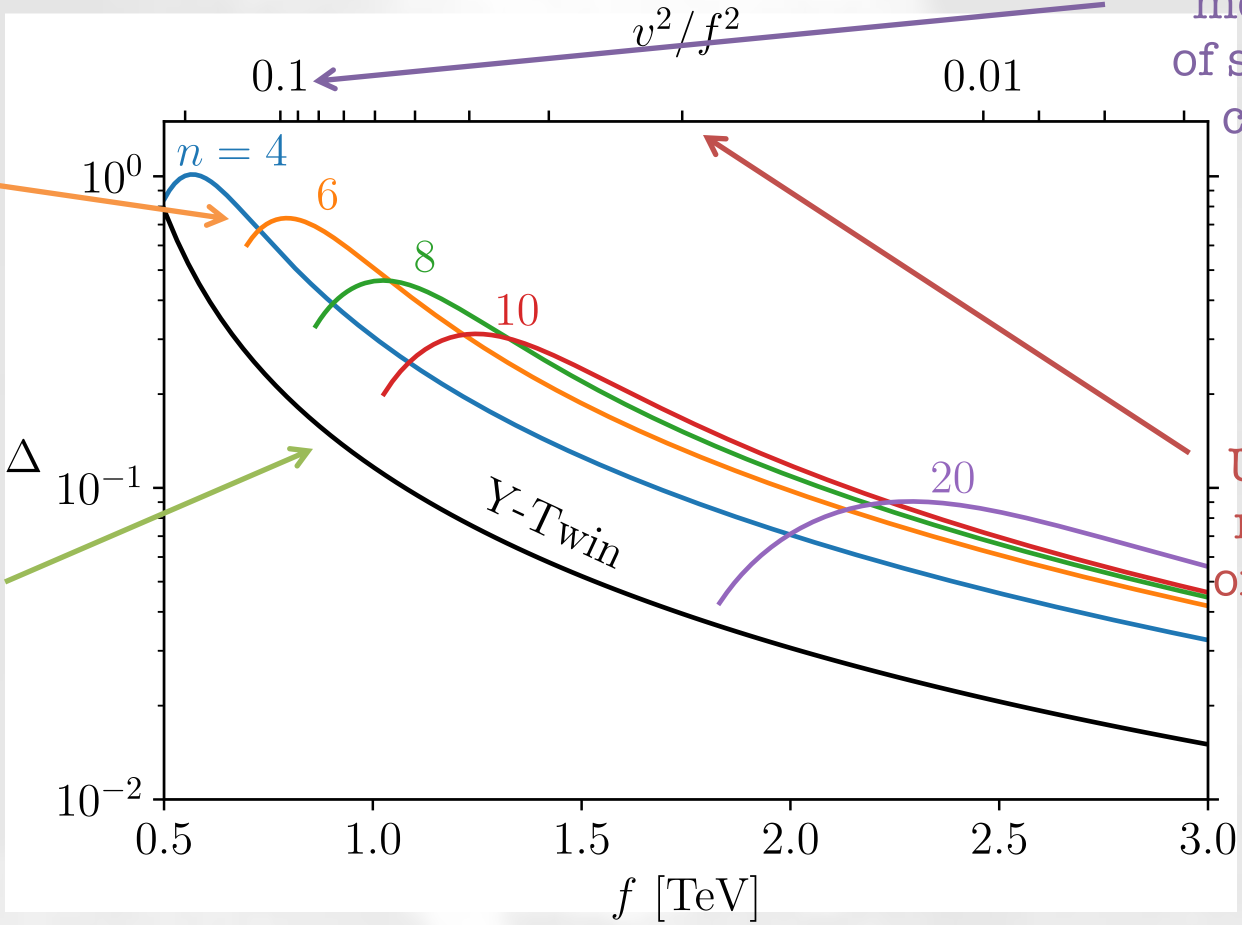
Two model parameters.

$$V_G^{(n)} = \bar{\epsilon} f^4 G_n^{3/2}\left(\cos 2h/f\right)$$

Again, this is radiatively stable at all scales.

Gegenbauer's Twin

Solving for the parameters a and $\bar{\epsilon}$ to get the observed Higgs vev and mass we may calculate the fine-tuning:



Universal 5% modification of single Higgs couplings.

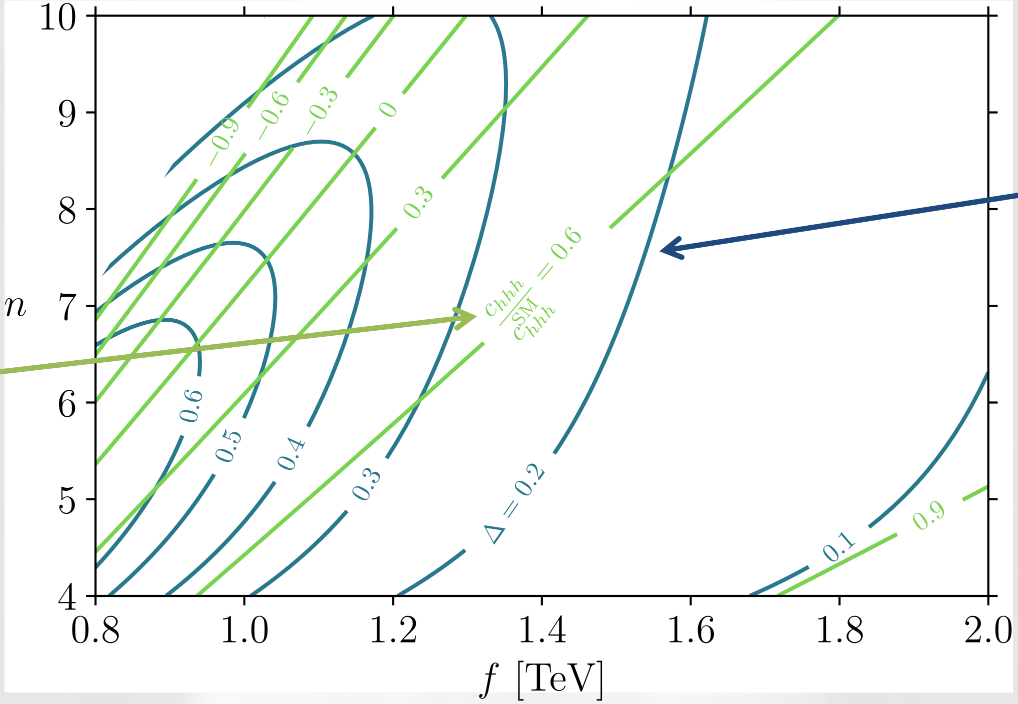
Essentially completely natural.

Normal Twin already pretty natural.

Universal 1% modification of single Higgs couplings.

Gegenbauer's Twin

While the single-Higgs coupling corrections are small, Higgs trilinear receives big corrections:



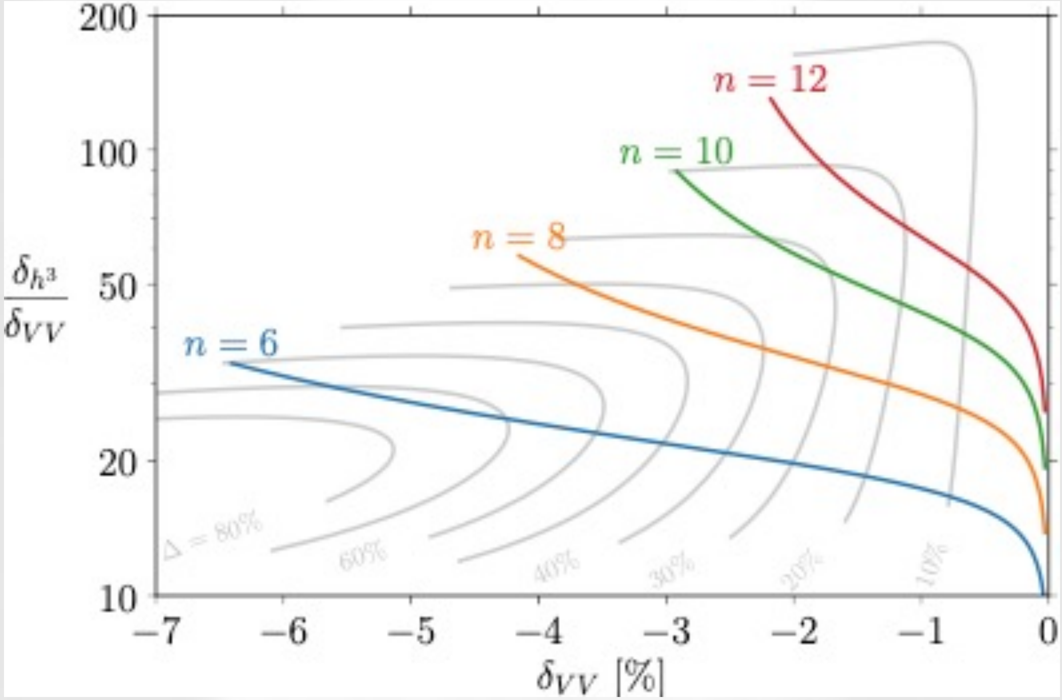
Tuning contours.

Contours of self-coupling Relative to SM prediction.

This is a smoking-gun signal of Gegenbauer's Twin and could be detected at the HL-LHC.

Gegenbauer’s Twin

Modifications to self-interaction relative to other couplings are huge:



Naturalness could show up in self-interaction!

Naturalness could be hiding in the Higgs potential.

Mini-Summary

Gegenbauer’s Twin is a symmetry-based model for a composite Higgs sector which is completely natural and consistent with LHC measurements.

Future signatures include a significantly modified Higgs self-coupling, but very SM-like single couplings.

Explicit counterexample to the expectation that you won’t discover natural new physics first in the Higgs self-coupling.

Conclusions

We don’t know if the Higgs boson propagates as predicted in the SM at LHC energies.

We don’t know if the Higgs interacts with itself as predicted in the SM. We don’t have a handle on the EW phase transition, when the Higgs gave mass to particles, without making severe assumptions about underlying physics.

The Higgs could be composite with no inconsistency with LHC measurements or fine-tuning.

Conclusions

We don’t know if the Higgs boson propagates as predicted in the SM at LHC energies.

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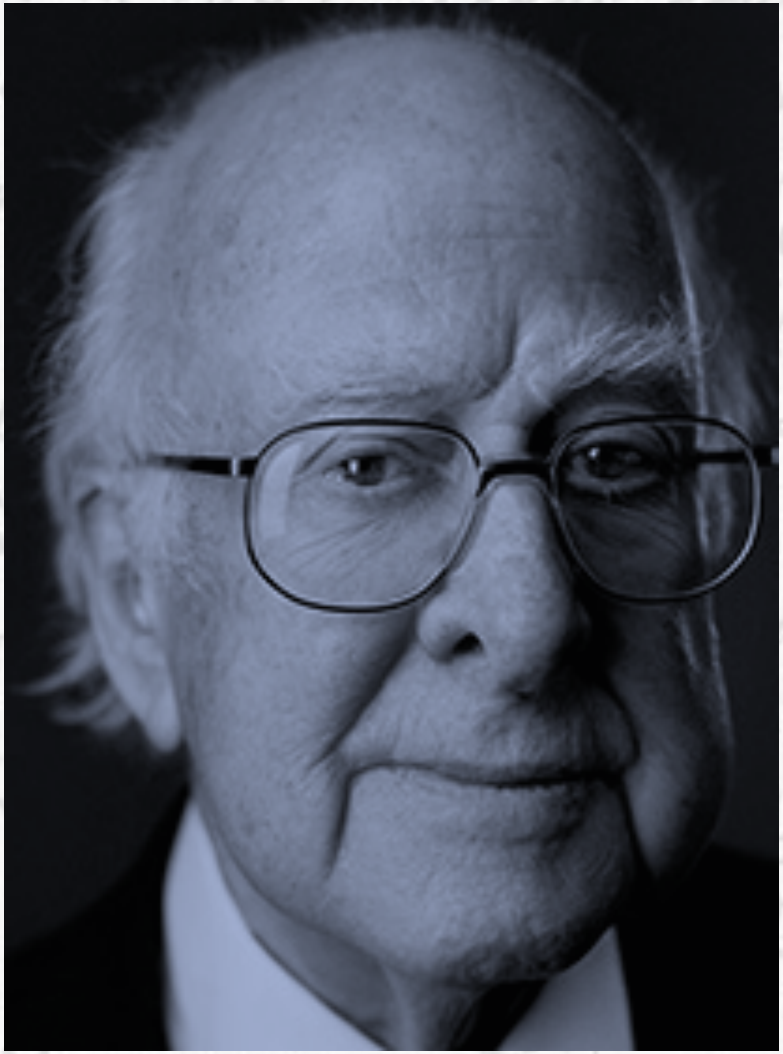
on the E
gave mass to partic
assumptions about underlying ph

Anyone who says the Higgs boson is Standard-Model-like doesn’t know what they are talking about.

The Higgs could be composite with no inconsistency with LHC measurements or fine-tuning.

How well do we know the Higgs?



Barely.

O·T'! R·U·ATOME?. $\iint$ Sphere^2 dS was done in the most general form in 1867. I have now bagged ξ & η from T & T' and done the numerical value of $\iint (Y_i^{(\sigma)})^2 dS$ in 4 lines, thus verifying T+T''s value of $\iint (\vartheta_i^{(s)})^2 dS$

Your plan seems indep^t of T + T' or of me. Publish! I am busy supplying the physical necessities of scientific life at Dom 11 Scroope Terrace, Cambridge. Prooves have got as far as grooves, corrugated plates, gratings and rings. If you have time for criticism then [illegible] and 1.

$$\iint \left(Y_i^{(\sigma)}\right)^2 dS = \frac{8\pi a^2}{2i+1} \frac{\underline{|i+s}\;\underline{|i-s}}{2^{2s}\;\underline{|i}\;\underline{|i}}$$

except when s = 0 when $\iint (Q_i)^2 dS = \frac{4\pi a^2}{2i+1}$

Hence

$$\int_{-1}^{+1} \left(\vartheta_i^{(s)}\right)^2 d\mu = \frac{2}{2i+1} \frac{2^{2s}\,\underline{|i-s}\;\underline{|s}\;\underline{|s}}{\underline{|i+s}}$$

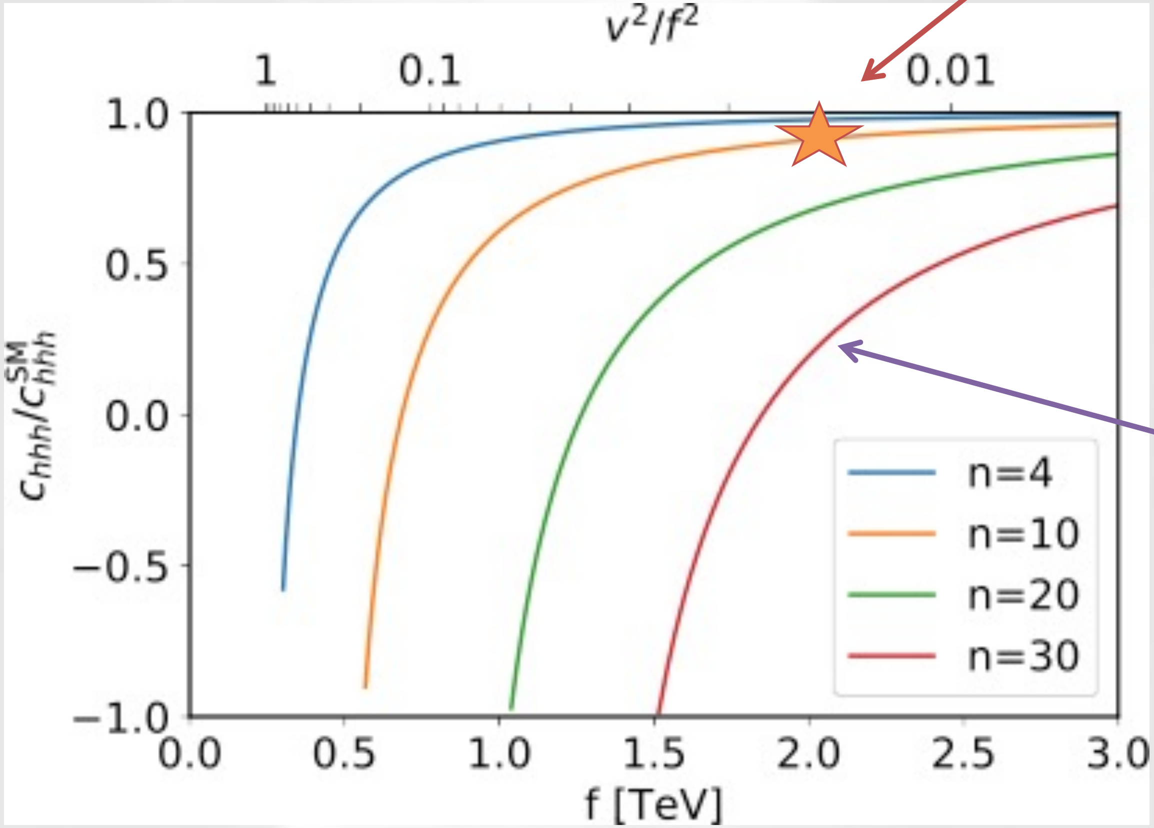
without exception

yrs $\frac{dp}{dt}$

Pheno?

Total modification less than 10%.

Higgs self-coupling:



But could be much greater.

But for this benchmark hVV coupling modifications below 1%, beyond LHC reach also.

Twin Higgs

Chacko, Goh, Harnik 2005

- Take two identical copies of the Standard Model:

$$\mathrm{SM}_A \xleftrightarrow{A \leftrightarrow B} \mathrm{SM}_B$$

- Enhance symmetry structure to global SO(8):

Desired quartic dictated by accidental symmetry:

$$V_{\text{Higgs}} = \lambda \left(|H_A|^2 + |H_B|^2\right)^2 - \Lambda^2 \left(|H_A|^2 + |H_B|^2\right)$$

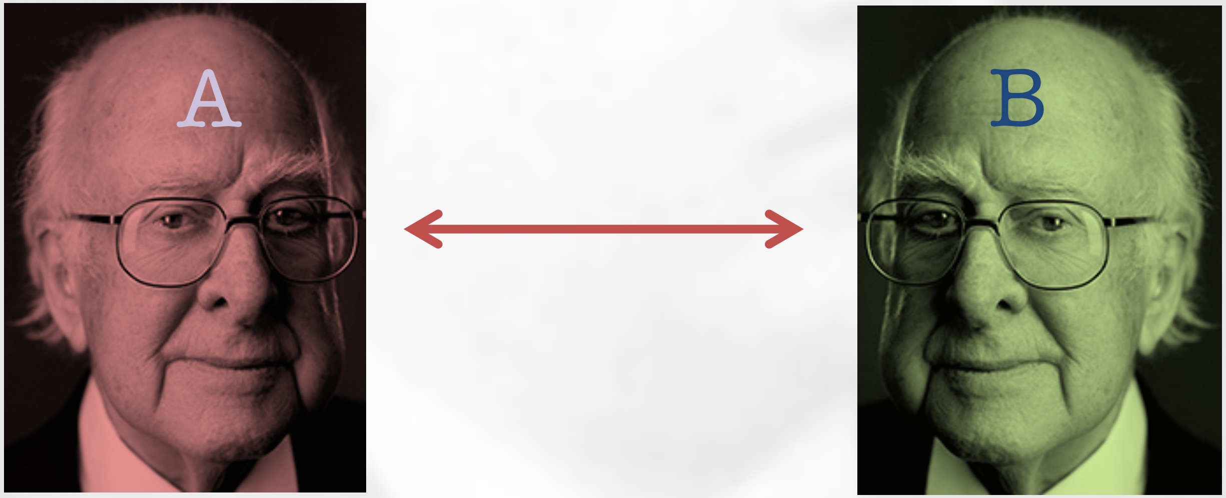
Exchange enforces equal quadratic corrections for each Higgs. Thus masses still respect SO(8) symmetry.

Composite Twin Higgs Recap

- Take two identical copies of the Standard Model:

$$\mathrm{SM}_A \xleftrightarrow{A \leftrightarrow B} \mathrm{SM}_B$$

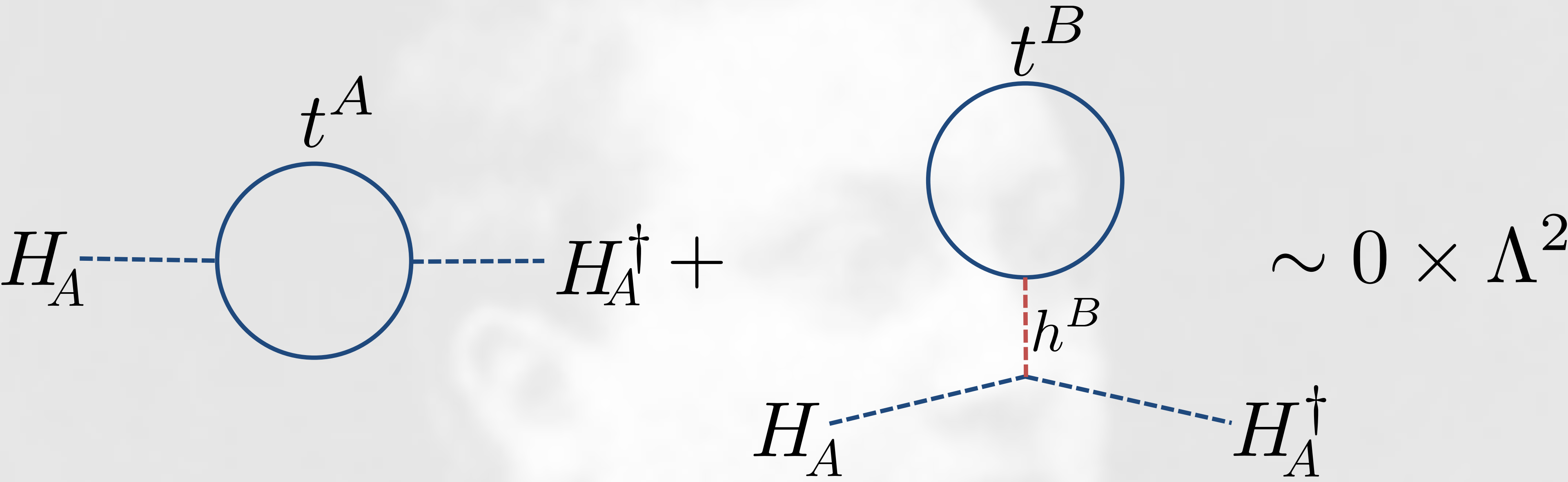
- Everything twinned.



Twin Higgs

Chacko, Goh, Harnik 2005

- In outdated “quadratic divergences” parlay:



Quadratic divergences from SM top quark loops cancelled by loops of “Twin” top quarks.

- Cancellation persists for all Twin particles: Twin W-bosons, Twin gluons, etc.

Twin Higgs

Chacko, Goh, Harnik 2005

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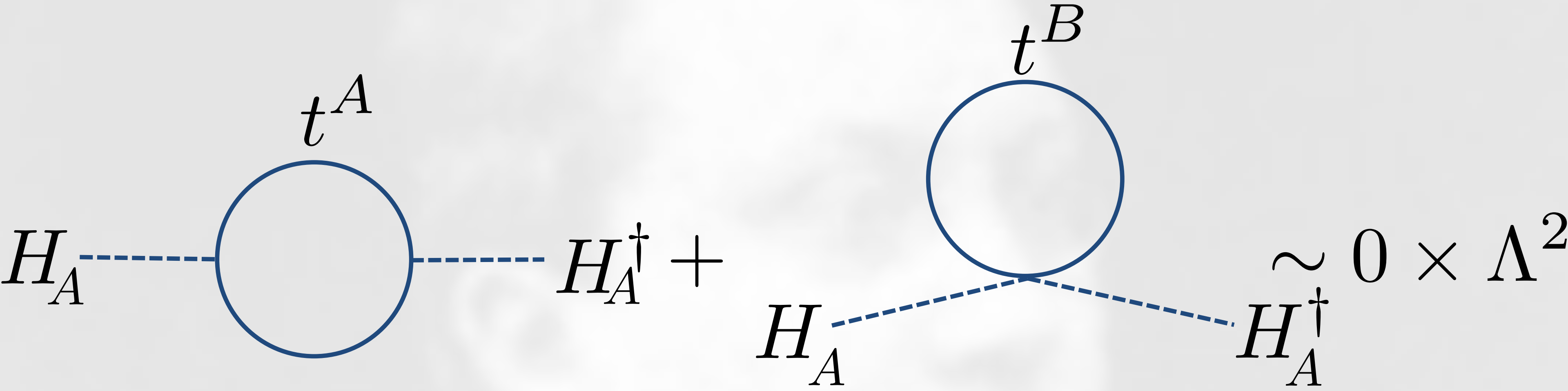

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Twin Higgs

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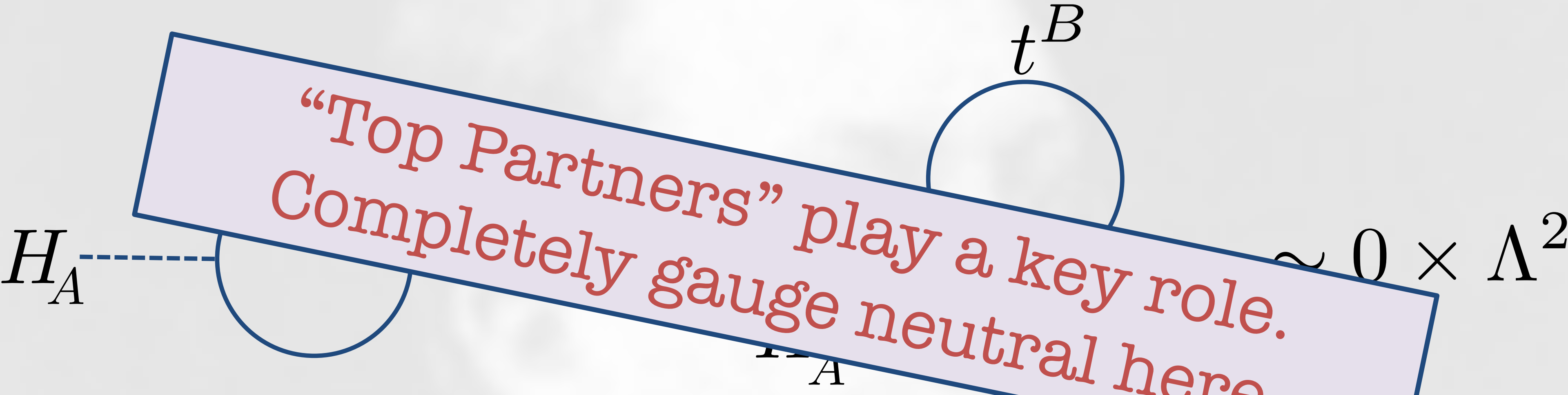

“Top Partners” play a key role.
Completely gauge neutral here.

Quadratic divergences from SM top quark loops cancelled by loops of “Twin” top quarks.

- Cancellation persists for all Twin particles: Twin W-bosons, Twin gluons, etc.

Twin Higgs

Chacko, Goh, Harnik 2005

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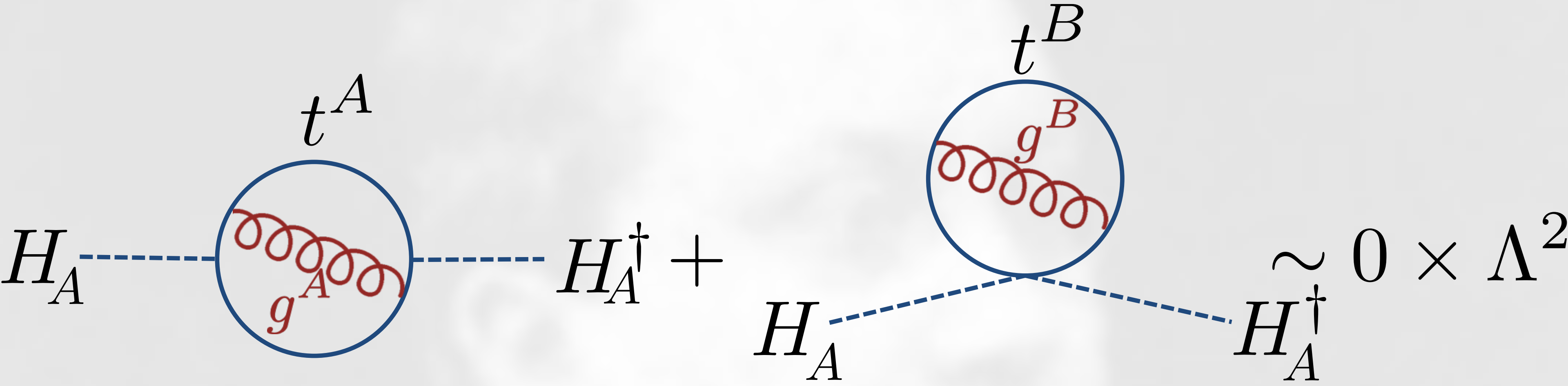

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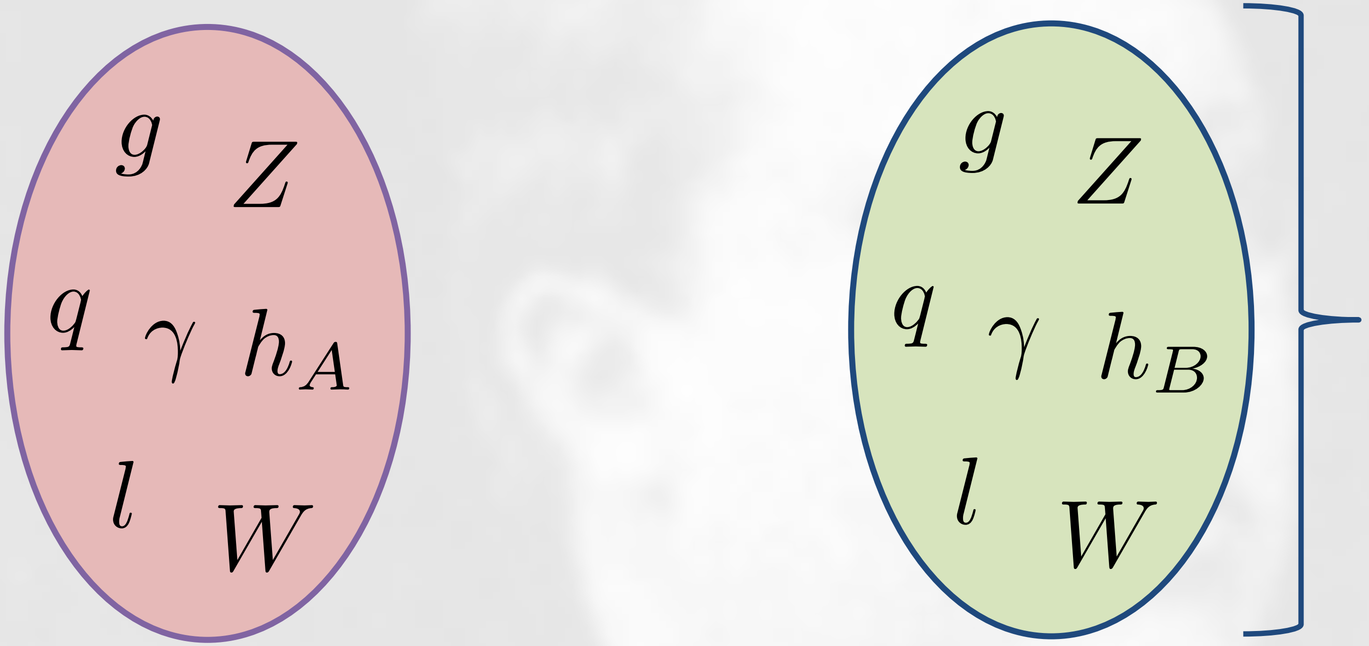
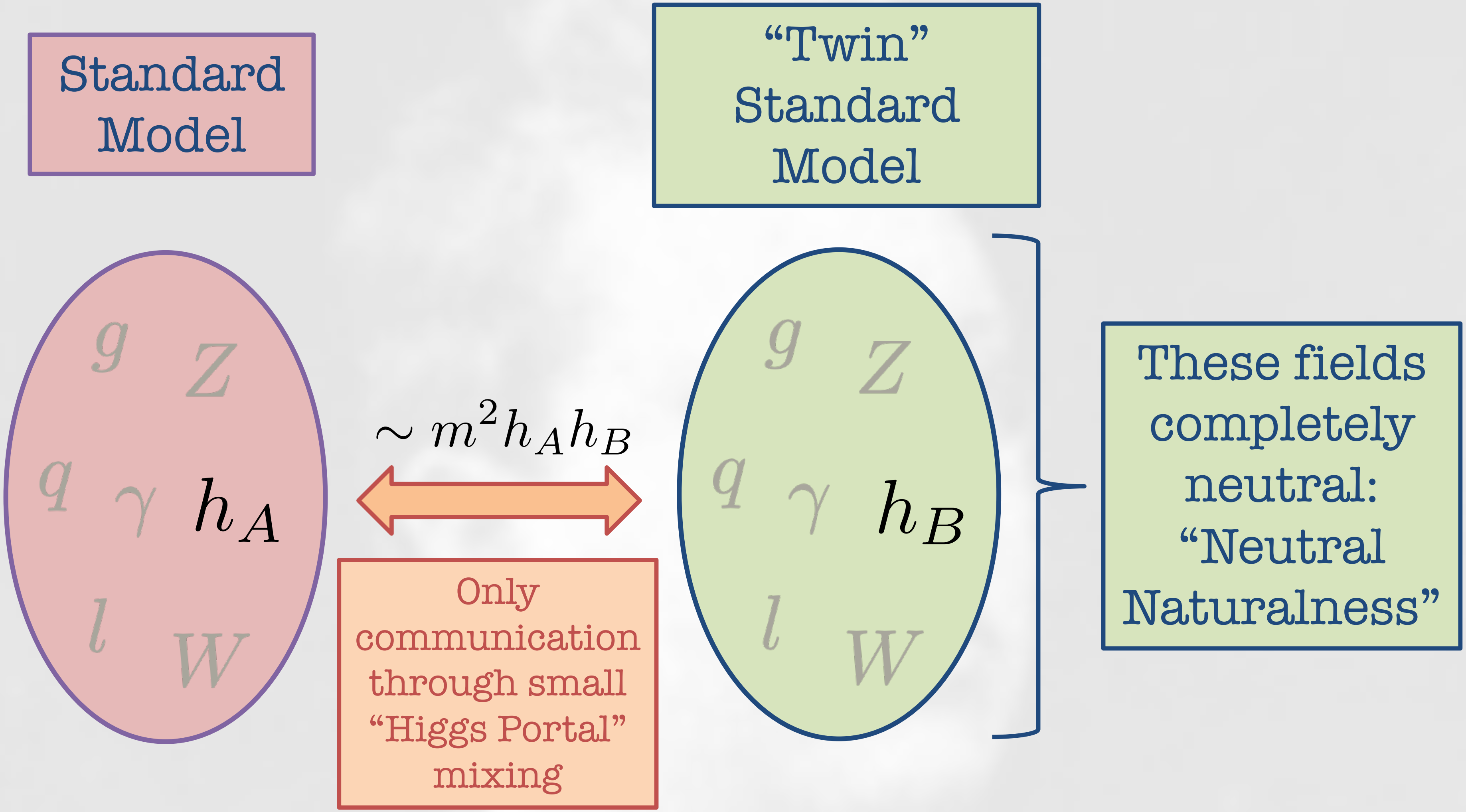
Standard Model

"Twin" Standard Model



These fields completely neutral: "Neutral Naturalness"

Predictions for Twin sector most robust for the Twins of the SM fields that couple most strongly to Higgs.



Standard Model

"Twin" Standard Model

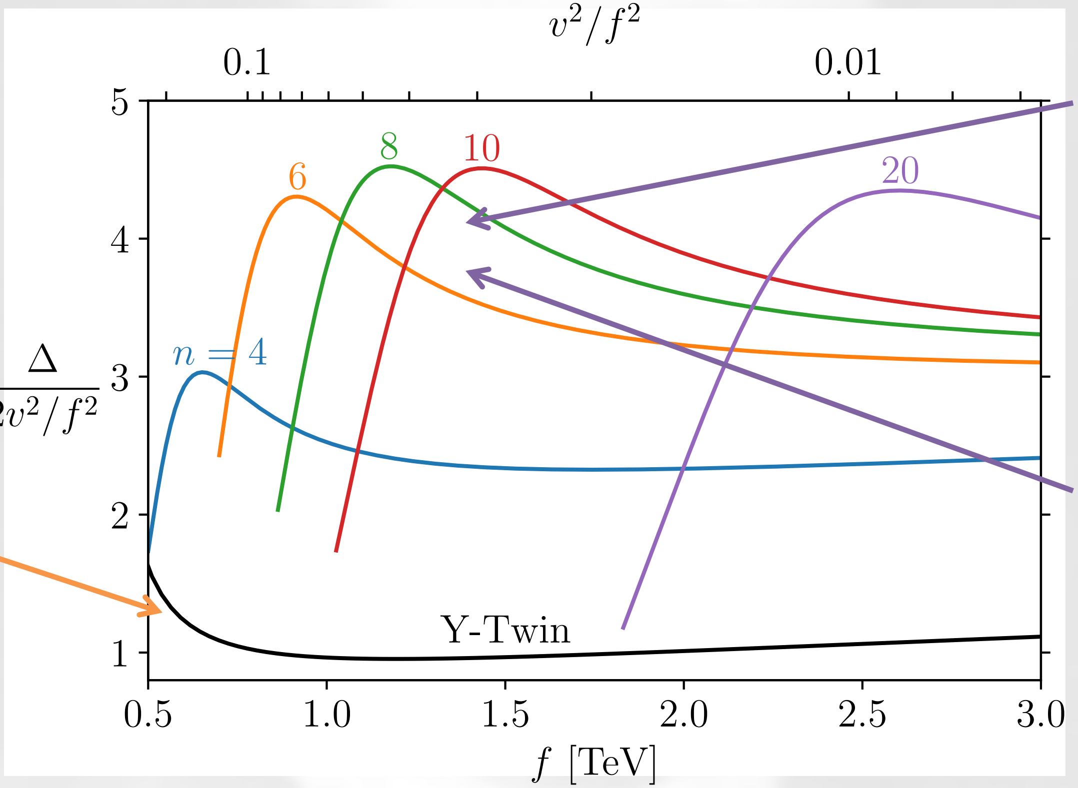
$\sim m^2 h_A h_B$

Only communication through small "Higgs Portal" mixing

These fields completely neutral: "Neutral Naturalness"

Gegenbauer’s Twin

Solving for the parameters a and $\bar{\epsilon}$ to get the observed Higgs vev and mass we may calculate the fine-tuning:



Standard Twin model that uses explicit breaking by not gauging Twin hypercharge.

Gegenbauer Twin models more natural by around a factor 4…

$$\frac{\Delta}{2v^2/f^2} \simeq \frac{4\pi^2 m_h^2}{3y_t^4 v^2} \approx 4$$

Gegenbauer's Twin

S for the parameters a and $\bar{\epsilon}$ to get the

c nd mass we may calculate

t

Gegenbauer

odels

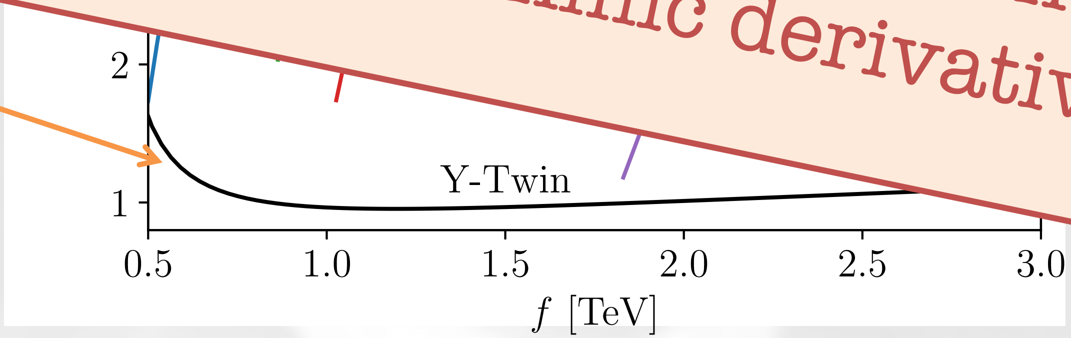
Fine-tuning calculated using most conservative definition.

$$\delta = \begin{pmatrix} \frac{\partial \log v^2}{\partial \log \bar{\epsilon}} & \frac{\partial \log v^2}{\partial \log a} \\ \frac{\partial \log m_h^2}{\partial \log \bar{\epsilon}} & \frac{\partial \log m_h^2}{\partial \log a} \end{pmatrix}$$

$$\Delta = \left(\sum \text{eigenvalues} \left(\delta^T \delta \right) \right)^{-1/2}$$

Always greater fine-tuning than choosing any single logarithmic derivative

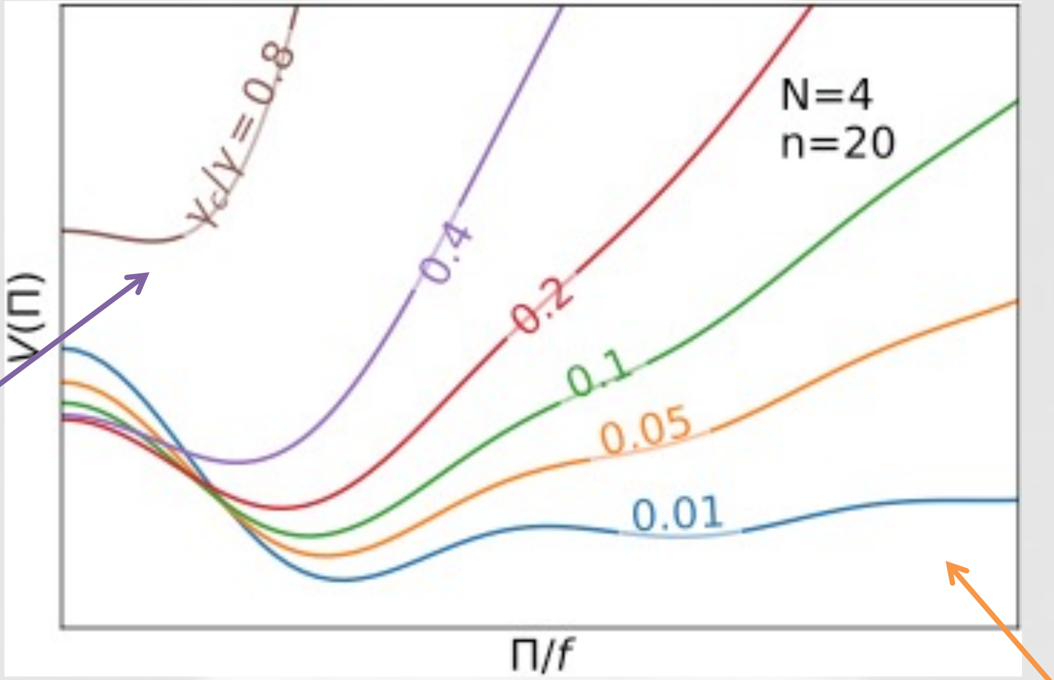
Tw model that uses explicit breaking by not gauging Twin hypercharge.



$\frac{m_h^2}{v^2} \approx 4$

Gegenbauer Higgs

$$V(\Pi) = \kappa \frac{N_c}{16\pi^2} y_t^2 f^2 M_T^2 \left[\sin^2 \Pi/f + \gamma\, G_n^{(N-1)/2}(\cos \Pi/f)\right]$$



The critical value beyond which origin is minimum.

More Gegenbauer-like

$$\gamma_c = -\frac{(\sin^2 \Pi/f)''}{(G_n^{(N-1)/2}(\cos \Pi/f))''}\Bigg|_{\Pi=0}$$

Gegenbauers from Irreps

Consider the Taylor expansion:

$$|t\phi - \tilde{\phi}|^{1-N} = \sum_{n=0}^{\infty} t^n K_n^{i_1 i_2 \ldots i_n}(\tilde{\phi})\phi^{i_1}\phi^{i_2}\ldots\phi^{i_n}$$

Irrep

Symmetric

Traceless
(Laplacian Vanishes)

Where: $K_n^{i_1 i_2 \ldots i_n}(\phi) = \frac{1}{n!}\frac{\partial^n \phi^{1-N}}{\partial\phi_{i_1}\partial\phi_{i_2}\ldots\partial\phi_{i_n}}$

But: $|t\phi - \tilde{\phi}| = \sqrt{1 - 2t\cos\Pi/f + t^2}$

and: $(1 - 2tx + t^2)^{-\lambda} = \sum_{n=0}^{\infty} t^n G_n^{\lambda}(x)$

How well should we know Higgs properties in the Standard Model?

OK: Claiming to have a measurement of something requires around 50% precision, to claim 2σ.

Better: Claiming to have discovered something requires around 20% precision, to claim 5σ.

Life goals: Quantum corrections* are around a few percent in the Higgs sector, so to claim to have probed the quantum nature, which we should, then aim for a few percent.

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* By quantum corrections, I mean an extra factor of $\hbar$ compared to leading result. Nothing to do with tree-versus-loop...

1. One-Loop Calculability

Suppose we insert O_6 into a one-loop diagram. In dim-reg operator dimensions don't mix. Thus we end up with a diagram scaling as:

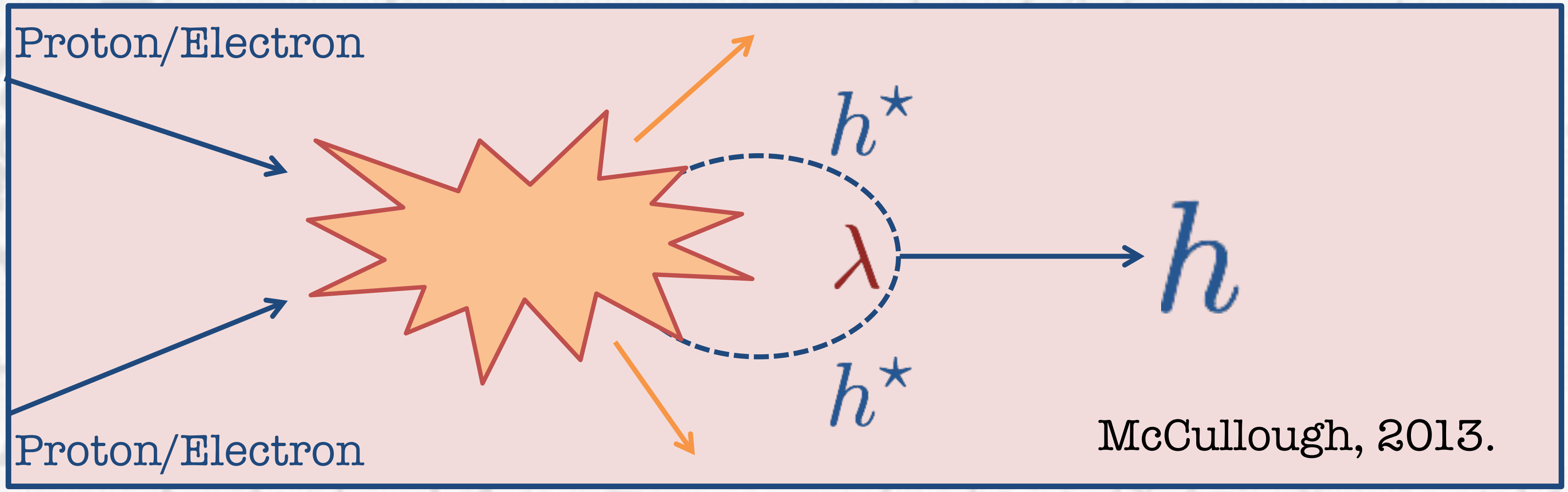
$$[\mathcal{A}] = [c_6] + [\hbar] + \ldots + [\lambda] + \ldots$$

So only way to get a contribution at $[g^2]$ is if only O_6 and no other couplings enter. But there is only one such diagram, which vanishes.

Hence, there can be no counter-terms at dim-6, thus the result must be finite!

1. One-Loop Calculability

In practise, self-coupling can be modified in one-loop contributions to Higgs single-production and result will be finite and IR-calculable, unlike modifications of any other coupling!



McCullough, 2013.