WHAT'S NEW IN PARTICLE THEORY

University of Birmingham

Particle Physics Seminar

29/09/2021

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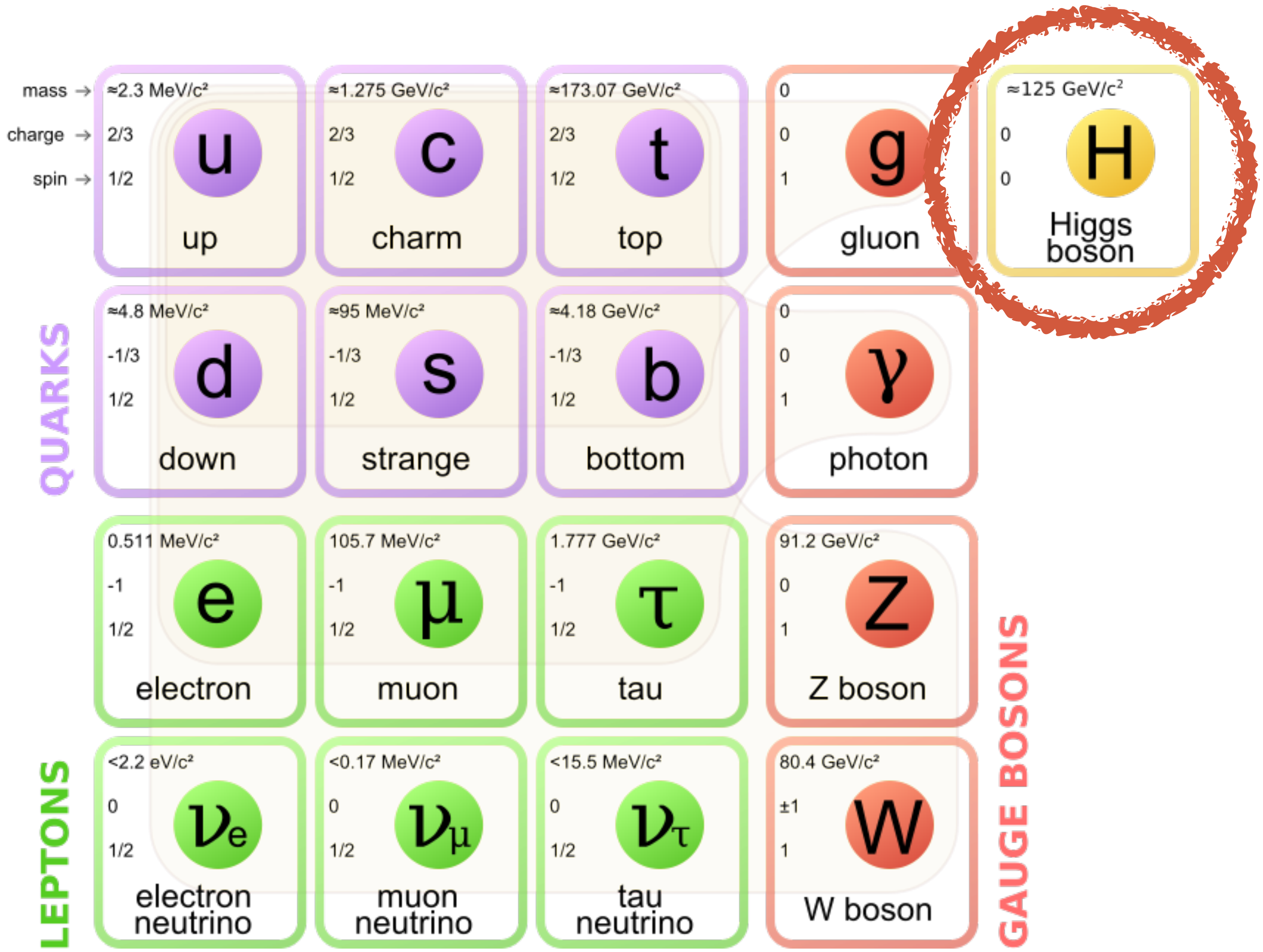
Technische Universität München

THE STANDARD MODEL AFTER THE HIGGS DISCOVERY



Higgs boson discovery in 2012 @ the LHC

Now the Standard Model is Complete!

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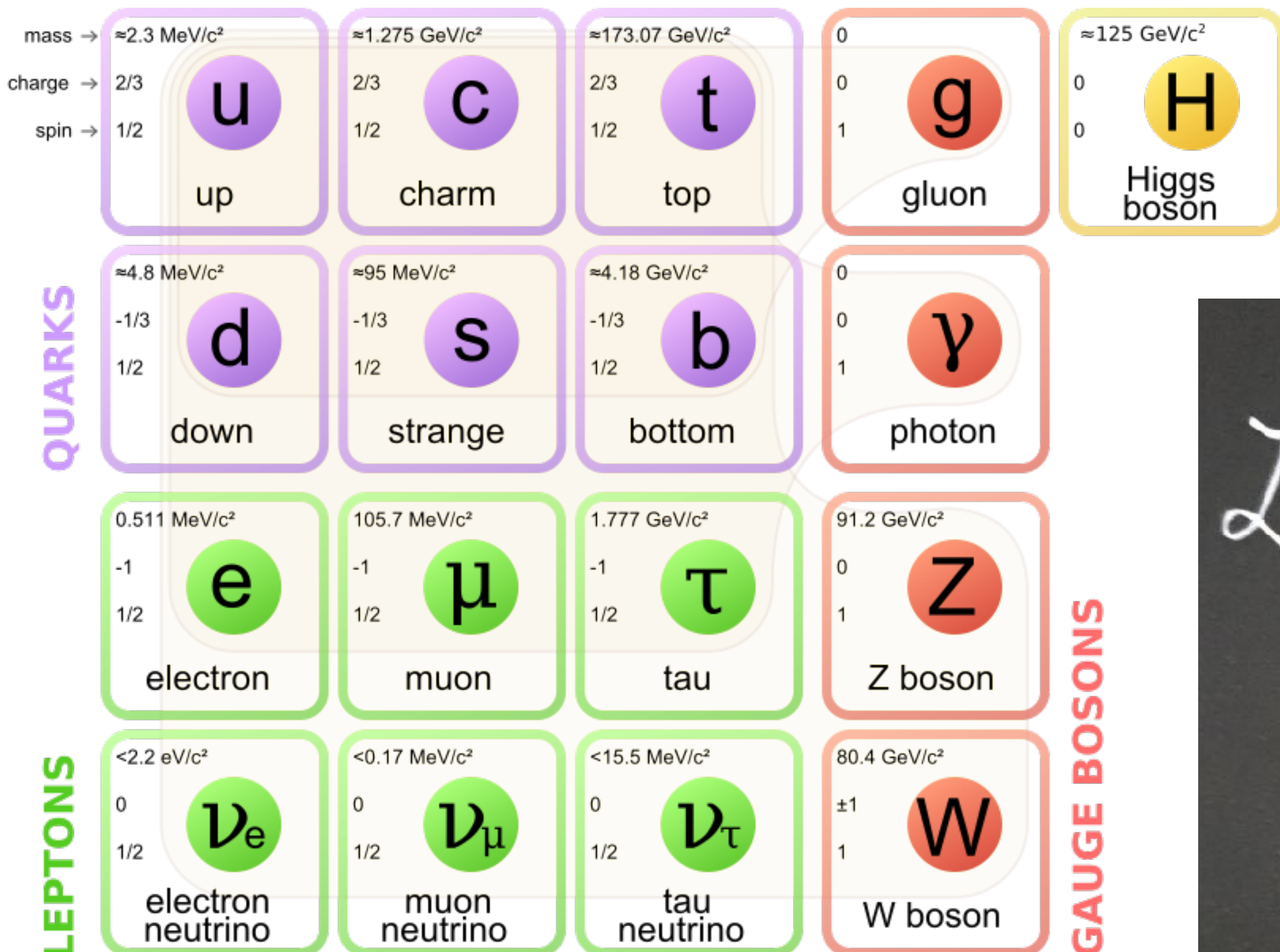
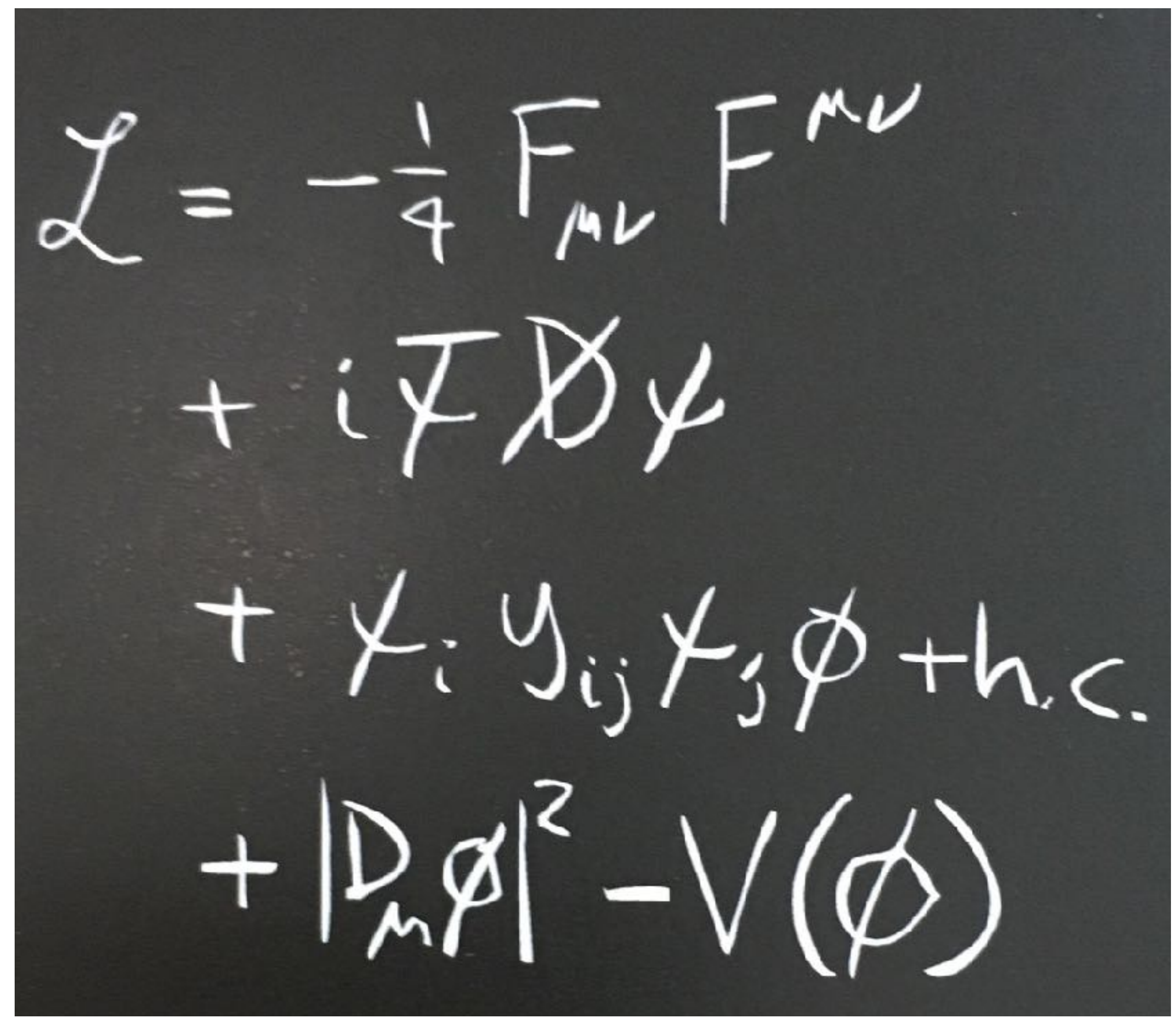


Interactions!



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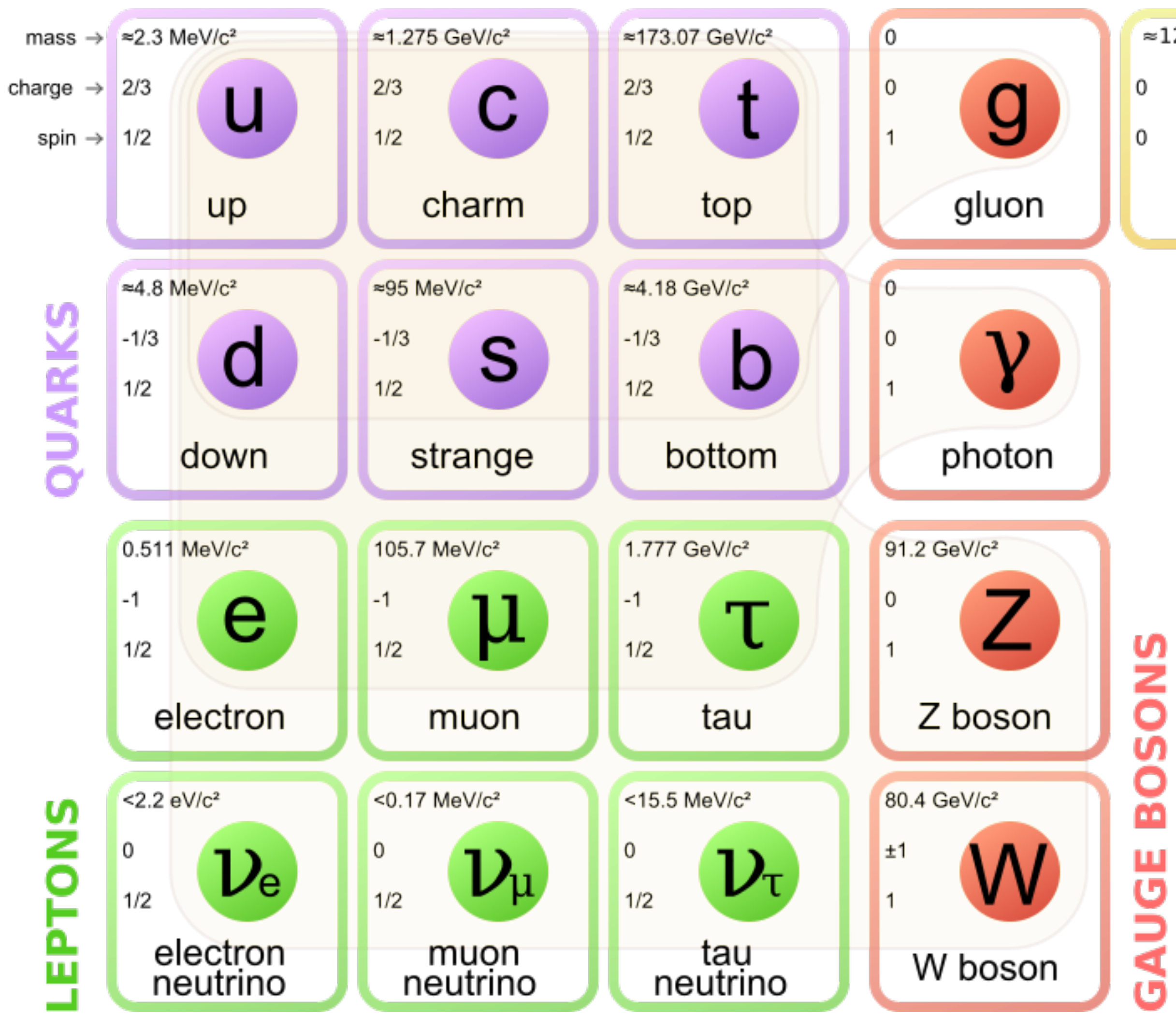

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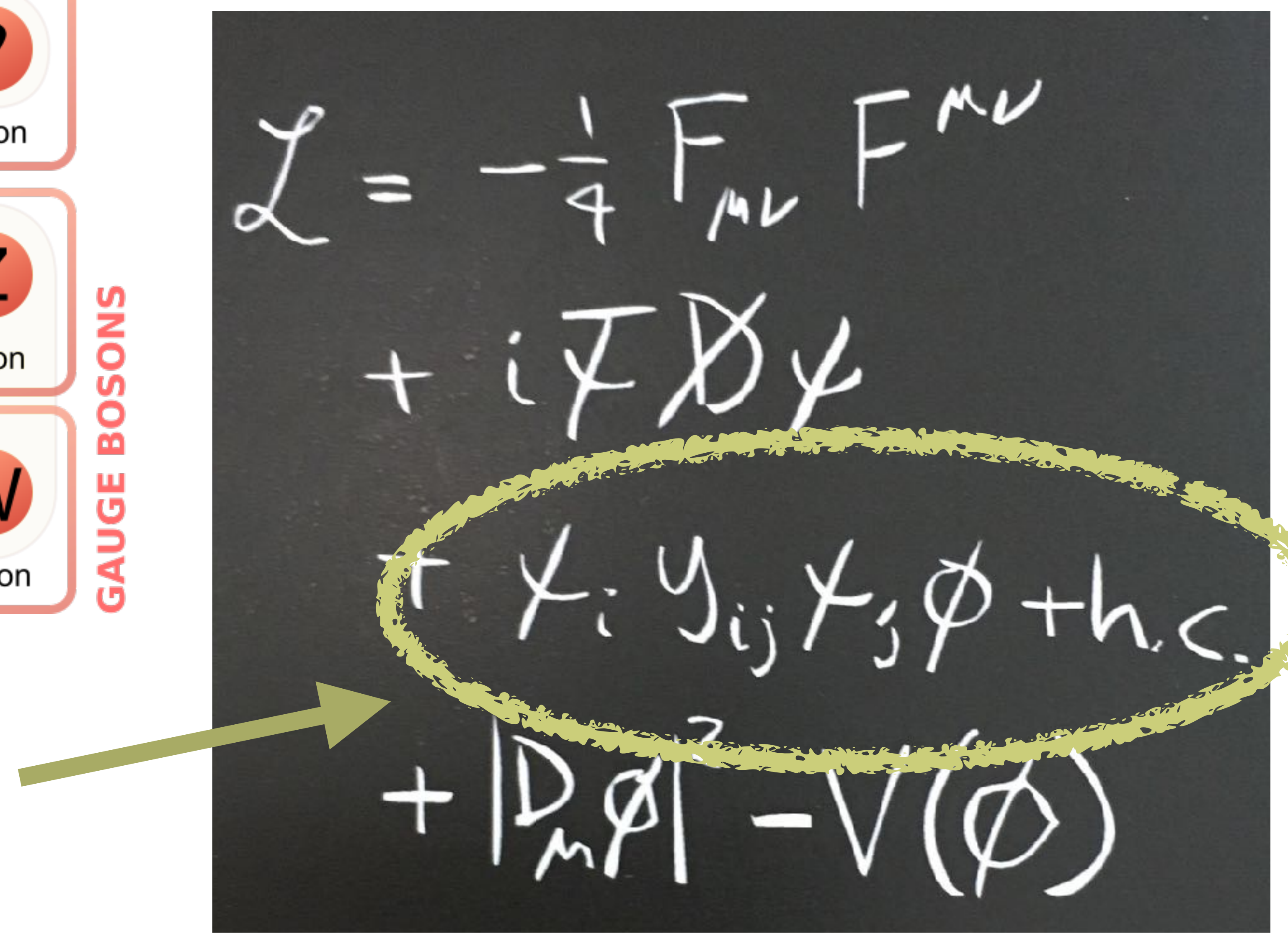
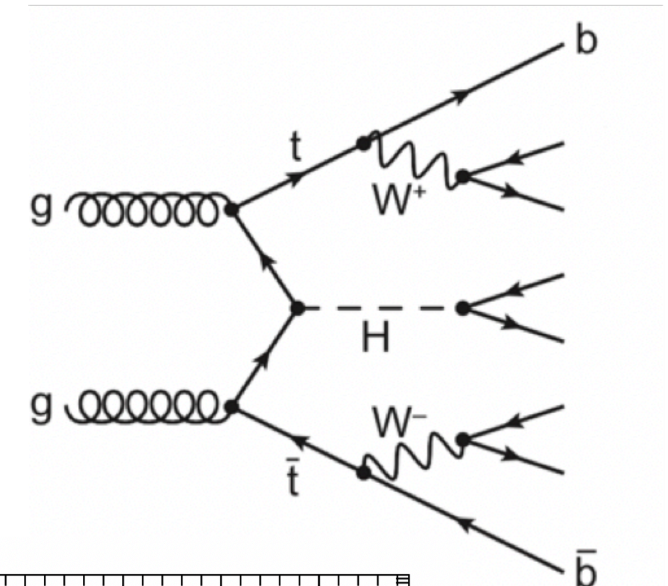


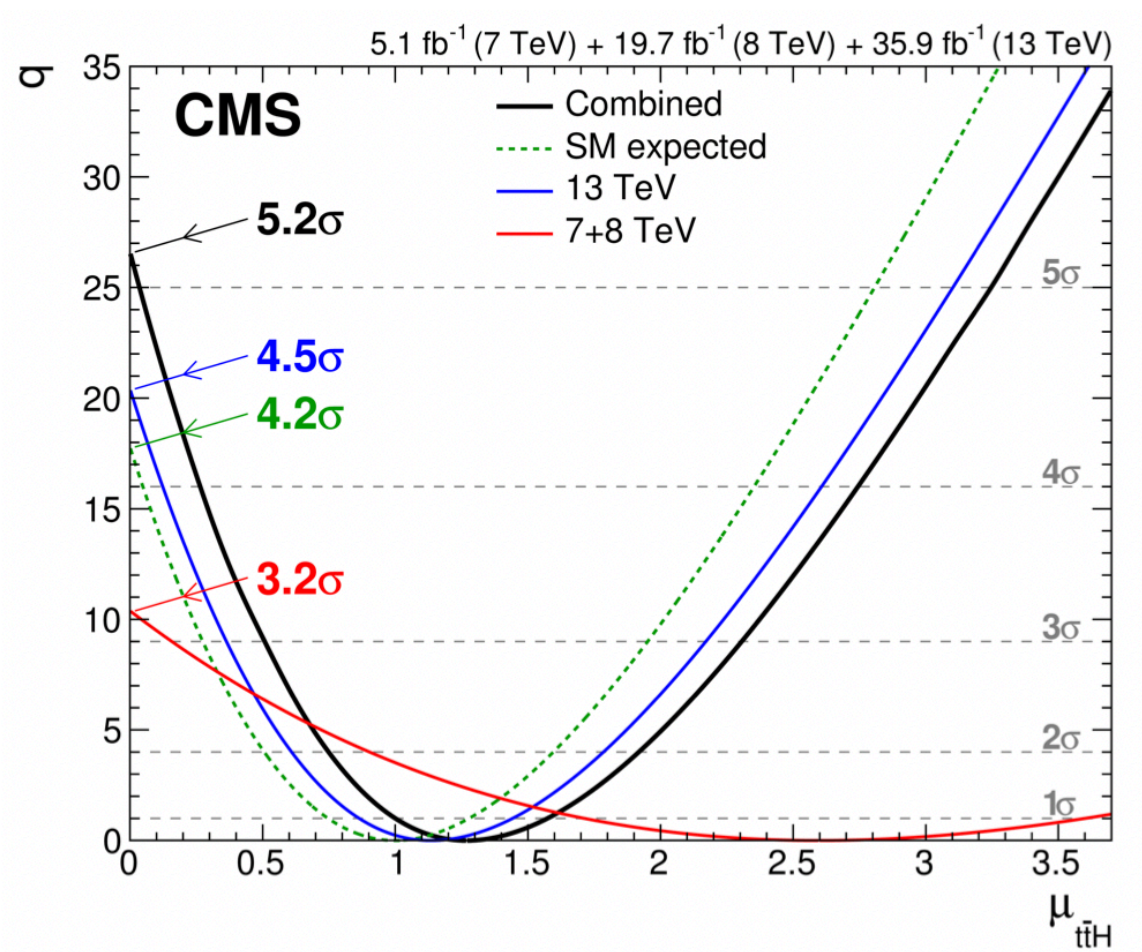
In particular the Yukawas!

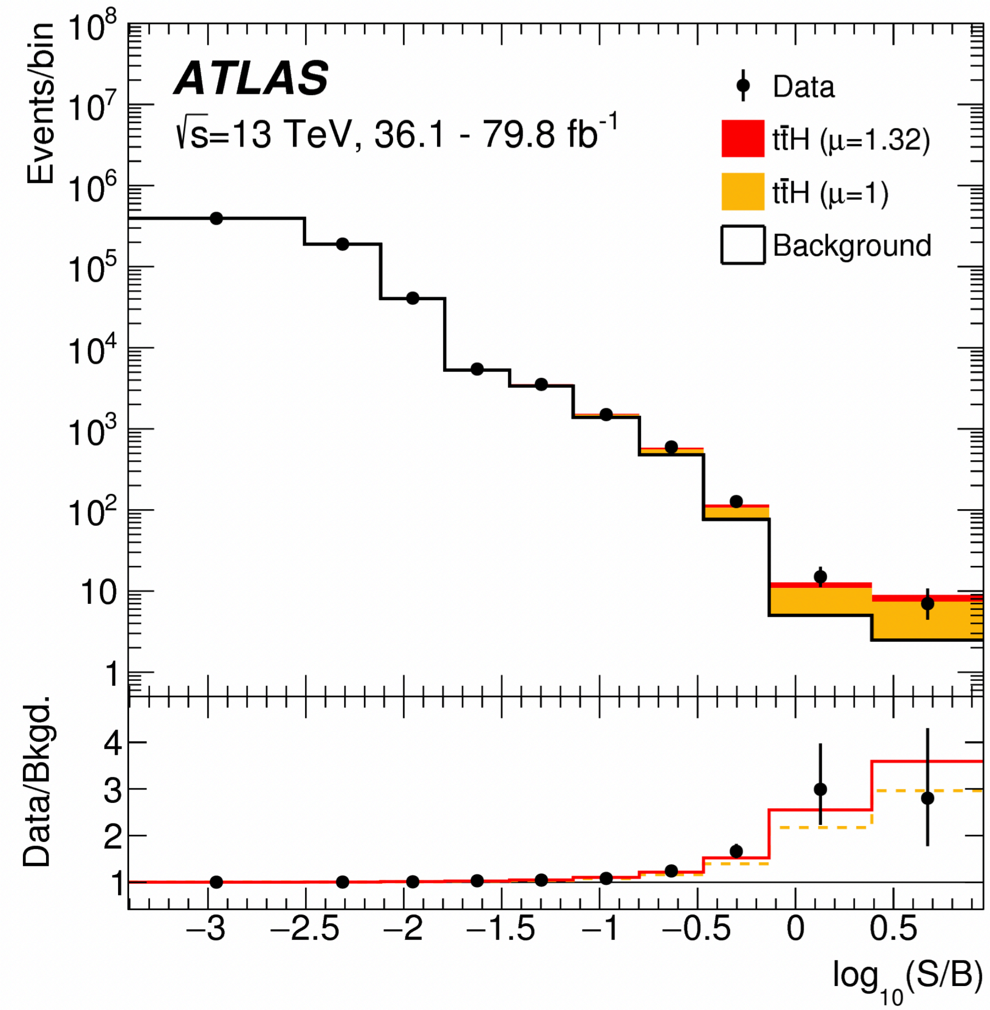
THE NEED OF PRECISION

The LHC is the first machine able to probe these couplings!



First direct observation of H coupling to quarks, $t\bar{t}H$ @ **LHC**

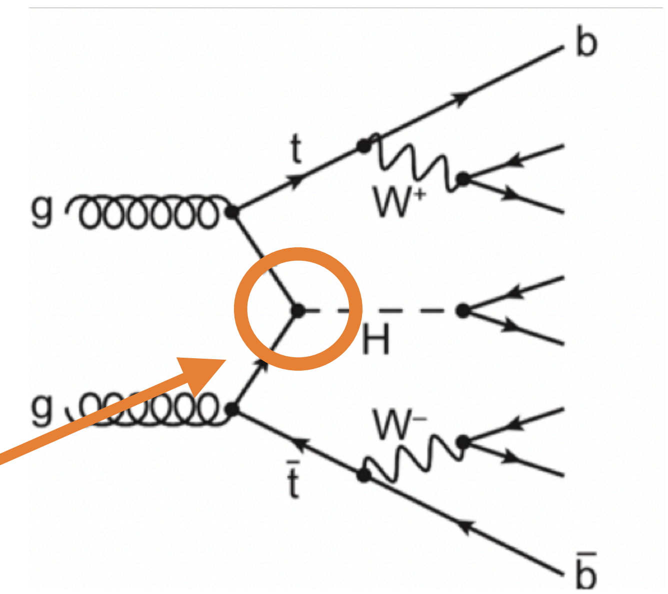




THE NEED OF PRECISION

We have just discovered a new fundamental interaction!

A lot of space for improvement in coming years



For Example, from CMS 1804:02610

		Uncertainty			
Parameter	Best fit	Stat	Expt	Thbgd	Thsig
$\mu_{t\bar{t}H}$	$1.26^{+0.31}_{-0.26}$	$^{+0.16}_{-0.16}$	$^{+0.17}_{-0.15}$	$^{+0.14}_{-0.13}$	$^{+0.15}_{-0.07}$

1. Theory uncertainty ~ statistical and experimental uncertainty ~ 15-20%
2. Statistical error could go down of a factor of 6 at HL-LHC ~ 2-5%

PRECISION PHYSICS AT THE LHC: HOW FAR CAN WE GO?

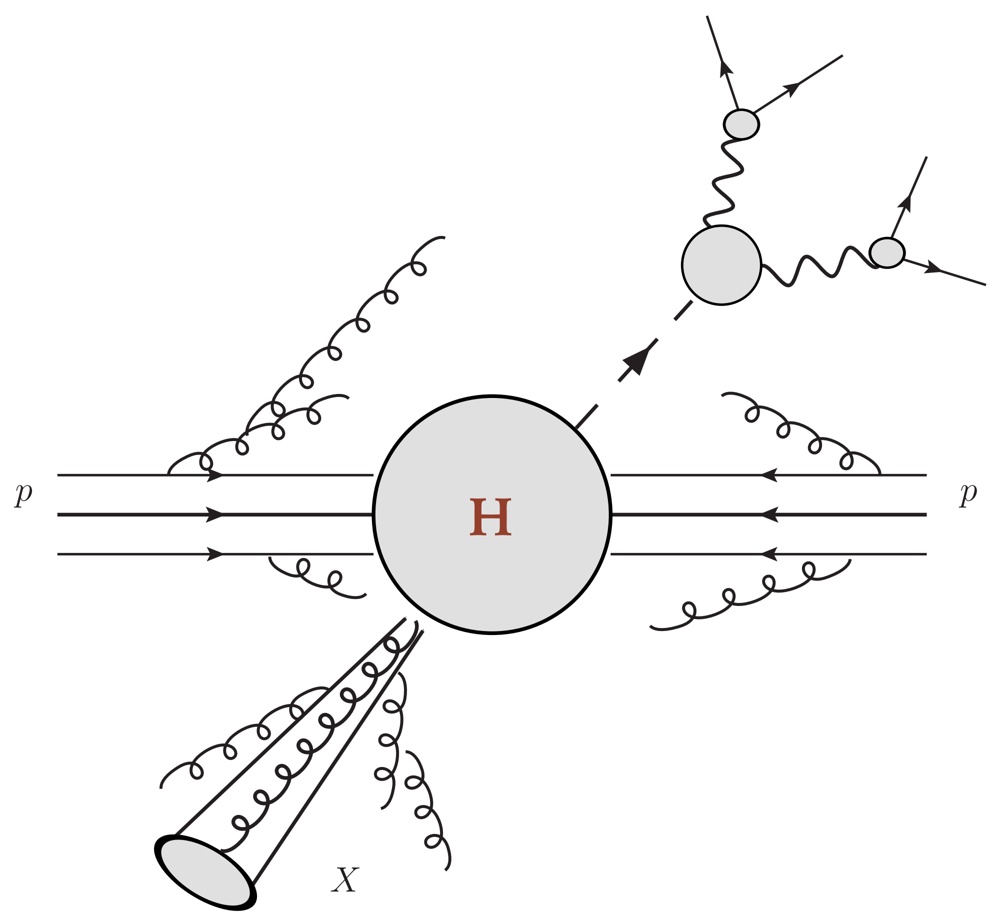
Reaching these precisions @ the LHC, is no piece of cake: **QCD is complicated**…!

$$pp \to HX \to l_1\bar{l}_1 + l_2\bar{l}_2 + X$$

Factorisation of long and short range physics

Non perturbative corrections

$\mathcal{O}\left(\frac{\Lambda_{QCD}}{Q}\right) \sim$ few percent?



Precise determination of **parton content of proton**

PDFs Currently known at level ~ **few % for LHC**

Parton Showers

Hadronisation

Detector Simulation

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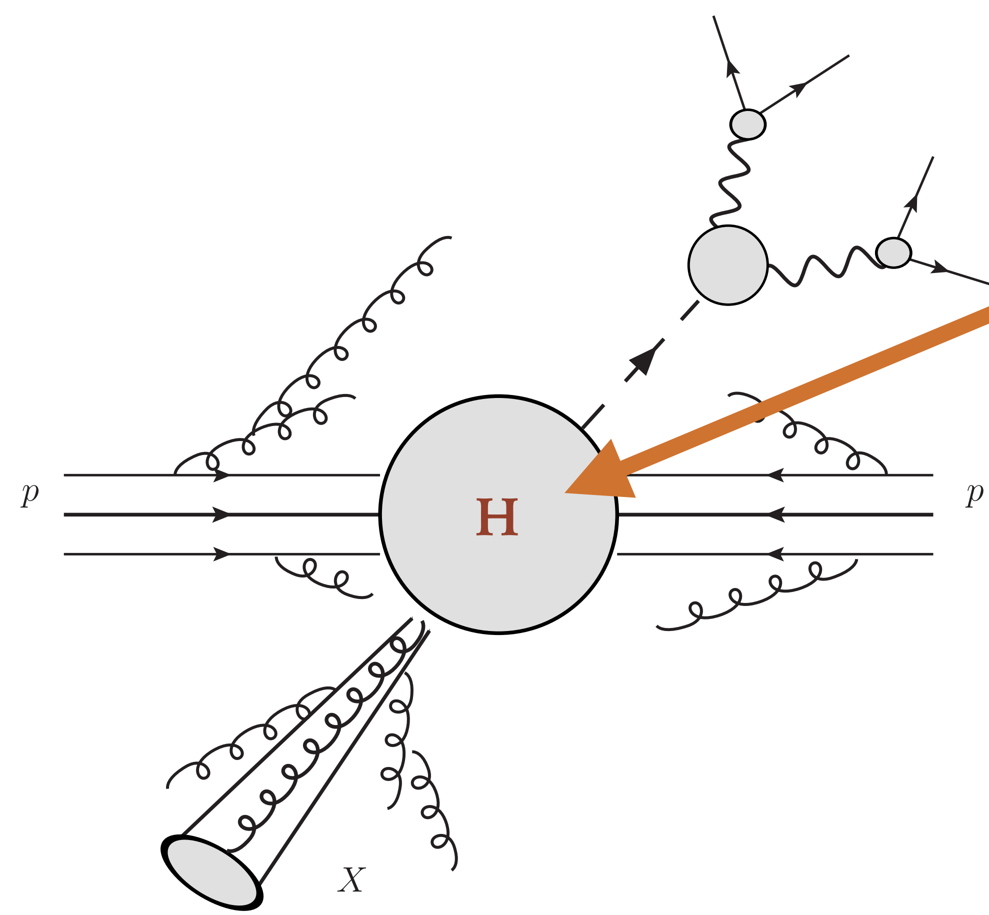

HARD SCATTERING

Aim to $\sim$ % precision

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FIXED ORDER CALCULATIONS

$$\sigma_{q\bar{q}\to gg} = \int [\mathrm{dPS}] \left|\mathcal{M}_{q\bar{q}\to gg}\right|^2$$

$$\left|\mathcal{M}_{q\bar{q}\to gg}\right|^2 = \left|\mathcal{M}^{LO}_{q\bar{q}\to gg}\right|^2 + \left(\frac{\alpha_s}{2\pi}\right)\left|\mathcal{M}^{NLO}_{q\bar{q}\to gg}\right|^2 + \left(\frac{\alpha_s}{2\pi}\right)^2\left|\mathcal{M}^{NNLO}_{q\bar{q}\to gg}\right|^2 + \ldots$$

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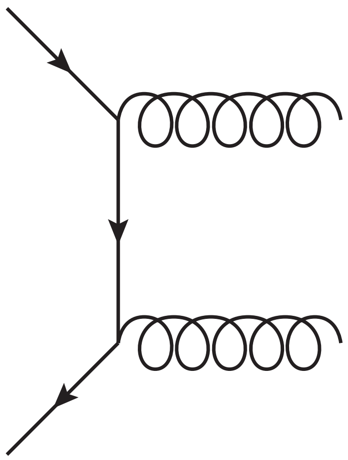

~ O(100%-50%)
precision

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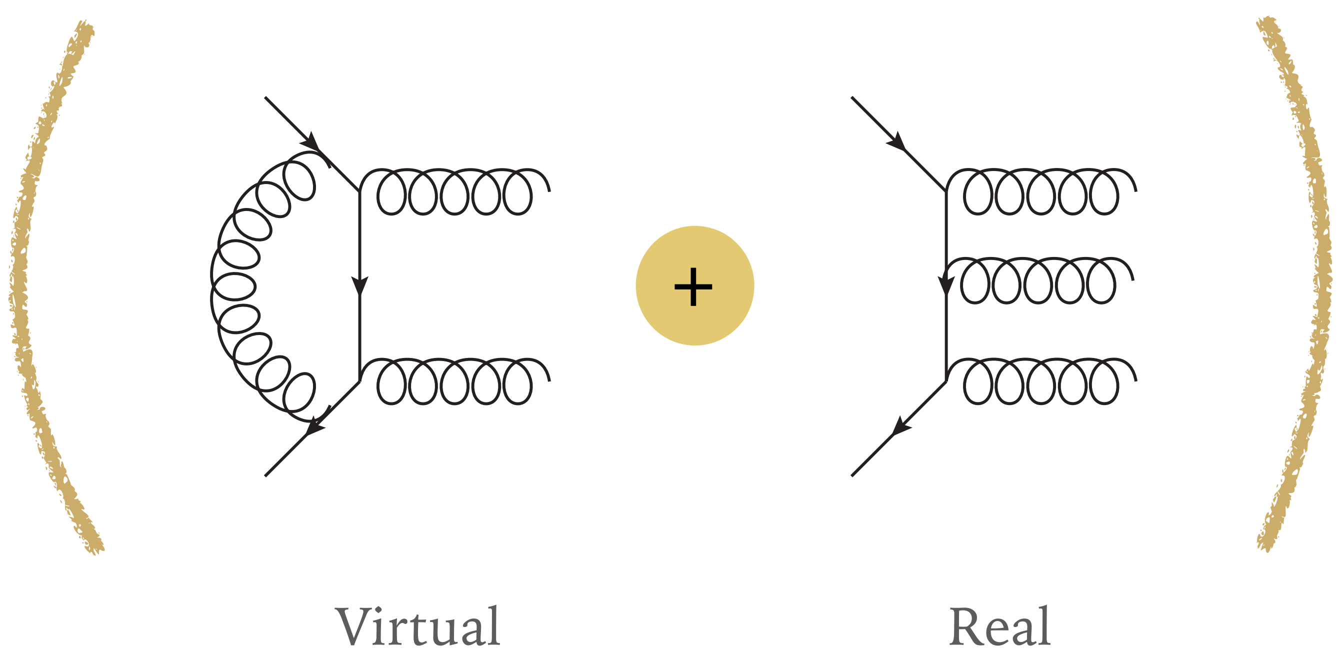


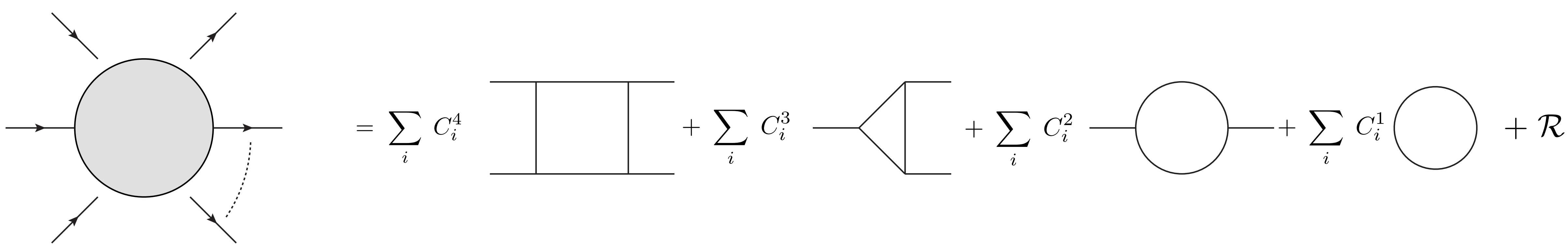
~ O(30%-10%) precision

THE NLO REVOLUTION (ONE-LOOP VIRTUAL AMPLITUDES)

Unitarity @ 1 loop

[Ossola, Papadopoulos, Pittau, '04]
[Bern Dixon, Kosover, '05]
[Ellis, Kunszt, Melnikov, Zanderighi, '08]

Every 1 loop amplitude can be decomposed in **boxes**, **triangles**, **bubbles** and **tadpoles**



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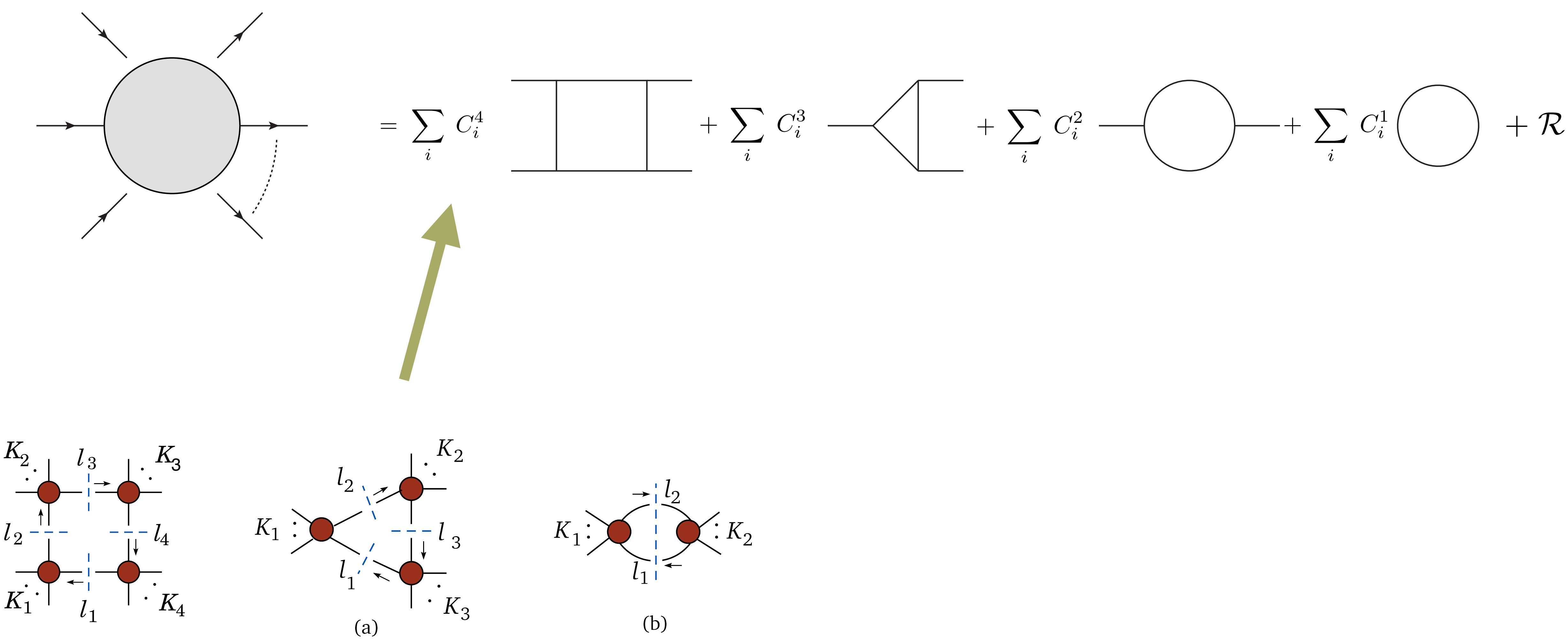

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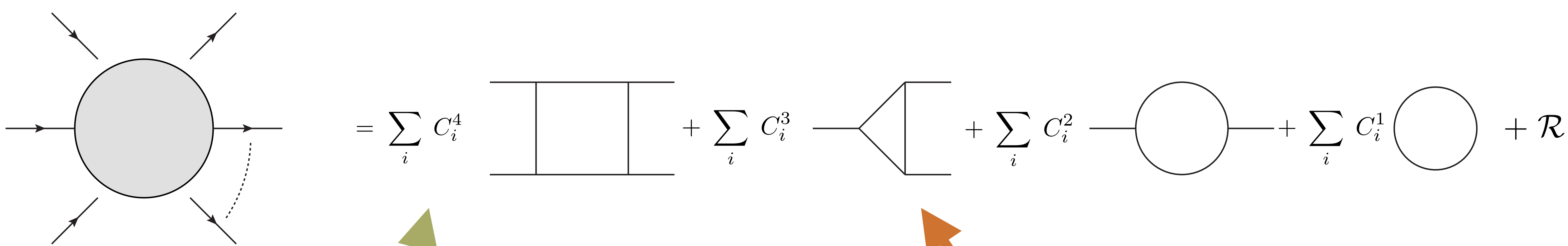

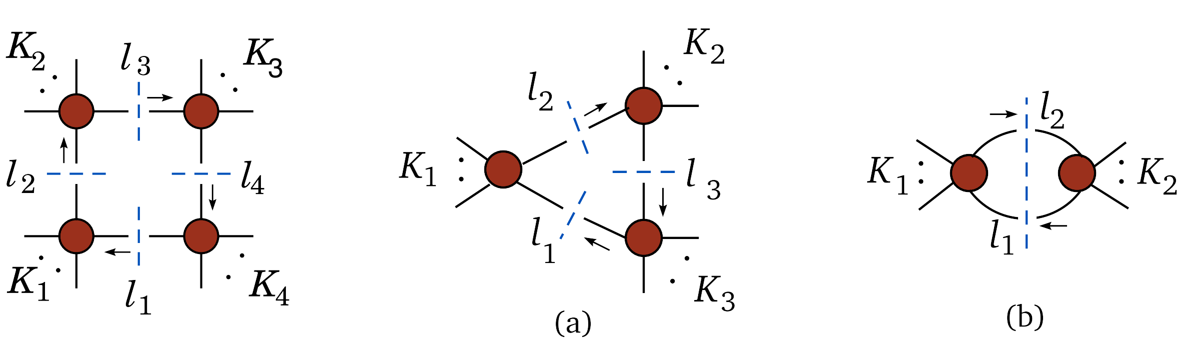

All **Master integrals** known analytically in terms of simple functions:

logarithms, **di-logarithms**

$$\mathrm{Li}_2(x) = -\int_0^x \frac{\ln(1-t)}{t}\,dt$$

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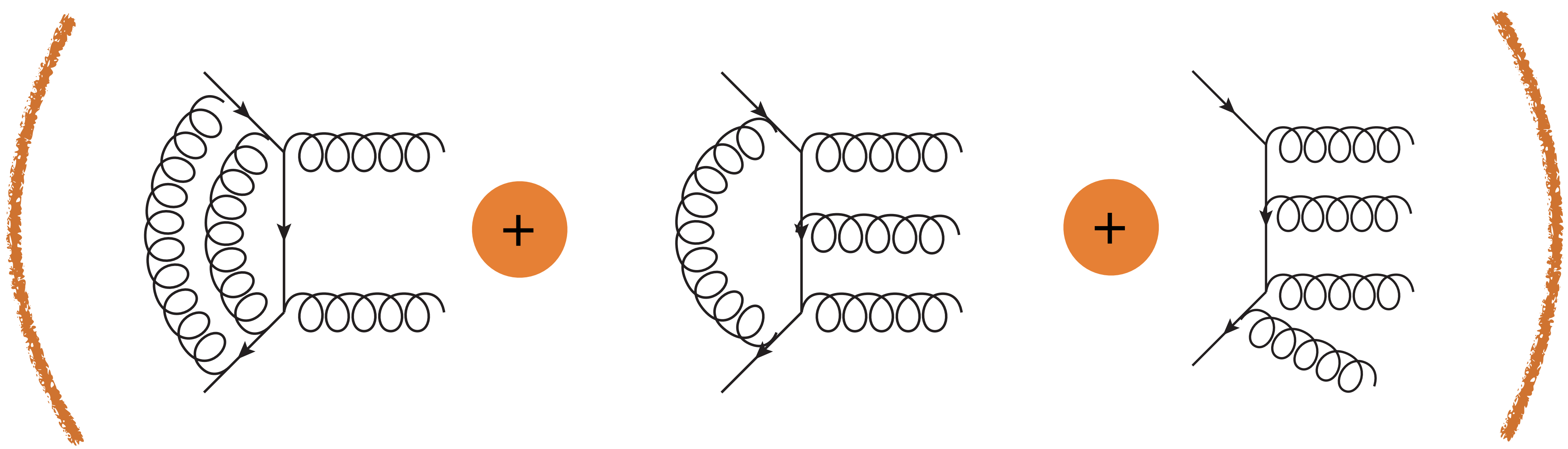

Double Virtual   Real Virtual   Double Real

~ O(5%) precision

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In the last two decades, effort dedicated to understand **two-loop scattering amplitudes**

Together with development of **IR subtraction schemes**

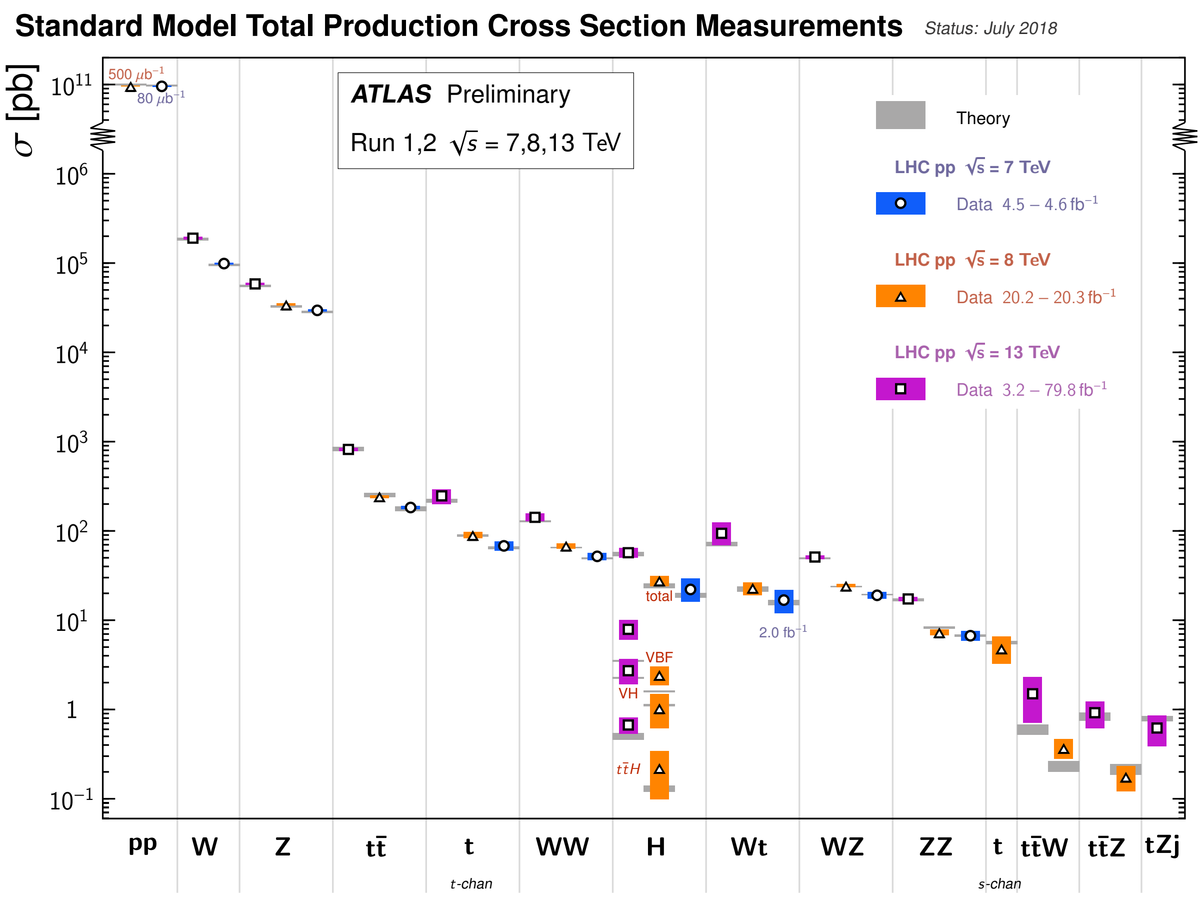
[Antennas, Stripper, ColorfulNNLO, Sector Improved, analytic sector subtraction,…]

Goal: **breaking the NNLO frontier for 2 → 2 processes**

In parallel, first impressive results for $\mathrm{N^3LO}$ **for 2 → 1** (Higgs and Drell-Yan)

WHAT HAVE WE LEARNED?

phenomenology and SM physics



Rediscovered the SM, **discovered the Higgs**, and started doing **precision physics**

Vector bosons, top quarks, Higgs couplings, jets, heavy flavours…

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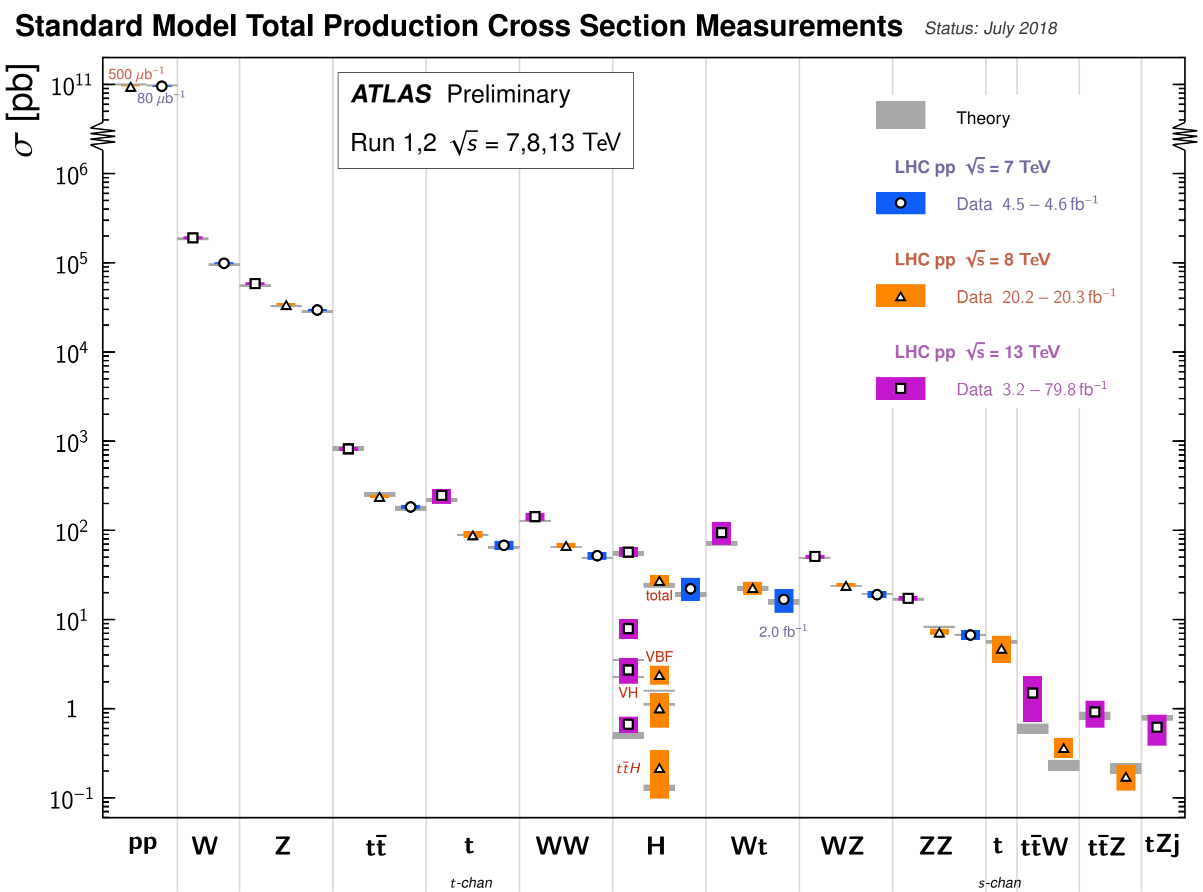

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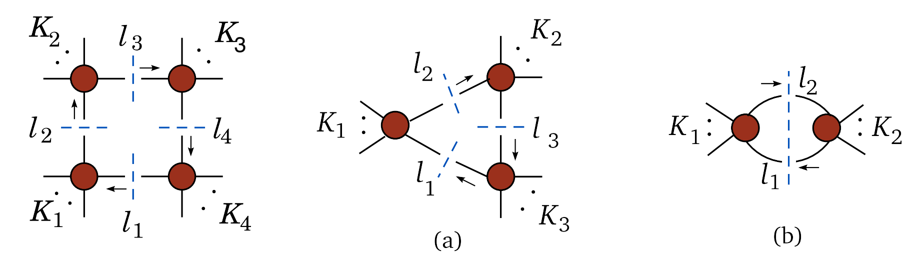
formal developments

structure of scattering amplitudes:

- unitarity, recursion relations, spinor helicity, color ordering, IBPs, DEs,…



Special functions in pQFT:

- connections with algebraic geometry and number theory, polylogs, elliptic stuff, CYs, iterated integrals…

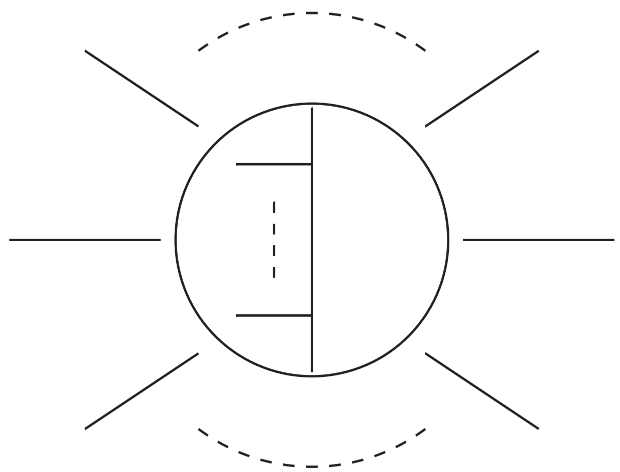
$$G(c_1, c_2, ..., c_n, x) = \int_0^x \frac{dt_1}{t_1 - c_1} G(c_2, ..., c_n, t_1)$$
$$= \int_0^x \frac{dt_1}{t_1 - c_1} \int_0^{t_1} \frac{dt_2}{t_2 - c_2} ... \int_0^{t_{n-1}} \frac{dt_n}{t_n - c_n}$$

IR divergences, factorisation in QCD, resummation, effective field theory…

“NEW” IDEAS TO SOLVE “OLD” PROBLEMS

PROBLEMS WITH PERTURBATIVE CALCULATIONS

To address typical calculation: **standard approach *(divide et impera)***



$$= \int \prod_{i=1}^{L} d^D k_i \; R_i(k_1, \ldots, k_L, p_1, \ldots, p_E, m_j)$$

$$= \sum_{i=1}^{N} R_i(x_1, \ldots, x_r)\, \mathcal{I}_i(x_1, \ldots, x_n)$$

Achieved in general by **integration-by-part identities**

[Chetyrkin, Tkachov '81]
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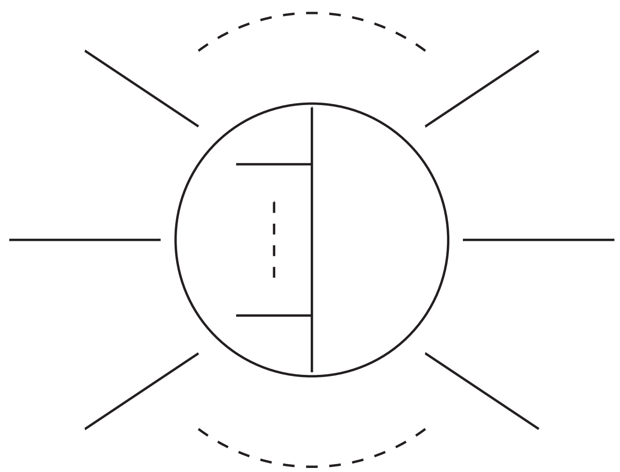

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Algebraic Complexity

Rational coefficients, factorial increase with number of particles and perturbative order

Analytic complexity

Master Integrals, Unitarity and Causality determine **analytic structure** of amplitude

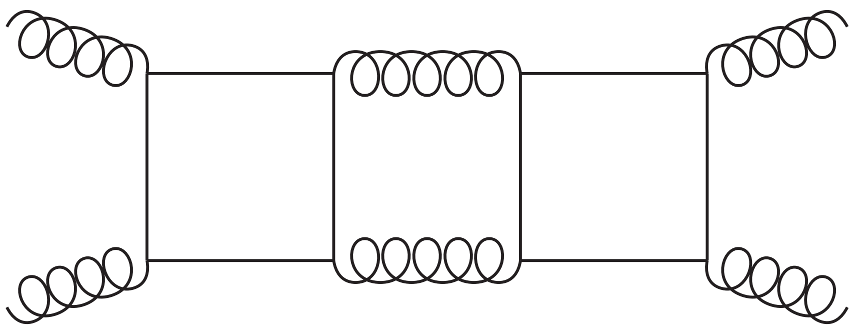
ALGEBRAIC COMPLEXITY

Tends to become **overwhelming** for

1) $2 \to 3$ at 2 loops

2) $2 \to 2$ at 3 loops

3) For massive internal/external particles



- Number of Feynman diagrams explodes: $gg \to gg$ **@ 3 loops** $\sim$ **50k Feyn. diag.**
- Each diagram produces ***thousands of terms***, compute $\sim \mathcal{O}(10^6)$ **integrals**

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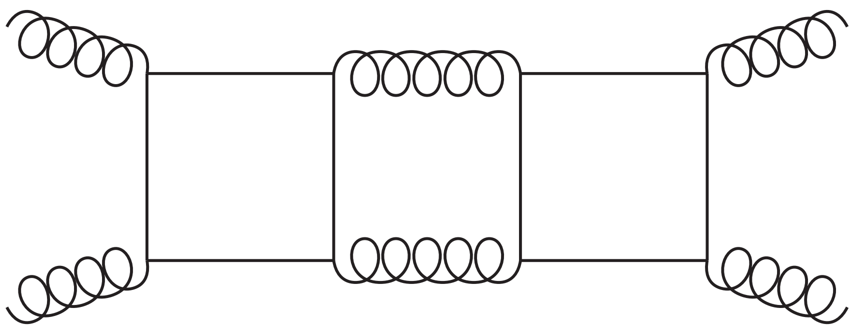

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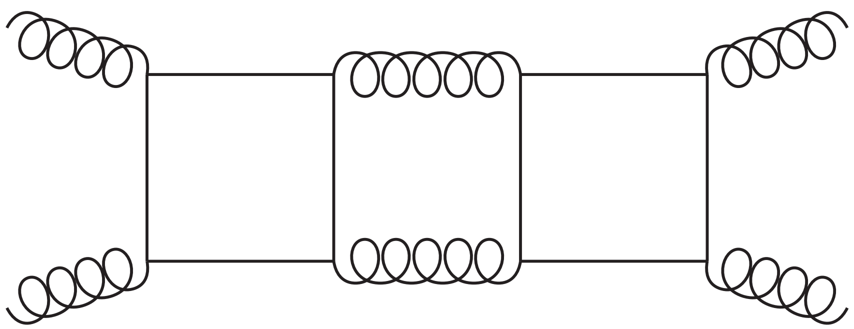

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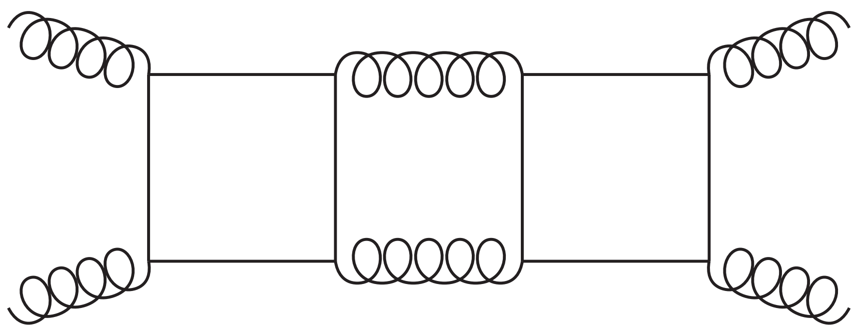

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Extract **helicity amplitudes** from **Feynman diagrams** through **"projector operators"** in *Conventional Dimensional Regularisation*

- **Cumbersome for** $2 \to 3$ **or multiple fermion lines (ex.** $q\bar{q} \to Q\bar{Q}$**)**
  [hundreds of projectors and form factors, spurious poles in $d \to 4$ *etc]*

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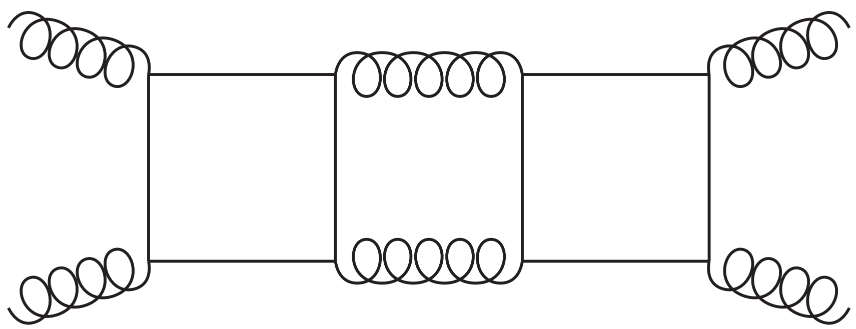

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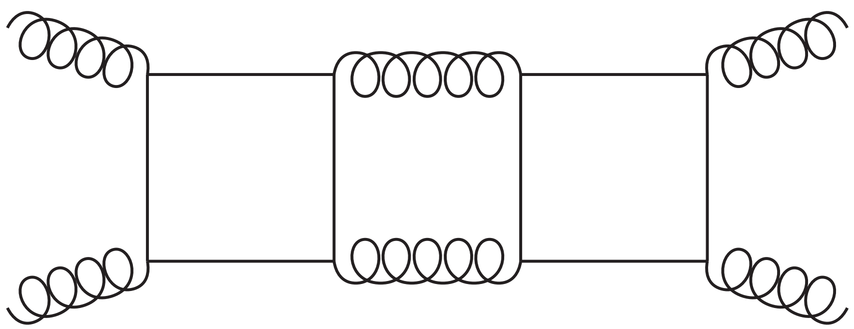

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- **Alternative:** integrand reduction [Mastrolia et al '10,…'16] [Badger et al '12,…'18]

ALGEBRAIC COMPLEXITY: DIAGRAMS

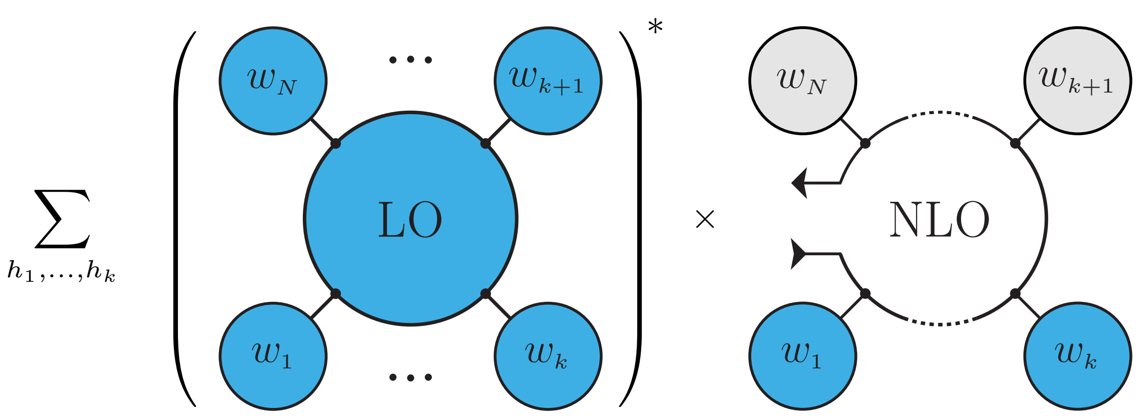
Modern techniques to handle # of Feynman Diagrams:

Alternative approaches

- attempts to **construct “integrand” iteratively**, generalising tree- and one-loop techniques

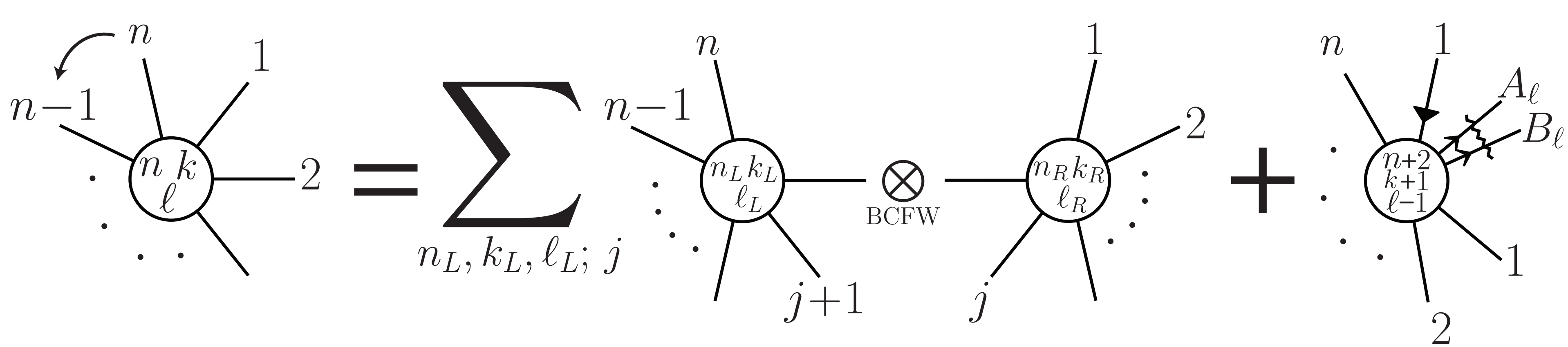
[Bern et al ’95; … BCFW ’05; …]

[Buccioni, Pozzorini, Zoller ’18]
[Lang, Pozzorini, Zhang, Zoller ’20, ’21]



- generalising **on-shell constructions** for **supersymmetric theories** (N=4 SUSY etc)

[Arkani-Hamed et al ’10; … Bourjaily et al ’20]



ALGEBRAIC COMPLEXITY: INTEGRALS

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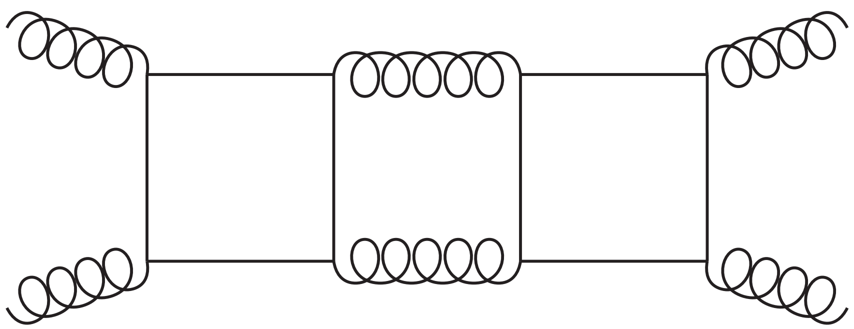

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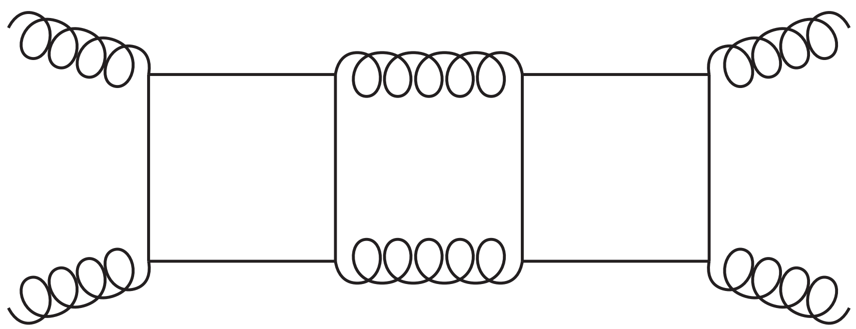

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"Reduction coefficients" R_i **extremely complicated.**

- Can be computed algorithmically solving IBPs [Laporta 2000]
- Each identity can be extremely complicated **[~GB]**
- *Final result for amplitude much simpler:* **just $\sim \mathcal{O}(500)$ integrals!**

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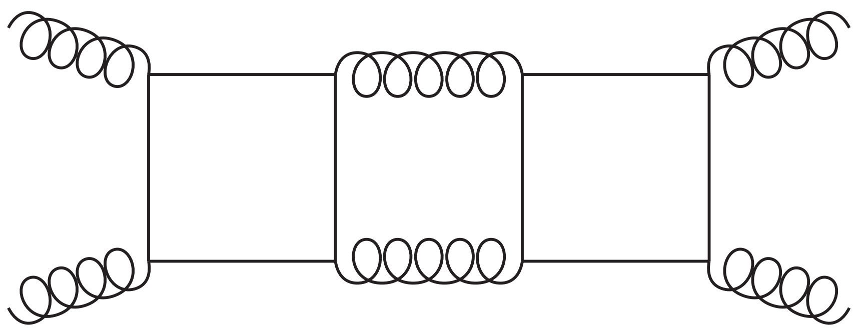

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Numerical methods (Finite Fields), avoid complexity in intermediate steps, reconstruct final result

[Manteuffel, Schabinger '14]
[Peraro '16, '19]

alternative representation for rational functions: multivariate partial-fractioning

[Remiddi,..., '99...] [Abreu et al '18] [Boehm, et al'20]
[Heller, Manteuffel '21]

NEW RESULTS @ TWO LOOPS / NNLO

New results for $2 \to 3$ scattering amplitudes @ 2 loop

Leading color $pp \to 3j$

Leading color $q\bar{q} \to \gamma\gamma\gamma$

Leading color $q\bar{q} \to \gamma\gamma g$

[Chawdhry, Czakon, Mitov, Ponchelet ’20]
[Abreu, Cordero, Ita, Page, Sotnikov ’17,…,’21]
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NEW RESULTS @ TWO LOOPS / NNLO

New results for $2 \to 3$ scattering amplitudes @ 2 loop

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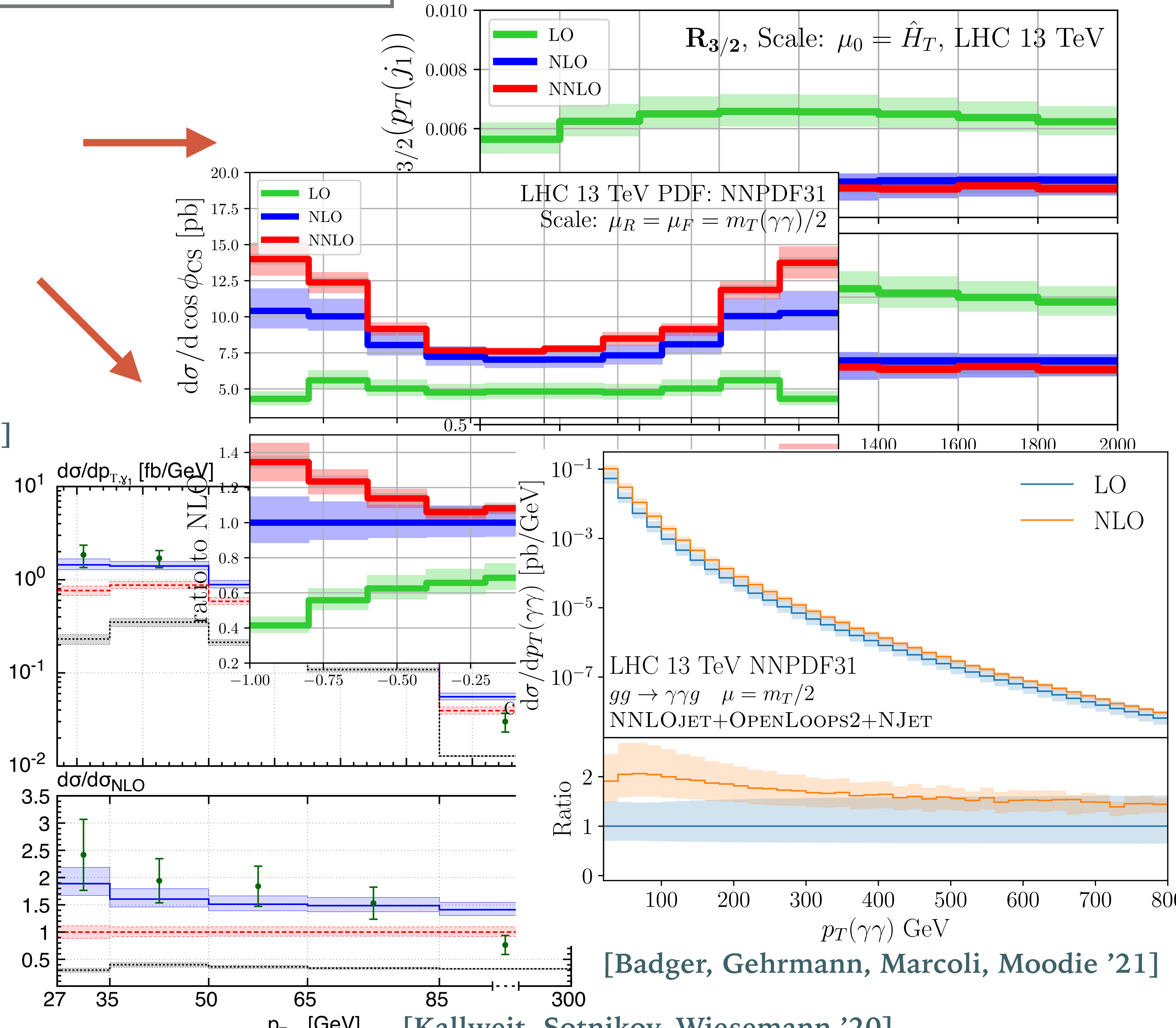

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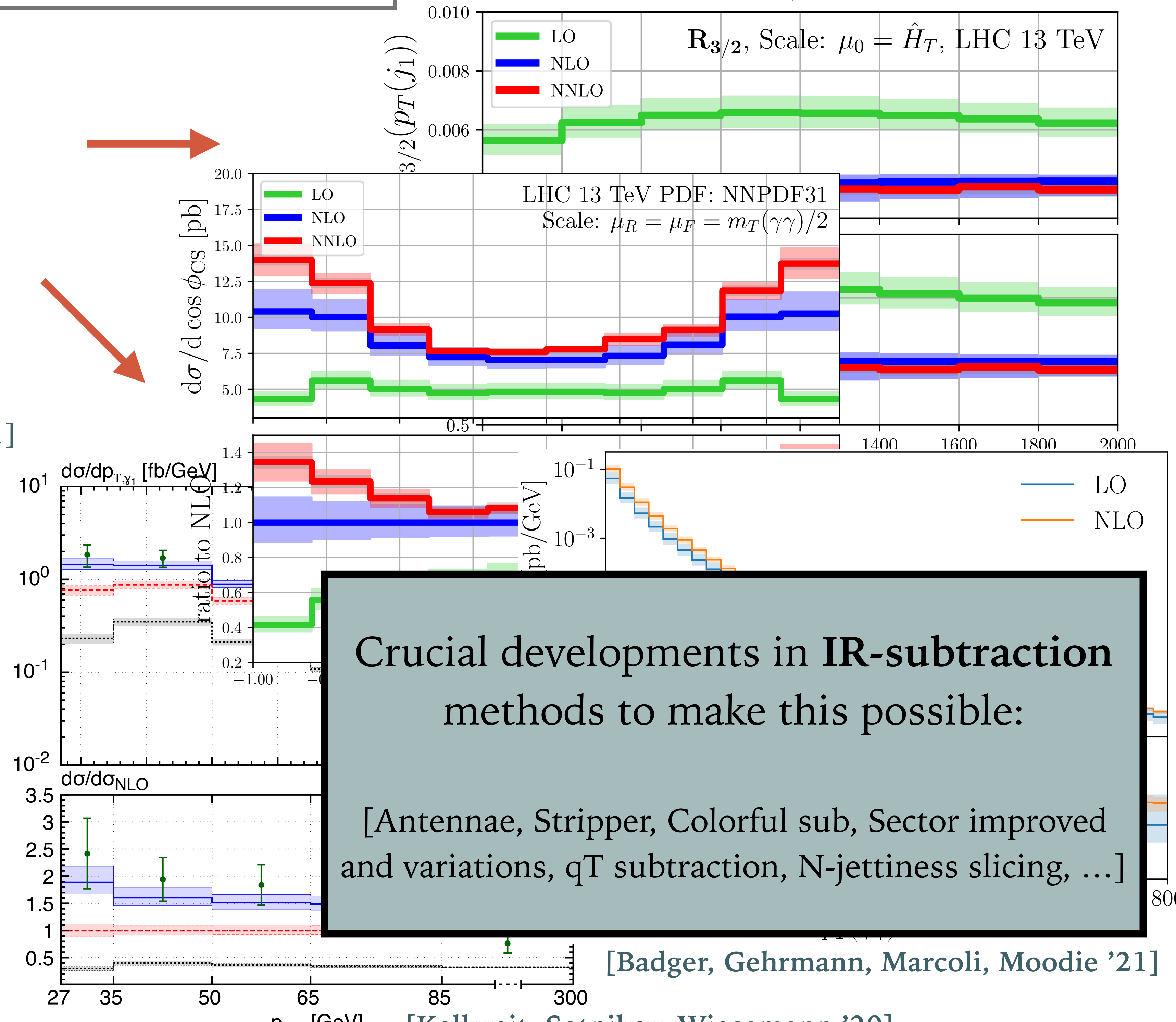

Crucial developments in **IR-subtraction** methods to make this possible:

[Antennae, Stripper, Colorful sub, Sector improved and variations, qT subtraction, N-jettiness slicing, …]

[Badger, Gehrmann, Marcoli, Moodie ʼ21]

[Kallweit, Sotnikov, Wiesemann ʼ20]

NEW RESULTS @ THREE LOOPS (TOWARDS N3LO)

- Subtraction methods not mature yet to address N3LO in full generality
- **Notable exceptions:**
  - Higgs Production [Anastasiou, Dulat, Duhr, Mistlberger et al '14,...,'20]
  - Drell Yan process (γ, $W^{\pm}$ mediated) [Dulat, Duhr, Mistlberger '19,...,'21]
  - VBF Higgs (Factorisable corrections) [Dreyer, Karlberg '16]

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($2 \to 1$ "special" does not require proper subtraction scheme)

Towards $2 \to 2$ @ N3LO

Recently progress on virtual **3 loop integrals** [Henn, Mistlberger, Wasser '19]

And **3 loop amplitudes**

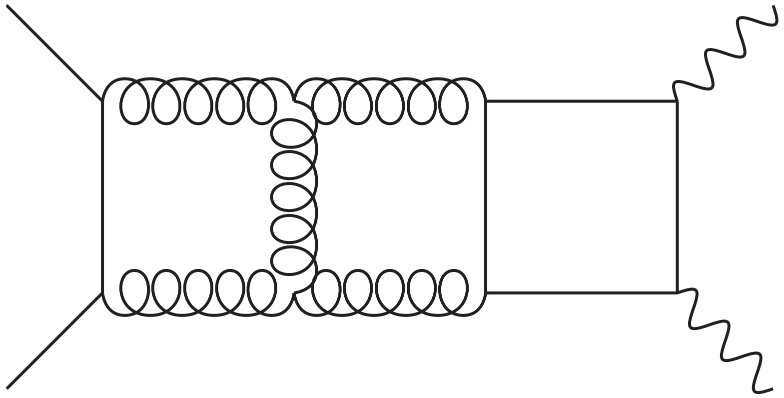
$q\bar{q} \to \gamma\gamma$ [Caola, Manteuffel, Tancredi '20]

$q\bar{q} \to Q\bar{Q}$ [Caola, Chakraborty, Gambuti, Manteuffel, Tancredi '21]

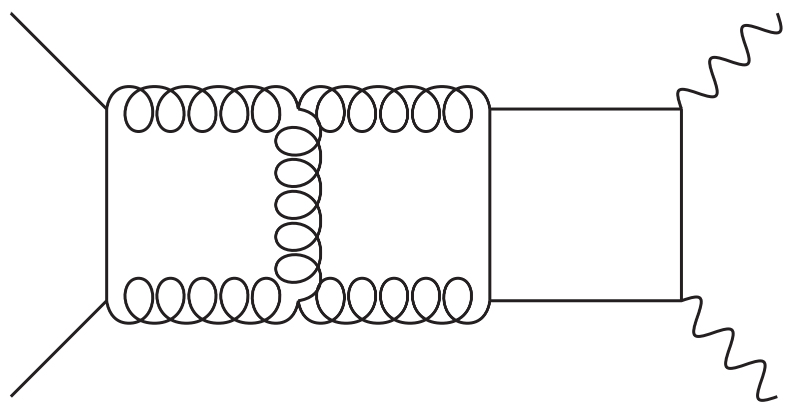
$gg \to \gamma\gamma$

$pp \to jj$

} [in the making...]



DIPHOTON PRODUCTION AT THREE LOOPS IN QCD



SCATTERING AMPLITUDES AT 3 LOOPS

Till recently, only results for 3 loop amplitudes in **SUSY** (N=4, N=8 SUGRA, etc..)

[Henn, Mistlberger '19,'20]

Diphoton is **simplest**, **non-trivial** place to start investigations of **three loop amplitudes in realistic theories as QCD**

$q\bar{q} \to \gamma\gamma$ non trivial for various reasons:

- Relatively large number of Feynman diagrams (~3000)
- Very non trivial IBP reduction needed *(rank-6 10 propagator NPL integrals)*

But still relatively simple

- *Simple functions:* 4 point massless @ 3 loops can be expressed in terms of **HPLs** [Henn, Mistlberger, Smirnov, Wasser '19]
- simpler color correlations and simpler IR structure than, say, $gg \to gg$

DI-PHOTON AS OF TODAY

The **production of two photons** has received lots of attention [Anastasiou et al ’00; Bern et al ’00,’01,’03; Glover et al. ’00,’01,’03, …]

- One- and Two-loop scattering amplitudes known for 20 years

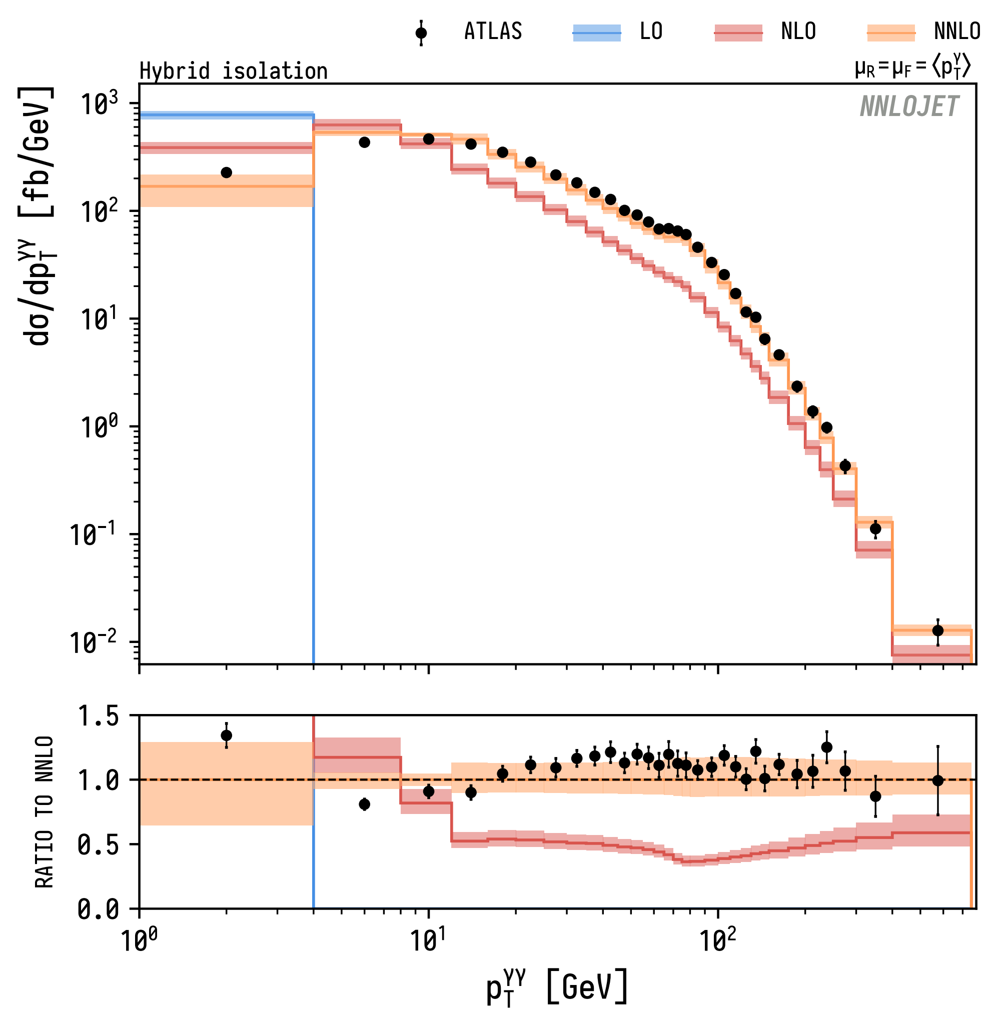
- NNLO *inclusive* and *recently exclusive* over final state radiation [Catani, et al ’11, ’13, Campbel et al ’16] [Chawdhry et al ’21]

- Various degrees of sophistication (resummation, PS, etc) [Alioli, et al ’10 …] [Gehrmann et al ’20]

Important background for **Higgs** + **New Physics**

<u>Clean final state</u>, <u>high production rate</u>, etc

Interesting theory/exp questions: (IR sensitivity cone isolation…) [Gehrmann et al ’20]



PUSHING UP TO THREE LOOPS

Consider the production of 2 photons in quark-antiquark annihilation

$$q(p_1) + \bar{q}(p_2) \to \gamma(p_3) + \gamma(p_4)\,, \quad \text{with} \quad p_i^2 = 0$$

$$s = (p_1 + p_2)^2\,, \quad t = (p_1 - p_3)^2\,, \quad \text{and} \quad x = -t/s \quad \longrightarrow \quad s > 0\,, \quad t < 0 \quad 0 < x < 1$$

Three-loop helicity amplitudes can be written, schematically in spinor helicity as

$$\mathcal{A}_{L--} = \frac{2[34]^2}{\langle 13\rangle[23]}\alpha(x)\,, \quad \mathcal{A}_{L-+} = \frac{2\langle 24\rangle[13]}{\langle 23\rangle[24]}\beta(x)\,,$$

$$\mathcal{A}_{L+-} = \frac{2\langle 23\rangle[41]}{\langle 24\rangle[32]}\gamma(x)\,, \quad \mathcal{A}_{L++} = \frac{2\langle 34\rangle^2}{\langle 31\rangle[23]}\delta(x)\,.$$

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Can be written in terms of simple functions

$$\mathrm{Li}_{3,2}(x,1),\ \mathrm{Li}_{3,2}(1-x,1),\ \mathrm{Li}_{3,2}(1,x),$$
$$\mathrm{Li}_{3,3}(x,1),\ \mathrm{Li}_{3,3}(1-x,1),\ \mathrm{Li}_{3,3}(x/(x-1),1),$$
$$\mathrm{Li}_{4,2}(x,1),\ \mathrm{Li}_{4,2}(1-x,1),\ \mathrm{Li}_{2,2,2}(x,1,1),$$

NUMERICAL RESULTS

Numerical evaluation of these functions is very well understood

$$\mathrm{Li}_{3,2}(x,1),\ \mathrm{Li}_{3,2}(1-x,1),\ \mathrm{Li}_{3,2}(1,x),$$
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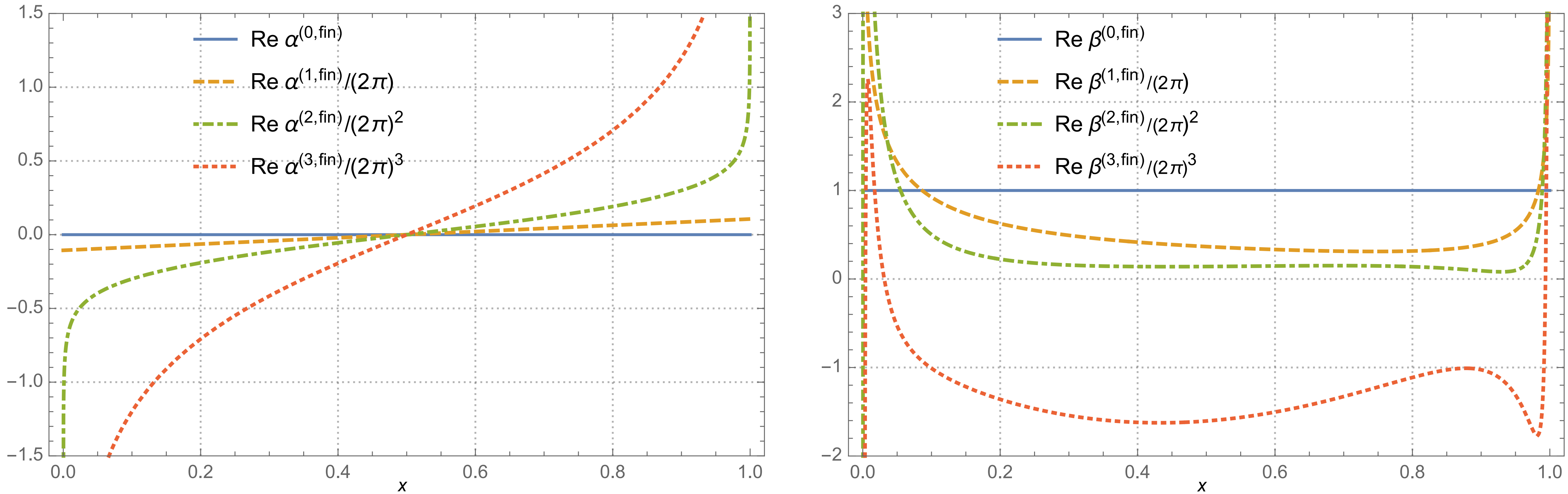
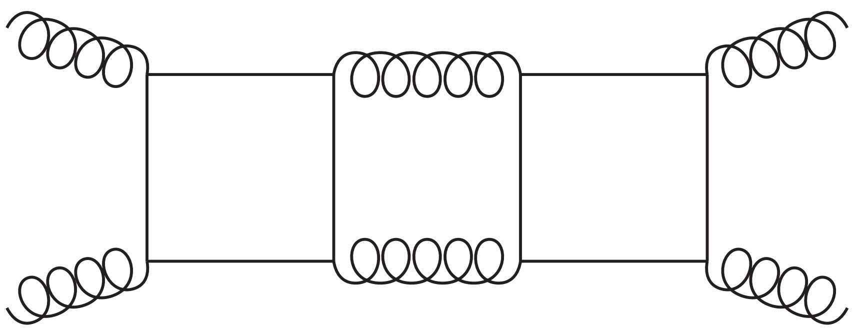


Crucial ingredient: understanding the integrals and the analytic structure of the functions involved!

ANALYTIC COMPLEXITY

ANALYTIC COMPLEXITY

Feynman integrals are building blocks of analytic complexity



?

$$= \sum_{i=1}^{N} R_i(x_1, \ldots, x_r)\, \mathcal{I}_i(x_1, \ldots, x_n)$$

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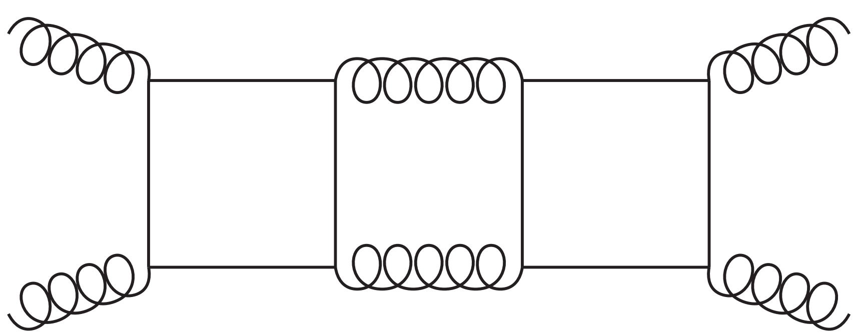

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$$= \sum_{i=1}^{N} R_i(x_1, \ldots, x_r)\, \mathcal{I}_i(x_1, \ldots, x_n)$$

Algebraic variety "broadly defined" by the Symanzik polynomials

$$\mathcal{I}(a_1, \ldots, a_n) = \frac{(-1)^{\omega+d}\Gamma(d/2)}{\Gamma((L+1)d/2-\omega)} \left(\prod_{k=1}^{n} \int_0^\infty \frac{x_k^{a_k-1} dx_k}{\Gamma(a_k)} \right) (\mathcal{U}+\mathcal{F})^{-d/2},$$

[Lee, Pomeransky '13]

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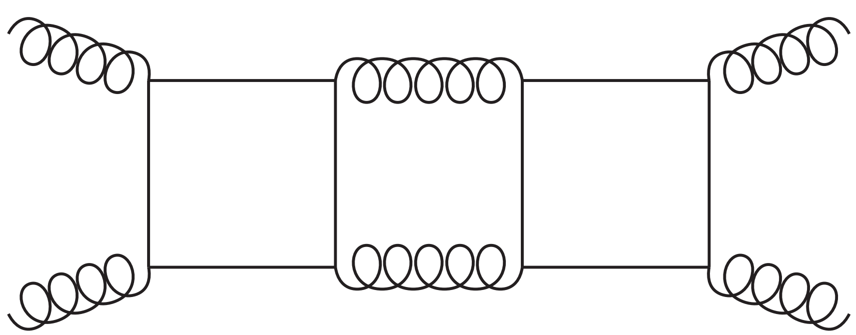
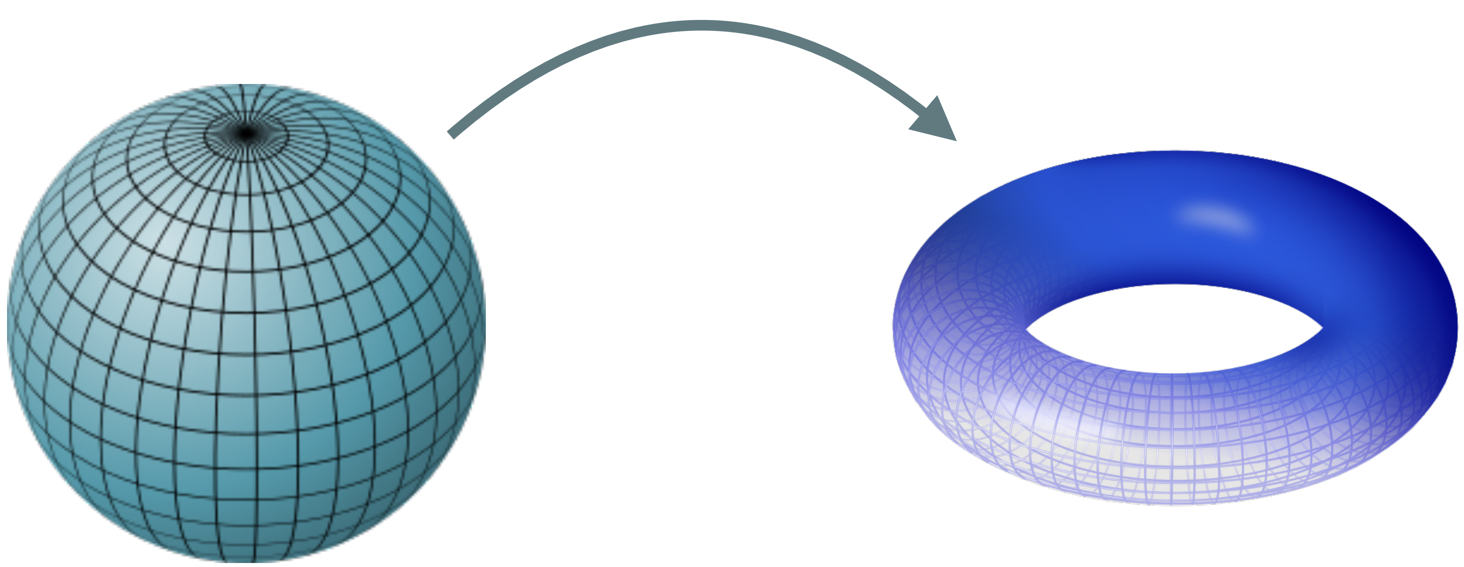


$$\stackrel{?}{=} \sum_{i=1}^{N} R_i(x_1, \ldots, x_r)\, \mathcal{I}_i(x_1, \ldots, x_n)$$

Topology of this variety determines complexity of the problem!

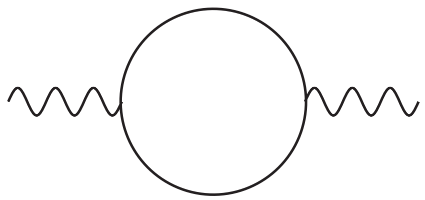


[Brown ’11, ‘13]
[Weinzierl et al ’13,...,’19]
[Sogaard, Zhang et al ’15,’16]
[Primo, Tancredi ’16,’17]
[Brödel, Duhr et al ’17,’18]
[Bourjaily et al ’18,’19]

fascinating connections to string theory and pure math

“ANALYTIC” VS “(SEMI-)NUMERICAL”

WHAT DOES IT MEAN TO BE ANALYTIC?



$$\sim \frac{1}{\sqrt{s(s-4m^2)}} \ln\left(\frac{\sqrt{s-4m^2}+\sqrt{s}}{\sqrt{s-4m^2}-\sqrt{s}}\right)$$

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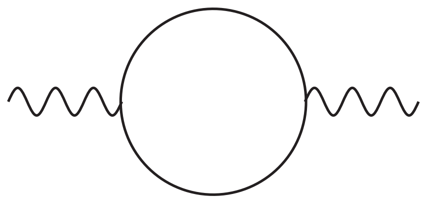

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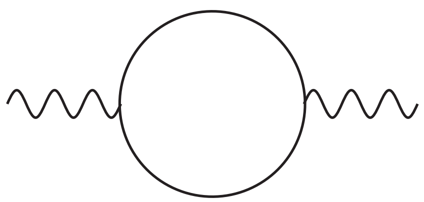

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No hidden zeros!

$$\log 1/x + \log x = 0$$

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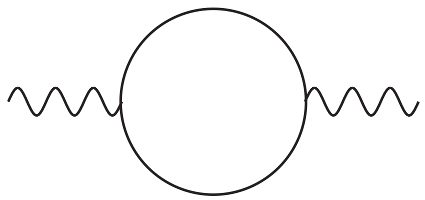

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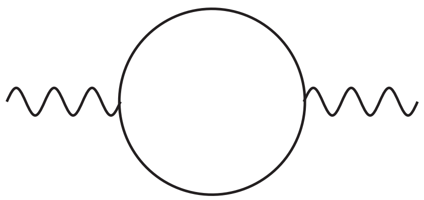

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Argument transformation and Series expansion for <u>numerical evaluation</u>

$$\log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + O\left(x^5\right)$$

ANALYTICAL METHODS

Computing the integrals analytically **exceptional effort**

Direct integration

Linear reducibility

[Brown, Panzer, '11,..,'14]

Differential Equations

[Kotikov '90, Remiddi '97]

[Kotikov '10, Henn '13]

[Argeri et al '13] [Lee '14]

[Papadopoulos '14]

[Primo, Tancredi '16,'17]

[Bosma, Sogaard Zhang '17]

[Frellesvig, Papadopoulos '17]

[Harley, Moriello, Schabinger '17]

Theory of special functions

(multiple polylogarithm, Elliptic Polylogarithms, Iterated integrals)

[Remiddi, Vermaseren '99]

[Goncharov, '95,'00]

[Brown, '10]

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[Brödel, Schlotterer, '17, '18]

[Brödel, Duhr, Dulat, Tancredi, '20]

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[Brödel, Schlotterer, '17, '18]
[Brödel, Duhr, Dulat, Tancredi, '20]

Latest success: all master integrals for **$2 \to 3$ massless scattering at 2 loops** — *"pentagon functions"* —

[Gehrmann, Lo Presti, Henn et al, '16,...,'20] [Abreu, Corder, Ita, Page, et al '18] [Chicherin, Sotnikov, '20]
[Papadopoulos, Tommasini, Wever '19] [Canko, Papadopoulos, Syrrakos '20]

(SEMI-) NUMERICAL METHODS

Numerically, one can avoid problem of studying special functions.

Problems are **numerical stability** and **treatment of divergences**

Sector Decomposition

[Binoth, Heinrich '04]
[Borowka et al '12,..,'18]
[Jones et al '16,...,'20]

Most notably:

$gg \to HH$ with top quarks

but also ZZ, H+j etc…

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Numerical differential equations

Old method (Frobenius expansion), revisited to solve **non trivial problems** $(Hj,\, V+jj, \ldots)$ [Moriello '19; Hidding '20] [Abreu, Page, Sotnikov '20]

Very interesting new method: **differential equations "in $i\epsilon$"** [Liu, Ma, Wang '17]

$$\to \frac{\partial}{\partial \epsilon} \frac{1}{p^2 - m^2 + i\epsilon}$$

Applied already to some very non-trivial cases

- $gg \to WW, ZZ$ [Brønnum-Hansen, Wang '20, '21]
- studies towards complex processes [Liu, Ma '21]

(SEMI-) NUMERICAL METHODS

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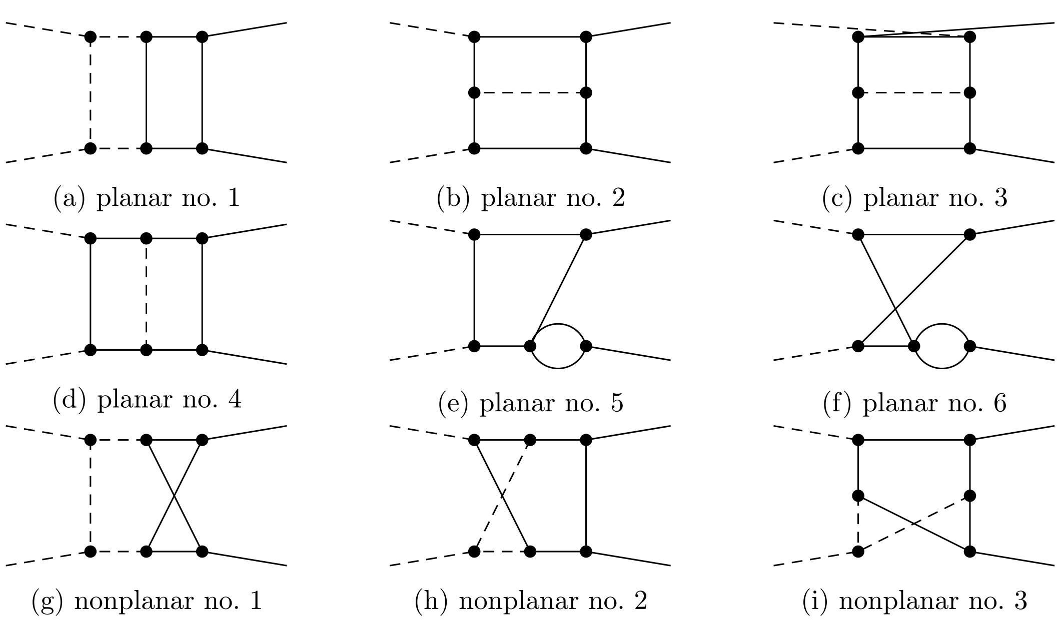
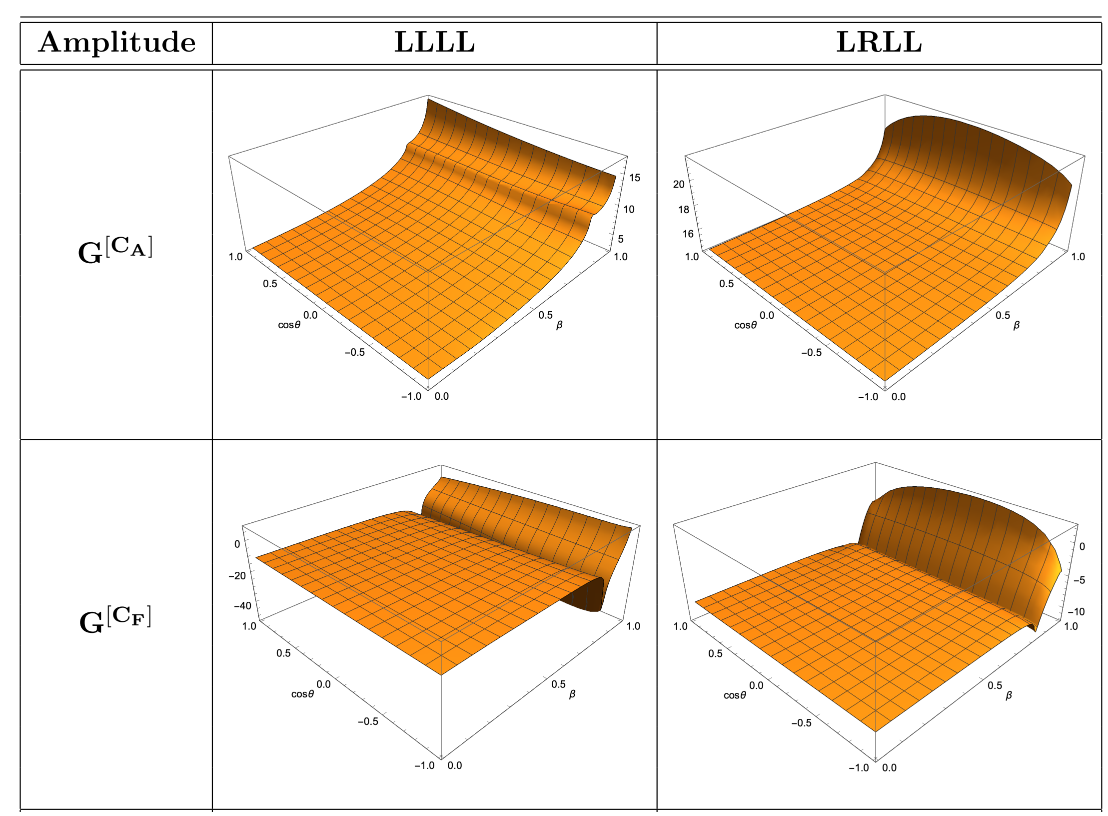
Semi-numerical **differential equations "in $i\epsilon$"**

$gg \to ZZ$ @ **2 loops with top quarks**





[Brønnum-Hansen, Wang '20, '21]

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Alternative: ***Series expansion in small/big parameters***

expansion for small external masses

gg **→** ***ZH*** **NLO with top quarks**

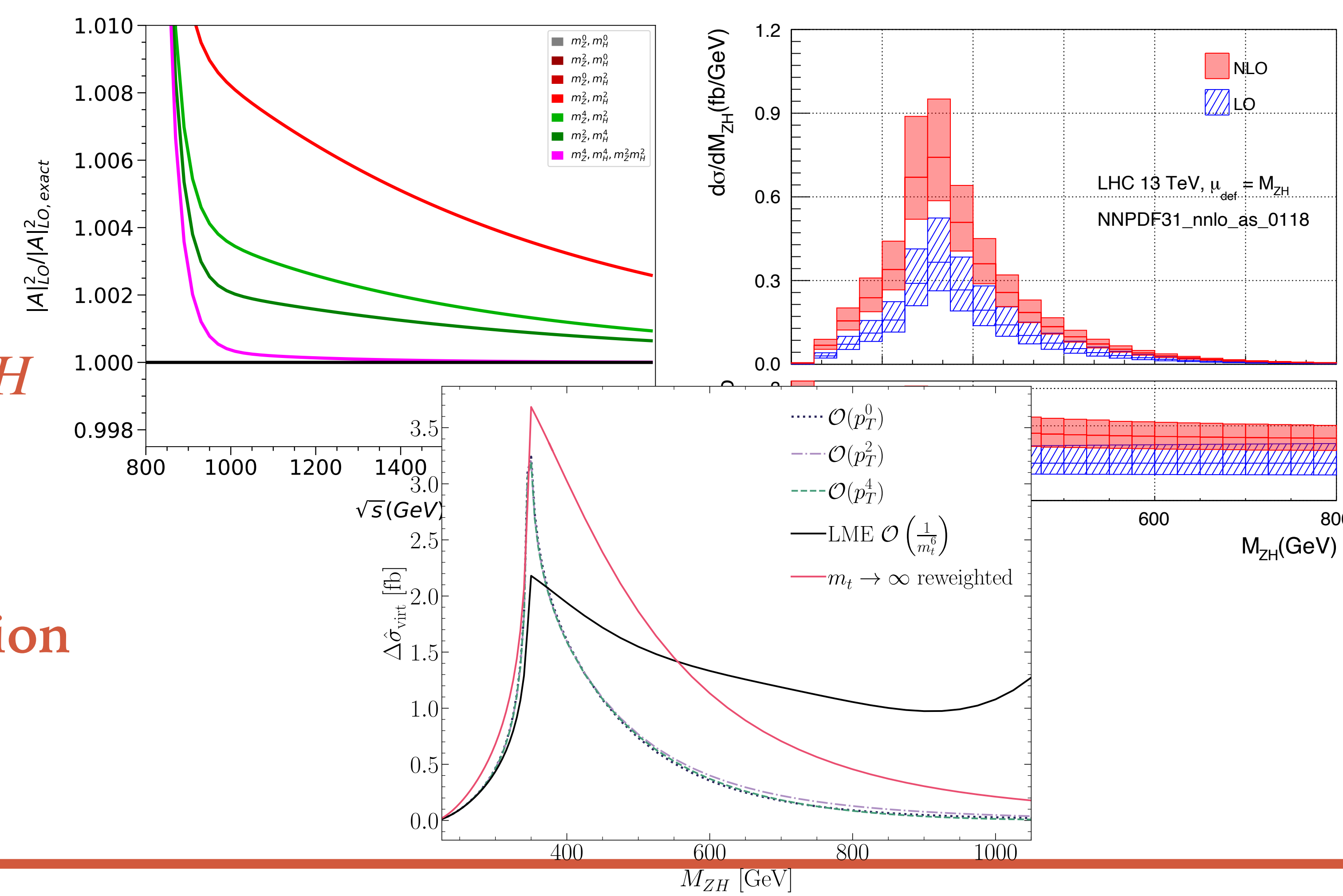
[Wang, Xu, Xu, Yang ’21]

expansion for large top mass $gg \to ZH$ **NLO with top quarks**

[Davis, Mishima, Steinhauser ’20]

Many other results: recently, **expansion in** p_T **for** $gg \to HH$ **and** $gg \to ZH$

[Bonciani et al ’19; Alasfar et al ‘21]



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MANY OTHER INTERESTING RESULTS

Impossible to do justice for **all the impressive results** achieved in the past years…

First results **2 → 3 with masses**: $Wb\bar{b}$ and $Hb\bar{b}$ production [Badger et al '20, '21]

QCD+EW corrections are becoming the standard, see for example

- DY QCD-EW [Heller et al '20,'21]
- W mass studies [Behring et al '21]
- W/Z, ZZ, WW,… production [Denner et al, …., Dittmaier et al '17…,'21]

4-loop QCD Form Factors, anomalous dimensions [Huber, Manteuffel et al '19,'20] [Brüser, Dlapa, Henn et al '18,…,'20]

Intersection numbers for Feynman integrals [Mizera, Mastrolia et al '18,…,'20]

Loop-Tree duality for local observables [Hirschi et al '19,….'21] [Bobadilla et al '18,….'21]

CONCLUSIONS

- past years have seen increasing interest for **methods for calculations in perturbative Quantum Field Theory**
- Learned a lot about QFT and also provided new pheno results
- **Advances in all directions**: higher orders (NNLO and N3LO), more masses (tops, W, Z, H etc), more final state particles, QCD+EW mixed corrections etc etc
- **Very new ideas**: **finite field arithmetic, algebraic geometry for special functions, intersection theory, multi-variate partial fractioning, new flexible semi-numerical methods…**
- Lots of new results, lots of pheno are awaiting, just in time for **run 3** and **HL-LHC** :-) !!!

THANK YOU VERY MUCH!